Module 4
Logarithmic Functions

What this module is about

This module deals with the definition, graph, properties, laws and application of the laws of the logarithmic function; and how to solve simple logarithmic equation. As you go over the discussion and exercises, you will appreciate the importance of this function. Find enjoyment in learning this module and go over the discussion and examples if you have not yet mastered a concept.

What you are expected to learn

This module is designed for you to:

1. define the logarithmic function $f(x) = \log_a x$ as the inverse of the exponential function $f(x) = a^x$,
2. draw the graph of the logarithmic function $f(x) = \log_a x$,
3. describe some properties of the logarithmic function from its graph;
4. apply the laws of logarithms; and
5. solve simple logarithmic equations.

How much do you know

1. What is the inverse of the exponential function?
   a. Linear function
   b. Quadratic function
   c. Polynomial function
   d. Logarithmic function

2. Which of the following is equivalent to $\log_2 16 = 4$?
   a. $2^4 = 16$
   b. $4^2 = 16$
   c. $16^2 = 4$
   d. $16^4 = 2$

2. Which of the following is equivalent to $(125)^{\frac{1}{3}} = 5$?
   a. $\log_{125} 5 = \frac{1}{3}$
   b. $\log_5 125 = \frac{1}{3}$
   c. $\log_{\frac{1}{3}} 5 = 125$
   d. $\log_5 \frac{1}{3} = 125$

4. The graph of $y = \log_5 x$ is asymptotic with which of the following lines?
   a. $y = x$
   b. x - axis
   c. y - axis
   d. $y = 1$

5. The graphs of $y = 10^x$ and $y = \log_{10} x$ are symmetric with respect to what line?
   a. $y = x + 1$
   b. $y = 2x$
   c. $y = x$
   d. $y = 2^x$

6. What is the point common to the graphs of functions in the form $y = \log_a x$?
   a. (0, 0)
   b. (1, 1)
   c. (0, 1)
   d. (1, 0)

7. Write $\log_3 x^2 + \log_3 y^3 - \log_3 z$ as a single logarithm.
   a. $\log_3\left(\frac{x^2 \bullet y^3}{z}\right)$
   b. $\log_3(x^2 + y^3 - z)$
   c. $\log_3\left(\frac{x^2 + y^3}{z}\right)$
   d. none of the above

8. Simplify $\log_3 18 + \log_3 2 - \log_3 4$ as a single number.
   a. 16
   b. 9
   c. 3
   d. 2

9. Given: $\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$, and $\log_{10} 5 = 0.6990$, what is $\log_{10} 1.2$?
   a. 0.2054
   b. 0.0791
   c. 1.1132
   d. -0.5553

10. Determine the value of x if $\log_7 x = -2$.
    a. 14
    b. $\frac{1}{14}$
    c. 49
    d. $\frac{1}{49}$

What you will do

Lesson 1

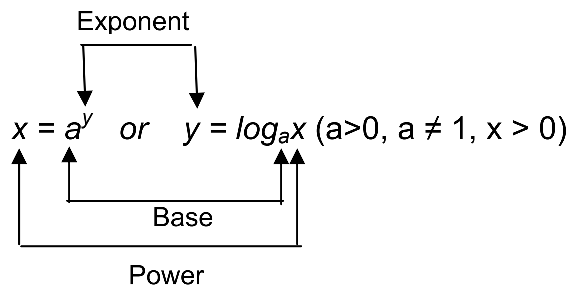
Logarithmic Function

In the previous module, you have learned about inverse functions. Recall that when the domain of one function is the range and the range is the domain of the other then they are inverses. Remember also that to determine the inverse of a function given an equation you have to interchange x and y then solve for y. Look at the illustration below on how to find the inverse of the exponential function, $y = a^x$.

Exponential Function	Inverse function
$y = a^x$	$x = a^y$

You will notice that the inverse of the exponential function shows that "*y* is the exponent to which the base *a* is raised in order to obtain the power *x*".

The inverse of the exponential function above is called *logarithmic function.* The function is defined by the equation -



The equation of a logarithmic function is read as "**y is the logarithm of x to the base a**". Take note that in the notation, *a* is the base, *x* is the power and *y* is the exponent to which *a* is raised in order to obtain *x*.

Example 1. The logarithmic equation $2 = \log_7 49$ is read as "2 is the logarithm of 49 to the base 7" or "the logarithm of 49 to the base 7 is 2" which means that the exponent of 7 in order to get 49 is 2.

Example 2. The logarithmic equation $\log_6 108 = 3$ is read as "the logarithm of 108 to the base 6 is 3" or "3 is the logarithm of 108 to the to the base 6" meaning the exponent of 6 is 3 to get 108.

Notice from the notation above that $y = \log_a x$ is equivalent to $x = a^y$. Thus, an equation in exponential form can be expressed in logarithmic form and vice-versa. Study the examples that follow.

Example 3*.* Transform the following equations in logarithmic form.

1. $2^6 = 64$
2. $3^{-2} = \frac{1}{9}$
3. $(128)^{\frac{1}{7}} = 2$

Solutions:

1. In $2^6 = 64$, the base is 2, the exponent is 6 and the power is 64.
   Thus, $2^6 = 64$ is equivalent to $6 = \log_2 64$ or $\log_2 64 = 6$.

2. In $3^{-2} = \frac{1}{9}$, the base is 3, the exponent in -2 and the power is $\frac{1}{9}$.

   Hence, $3^{-2} = \frac{1}{9}$ is equivalent to -2 = $\log_3 \frac{1}{9}$ or $\log_3 \frac{1}{9}$ = -2.

   .

3. In $(128)^{\frac{1}{7}} = 2$, the base is 128, the exponent is $\frac{1}{7}$ and the power is 2.

   Therefore, $(128)^{\frac{1}{7}} = 2$ is equivalent to $\frac{1}{7}$ = $\log_{128}2$ or $\log_{128}2 = \frac{1}{7}$.

Example 4. Transform the following logarithmic equations to exponential form.

1. $\log_5 1 = 0$
2. $\log_{10}.0001 = -4$
3. $\log_c a = -b$

Solutions:

1. In $\log_5 1 = 0$, the base is 5, the exponent is 0, and the power is 1. Therefore, $\log_5 1 = 0$ is equivalent to $5^0 = 1$.

2. In $\log_{10}0.0001 = -4$, the base is 10, the exponent is -4, and the power is 0.0001. Thus, $\log_{10}0.0001 = -4$ is equivalent to $10^{-4} = 0.0001$.

3. In $\log_c a = -b$, the base is c, the exponent is -b, and the power is a. Hence, $\log_c a = -b$ is equivalent to $c^{-b} = a$.

Logarithms can be obtained by considering the corresponding exponential form of the expression.

Example 5. Evaluate the logarithms of the following:

1. $\log_4 64$
2. $\log_9 27$
3. $\log_5 \frac{1}{625}$

Solutions:

1. Let $\log_4 64 = x$. Transform it in exponential form then solve for x.

$$4^x = 64$$
$$4^x = 4^3$$
$$x = 3$$
$$\therefore \log_4 64 = 3$$

2. Let $\log_9 27 = x$. Transform it in exponential form then solve for x.

$$\log_9 27 = x$$
$$9^x = 27$$
$$(3^2)^x = 3^3$$

$$3^{2x} = 3^3$$
$$2x = 3$$
$$x = \frac{3}{2}$$
$$\therefore \log_9 27 = \frac{3}{2}$$

3. Let $\log_5 \frac{1}{625}$ = x. Transform it in exponential form then solve for x.

$$\log_5 \frac{1}{625} = x$$
$$5^x = \frac{1}{625}$$
$$5^x = 5^{-4}$$
$$x = -4$$
$$\therefore \log_5 \frac{1}{625} = -4$$

Try this out

A. Write the equivalent exponential form of the following.

1. $\log_2 32 = 5$
2. $\log_3 81 = 4$
3. $\log_{12} 12 = 1$
4. $\log_{10} 100000 = 5$
5. $\log_{1/5} 125 = -3$
6. $\log_4 64 = 3$
7. $\log_2 \frac{1}{16} = -4$
8. $\log_r t = p$
9. $\log_a p = s$
10. $\log_q p = m$

B. Write the equivalent logarithmic form.

1. $3^5 = 243$
2. $9^0 = 1$
3. $11^2 = 121$
4. $\left(\frac{2}{3}\right)^4 = \left(\frac{16}{81}\right)$
5. $\left(\frac{16}{9}\right)^{½} = \frac{4}{3}$
6. $6^{-2} = \frac{1}{36}$
7. $a^{-5} = \frac{1}{a^5}$
8. $16^{\frac{1}{2}} = 4$
9. $125^{2/3} = 25$
10. $64^{2/3} = 16$

C. Evaluate the logarithm of each of the following

1. $\log_9 81$
2. $\log_2 128$
3. $\log_7 343$
4. $\log_{25} 625$
5. $\log_8 1$
6. $\log_4 8$
7. $\log_{64} 4$
8. $\log_6 \frac{1}{216}$
9. $\log_4 \frac{1}{256}$
10. $\log_8 \sqrt[5]{64}$

Lesson 2

Graphs of Logarithmic Function

From a previous module, you have learned about the graphs of inverse functions, that is, the graphs of inverse functions are reflections of each other and that they are symmetrical about the line y = x. Thus, the graph of the logarithmic function $y = \log_a x$ can be obtained from the graph of the exponential function $y = a^x$. To do this, simply flip the graph of $y = a^x$ along the line y = x.

Example 1. Draw the graph of $y = \log_2 x$.

Solution:

To draw the graph of $y = \log_2 x$, recall the graph of $y = 2^x$. Flip the graph of $y = 2^x$ about the line y = x. You should be able to observe that the two graphs contain the following integral values.

x	-3	-2	-1	0	1	2	3
$y = 2^x$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

x	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
$y = \log_2 x$	-3	-2	-1	0	1	2	3

For example, the point (1, 2) is on the graph of $y = 2^x$ and the point (2, 1) is on the graph of $y = \log_2 x$. Observe now that the graph of $y = \log_2 x$ is a reflection of the graph of the exponential function $y = 2^x$ along the line y = x which is the axis of symmetry. This can be seen in the figure below.

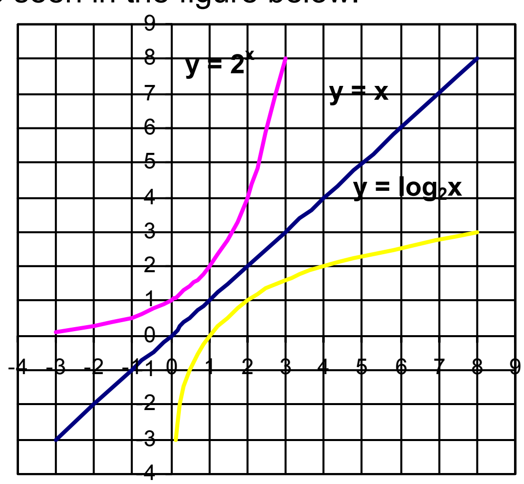


Figure 1

To draw the graph of $y = \left(\frac{1}{2}\right)^x$ and its inverse $y = \log_{\frac{1}{2}} x$, follow the steps in the previous example, that is, draw the graph of $y = \left(\frac{1}{2}\right)^x$ then flip it along the line y = x. Observe that the graph of $y = \log_{\frac{1}{2}} x$ contain the following integral values shown below.

x	-3	-2	-1	0	1	2	3
$y = 2^x$	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

x	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$
$y = \log_2 x$	-3	-2	-1	0	1	2	3

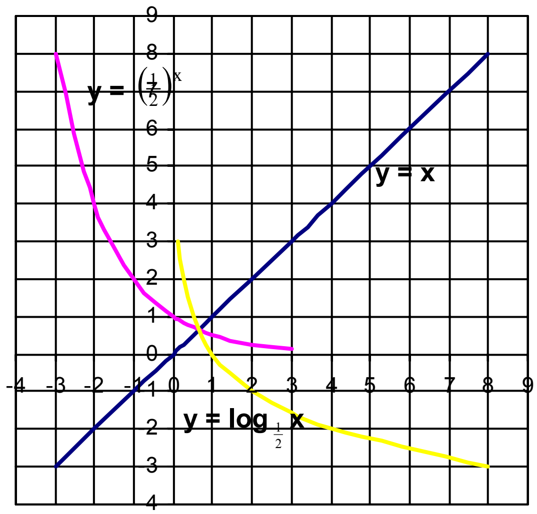


Figure 2

Notice that just like in Figure 1, the graph of $y = \log_{\frac{1}{2}} x$ is a reflection of the graph of $y = \left(\frac{1}{2}\right)^x$. The two graphs are also symmetrical about the line y = x.

Try this out

A. Sketch the graph of each of the following logarithmic functions using the graph of its inverse.
   1. $f(x) = \log_3 x$
   2. $f(x) = \log_4 x$
   3. $f(x) = \log_5 x$
   4. $f(x) = \log_{1/3} x$
   5. $f(x) = \log_{1/4} x$
   6. $f(x) = \log_{1/5} x$

B. Compare and contrast the graphs of numbers 1 and 4, numbers 2 and 5, and numbers 3 and 6.

Lesson 3

Properties of the Graph of a Logarithmic Function

From Figure 1 in Lesson 2, you can see the properties of the graph of the logarithmic function $y = \log_2 x$. Observe that the domain is the set of all positive real numbers and the range are all real numbers. It has an x-intercept (1. 0) and its asymptote is the y-axis. Notice also that the function is positive for all x greater than 1 and negative for all x less than 1. Thus, this function is increasing.

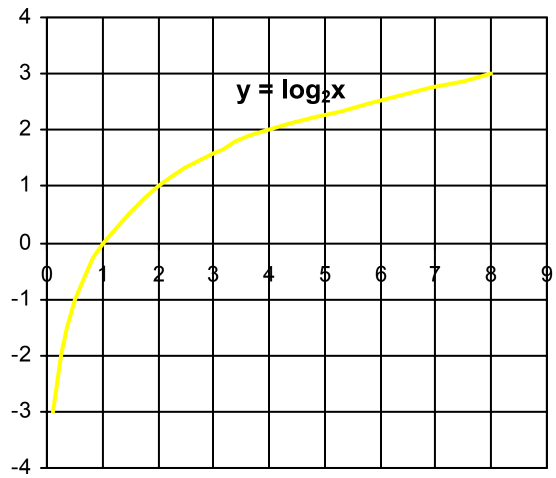
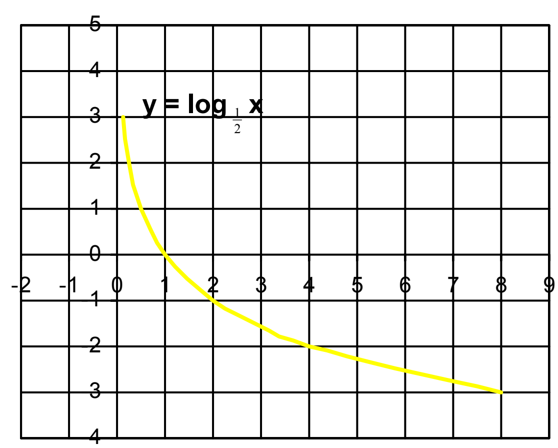


Figure 1

The properties of the graph in figure 1 illustrates the properties of logarithmic functions in the form $y = a^x$, $a > 1$.

1. The domain is the set of all positive real numbers and the range are all real numbers.
2. The x-intercept is (1. 0) and its asymptote is the y-axis.
3. The function is positive for all x greater than 1 and negative for all x less than 1.
4. The function is increasing.

From Figure 2 in Lesson 2, the properties of the graph of the logarithmic function $y = \log_{1/2} x$ can be observed. The domain is the set of all positive real numbers and the range are all real numbers. It has an x-intercept (1. 0) and its asymptote is the y-axis. Notice also that the function is negative for all x greater than 1 and positive for all x less than 1. Thus, this function is decreasing.



The properties of the graph in figure 1 illustrates the following properties of logarithmic functions in the form $y = a^x$, $0 < a < 1$.

1. The domain is the set of all positive real numbers and the range are all real numbers.
2. The x-intercept is (1. 0) and its asymptote is the y-axis.
3. The function is negative for all x greater than 1 and positive for all x less than 1.
4. The function is decreasing.

Try this out

A. Enumerate the properties of the graphs of the following logarithmic functions. Refer to the graphs drawn in Lesson 2.

1. $f(x) = \log_3 x$
2. $f(x) = \log_4 x$
3. $f(x) = \log_5 x$
4. $f(x) = \log_{1/3} x$
5. $f(x) = \log_{1/4} x$
6. $f(x) = \log_{1/5} x$

B. Give the properties common to the graphs in A.

Lesson 4

Laws of Logarithms

Since logarithmic function and exponential function are inverse functions, the laws of exponents will be used to derive the laws of logarithms. The laws of logarithms were important tools in shortening complicated computations long before the use of scientific calculators and computers. Nowadays, logarithms are used for different purposes specifically in sciences.

A. The Logarithm of a Product

Let $M = a^x$ and $N = a^y$.

By Law of Exponents for Products, $MN = a^x \bullet a^y = a^{x+y}$.

By definition of logarithmic function,

$$M = a^x \leftrightarrow \log_a M = x,$$
$$N = a^y \leftrightarrow \log_a N = y, \text{ and}$$

$$MN = a^{x+y} \leftrightarrow \log_a MN = x + y.$$

By substitution, $\log_a MN = \log_a M + \log_a N$.

From the derivation, the logarithm of the product of two numbers is the sum of the logarithms of the two factors.

B. The Logarithm of a Quotient

Use the given in A and the Law of Exponents for Quotients, $\frac{M}{N} = \frac{a^x}{a^y} = a^{x-y}$.

By definition of logarithmic function,

$$M = a^x \leftrightarrow \log_a M = x,$$
$$N = a^y \leftrightarrow \log_a N = y, \text{ and}$$
$$\frac{M}{N} = a^{x-y} \leftrightarrow \log_a\left(\frac{M}{N}\right) = x - y$$

By substitution, $\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$

From the derivation, the logarithm of the quotient of two numbers is the difference of the logarithms of the dividend and the divisor.

C. The logarithm of a Power

Let $M = a^x$.

By the Law of Exponents for a Power, $M^k = (a^x)^k$.

By definition of logarithmic function,

$$M = a^x \leftrightarrow \log_a M = x, \text{ and}$$
$$M^k = (a^x)^k \leftrightarrow \log_a M^k = xk.$$

By substitution, $\log_a M^k = k\log_a M$

From the derivation, the logarithm of the k^{th} power of a number is k times the logarithm of the number.

Below are examples of the application of the laws of logarithms.

Example 1. Simplify the following using the laws of logarithms.

1. $\log_a 5PQ$
2. $\log_a\left(\frac{32}{5}\right)$
3. $\log_b(24)^3$
4. $\log_b \sqrt[3]{\frac{x^7}{y}}$

Solutions:

1. Apply the logarithm of a product.
   $\log_a 5PQ = \log_a 5 + \log_a P + \log_a Q$

2. Apply the logarithm of a quotient.
   $\log_a\left(\frac{32}{5}\right) = \log_a 32 - \log_a 5$

3. Apply the logarithm of a power.
   $\log_b(24)^3 = 3\log_b 24$

4. Apply the logarithm of a power and logarithm of a quotient and distributive property.

$$\log_b \sqrt[3]{\frac{x^7}{y}} = \log_b\left(\frac{x^7}{y}\right)^{\frac{1}{3}}$$
$$= \frac{1}{3}\log_b\left(\frac{x^7}{y}\right)$$
$$= \frac{1}{3}(\log_b x^7 - \log_b y)$$
$$= \frac{1}{3}\log_b x^7 - \frac{1}{3}\log_b y$$
$$= \frac{1}{3}(7\log_b x) - \frac{1}{3}\log_b y$$
$$= \frac{7}{3}\log_b x - \frac{1}{3}\log_b y$$

Example 2: Write the following as a single logarithm. Simplify if possible.

1. $\log_4 12p + 9\log_4 q$
2. $\log_d(h^2 - 16) - \log_4(h + 4)$

Solution:

1. The expression is an application of the logarithm of a power and logarithm of a product.

$$\log_4 12p + 9\log_4 q = \log_4 12p + \log_4 q^9$$
$$= \log_4(12pq^9)$$

2. The expression is an application of the logarithm of a quotient.

$$\log_d(h^2 - 16) - \log_d(h + 4) = \log_d\left(\frac{h^2 - 16}{h + 4}\right)$$

$$= \log_d \left[\frac{(h-4)(h+4)}{h+4} \right]$$
$$= \log_d(h-4)$$

Example 3: Given that log 2 = 0.3010, log 3 = 0.4771, and log 5 = 0.6990, determine-

1. log 15
2. log 0.4
3. log 3.6
4. $\log \sqrt[4]{625}$
5. $\log \frac{\sqrt[3]{30}}{2}$

Solution:

Observe that no base was indicated in example 3. When no base is indicated, it means that the base is 10. By convention, when base 10 is used we write log x instead of $\log_{10}x$. We call this common logarithm. Thus, we will use this convention from now on.

1. log 15 = log (3•5) = log 3 + log 5 = 0.4771 + 0.6990 = 1.1761

2. $\log 0.6 = \log \frac{3}{5}$ = log 3 – log 5 = 0.4771 – 0.6990 = -0.2219

3. $\log 3.6 = \log \frac{18}{5}$

$$= \log \frac{2 \bullet 3^2}{5}$$
$$= \log 2 + \log 3^2 - \log 5$$
$$= \log 2 + 2\log 3 - \log 5$$
$$= 0.3010 + 2(0.4771) - 0.6990$$
$$= 0.3010 + 0.9542 - 0.6990$$
$$= 0.5562$$

4. $\log \sqrt[4]{625} = \log(125)^{\frac{1}{4}}$

$$= \log\left(5^3\right)^{\frac{1}{4}}$$
$$= \log 5^{\frac{3}{4}}$$
$$= \frac{3}{4}\log 5$$
$$= \frac{3}{4}(0.6990)$$
$$= 0.52425$$

5. $\log \frac{\sqrt[3]{30}}{2} = \log \sqrt[3]{30} - \log 2$

$= \log(2 \bullet 3 \bullet 5)^{\frac{1}{3}} - \log 2$

$= \frac{1}{3}(\log 2 + \log 3 + \log 5) - \log 2$

$= \frac{1}{3}(0.3010 + 0.4771 + 0.6990) - 0.3010$

$= \frac{1}{3}(1.4771) - 0.3010$

$= 0.4924 - 0.3010$

$= 0.1914$

Try this out

A. Express the following as a single logarithm and simplify:

1. $\log_2 192 + \log_2 6$
2. $\log_3 36 + \log_3 4 - \log_3 16$
3. $\log_5 \sqrt[3]{625}$
4. $\log_7 98 + 2\log_7 7 - \log_7 2$
5. $\frac{1}{6}\log_3 27 + \log_3 18 - \log_3 2$
6. $\log_2 x + 3\log_2 y - \log_2 z$
7. $\log(3x^2 + 11x - 20) - \log(3x - 4)$
8. $\log_b(2x - 5) + \log_b(x + 1)$
9. $2\log_a x^3 + 3\log_a y + 4\log_a z - 3\log_a w$
10. $\frac{1}{2}\log(x^2 - 4) + 3\log(x + 2)$

B. Given log 3 = 0.4771, log 5 = 0.6990, and log 7 = 0.8451, evaluate the following applying the laws of logarithms:

1. $\log 21$
2. $\log 63$
3. $\log 4.2$
4. $\log 49^2$
5. $\log \sqrt[4]{45}$
6. $\log \frac{75}{7}$
7. $\log \frac{\sqrt{49}}{9}$
8. $\log \frac{75\sqrt[4]{21}}{5}$
9. $\log \frac{(21)^2}{27}$
10. $\log \frac{147}{\sqrt{5}}$

C. If z = log 5, write the following expressions in terms of z:

1. log 25
2. $\log 5^6$
3. $\log \frac{1}{125}$
4. $\log \sqrt{25}$
5. log 5000

Lesson 5

Logarithmic Equations

Logarithmic equations are equations involving logarithmic functions.

From previous lesson, you learned that exponential functions and logarithmic functions are inverses. Hence, the properties of these functions can be used to solve equations involving these two functions.

Study the following examples illustrating how logarithmic equations are solved.

Example 1*:* Solve the missing terms in the following logarithmic equations.

1. $\log_6 216 = x$
2. $\log_x 81 = \frac{2}{3}$
3. $\log_{25} x = \frac{3}{2}$
4. $\log_4(5x + 4) = 3$
5. $\log (2x^2 + 4x - 3) = \log (x^2 + 2x + 12)$

Solutions:

1. Transform $\log_6 216 = x$ in exponential form, then solve for x.

$$6^x = 216$$
$$6^x = 6^3$$
$$x = 3$$

Check:

If x = 3, then $6^3 = 216$.

$\therefore \log_6 216 = 3$ and x = 3.

2. Transform $\log_x 81 = \frac{2}{3}$ in exponential form, then solve for x using the properties of exponents.

$$x^{\frac{2}{3}} = 81$$

$$x^{\frac{2}{3}} = 3^4$$

$$\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = \left(3^4\right)^{\frac{3}{2}}$$ Raise both sides to the same power and simplify

$$x = 3^6$$

$$x = 729$$

Check:

If x = 729, then $729^{\frac{2}{3}} = \left(3^6\right)^{\frac{2}{3}} = 3^4 = 81$.

$\therefore x = 729$.

3. Transform $\log_{25}x = \frac{3}{2}$ in exponential form, then solve for x using the properties of exponents.

$$x = 25^{\frac{3}{2}}$$

$$x = \left(5^2\right)^{\frac{3}{2}}$$

$$x = 5^3$$

$$x = 125$$

Check:

$25^{\frac{3}{2}} = \left(5^2\right)^{\frac{3}{2}} = 5^3 = 125$

$\therefore x = 125$.

4. Transform $\log_4(5x + 4) = 3$ in exponential form, then solve for x.

$$5x + 4 = 4^3$$

$$5x + 4 = 64$$

$$5x = 64 – 4$$

$$5x = 60$$

$$x = \frac{60}{12}$$

$$x = 5$$

Check:

If x = 5, then $\log_4[5(12) + 4] = \log_4(60 + 4) = \log_4 64 = 3$.

$\therefore x = 5$

5. $\log(2x^2 - 7x – 9) = \log(x^2 - 9x + 6)$

Since both sides of the equation are in base 10, then

$2x^2 - 7x – 9 = x^2 - 9x + 6$

$2x^2 - 7x – 9 – (x^2 - 9x + 6) = 0$ Addition Property of Equality

$x^2 + 2x - 15 = 0$ Simplify

$(x + 5)(x - 3) = 0$ Factoring

$x + 5 = 0$ or $x - 3 = 0$

$x = -5$ or $x = 3$

Check:

If x = -5, then $\log(2x^2 - 7x - 9) = \log[2(-5)^2 - 7(-5) - 9]$
$= \log[2(25) + 35 - 9]$
$= \log(50 + 35 - 9)$
$= \log 76$

and $\log(x^2 - 9x + 6) = \log[(-5)^2 - 9(-5) + 6]$
$= \log(25 + 45 + 6)$
$= \log 76.$

Hence, x = -5 is a solution

If x = 3, then $\log_5 \log(2x^2 - 7x - 9) = \log[2(3)^2 - 7(3) - 9]$
$= \log[2(9) - 21 - 9]$
$= \log(18 - 21 - 9)$
$= \log(-12)$

Hence, x = 3 is not a root since logarithms is defined only for positive number.

$\therefore x = -5$ is the only solution.

Notice that to solve problems 1 to 4 the logarithmic equations were first transformed to exponential form.

Example 2*:* Solve for x in the following exponential equations:

1. $5^x = 9$
2. $3^{x+7} = 2^x$

Solutions:

Exponential equations involving different bases can be solved with the use of logarithms and the aid of a scientific calculator or a table of logarithms. In the case of the example 2, a scientific calculator will be used.

1. $5^x = 9$

$\log 5^x = \log 9$	Get the logarithm of both sides.
$x\log 5 = \log 9$	Logarithm of a Power
$x = \frac{\log 9}{\log 5}$	Multiplication Property of Equality
$x = \frac{0.9542}{0.6990}$	Replacement of value from the table or calculator
x = 1.3651	Division Fact

Check:

Using a scientific calculator,

$5^{1.3651} = 9$ (rounded to the nearest whole number)

2. $3^{x+2} = 2^x$

$\log 3^{x+2} = \log 2^x$	Get the logarithm of both sides
$(x + 2)\log 3 = x \log 2$	Logarithm of a Power
$x \log 3 + \log 3 = x \log \quad 2$	Distributive Property
$x \log 3 – x \log 2 = -2\log 3$	Addition Property of Equality
$x(\log 3 – \log 2) = -2\log 3$	Factoring
$x = \frac{-2\log 3}{\log 3 - \log 2}$	Multiplication Property of Equality
$x = \frac{-2(0.4771)}{0.4771 - 0.3010}$	Replacement of value from the table or calculator
$x = \frac{-0.9542}{.1761}$	Multiplication Fact
$x = -5.4185$	Division Fact

Check:

Using a scientific calculator,

$3^{-5.4185 + 2} = 2^{-5.4185}$

$3^{-3.4185} = 2^{-5.4185}$

$0.0234 = 0.0234$

Try this out

A. Solve for the missing term:

1. $\log_7 n = -3$
2. $\log_4 512 = x$
3. $\log_c 36 = -2$
4. $\log_{25} r = \frac{7}{2}$
5. $\log_p 256 = -7$
6. $\log_8 128 = y$
7. $\log x - \log 12 = 3$
8. $\log_5 x^5 - \log_5 x^3 = 2$
9. $\log p – \log 3 = 1$
10. $\log_2 z + \log_2 2z = 3$
11. $\log_7 a + \log_7 (a - 1) = \log_7 12$
12. $\log_3 y – \log_3 (y – 2) = \log_3 2$
13. $\log x + \log (x + 1) = \log 20$
14. $\log (3x – 5) + \log 2x = \log 4$
15. $\log 25x – \log 5 = 2$
16. $\log_3 (x^2 – 16) – \log_3 (x + 4) = 2$
17. $\log_5 (m + 5) – l0g_5 (m + 1) = \log_5 4$
18. $\log_7 (q + 4) – \log_7 3 = \log_7 (q – 4)$
19. $\log (14v^2 + 35v) = \log 3 + \log (2v + 5)$

20. $\log(21h^2 + 50h - 14) = \log(3h^2 + 25h + 14)$

B. Solve the following exponential equations using logarithms:

1. $4^x = 5$
2. $3^{2x} = 18$
3. $5^x + 1 = 9$
4. $5^{x-2} = 4^{x-1}$
5. $2^{3x+5} = 3^x$

Let's Summarize

1. The inverse of the exponential function above is called *logarithmic function.* The function is defined by the equation,

$$x = a^y \quad or \quad y = \log_a x \ (a>0,\ a \neq 1,\ x > 0)$$

   read as "**y is the logarithm of x to the base a**" where *a* is the base, *x* is the power and *y* is the exponent to which *a* is raised in order to obtain *x.*

2. The graph of the logarithmic function **$y = \log_a x$** can be obtained from the graph of the exponential function **$y = a^x$**. To do this, flip the graph of $y = a^x$ along the line $y = x$.

3. The properties of the graph logarithmic functions in the form $y = a^x$, $a > 1$ are as follow:

   a. The domain is the set of all positive real numbers and the range are all real numbers.
   b. The x-intercept is (1. 0) and its asymptote is the y-axis.
   c. The function is positive for all x greater than 1 and negative for all x less than 1.
   d. The function is increasing.

4. The properties of the graph of logarithmic functions in the form $y = a^x$, $0 < a < 1$are as follow:

   a. The domain is the set of all positive real numbers and the range are all real numbers.
   b. The x-intercept is (1. 0) and its asymptote is the y-axis.
   c. The function is negative for all x greater than 1 and positive for all x less than 1.
   d. The function is decreasing.

5. The Law of Logarithms are as follows:

   a. Logarithm of a Product: $\log_a MN = \log_a M + \log_a N$
   b. Logarithm of a Quotient: $\log_a \left(\frac{M}{N}\right) = \log_a M - \log_a N$
   c. Logarithm of a Power: $\log_a M^k = k\log_a M$

6. To solve simple logarithmic equations, transform the equation in exponential form, then solve for the missing term.

7. To solve exponential equations involving different bases, find the logarithm of both sides of the equation with the aid of a scientific calculator or a table of logarithms then solve for the missing term applying the properties of equality.

What have you learned

1. Which of the following is the inverse of $y = a^x$?
   a. $y = ax$
   b. $y = x^a$
   c. $y = \log_a x$
   d. $a = y^x$

2. Which of the following is equivalent to $\log_4 64 = 3$?
   a. $3^4 = 64$
   b. $4^3 = 64$
   c. $64^4 = 3$
   d. $16^3 = 4$

3. Which of the following is equivalent to $(27)^{\frac{1}{3}} = 3$?
   a. $\log_{27} 3 = \frac{1}{3}$
   b. $\log_3 27 = \frac{1}{3}$
   c. $\log_{\frac{1}{3}} 3 = 27$
   d. $\log_3 \frac{1}{3} = 27$

4. At what point does the graph of $y = \log_2 x$ intersect the x-axis?
   a. (0, 1)
   b. (0, 0)
   c. (-1, 0)
   d. (1, 0)

5. The graphs of $y = 4^x$ and $y = \log_4 x$ are symmetric with respect to what line?
   a. $y = x + 1$
   b. $y = 4x$
   c. $y = x$
   d. $y = 4^x$

6. What is the point common to the graphs of functions $y = \log_3 x$ and $y = \log_{10} x$?
   a. (0, 0)
   b. (1, 1)
   c. (0, 1)
   d. (1, 0)

7. Write 3 log a + 5 log b – 4 log c as a single logarithm.

a. $\log\left(\frac{a^3 \bullet b^5}{c^4}\right)$

c. $\log\left(\frac{a^3 + b^5}{c^4}\right)$

b. $\log(a^3 + b^5 - c^4)$

d. none of the above

8. Simplify $\log_3 81 + \log_3 3 - \log_3 9$ as a single number.

a. 27

c. 3

b. 5

d. 2

9. Given that $\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$, and $\log_{10} 5 = 0.6990$, what is $\log_{10} 7.5$?

a. 0.5229

c. 0.8751

b. 0.0791

d. 1.4471

10. Determine the value of x if $\log_5 (8x - 7) = 2$

a. 4.875

c. 3.125

b. 4

d. 2.25

Answer Key

How much do you know

1. d
2. a
3. a
4. c
5. c
6. d
7. a
8. d
9. b
10. d

Try this out

Lesson 1

A.
1. $2^5 = 32$
2. $3^4 = 81$
3. $12^1 = 12$
4. $10^5 = 100000$
5. $\left(\frac{1}{5}\right)^{-3} = 125$
6. $4^3 = 64$
7. $2^{-4} = \frac{1}{16}$
8. $r^p = t$
9. $a^s = p$
10. $q^m = p$

B.
1. $\log_3 243 = 5$
2. $\log_9 1 = 0$
3. $\log_{11} 121 = 2$
4. $\log_{\frac{2}{3}} \frac{16}{81} = 4$
5. $\log_{\frac{16}{9}} \frac{4}{3} = \frac{1}{2}$
6. $\log_6 \frac{1}{36} = -2$
7. $\log_a \frac{1}{a^5} = -5$
8. $\log_{16} 4 = \frac{1}{2}$

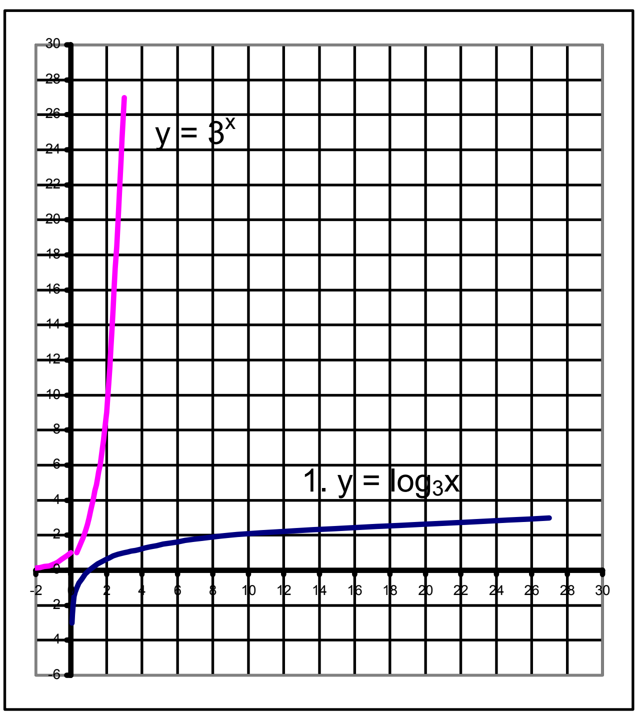
9. $\log_{125} 25 = \frac{2}{3}$

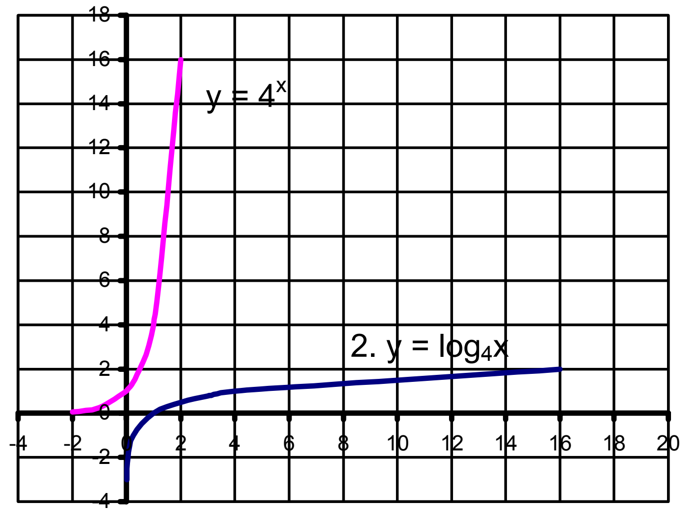
10. $\log_{64} 16 = \frac{2}{3}$

C.
1. 2
2. 7
3. 3
4. 2
5. 0
6. $\frac{3}{2}$
7. $\frac{1}{3}$
8. -3
9. -4
10. $\frac{2}{5}$

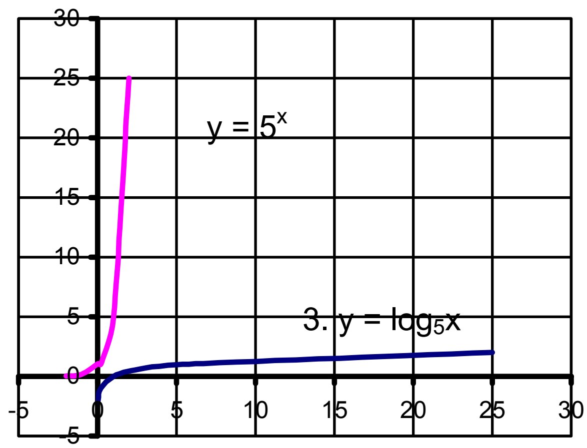
Lesson 2

A.

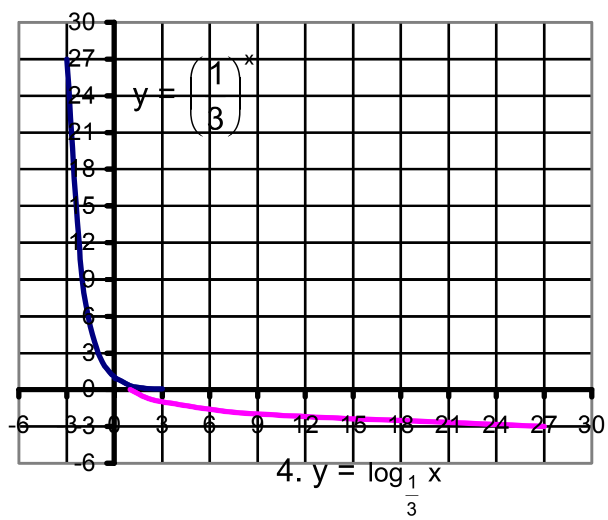




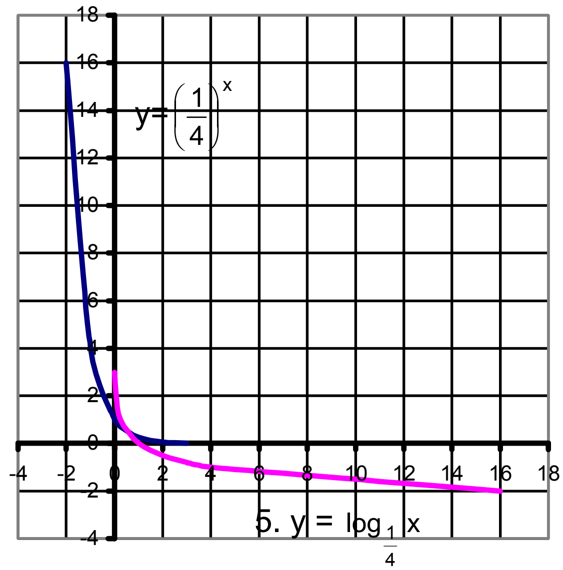
y = 4^x
2. y = log4x

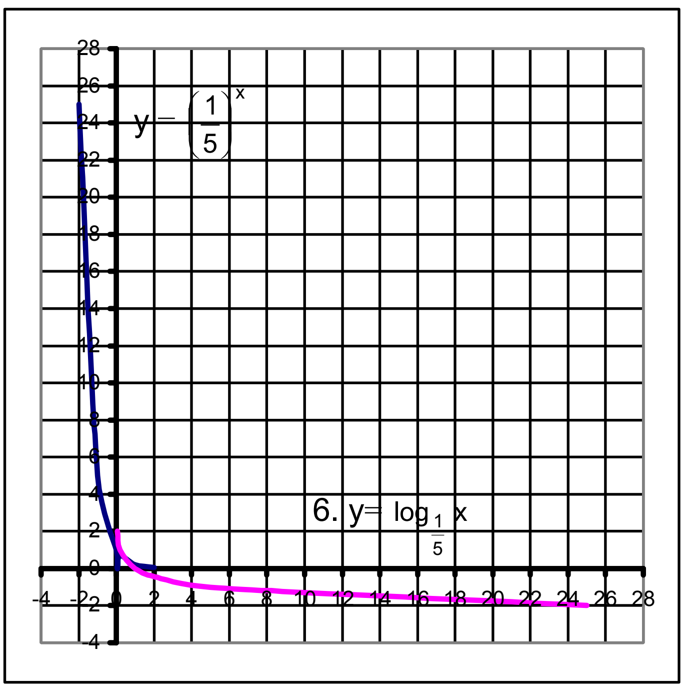


y = 5^x
3. y = log5x



y = (1/3)^x
4. y = log 1/3 x





B. The graph of $y = \log_3 x$ has the same shape and size as the graph of $y = \log_{\frac{1}{3}} x$. The same is true for the graphs of $y = \log_4 x$ and $y = \log_{\frac{1}{4}} x$, and $y = \log_5 x$ and $y = \log_{\frac{1}{5}} x$. The graphs of $y = \log_3 x$, $y = \log_4 x$ and $y = \log_5 x$ are increasing while the graphs of $y = \log_{\frac{1}{3}} x$, $y = \log_{\frac{1}{4}} x$, and

$y = \log_{\frac{1}{5}} x$ are decreasing. The six graphs have a common point (1, 0).

Lesson 3

A.

No.	Equation	Domain	Range	x-intercept	Asymptote	Trend
1	$y = \log_3 x$	+ Real Nos.	Real Nos.	(1, 0)	y-axis	increasing
2	$y = \log_4 x$	+ Real Nos.	Real Nos.	(1, 0)	y-axis	increasing
3	$y = \log_5 x$	+ Real Nos.	Real Nos.	(1, 0)	y-axis	increasing
4	$y = \log_{\frac{1}{3}} x$	+ Real Nos.	Real Nos.	(1, 0)	y-axis	decreasing
5	$y = \log_{\frac{1}{4}} x$	+ Real Nos.	Real Nos.	(1, 0)	y-axis	decreasing
6	$y = \log_{\frac{1}{5}} x$	+ Real Nos.	Real Nos.	(1, 0)	y-axis	decreasing

B. Common Properties: Domain, range, x-intercepts, and asymptotes.

Lesson 4

A.

1. $\log_2 32 = 5$
2. $\log_3 9 = 2$
3. $\frac{4}{3}\log_5 5 = \frac{4}{3}$
4. $4\log_7 7 = 4$
5. $\frac{5}{2}\log_3 3 = \frac{5}{2}$
6. $\log_2 \frac{xy^3}{z}$
7. $\log (x+5)$
8. $\log_b(2x^2 - 3x - 5)$
9. $\log_a \frac{x^6y^3z^4}{w^3}$
10. $\log \sqrt{x^2-4}(x+2)^3$

B.

1. 1.3222
2. 1.7993
3. 0.6232
4. 3.3804
5. 0.4133
6. 1.03
7. -0.1091
8. 1.5066
9. 1.2131
10. 1.8178

C.

1. 2z
2. 6z
3. -3z
4. z
5. z + 3

Lesson 5

A.

1. $\frac{1}{343}$
2. $\frac{9}{2}$
3. $\frac{1}{6}$
4. 78125
5. $\frac{1}{2}$
6. $\frac{7}{3}$
7. 12000
8. 5
9. 30
10. 2
11. 4
12. 4
13. 4
14. 2
15. 20
16. 13
17. $\frac{1}{3}$
18. 8
19. $\frac{3}{7}$
20. 3

B.

1. 1.16
2. 1.32
3. 1.29
4. 8.21
5. -3.53

What have you learned

1. c
2. b
3. a
4. d
5. c
6. d
7. a
8. c
9. c
10. b