Module 5
Exponential Functions

What this module is about

This module is about problems involving exponential functions. As you go over this material, you will develop the skills in solving different problems involving exponential functions.

What you are expected to learn

This module is designed for you to solve problems involving exponential and functions.

How much do you know

1. An amount of P5,000 is deposited at 6% interest compounded annually. Find the total amount after 3 years.

2. The population of a city at the start of year 1998 is 3, 200,000 and is steadily increasing at the rate of 2% annually. At this rate, what is the population of the city at the end of year 2005?

3. The half-life of a certain radioactive substance is 2 years. If this substance has an initial amount of 128 grams, how much of the substance will be left after a decade?

4. Suppose a culture of 100 bacteria is put in a Petri dish and the culture doubles every minute. How many bacteria will there be after half an hour?

5. The present population of a city is 10 million with a growth rate of 2.5% annually. At this rate, in what year will the city reach the 100 million populations?

6. The original weight of a certain radioactive substance is 100 grams. After 2 hours it decays to 12.5 grams. Find the half-life of this substance.

7. If P100,000 is invested at 10% compounded annually, when will the amount of investment be P121,000?

8. A certain investment earns interest at an annual rate of 12% compounded semiannually. After 3 year, the total amount is P110,255.09. Approximately, how much money was originally invested?

9. A certain radioactive substance decays a 25% of itself every hour. Initially, there are 120 milligrams of this substance. How much substance will be left after a day?

10. A car bought for P700,000 depreciates by 8% a year. After how many years one can buy the car at P644,000?

What you will do

Lesson 1

Population Growth

Suppose that the population *P* of a certain town is increasing at a constant rate *r* every year. The table below shows the increase in population and the population written in exponential form.

Number of Years After	Previous Year's Population	Increase in Population	Current Population	Exponential Form
0	-	-	P	$P_0 = P(1 + r)^0$
1	P	Pr	$P + Pr$	$P_1 = P(1 + r)^1$
2	$P(1 + r)$	$P(1 + r)r$	$P(1 + r) + P(1 + r)r = P(1 + r)(1 + r)$	$P_2 = P(1 + r)^2$
3	$P(1 + r)^2$	$P(1 + r)^2r$	$P(1 + r)^2 + P(1 + r)^2r = P(1 + r)^2r(1 + r)$	$P_3 = P(1 + r)^3$
. . .	. . .	. . .	. . .	. . .
n	$P(1 + r)^{n-1}$	$P(1 + r)^{n-1}r$	$P(1 + r)^{n-1} + P(1 + r)^{n-1}r = P(1 + r)^{n-1}r(1 + r)$	$P_n = P(1 + r)^n$

In general, in the absence of factors affecting population growth, the population after n years at a constant rate r is $P(1 + r)^n$.

If the population P is decreasing at a constant rate r, at the end of n years the population would be $P(1 – r)^n$.

Examples:

Solve each problem.

1. A town has a population of 40,000 that is increasing at the rate of 5% each year. Find the population of the town after 6 years.

   Given: P = 40,000, r = 5% or 0.05 , n = 6

   Find: P_6

   $$P_6 = P(1 + r)^6$$
   $$= 40{,}000(1 + 0.05)^6$$
   $$= 40{,}000(1.05)^6$$
   $$= 40{,}000(1.34010)$$
   $$= 53{,}603$$

   The population of the town after 6 years is 53,604.

2. Suppose a culture of 300 bacteria is put in a Petri dish and the culture doubles every hour. How many bacteria will there be after 9 hours?

   Given: P = 300, r = 100% or 1, n = 9

   Find: P_9

   $$P_9 = P(1 + r)^9$$
   $$= 300(1 + 1)^9$$
   $$= 300(2)^9$$
   $$= 300(512)$$
   $$= 153{,}600$$

   After 9 hours, there will be 153,600 bacteria.

3. A certain radioactive substance decays half of itself everyday. Initially, there are 10 grams of this substance. How much substance will be left after 8 days?

Given: P = 10, r = $\frac{1}{2}$, n = 8

Find: P_8

$P_8 = P(1 - r)^8$

$= 10\left(1 - \frac{1}{2}\right)^8$

$= 10\left(\frac{1}{2}\right)^8$

$= 10\left(\frac{1}{256}\right)$

$= 0.04$

After 8 hours, only o.04 grams of the substance will be left.

Try this out

Solve the following problems:

Set A

1. A town has a population of 50,000 that is increasing at the rate of 3.2% each year. Find the population of the town after 5 years.

2. Suppose a culture of 100 bacteria is put in a Petri dish and the culture triples every hour. How many bacteria will there be after 6 hours?

3. A certain radioactive substance decays a quarter of itself every minute. Initially, there are 50 grams of this substance. How much substance will be left after 4 minutes?

4. A biologist wants to determine the number of hours it needed for a given culture to grow to 512 bacteria. If the number of N bacteria in the culture is given by the formula $N = 8(2)^t$, find t.

5. In the above problem, how many bacteria in the culture will there be after 10 hours?

Set B

1. The population of a province is 75,000 and has been increasing at the rate of 2.5% a year for the past 15 years. What was the population 15 years ago?

2. In the early stages of SARS outbreak in China, there were 100 persons infected and each day the number rose by 5%. If the spread of the syndrome was not controlled, how many persons would have been affected after 15 days?

3. At the start of year 2002, the population of a city is 2 million with a growth rate of 2% annually. At this rate, in what year will the city reach the 4 million populations?

4. In January 1, 1998, the population of a town is 8,000.In December 31, 2003, the population of the said town is 10,000. At what rate did the population grow?

5. In a particular time, a certain bacteria numbers about 160 and its number grow at the rate of 50% in a day. After how many hours will the number reach 810?

Set C

1. The Philippines has a population of approximately 73 million in January, 1995. The population steadily increases at the rate of 2.3% each year. At this rate, find the population of the country in January, 2003.

2. A dengue epidemic occurred in a town in Africa. There were 500 persons initially infected and each week the number doubles. If the epidemic was not controlled, how many persons would have been affected after 7 days?

3. At the start of year 2002, the population of a municipality is 1,000,000 with a growth rate of 3% annually. At this rate, in what year will the municipality reach the 1,092,727 populations?

4. In January 1, 1999, the population of a town is 15,000. In December 31, 2002, the population of the said town is 105,000. At what rate did the population grow?

5. A certain radioactive substance decays a 5% of itself every minute. Initially, there are 70 grams of this substance. How much substance will be left after half an hour?

Lesson 2

Half-life

A radioactive element is one whose atoms give off radiation as it decays, or break down into the atoms of a different element. Some radioactive elements decay in a few seconds. Some take thousands, millions, or even billions of years to decay. But the rate of decay for each element is steady. Scientists measure the decay rate of a radioactive element by a unit called half-life. The half-life is the time it takes for half of a radioactive element to decay.

To find the remaining amount P_n of the radioactive substance P with half-life k in n period of time, the formula to be used is

$$P_n = P\left(\frac{1}{2}\right)^{\frac{n}{k}}.$$

The unit of k and n must be the same.

Examples:

Solve each problem:

1. Carbon-14 is the radioactive form of carbon. The half-life of carbon-14 is 5,770 years. Carbon-14 is present in all living things. When a living thing dies, the carbon-14 begins to decay. Suppose a newly excavated fossil of an organism contained 10 grams and the amount of carbon-14 in a similar living organism is 40 grams, how old is the fossil?

   Given: P_n = 10 grams, P = 40 grams, k = 5,770 years

   Find: n

$$P_n = P\left(\frac{1}{2}\right)^{\frac{n}{k}}$$

$$10 = 40\left(\frac{1}{2}\right)^{\frac{n}{5,770}}$$

$$\frac{10}{40} = \left(\frac{1}{2}\right)^{\frac{n}{5,770}}$$

$$\frac{1}{4} = \left(\frac{1}{2}\right)^{\frac{n}{5,770}}$$

$$\left(\frac{1}{2}\right)^{2} = \left(\frac{1}{2}\right)^{\frac{n}{5,770}}$$

$$2 = \frac{n}{5,770}$$

$$n = 11,540$$

The newly excavated fossil is 11,540 years old.

2. Suppose that the half-life of a certain substance is 20 days and there are 10 grams of the substance initially. Determine the amount of the substance remaining after 75 days.

   Given: k = 20 days, P = 10 grams, n = 75 days

   Find: P_{75}

$$P_n = P\left(\frac{1}{2}\right)^{\frac{n}{k}}$$

$$P_{75} = 10\left(\frac{1}{2}\right)^{\frac{75}{20}}$$

$$P_{75} = 10\left(\frac{1}{2}\right)^{3.75}$$

$$P_{75} = 10(0.07433)$$

$$P_{75} = 0.74$$

   After 75 days, the there will be only 0.74 grams remaining.

Try this out

Solve the following problems:

Set A

1. The original weight of a certain radioactive substance is 112 grams. After 3 hours it decays to 56 grams. Find the half-life of this substance.

2. The half-life of a certain radioactive element is 5 days. Initially, there is a gram of this element. How many grams are left after 15 days?

3. The half-life of a substance is 30 days. How much of a 50-gram substance will remain after 90 days?

4. If the half-life of a substance is 50 years, what fraction of a 200-gram substance remains after 200 years?

5. Find the half-life of a 5 kg-substance if after 90 days, only 625 g of the substance remains.

Set B

1. After 5 hours, a 90-gram radioactive substance decays to 22.5 grams. Find the half-life of this substance.

2. The half-life of a certain radioactive element is 4 days. After 20 days, only 0.128 g is left. How much of the substance are initially?

3. The half-life of Carbon A is 5,500 years. A chemist found that a mummified cadaver was found to contain only 100 grams of Carbon A. Originally, how much of the substance was present if the body died 1,375 years ago?

4. If the half-life of a substance is 2,000 years, what fraction of a 5-kg substance remains after 800 years?

5. Find the half-life of a 1,500 g-substance if after 1 year, only 375 g of the substance remains.

Set C

1. The original weight of a certain radioactive substance is 144 grams. After 5 hours it decays to 9 grams. Find the half-life of this substance.

2. The half-life of a certain radioactive element is 400 years. After 20 years, only 1 g is left. How much of the substance are initially?

3. The half-life of a substance is 300 days. How much of a 6.25-kg substance will remain after 90 days?

4. If the half-life of a substance is 200 years, what fraction of a 81-g substance remains after 80 years?

5. Find the half-life of a 5,000 mg-substance if after 15 days, only 625 g of the substance remains.

Lesson 3

Compound Interest

Compound interest is the amount paid by business entrepreneur on money invested. It may be a charge imposed on a loan or interest earned on a bank deposit. Compound interest is computed periodically. It may be computed annually, semiannually, quarterly, or even monthly, and is added to the principal amount. Compound interest behaves exponentially. This is explained by its formula

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

where A is the total amount after t years of investing the principal amount P at r interest rate which is compounded m times a year.

Examples:

Solve each problem.

1. Immanuel deposited P10,000 in a bank that pays 3% compound interest annually. How much money will he have after 11 years?

   Given: P = P10,000, r = 3% or 0.03, m = 1, t = 11

   Find: A

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

$$A = 10,000\left(1 + \frac{0.03}{1}\right)^{(1)(11)}$$

$$A = 10,000\left(1 + 0.03\right)^{11}$$

$$A = 10,000\left(1.03\right)^{11}$$

$$A = 10,000\left(1.384234\right)$$

$$A = P13,842.34$$

   After 11 years, Immanuel will have the there will P13,842.34.

2. An amount of P200,000 is invested in a business establishment that pays 12.5% compounded quarterly. Find the total investment after 2 years.

Given: P = P200,000, r = 12.5% or 0.125, m = 4, t = 2

Find: A

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

$$A = 200,000\left(1 + \frac{0.125}{4}\right)^{(4)(2)}$$

$$A = 200,000\left(1 + 0.03125\right)^{8}$$

$$A = 200,000\left(1.03125\right)^{8}$$

$$A = 200,000\left(1.279121187\right)$$

$$A = P255,824.2374$$

After 2 years, the total investment is P255,824.2374

Try this out

Solve the following problems:

Set A

1. Mr. Seniro time deposited P150,000 in a bank that pays 7.5% compound interest monthly. How much money will he have after 3 months?

2. An amount of 1 million pesos is invested in a business establishment that earns 8% compounded quarterly. Find the total investment after 3 years.

3. If P10,000 is invested at 10% compounded quarterly, when will the amount of investment be P10,250?

4. Bank of Manila offers 6% interest rate compounded semiannually to its depositors. Philippine Bank, on the other hand, offers 5% interest rate compounded quarterly to its depositor. If you have a million pesos for deposit, in which bank will you place your money if you do not have the intention of withdrawing a single centavo in 5 years?

5. A certain investment earns interest at an annual rate of 12% compounded quarterly. After one year, the total amount is P11,255.09. Approximately, how much money was originally invested?

Set B

1. Mrs. Santos deposited P500,000 in a bank that pays 15% interest compounded quarterly. How much money will he have after 4 years?

2. An amount of P125,000 is invested in a money lending institution that pays 10% interest compounded quarterly. Find the total investment after 5 years.

3. If P450,000 is invested at 12% compounded semiannually. When will the amount of investment be P477,000?

4. A consumers' cooperative offers 8.5% interest rate compounded semiannually to its prospective member. A bank, on the other hand, offers 10% interest rate compounded annually to its prospective depositor. If you have a P100,000 for deposit, in which investment will you place your money if you do not have the intention of withdrawing a single centavo in 3 years?

5. A certain investment earns interest at an annual rate of 15% compounded semiannually. After one year, the total amount is P150,000 rounded to the nearest thousand pesos. Approximately, how much money was originally invested?

Set C

1. Find the amount and the compound interest earned for a principal amount of P20,000 if it is allowed to accumulate for one and a half years at 30% interest compounded monthly.

2. An amount of P123,500 is invested in a money lending institution that pays 7.5% interest compounded annually. Find the total investment after 3 years.

3. An OFW deposited P750,000 in a bank that pays 8.5% compound interest quarterly. How much money will he have after 7 quarters?

4. An investment of P10,000 earns a certain interest rate compounded quarterly. After one year, the total amount reached P11,255.09. Approximately, at what rate is the money invested?

5. A certain amount of money is invested at an annual rate of 10% compounded quarterly. How long will it take for the investment to increase its value to 102.5% of its original value?

Let's summarize

1. In the absence of factors affecting population growth, the population after n years at a constant rate r is $P(1 + r)^n$.

2. If the population P is decreasing at a constant rate r, at the end of n years the population would be $P(1 – r)^n$.

3. The half-life is the time it takes for half of a radioactive element to decay.

4. To find the amount P_n of the radioactive substance with half-life k in n period of time, the formula to be used is

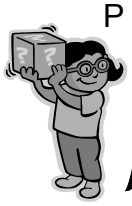
$$P_n = P\left(\frac{1}{2}\right)^{\frac{n}{k}}.$$

5. The unit of k and n must be the same.

6. Compound interest behaves exponentially. This is explained by its formula

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

7. where a is the total amount after t years of investing the principal amount P at r interest rate which is compounded m times a year.



What have you learned

1. An amount of P50,000 is deposited at 8.5% interest compounded annually. Find the total amount after 5 years.

2. The population of a town at the start of year 1995 is 200,000 and is steadily increasing at the rate of 3% annually. At this rate, what is the population of the town at the end of year 2002?

3. The half-life of a certain radioactive substance is 5 years. If this substance has an initial amount of 1.2 kilograms, how much of the substance will be left after a 2 decades?

4. Suppose a culture of 500 bacteria is put in a petri dish and the culture grow 5 times its original number every minute. How many bacteria will there be after 10 minutes?

5. The present population of a barangay is 4,000 with a growth rate of 3% annually. At this rate, in what year will the barangay reach the 4,120 populations?

6. The original weight of a certain radioactive substance is 600 grams. After 2 and a half hours it decays to 300 grams. Find the half-life of this substance.

7. If P75,000 is invested at 12% compounded quarterly, when will the amount of investment be P77,250?

8. A certain investment earns interest at an annual rate of 9% compounded semiannually. After 2 years, the total amount is P100,251.28. Approximately, how much money was originally invested?

9. A certain radioactive substance decays a 12.5% of its weight every day. Initially, there are 450 milligrams of this substance. How much substance will be left after a week?

10. A car bought for P1,200,000 depreciates by 5% a year. After how many years one can buy the car at P1,140,000?

Answer Key

How much do you know

1. P5,955.08
2. 3,749,310
3. 4 grams
4. 107,374,182,400
5. 2 years
6. $\frac{2}{3}\ hours$
7. 2 years
8. P77,725.49
9. 25,411
10. 1 year

Try this out

Lesson 1

Set A

1. 58,528
2. 72,900
3. 15.82 grams
4. 6
5. 8,192

Set B

1. 51,784
2. 207
3. after 36 years, or in the year 2038
4. 3.79%
5. 4 years

Set C

1. 89,578,454
2. 64,000

3. after 3 years, or in the year 2005
4. 62.66%
5. 15.02 grams

Lesson 2

Set A

1. 3 hours
2. $\frac{2}{3}$ grams
3. 6.25 grams
4. 12.5 grams
5. 20 days

Set B

1. 2.5 hours
2. 4.096 grams
3. 118.92 grams
4. 3.789 grams
5. $\frac{1}{2}$ years

Set C

1. 1.25 hours
2. 1.04 grams
3. 5.08 grams
4. 61.39 grams
5. 5 days

Lesson 3

Set A

1. P187,716.92
2. P1,268,241.80
3. after 3 months
4. In Bank of Manila. I will gain P61,879.15 more.
5. P10,000

Set B

1. P901,113.90
2. P204,825.055
3. After 6 months
4. In the bank. I will gain P4,732.12 more.
5. P129,799.89

Set C

1. P31.193.17; P11,193.17
2. P153,423.66
3. P868,931.92
4. 12%
5. 3 months

What have you learned

1. P75,182.83
2. 253,354
3. 0.075 kg
4. 4,882,812,500
5. 1 year
6. $2\frac{1}{2}$ hours
7. After 3 months
8. P84,066.85
9. 176.71 grams
10. 1 year