Module 3
Circular Functions and Trigonometry

What this module is about

This module will teach you about how to define and find the values of the six trigonometric functions of an acute angle which is in standard position.

What you are expected to learn

This module is designed for you to :

1. Define the six trigonometric functions of an angle on the unit cirlcle.

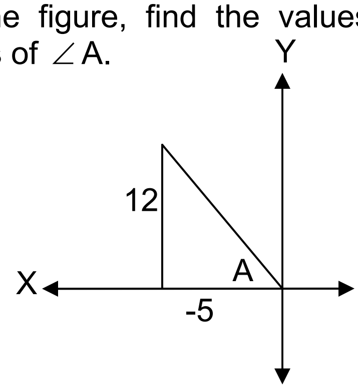
2. Define the six trigonometric functions of an angle in standard position whose terminal point is not on the unit circle

3. Find the values of six trigonometric functions of an angle , given some conditions.

How much do you know

1. What is the value of y = $4\sin\frac{1}{2}x$, if x = $\frac{5\pi}{3}$?

Given the figure, find the values of the radius and the six trigonometric functions of $\angle$A.



2. r
3. Sin A
4. Cos A
5. Tan A
6. Cot A
7. Csc A
8. Sec A

Find the values of the other five trigonometric function for $\angle$S if Tan S = -6/8, Sin S < 0

9. r
10. Sin S
11. Cos S
12. Cot S
13. Csc S
14. Sec S

What you will do

Lesson 1

Six Trigonometric Function of a Point on the Unit Circle

Recall in the previous module, we have defined the six circular functions of an angle θ whose terminal point (x, y) is on the unit circle as:

sin θ = y

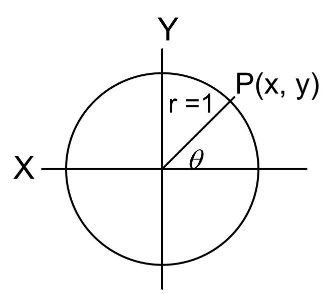
cos θ = x

tan θ = $\frac{y}{x}$

csc θ = $\frac{1}{y}$

sec θ = $\frac{1}{x}$

cot θ = $\frac{x}{y}$



Unit Circle

Since the reference for the definition of the six trigonometric function is the unit circle, they are also called circular functions.

Examples:

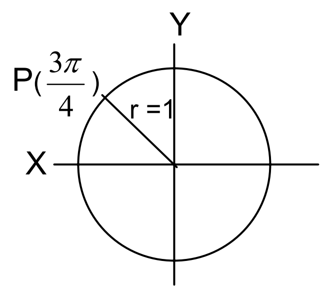
1. Find the six circular function of $\frac{3\pi}{4}$.

   Since $\frac{3\pi}{4}$ is in QII, its coordinates are $(-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2})$. x = $-\frac{\sqrt{2}}{2}$ and y = $\frac{\sqrt{2}}{2}$.

   Then, the six circular functions are:

   $\sin\frac{3\pi}{4}$ = y = $\frac{\sqrt{2}}{2}$

   $\cos\frac{3\pi}{4}$ = x = $-\frac{\sqrt{2}}{2}$



   $\tan\frac{3\pi}{4} = \frac{y}{x} = \frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}$ = -1

   $\csc\frac{3\pi}{4} = \frac{1}{y} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

   $\sec\frac{3\pi}{4} = \frac{1}{x} = \frac{1}{-\frac{\sqrt{2}}{2}} = -\frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = -\frac{2\sqrt{2}}{2} = -\sqrt{2}$

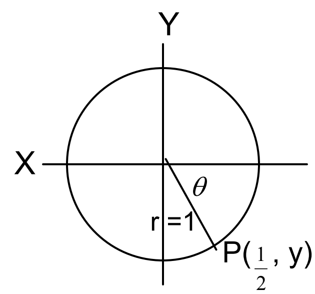
   $\cot\frac{3\pi}{4} = \frac{x}{y} = \frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$ = -1

2. Find the six circular functions of angle θ whose coordinates is defined by $(\frac{1}{2}$, y) and θ is in QIV.

Using the equation of a unit circle $x^2 + y^2 = 1$,

$$\left(\frac{1}{2}\right)^2 + y^2 = 1$$

$$\frac{1}{4} + y^2 = 1$$



$$y^2 = 1 - \frac{1}{4}$$

$$y^2 = \frac{3}{4}$$

$$y = \sqrt{\frac{3}{4}}$$

$$y = \frac{\sqrt{3}}{2}$$

Since θ is in QIV, then $y = -\frac{\sqrt{3}}{2}$ and has coordinates $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

The six circular functions or trigonometric ratios of angle θ are:

$$\sin\theta = y = -\frac{\sqrt{3}}{2}$$

$$\cos\theta = x = \frac{1}{2}$$

$$\tan\theta = \frac{y}{x} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}$$

$$\csc\theta = \frac{1}{y} = \frac{1}{-\frac{\sqrt{3}}{2}} = -\frac{2}{\sqrt{3}} = -\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

$$\sec\theta = \frac{1}{x} = \frac{1}{\frac{1}{2}} = 2$$

$$\cot\theta = \frac{x}{y} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Try this out

Find the six circular functions of the following angles:

1. $\frac{3\pi}{2}$
2. $\frac{5\pi}{6}$
3. $\frac{5\pi}{3}$
4. lying on point $(-\frac{12}{13}, y)$ and is in Q III.
5. lying on point (x, -0.6) and is in QIV.

Lesson 2

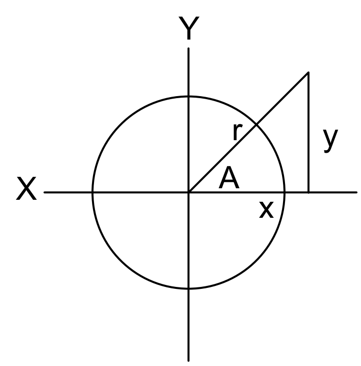
Six Trigonometric Function of an Angle in Standard Position Where the Terminal Point is Not on the Unit Circle

The six trigonometric functions of $\angle$A if the terminal side is not on the unit circle are defined as:

sin A = $\frac{y}{r}$ csc A = $\frac{r}{y}$

cos A = $\frac{x}{r}$ sec A = $\frac{r}{x}$

tan A = $\frac{y}{x}$ cot A = $\frac{x}{y}$

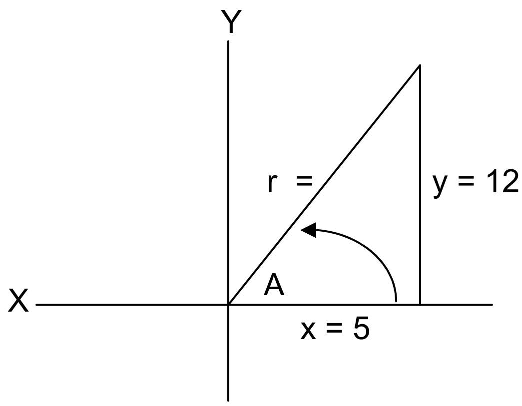


Examples:

1. Find the six trigonometric functions of $\angle$A if the coordinates of P(5,12) lies on its terminal side.

Solution:

x = 5 ; y = 12



Solve for r :

Using Pythagorean Theorem

$$r = \sqrt{x^2 + y^2}$$

$$= \sqrt{5^2 + 12^2}$$

$$= \sqrt{25 + 144}$$

$$= \sqrt{169}$$

r = 13

The ratio of the six functions are :

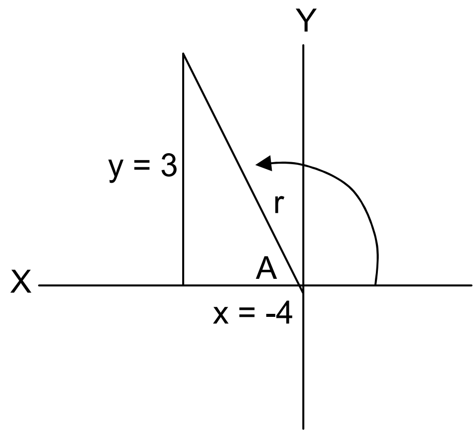
$\sin A = \dfrac{y}{r} = \dfrac{12}{13}$ $\qquad$ $\csc A = \dfrac{r}{y} = 12$

$\cos A = \dfrac{x}{r} = \dfrac{5}{13}$ $\qquad$ $\sec A = \dfrac{r}{x} = \dfrac{13}{5}$

$\tan A = \dfrac{y}{x} = \dfrac{12}{5}$ $\qquad$ $\cot A = \dfrac{x}{y} = \dfrac{5}{12}$

2. If P(-4,5), find the values of radius and six trigonometric functions of $\angle A$.

Solution: x = -4 ; y = 3



You need to find r:

By Pythagorean Theorem

$$r = \sqrt{x^2 + y^2}$$

$$= \sqrt{-4^2 + 3^2}$$

$$= \sqrt{16 + 9}$$

$$= \sqrt{25}$$

r = 5

The ratio of the six functions are:

sin A = $\frac{y}{r} = \frac{3}{5}$

csc A = $\frac{r}{y} = \frac{5}{3}$

cos A = $\frac{x}{r} = -\frac{4}{5}$

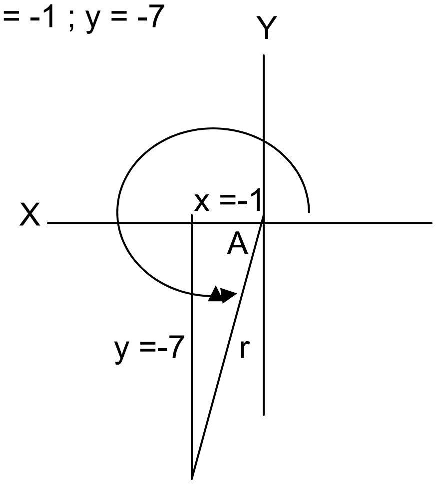
sec A = $\frac{r}{x} = -\frac{5}{4}$

tan A = $\frac{y}{x} = -\frac{3}{4}$

cot A = $\frac{x}{y} = -\frac{4}{3}$

3. If P(-1,-7), find the values of the radius and six trigonometric function of $\angle$A.

Solution: x = -1 ; y = -7



Solve for r:

By Pythagorean Theorem

$$r = \sqrt{x^2 + y^2}$$

$$= \sqrt{(-1)^2 + (-7)^2}$$

$$= \sqrt{1 + 49}$$

$$= \sqrt{50}$$

$$= \sqrt{25 \cdot 2}$$

$$r = 5\sqrt{2}$$

The ratios of the six functions are:

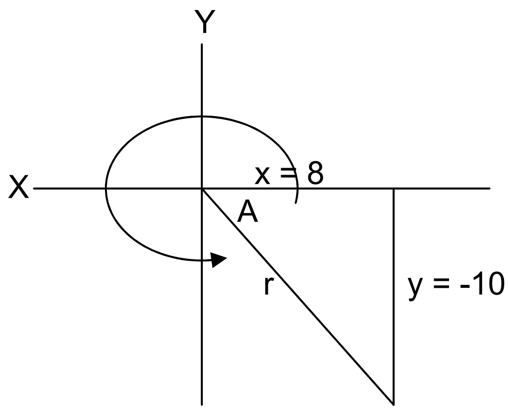
$\sin A = \frac{y}{r} = \frac{-7}{5\sqrt{2}} = \frac{-7}{5\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-7\sqrt{2}}{10}$ $\quad \csc A = \frac{r}{y} = -\frac{5\sqrt{2}}{7}$

$\cos A = \frac{x}{r} = \frac{-1}{5\sqrt{2}} = \frac{-1}{5\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-\sqrt{2}}{10}$ $\quad \sec A = \frac{r}{x} = -5\sqrt{2}$

$\tan A = \frac{y}{x} = \frac{-7}{-1} = 7$ $\quad \cot A = \frac{x}{y} = \frac{1}{7}$

4. If P(8,-10), find the values of the radius and six trigonometric function of $\angle$A.

Solution: x = 8 ; y = -10



Solve for r:

Using Pythagorean Theorem

r = $\sqrt{x^2 + y^2}$

= $\sqrt{8^2 + (-10)^2}$

= $\sqrt{64 + 100}$

= $\sqrt{164}$

= $\sqrt{4 \cdot 41}$

r = 2$\sqrt{41}$

The ratios of the six functions are:

sin A = $\frac{y}{r}$ = $\frac{-10}{2\sqrt{41}}$ = $\frac{-10}{2\sqrt{41}} \cdot \frac{\sqrt{41}}{\sqrt{41}} = \frac{-5\sqrt{41}}{41}$ csc A = $\frac{r}{y}$ = $-\frac{2\sqrt{41}}{10} = -\frac{\sqrt{41}}{5}$

cos A = $\frac{x}{r}$ = $\frac{8}{2\sqrt{41}}$ = $\frac{8}{2\sqrt{41}} \cdot \frac{\sqrt{41}}{\sqrt{41}} = \frac{4\sqrt{41}}{41}$ sec A = $\frac{r}{x}$ = $\frac{2\sqrt{41}}{8} = \frac{\sqrt{41}}{4}$

tan A = $\frac{y}{x}$ = $\frac{\frac{-10}{2\sqrt{41}}}{\frac{8}{2\sqrt{41}}} = \frac{-10}{8} = \frac{-5}{4}$ cot A = $\frac{x}{y}$ = $-\frac{4}{5}$

Try this out

Given the coordinates of the terminal point, sketch the figure and find the value of r and the six trigonometric functions.

1. (-5, 7)
2. (-8, -15)
3. (24, - 7)
4. (2, 3)
5. (-9, 40)

Lesson 3

Find the Values of Six Trigonometric Functions of $\angle A$ Under Some Given Conditions

You can determine the five other trigonometric functions if one of the trigonometric function is given.

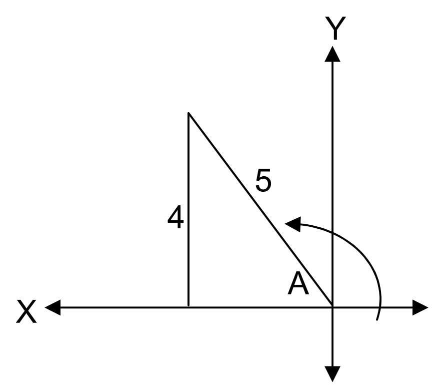
Examples:

1. if sin A = $\frac{4}{5}$, $\angle A$ is not in QI, find the other functions.

Solution: Since the sin function is positive in QI & II and $\angle A$ is not in QI, then $\angle A$ is in QII.

Since Sin A = $\frac{y}{r}$ and y = 4, r = 5,

solve for x:



Find x using Pythagorean theorem

$$r^2 = x^2 + y^2$$

$$r^2 - y^2 = x^2$$

$5^2 - 4^2 = x^2$

$25 - 16 = x$

$9 = x^2$

$\pm 3 = x$

Since A is in QII, consider the value of x = -3.

The five trigonometric functions are:

cos A = $\frac{y}{r}$ = $-\frac{3}{5}$ csc A = $\frac{r}{y}$ = $\frac{5}{4}$

tan A = $\frac{y}{x}$ = $-\frac{4}{3}$ sec A = $\frac{r}{x}$ = $-\frac{5}{3}$

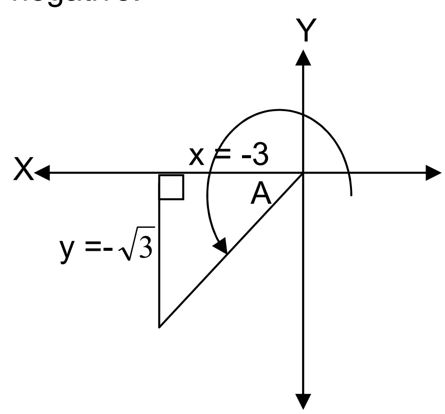
cot A = $\frac{x}{y}$ = $-\frac{3}{4}$

2. Tan A = $\frac{\sqrt{3}}{3}$, cos A < 0 and sin A < 0, find the values of the five other trigonometric functions of $\angle$A.

Solution: The terminal point lies in quadrant III since the sine and cosine functions are both negative. Considering the algebraic sign of the function in QIII, the tan and cot functions are the only positive functions while the rest are negative.

Since Tan A = $\frac{\sqrt{3}}{3}$,

Then, y = $-\sqrt{3}$, x = -3



Solve for r:

By Pythagorean theorem

$r^2 = x^2 + y^2$

$r^2 = (-3)^2 + (-\sqrt{3})^2$

$r^2 = 9 + 3$

$r^2 = 12$

$r = \sqrt{12}$

$r = \sqrt{4 \cdot 3}$

$r = 2\sqrt{3}$

The five other trigonometric functions are:

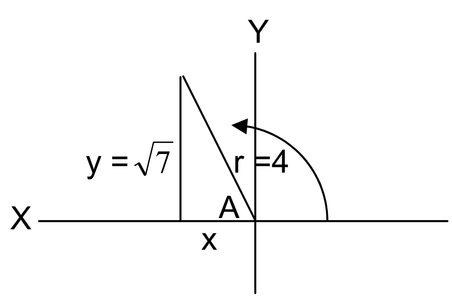
$\sin A = \frac{y}{r} = -\frac{1}{2}$ $\qquad$ $\csc A = \frac{r}{y} = -2$

$\cos A = \frac{x}{r} = -\frac{\sqrt{3}}{2}$ $\qquad$ $\sec A = \frac{r}{x} = -\frac{2\sqrt{3}}{3}$

$\cot A = \frac{x}{y} = \sqrt{3}$

3. If $\sin A = \frac{\sqrt{7}}{4}$, and $\frac{\pi}{2} < A < \pi$. Find the values of the five other trigonometric functions of $\angle A$.

Solution: Sin A = $\frac{\sqrt{7}}{4}$. We define $\sin A = \frac{y}{r}$, and $y = \sqrt{7}$, $r = 4$ and $\angle A$ lies in QII. If Sin A and csc A are positive, then the five other functions are negative.



You need to find x:

$r^2 = x^2 + y^2$

$r^2 - y^2 = x^2$

$4^2 - (\sqrt{7})^2 = x^2$

$16 - 7 = x^2$

$9 = x^2$

$\sqrt{9} = x^2$

$-3 = x$

The five other trigonometric functions are:

$\cos A = \frac{x}{r} = -\frac{3}{4}$ $\qquad$ $\csc A = \frac{r}{y} = \frac{4}{\sqrt{7}} = \frac{4}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{4\sqrt{7}}{7}$

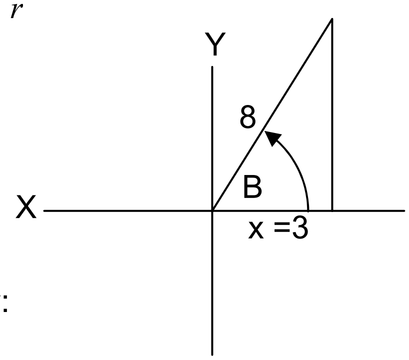
$\tan A = \frac{y}{x} = -\frac{\sqrt{7}}{3}$ $\qquad$ $\sec A = \frac{r}{x} = -\frac{4}{3}$

$\cot A = \frac{x}{y} = -\frac{3}{\sqrt{7}} = -\frac{3}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{3\sqrt{7}}{7}$

4. If $\sec B = \frac{8}{3}$ and $0 < B < 90$, find the other trigonometric functions of $\angle B$.

Solution: Since sec B and cos B are reciprocals, then $\cos B = \frac{3}{8}$.

We define $\cos B = \frac{x}{r}$, so $x = 3$; $r = 8$; B lies in QI, all the functions have positive sign.



You need to solve for y:

$r^2 = x^2 + y^2$

$r^2 - x^2 = y^2$

$8^2 - 3^2 = y^2$

$64 - 9 = y^2$

$55 = y^2$

$\sqrt{55} = y$

The trigonometric functions are:

sin B = $\frac{y}{r}$ = $\frac{\sqrt{55}}{8}$

tan B = $\frac{y}{x}$ = $\frac{\sqrt{55}}{3}$ = $\frac{3}{\sqrt{55}} = \frac{3}{\sqrt{55}} \cdot \frac{\sqrt{55}}{\sqrt{55}} = \frac{3\sqrt{55}}{55}$

cot B = $\frac{x}{y}$ = $\sqrt{3}$

csc B = $\frac{r}{y}$ = $\frac{8}{\sqrt{55}} = \frac{8}{\sqrt{55}} \cdot \frac{\sqrt{55}}{\sqrt{55}} = \frac{8\sqrt{55}}{55}$

Try this out

A. Find the value of each of the remaining functions of the acute angle A :

1. If cos A = $\frac{5}{13}$ and Sin A < 0.

2. If sin A = $\frac{\sqrt{3}}{4}$ and $\frac{\pi}{2}$ < A < π.

3. If sin A = $\frac{2}{3}$ and cos A > 0

4. If cos A = -$\frac{2\sqrt{3}}{3}$ and Tan A > 0

5. If tan A = -$\frac{3}{4}$, and 270 < A< 360

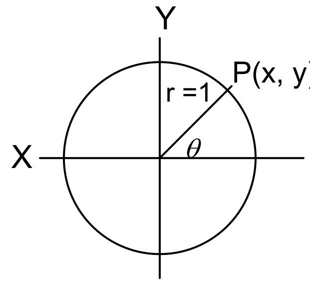
Let's summarize

The six circular functions of an angle θ whose terminal point (x, y) is on the unit circle:

$\sin\theta = y$ $\qquad\qquad$ $\csc\theta = \dfrac{1}{y}$

$\cos\theta = x$ $\qquad\qquad$ $\sec\theta = \dfrac{1}{x}$

$\tan\theta = \dfrac{y}{x}$ $\qquad\qquad$ $\cot\theta = \dfrac{x}{y}$



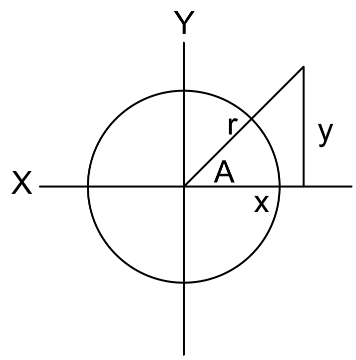
Unit Circle

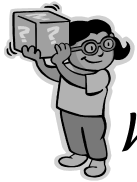
The six trigonometric functions of $\angle$A if the terminal side is not on the unit circle:

$\sin A = \dfrac{y}{r}$ $\qquad\qquad$ $\csc A = \dfrac{r}{y}$

$\cos A = \dfrac{x}{r}$ $\qquad\qquad$ $\sec A = \dfrac{r}{x}$

$\tan A = \dfrac{y}{x}$ $\qquad\qquad$ $\cot A = \dfrac{x}{y}$

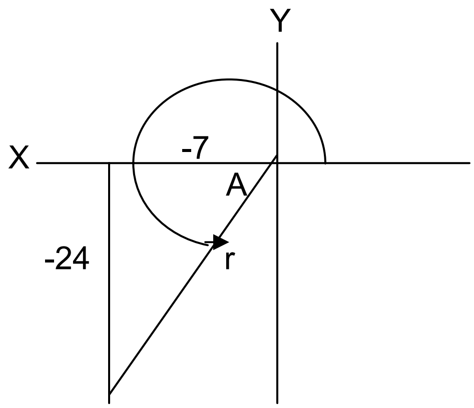




What have you learned

1. What is the value of y = $\frac{1}{2}\cos x$ if x = $\frac{\pi}{2}$.

Given the figure, find the value of r and the six trigonometric functions of $\angle$A.



2. r
3. Sin A
4. Cos A
5. Tan A
6. Cot A
7. Csc A
8. Sec A

Sketch the figure and find the values of r and the other five trigonometric functions of $\angle$C if tan C = $-\frac{4}{3}$ and sin C > 0.

9. r
10. Sin C
11. Cos C
12. Cot C
13. Sec C
14. Csc C

Answer key

How much do you know

1. sol: $y = 4 \sin \frac{1}{2}(\frac{5\pi}{3})$

   $y = 4 \sin \frac{5\pi}{6}$

   $y = 4(\frac{1}{2})$

   $y = 2$

2. r = 13
3. $\sin A = \frac{12}{13}$
4. $\cos A = -\frac{5}{13}$
5. $\cot A = -\frac{5}{12}$
6. $\csc A = \frac{13}{12}$
7. $\tan A = -\frac{12}{5}$
8. $\sec A = -\frac{13}{5}$
9. r = 10
10. $\sin k = -\frac{3}{5}$
11. $\cos S = \frac{4}{5}$
12. $\cot S = -\frac{4}{3}$
13. $\csc S = -\frac{13}{12}$

14. sec S = $\frac{5}{4}$

Try this out

Lesson 1

1. a. $\sin\frac{3\pi}{2} = -1$ d. $\csc\frac{3\pi}{2} = -1$

b. $\cos\frac{3\pi}{2} = 0$ e. $\sec\frac{3\pi}{2}$ = undefined

c. $\tan\frac{3\pi}{2}$ = undefined f. $\cot\frac{3\pi}{2} = 0$

2. a. $\sin\frac{5\pi}{6} = \frac{1}{2}$ d. $\csc\frac{5\pi}{6} = 2$

b. $\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$ e. $\sec\frac{5\pi}{6} = -\frac{2\sqrt{3}}{3}$

c. $\tan\frac{5\pi}{6} = -\frac{\sqrt{3}}{3}$ f. $\cot\frac{5\pi}{6} = -\sqrt{3}$

3. a. $\sin\frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$ d. $\csc\frac{5\pi}{3} = -\frac{2\sqrt{3}}{3}$

b. $\cos\frac{5\pi}{3} = \frac{1}{2}$ e. $\sec\frac{5\pi}{3} = 2$

c. $\tan\frac{5\pi}{3} = -\sqrt{3}$ f. $\cot\frac{5\pi}{3} = -\frac{\sqrt{3}}{3}$

4. $y = -\frac{1}{13}$

a. $\sin A = -\frac{1}{13}$ d. $\csc A = -13$

b. cos A = $-\frac{12}{13}$ e. sec A = $-\frac{13}{12}$

c. tan A = $\frac{1}{12}$ f. cot A = 12

5. x = 0.8

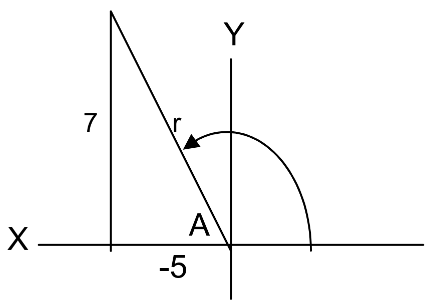
a. sin A = -0.6 d. csc A = -$\frac{5}{3}$

b. cos A = 0.8 e. sec A = $\frac{5}{4}$

c. tan A = -$\frac{3}{4}$ f. cot A = -$\frac{4}{3}$

Lesson 2

1. (-5, 7)



Solve for r:
By Pythagorean Theorem

$r^2 = (x)^2 + (y)^2$

$= (5)^2 + (7)^2$

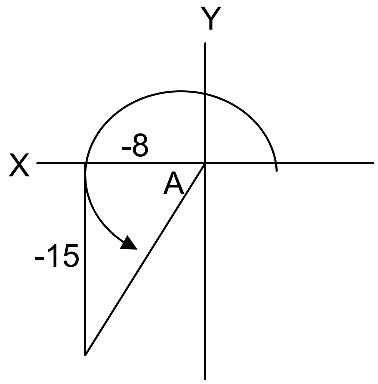
$= 25 + 49$

$r = \sqrt{74}$

a. Sin A = $\frac{7\sqrt{74}}{74}$ c. Tan A = -$\frac{7}{5}$ e. Cot A = -$\frac{5}{7}$

b. Cos A = $\frac{-5\sqrt{74}}{74}$ d. Sec A = $\frac{\sqrt{74}}{5}$ f. Csc A = $\frac{\sqrt{74}}{7}$

2. (-8, -15)



Solve for r
By Pythagorean Theorem:

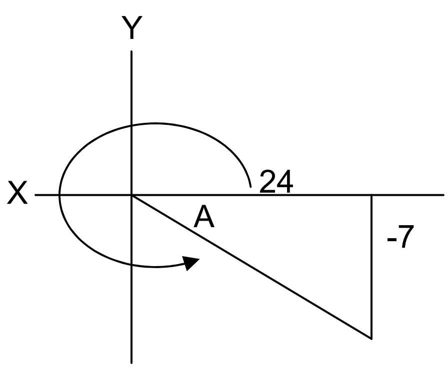
$$r^2 = (x)^2 + (y)^2$$
$$= (-8)^2 + (-15)^2$$
$$= 64 + 225$$
$$= 289$$
$$r = 17$$

$\text{Sin } A = -\frac{15}{17}$ $\text{Csc } A = -\frac{17}{15}$

$\text{Cos } A = -\frac{8}{17}$ $\text{Sec } A = -\frac{17}{8}$

$\text{Tan } A = \frac{15}{8}$ $\text{Cot } A = \frac{8}{15}$

2. (24, - 7)



By Pythagorean Theorem:

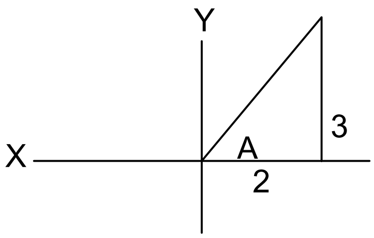
$$r^2 = x^2 + y^2$$
$$= (24)^2 + (-7)^2$$
$$= 576 + 49$$
$$= 625$$
$$r = 25$$

$\text{Sin } B = -\frac{7}{25}$ $\text{Csc } B = -\frac{25}{7}$

$\text{Cos } B = \frac{24}{25}$ $\text{Sec } B = \frac{25}{24}$

$\text{Tan } B = -\frac{7}{24}$ $\text{Cot } B = -\frac{24}{7}$

3. (2, 3)



Solve for r:

By Pythagorean theorem:

$r^2 = x^2 + y^2$

$= 2^2 + 3^2$

$= 4 + 9$

$= 13$

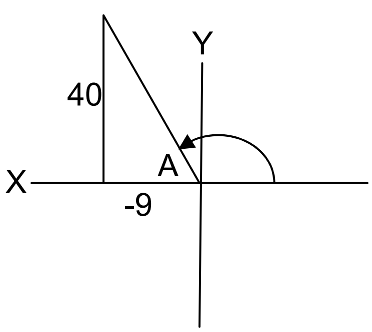
$r = \sqrt{13}$

Sin A = $\frac{3\sqrt{13}}{13}$ Csc A = $\frac{\sqrt{13}}{3}$

Cos A = $\frac{2\sqrt{13}}{13}$ Sec A = $\frac{\sqrt{13}}{2}$

Tan A = $\frac{3}{2}$ Cot A = $\frac{2}{3}$

4. (-9, 40)



Solve for r

By Pythagorean Theorem

$r^2 = x^2 + y^2$

$= (-9)^2 + (40)^2$

$= 81 + 1600$

$= 1681$

$r = 41$

Sin A = $\frac{40}{41}$ Csc A = $\frac{41}{40}$

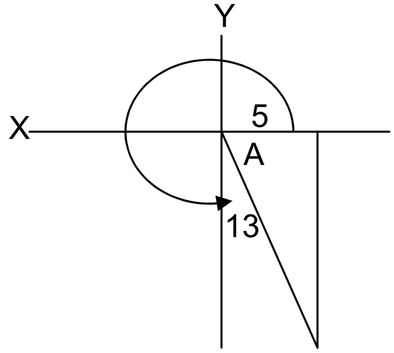
Cos A = $-\frac{9}{41}$ Sec A = $-\frac{41}{9}$

Tan A = $-\frac{40}{9}$ Cot A = $-\frac{91}{40}$

Lesson 3

1. Cos A $= \frac{5}{13}$, If A is in Q IV

   Solution: In Q IV the only positive functions are cos A and sec A .



By Pythagorean Theorem:

$x^2 + y^2 = r^2$

$y^2 = (13)^2 – (5)^2$

$y^2 = 169 – 25$

$y^2 = 144$

$y = 12$

a. Sin A $= -\frac{12}{13}$

b. Tan A $= -\frac{12}{5}$

c. Sec A $= \frac{13}{5}$

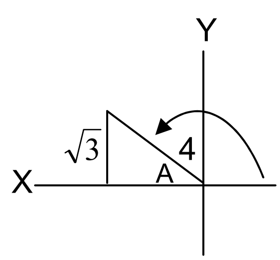
d. Csc A $= -\frac{13}{12}$

e. Cot A $= -\frac{5}{12}$

2. Sin A $= \frac{\sqrt{3}}{4}$, If A is in Q II

   Solution: In Q II sin A and csc A are positive and the rest of the functions are negative.

   By Pythagorean Theorem



$x^2 + y^2 = r^2$

$x^2 = r^2 – y^2$

$x^2 = 4^2 – (\sqrt{3})^2$

$x^2 = 16 – 3$

$x = \pm\sqrt{13}$, since A is in QII

$x = -\sqrt{13}$

a. Cos A = $-\frac{\sqrt{13}}{4}$

b. Tan A = $-\frac{\sqrt{39}}{13}$

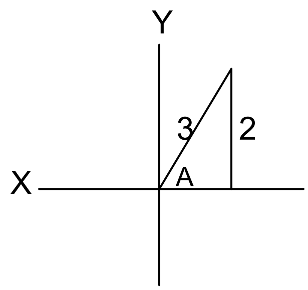
c. Csc A = $-\frac{4\sqrt{3}}{3}$

d. Sec A = $-\frac{4\sqrt{13}}{13}$

e. Cot A = $-\frac{\sqrt{39}}{3}$

3. Sin A = $\frac{2}{3}$, Where A is in Q I

Solution: Since A is in Q I, all the functions are positive.



By Pythagorean Theorem

$x^2 + y^2 = r^2$

$x^2 = (3)^2 - (2)^2$

$x^2 = 9 - 4$

$x^2 = 5$

$x = \pm\sqrt{5}$ but A is in Q I then $x = +\sqrt{5}$

a. Cos A = $\frac{\sqrt{5}}{3}$

b. Tan A = $\frac{2\sqrt{5}}{5}$

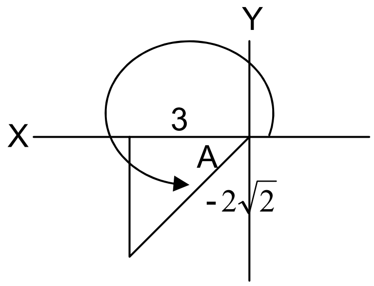
c. Sec A = $\frac{3\sqrt{5}}{5}$

d. Csc A = $\frac{3}{2}$

e. Cot A = $\frac{\sqrt{5}}{2}$

4. Cos A = $-\frac{2\sqrt{2}}{3}$, A is in QIII

Solution: Since a is in QIII, tan and cot are the only positive and the rest are negative.



By Pythagorean theorem

$x^2 + y^2 = r^2$

$y^2 = (3)^2 - (2\sqrt{2})^2$

$= 9 - 8$

$y = 1$

a. Sin A = $-\frac{1}{3}$

a. Tan A = $\frac{\sqrt{2}}{4}$

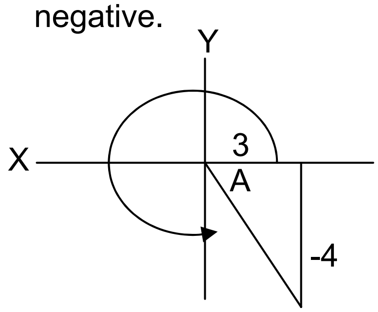
b. Csc A = -3

d. Sec A = $-\frac{2\sqrt{2}}{3}$

e. Cot A = $2\sqrt{2}$

5. Tan A = $-\frac{3}{4}$, A is in QIV

Solution: Since A is in QIV, cos and sec are the only positive and the rest are negative.



By Pythagorean Theorem

$r^2 = x^2 + y^2$

$r^2 = (4)^2 + (-3)^2$

$= 16 + 9$

$= 25$

$r = 5$

a. Sin A = -3/5

b. Cos A = 4/5

c. Cot A = -4/3

d. Csc A = -5/3

e. Sec A = 5/4

What have you learned

1. y = 0
2. r = 25
3. Sin A = $-\frac{24}{25}$
4. Cos A = $-\frac{75}{25}$
5. Tan A = $\frac{24}{7}$
6. Csc A = $-\frac{25}{24}$
7. Sec A = $-\frac{25}{7}$
8. Cot A = $\frac{7}{24}$
9. r = 5
10. Sin C = $-\frac{4}{5}$
11. Tan C = $-\frac{4}{3}$
12. Cot C = $-\frac{3}{4}$
13. Sec C = $\frac{5}{3}$
14. Csc C = $-\frac{5}{4}$