

Module 4

Quadratic Functions



What this module is about

This module is about the application of quadratic equations and functions in everyday situations. As you go over the different problems you will apply your knowledge and skills related to quadratic equations and functions in solving problems. Frequently, only one solution of the equation is relevant to the problem at hand. The root which does not satisfy the conditions of the problem must be rejected. It is always a good practice, then, to check the solutions to determine if one or both may be used.



What you are expected to learn

This module is designed for you to:

1. recall the different steps in solving word problems
2. translate verbal statements into symbols
3. apply knowledge and skills related to quadratic equations and functions in solving problems.



How much do you know

A. Write the letter of the correct answer.

1. The roots of a quadratic equation are 8 and -8. What is the equation?
a. $x^2 + 64 = 0$ b. $x^2 - 64 = 0$ c. $x^2 - 8 = 0$ d. $x^2 + 8 = 0$
2. What are the values of a, b, and c in the equation $6x = 2x^2 + 1$
a. $a = -2$ b. $b = -6$ c. $c = 1$

- b. $a = 2$ $b = -6$
- c. $a = -2$ $b = 6$ c. $= -1$
- d. $a = 2$ $b = -6$ c. $= 1$

3. Which of the following is a solution of $(x - 3)(x - 4) = 20$

- a = -5 b = 4 c = 8 d = 20

4. Which of the following has roots of -3 and 7?

- a. $x^2 + 4x - 21 = 0$ b. $x^2 - 4x - 21 = 0$
- c. $x^2 - 4x + 21 = 0$ d. $x^2 + 4x + 21 = 0$

5. Find the product of the roots of $2x^2 = x + 3$

- a. $\frac{1}{2}$ b. $\frac{3}{2}$ c. -1 d. $\frac{-3}{2}$

B. Write an equation to show the functional relationship between the quantities involved in the problems using the indicated variables.

1. The area (A) of a square of side s is s^2 .
2. The perimeter of a rectangle is equal to twice the Area.
3. $12x$ plus seven is equal to $3x$ reduced by 4
4. The circumference (C) of a circle is twice the product of π and the radius (r).
5. The total distance (d) covered is equal to the product of the rate (r) and the time (t).

C. Solve the following problems.

1. The product of two consecutive odd integers is 143. Find the integers.
2. One side of a rectangle is 10 cm longer than its width. The area of the rectangle is 96 sq. cm. Find the dimensions of the rectangle.
3. The sum of two numbers is 22 and the sum of their squares is 250. Find the numbers.

4. The sum of two numbers is 24. What is the maximum possible product of the two numbers?



Lesson 1

Number Problems

Many students could not solve this type of problem easily because they fail to translate correctly the different expressions given by the word problem into a correct equation. In the following examples, you will see that before you solve a word problem, you must first translate the word problem into an equation.

Steps in Solving word problems involving quadratic equation and function:

1. Identify your unknowns.
2. Write your equation.
3. Operate on the numbers.
4. Convert the equation into the standard form. Remember to change signs.
5. Factor the equation into its two corresponding linear equations. By factoring, you reduce the quadratic equation into two equivalent linear equations.

Examples:

1. Find two consecutive odd integers whose product is 195.

Solution:

1st step: Identify your unknowns.

Let x = one odd number,
so $x + 2$ = next consecutive odd number.

2nd step: Write your equation.

$$\begin{array}{ccc} x & (x + 2) & = & 195 \\ \downarrow & \downarrow & & \downarrow \\ \text{one odd} & \text{consecutive} & & \text{their} \\ \text{number} & \text{odd number} & & \text{product} \end{array}$$

3rd step: Operate on the numbers. Multiply the two quantities.

Thus:

$$x^2 + 2x = 195$$

4th step: Bring all terms to the left side of the equal sign and equate to zero. Remember to change signs.

$$\text{So } x^2 + 2x - 195 = 0$$

5th step: Factor the equation.

$$\text{Thus: } (x + 15)(x - 13) = 0$$

6th step: Solve for x.

$$x + 15 = 0 \quad \text{or} \quad (x - 13) = 0$$

$$x = -15 \quad \text{or} \quad x = 13$$

The consecutive numbers are :

$$x = 13 \quad \text{and} \quad x + 2 = 15, \quad 13 \quad \text{and} \quad 15$$

or

$$x = -15 \quad \text{and} \quad x + 2 = -15 + 2 = -13, \quad -13 \quad \text{and} \quad -15$$

Check:

$$x(x + 2) = 195 \quad \text{or} \quad x(x + 2) = 195$$

$$(13)(15) = 195 \quad \text{or} \quad (-13)(-15) = 195$$

$$195 = 195 \quad \text{or} \quad 195 = 195$$

2. The sum of two numbers is 19 and their product is 60. Find the numbers.

Solution:

1st step: Identify the unknown.

$$\begin{aligned} \text{Let } x &= \text{one number} \\ \text{So } 19 - x &= \text{the other number} \\ \text{And } x(19 - x) &= \text{their product.} \end{aligned}$$

2nd step: Write the equation.

$$x(19 - x) = 60$$

3rd step: Operate the numbers to remove the parentheses grouping symbol.

$$19x - x^2 = 60$$

4th step: Convert the equation to its standard form.

$$\begin{aligned} \text{Thus:} \\ 19x - x^2 - 60 = 0 \quad \text{or} \quad x^2 - 19x + 60 = 0 \end{aligned}$$

5th step: Factor the equation.

$$(x - 15)(x - 4) = 0$$

6th step: Solve for x.

$$\begin{aligned} x - 15 = 0 \quad \text{and} \quad x - 4 = 0 \\ x = 15 \quad \quad \quad x = 4 \end{aligned}$$

$$19 - x \rightarrow 19 - 15 = 4 \quad 19 - x \rightarrow 19 - 4 = 15$$

The numbers are therefore 15 and 4

Check:

$$\begin{aligned} \text{Their sum:} \quad 15 + 4 &= 19 \\ \quad \quad \quad 19 &= 19 \text{ check.} \end{aligned}$$

$$\begin{aligned} \text{Their product:} \quad (15)(4) &= 60 \\ \quad \quad \quad 60 &= 60 \text{ check.} \end{aligned}$$

3. The sum of two numbers is 22 and the sum of their squares is 250. Find the numbers.

Solution:

1st step: Identify your unknown

$$\begin{aligned} \text{Let } x &= \text{ one of the two numbers} \\ 22 - x &= \text{ the other number} \end{aligned}$$

2nd step: Write the equation

$$x^2 + (22 - x)^2 = 250$$

3rd step: Operate on the numbers to remove the parentheses grouping symbol.

$$x^2 + (22 - x)^2 = 250$$

$$x^2 + 484 - 44x + x^2 = 250$$

$$2x^2 - 44x + 484 = 250$$

4th step: Convert the equation to its standard form.

$$2x^2 - 44x + 484 - 250 = 0$$

$$2x^2 - 44x + 234 = 0 \quad \text{divide the equation by 2}$$

$$x^2 - 22x + 117 = 0$$

5th step: Factor the quadratic equation to obtain the two equivalent linear equations.

$$(x - 13)(x - 9) = 0$$

6th step: Solve for x.

$$\begin{aligned} x - 13 &= 0 & \text{and} & & x - 9 &= 0 \\ x &= 13 & \text{and} & & x &= 9 \end{aligned}$$

$$\text{If } x = 13, \quad 22 - 13 = 9.$$

$$\text{If } x = 9, \quad 22 - 9 = 13.$$

The numbers are 9 and 13.

Check:

$$9 + 13 = 22;$$

$$9^2 + 13^2 = 81 + 169 = 250.$$

Try this out

Analyze and solve.

1. Find 2 consecutive integers such that three times the square of the first is equal to seven more than five times the second.
2. Find 2 consecutive even integers whose product is 224.
3. Find an integer such that the square of the integer is eighty one less than eighteen times the integer.
4. One number is three times another and their product is equal to their sum increased by fifty five. Find the numbers.
5. The sum of two numbers is 16 and the sum of their squares is 146. Find the numbers.
6. A number is $\frac{16}{15}$ less than its multiplicative inverse. Find the number.
7. Find two numbers whose difference is 5 and the difference of their squares is 65.
8. One number is three more than a second number. The sum of their squares is 37 more than the product of the numbers. Find the two numbers.
9. The product of two consecutive integers is 47 more than the next consecutive integer. Find the two numbers.
10. The tens digit of a certain number is 7 less than the units digit. The sum of the squares of the two digits is 85. Find the numbers.

Lesson 2

Geometry Problems

Many government and classroom examinations deals with problems involving measurements of different geometric figures. These problems are also called geometry problems.

Examples:

1. The altitude of a triangle is 3 cm. less than the base. The area of the triangle is 35 square centimeters. What are its dimensions?

Solution:

1st step: Identify the unknown.

Let x cm = length of the base
So $(x - 3)$ cm = length of the altitude

$$\text{Area of a triangle} = \frac{(\text{base})(\text{altitude})}{2}$$

2nd step: Write the equation.

$$35 = \frac{x(x-3)}{2}$$

3rd step: Operate the numbers to remove the parentheses grouping symbol.

$$\begin{aligned}x(x-3) &= 70 \\x^2 - 3x &= 70\end{aligned}$$

4th step: Convert the equation to its standard form.

$$x^2 - 3x - 70 = 0$$

5th step: Factor the equation.

$$(x - 10)(x + 7) = 0$$

6th step: Solve for x.

$$\begin{array}{l} x - 10 = 0 \\ x = 10 \end{array} \quad \text{and} \quad \begin{array}{l} x + 7 = 0 \\ x = -7 \end{array}$$

We can not accept -7 as a value for the base of the triangle. Therefore the base of the triangle is 10 cm while the altitude is $x - 7 = 7$ cm.

2. The perimeter of a rectangle is 138 meters and the area is 1080 square meters. Find the length and the width of the rectangle.

Solution:

1st step: Identify the unknown.

$$\begin{array}{l} \text{Let } x \text{ cm} = \text{length of the rectangle} \\ 138 \text{ cm} = \text{Perimeter} \\ P = 2l + 2w \\ 138 = 2x + 2w \quad \text{express perimeter in terms of } x. \\ 138 = 2(x + w) \\ 69 = x + w \\ w = 69 - x \quad \text{the width} \end{array}$$

Area of a rectangle = length x width

2nd step: Write the equation.

$$1080 = x(69 - x)$$

3rd step: Operate the numbers to remove the parentheses grouping symbol.

$$\begin{array}{l} x(69 - x) = 1080 \\ 69x - x^2 = 1080 \end{array}$$

4th step: Convert the equation to its standard form.

$$x^2 - 69x + 1080 = 0$$

5th step: Factor the equation.

$$(x - 45)(x - 24) = 0$$

6th step: Solve for x.

$$\begin{aligned}x - 45 &= 0 & \text{and} & & x - 24 &= 0 \\x &= 45 & & & x &= 24 \\x &= 45 \text{ m is the length of the rectangle} \\69 - 45 &= 24 \text{ m is the width}\end{aligned}$$

$$\begin{aligned}\text{Perimeter} &= 2(45) + 2(24) = 138 \text{ m} \\ \text{Area} &= 45(24) = 1080 \text{ square meters}\end{aligned}$$

3. If the hypotenuse of a right triangle measures 13 cm and one leg is 7 cm more than the other, what are the lengths of the two legs?

Solution:

1st step: Identify the unknown.

$$\begin{aligned}\text{Let } x \text{ cm} &= \text{length of shorter leg} \\(x + 7) \text{ cm} &= \text{length of other leg}\end{aligned}$$

$$\text{Formula: } a^2 + b^2 = c^2$$

2nd step: Write the equation.

$$x^2 + (x + 7)^2 = 13^2$$

3rd step: Operate the numbers to remove the parentheses grouping symbol.

$$\begin{aligned}x^2 + x^2 + 14x + 49 &= 169 \\2x^2 + 14x + 49 &= 169\end{aligned}$$

4th step: Convert the equation to its standard form.

$$\begin{aligned}2x^2 + 14x + 49 - 169 &= 0 \\2x^2 + 14x - 120 &= 0\end{aligned}$$

simplify by dividing by 2

$$x^2 + 7x - 60 = 0$$

5th step: Factor the equation.

$$(x + 12)(x - 5) = 0$$

6th step: Solve for x.

$$\begin{array}{l} x + 12 = 0 \\ x = -12 \end{array} \quad \text{and} \quad \begin{array}{l} x - 5 = 0 \\ x = 5 \end{array}$$

Disregard -12 as a solution because it has no real meaning. You cannot have a length of negative 12.

$x = 5$ m is the length of shorter leg
 $x + 7 = 12$ m is the length of the other leg

Check: $a^2 + b^2 = c^2$

$$5^2 + (5 + 7)^2 = 13^2$$

$$25 + 144 = 169$$

$$169 = 169$$

Try this out

Solve:

1. The length of a rectangular floor is twice the width. The area of the floor is 32 m^2 . What are the dimensions of the room?
2. If the perimeter of a rectangular garden is 76m and the area is 360 square meters, what are the dimensions of the garden?
3. The base of a triangle is 7 cm more than its height. If the area is 294 square centimeters, find the base and the height of the triangle.
4. A square pool was surrounded by 3 meters wide Bermuda grass. If the total area of the sidewalk and the square pool is 196 sq. m , how long is each side of the pool? What is the area of the Bermuda grass sidewalk?
5. A rectangular flower garden with dimensions 3m by 7m is surrounded by the walk of uniform width. If the area of the walk is 11 sq. m , what is the width in meters?

Lesson 3 Motion Problems

Motion problems deal with three quantities. They are:

Distance
Rate or speed
Time

All uniform motion problems are tied-up with the formula:

$$\begin{aligned} \text{Distance} &= \text{Time} \times \text{Rate} \\ \text{or } D &= rt \end{aligned}$$

Examples:

1. A car travels 10 kilometers per hour faster than a truck. The car goes 600 kilometers in 5 hours less time than it takes the truck to travel the same distance. Find the rate of each vehicle in kilometers per hour.

Solution:

1st step: Identify the unknown.

	Rate	Time	Distance
Truck	x	$\frac{600}{x}$	600
Car	$x + 10$	$\frac{600}{x+10}$	600

2nd step: Write the equation.

$$\frac{600}{x} = \frac{600}{x+10} + 5$$

3rd step: Operate the numbers to remove the parentheses grouping symbol.
Multiply both sides by the least common denominator which is $x(x + 10)$

$$600(x + 10) = 600x + 5(x^2 + 10x)$$

$$600x + 6000 = 600x + 5x^2 + 50x$$

4th step: Convert the equation to its standard form.

Thus:

$$5x^2 + 600x - 600x + 50x - 6000 = 0$$

Simplify by dividing the equation by 5

$$x^2 + 10x - 1200 = 0$$

5th step: Factor the equation.

$$(x + 40)(x - 30) = 0$$

6th step: Solve for x.

$$\begin{array}{l} x + 40 = 0 \\ x = -40 \end{array} \quad \text{and} \quad \begin{array}{l} x - 30 = 0 \\ x = 30 \end{array}$$

We must disregard the negative value. Thus, the rate of the truck is 30 kilometers per hour. The rate of the car is $x + 10 = 40$ kilometers per hour.

2. A man drives 500 km to a business convention. On the return trip, he increases his speed by 25 km per hour and saves 1 hour of driving time. How fast did he go in each direction?

Solution:

1st step: Identify the unknown.

Let r = speed in going to the convention
 $r + 25$ = speed of the return trip

$$\frac{500}{r} = \text{length of time to the convention}$$

$$\frac{500}{r + 25} = \text{length of time in returning from the convention}$$

$$(\text{length of time in going}) = (\text{length of time in returning}) + 1$$

2nd step: Write the equation.

$$\frac{500}{r} = \frac{500}{r+25} + 1$$

3rd step: Operate the numbers to remove the parentheses grouping symbol. Multiply both sides by the least common denominator which is $r(r+25)$

$$500(r+25) = 500r + r(r+25)$$

$$500r + 12,500 = 500r + r^2 + 25r$$

$$500r + 12,500 = 525r + r^2$$

4th step: Convert the equation to its standard form.

Thus:

$$0 = r^2 + 525r - 500r - 12,500$$

5th step: Factor the equation.

$$0 = (r+125)(r-100)$$

6th step: Solve for x.

$$r+125 = 0 \quad \text{or} \quad r-100 = 0$$

$$r = -125 \quad r = 100$$

We must disregard the negative value. Thus, the rate of the man is 100 kilometers per hour in going to the convention and returns at $100 + 25 = 125$ kilometers per hour.

2. A man can row 8 kilometers downstream and back in 6 hours. If the rate of the stream is 1 kilometer per hour, what is the rate of the man rowing in still water?

Solution:

1st step: Identify the unknown.

Let x = rate of the man in still water

	D	r	t
Downstream	8	$x + 1$	$\frac{8}{x+1}$
Upstream	8	$x - 1$	$\frac{8}{x-1}$

2nd step: Write the equation.

$$\frac{8}{x+1} + \frac{8}{x-1} = 6$$

3rd step: Operate the numbers to remove the parentheses grouping symbol.
Multiply both sides by the least common denominator which is
 $(x + 1)(x - 1)$

$$8(x - 1) + 8(x + 1) = 6(x + 1)(x - 1)$$

$$8x - 8 + 8x + 8 = 6(x^2 - 1)$$

$$16x = 6x^2 - 6$$

4th step: Convert the equation to its standard form.

Thus:

$$0 = 6x^2 - 16x - 6$$

Simplify by dividing the equation by 2

$$0 = 3x^2 - 8x - 3$$

5th step: Factor the equation.

$$0 = (3x + 1)(x - 3)$$

6th step: Solve for x.

$$3x + 1 = 0 \quad \text{or} \quad x - 3 = 0$$

$$x = \frac{-1}{3} \quad \quad \quad x = 3$$

We must disregard the negative value. Thus, the rate of the man is 3 kilometers per hour.

Try this out

Answer the following problems:

1. Two cyclist A and B traveled the same distance of 120 kilometers. A traveled 4 kilometers per hour faster than B and covered the distance in one hour less than B. Find the rate of each.
2. If Mr. Cruz will increase his usual rate by 10 kilometers per hour, it will take him one hour shorter to cover a distance of 200 kilometers. What is his usual speed in driving?
3. If you can row upstream to a landing 5 kilometers away and then row back to your starting point all in three hours and 20 minutes and if the river has an average current of 2 kilometers per hour, at what rate are you able to row in still water?
4. A horse travels 30 kilometers per hour faster than a mule. The horse goes 360 kilometers in two hours less time than the mule goes 360 kilometers. Find the speed of each animals?
5. An airplane flies 900 miles against a headwind of 25 miles per hour. The plane took 15 minutes longer for this flight than with a tailwind of 25 miles per hour. How fast could the plane fly in still air?

Lesson 4

Work Problems

Work problems are important not only to students but also to industrialists, engineers, production executives and managers as well. This is because everyone often meets work problems in real life.

Common work problems have the following characteristics

1. They ask you how long individual can finish a job alone, when working with others when his rate of working is given; and
2. They ask you how long a group of workers can finish a job, if rate of work of each member of the group is given.

Examples:

1. Mario takes 5 days more to do a job than Jose. Together they can do it in 6 days. How long does it take each alone to do the job?

Solution:

1st step: Identify the unknown.

	No. of days to do job	Amount of work done in 1 day
Jose	x	$\frac{1}{x}$
Mario	$x + 5$	$\frac{1}{x + 5}$
Together	6	$\frac{1}{6}$

2nd step: Write the equation.

$$\frac{1}{x} + \frac{1}{x+5} = \frac{1}{6}$$

3rd step: Operate the numbers to remove the parentheses grouping symbol.
Multiply both sides by the least common denominator, $6x(x + 5)$.

$$6(x + 5) + 6x = x(x + 5)$$

$$6x + 30 + 6x = x^2 + 5x$$

4th step: Convert the equation to its standard form.

Thus:

$$0 = x^2 + 5x - 12x - 30$$

$$0 = x^2 - 7x - 30$$

5th step: Factor the equation.

$$0 = (x + 3)(x - 10)$$

6th step: Solve for x .

$$x + 3 = 0 \quad \text{or} \quad x - 10 = 0$$

$$x = -3 \quad \quad \quad x = 10$$

The solutions of the fractional equation are 10 and -3.

However, the solution of the problem is $x = 10$ and x stands for the number of days it takes Jose to do the work alone $x + 5 = 15$ is the number of days it takes Mario to do the work alone.

2. tank has a supply pipe and an exhaust pipe. The exhaust pipe takes 5 minutes longer to empty the tank than for the supply pipe to fill it. If both are open, it takes the supply pipe 30 minutes to fill the tank. Find how long it takes the supply pipe to fill the tank when the exhaust pipe is closed.

Solution:

1st step: Identify the unknown.

Let x = no. of minutes to fill in the tank

$$\frac{1}{x} = \text{part of the tank filled in one minute}$$

$x + 5$ = no. of minutes to empty the tank

$\frac{1}{x+5}$ = part of the tank emptied in one minute

It takes 30 minutes to fill the tank with both pipes open

2nd step: Write the equation.

$$\frac{30}{x} - \frac{30}{x+5} = 1$$

3rd step: Operate the numbers to remove the parentheses grouping symbol.
Multiply both sides by the least common denominator which is $x(x + 5)$.

$$30(x + 5) - 30x = x(x + 5)$$

$$30x + 150 - 30x = x^2 + 5x$$

4th step: Convert the equation to its standard form.

Thus:

$$0 = x^2 + 5x - 150$$

5th step: Factor the equation.

$$0 = (x + 15)(x - 10)$$

6th step: Solve for x .

$$x + 15 = 0 \quad \text{or} \quad x - 10 = 0$$

$$x = -15 \quad \quad \quad x = 10$$

It takes the supply pipe 10 minutes to fill the tank when the exhaust pipe is closed.

Try this out

Answer the following problems:

1. Charlie can construct 100 identical boxes in two days less than Cholo. If they work together they can finish the work in $2\frac{11}{12}$ days. How long will it take each of them to finish 100 boxes?
2. It takes Peter 6 hours longer than Luis to do a certain job. Together they can do it in 4 hours. How long would it take each working alone to do the job?
3. Mang Tano and his son Mario can finish planting their crops in 4 days if they work together. Working alone, Mario will need $2\frac{1}{3}$ days more than the number of days that it will take his father to finish the work alone. How many days will each of the father and son need to finish the work if each will work alone?
4. Mr. Cruz can paint a house in 2 days less than his son. When they work together they can do the job in $4\frac{4}{9}$ days. How long would it take each working alone to do job?

Lesson 5

Problems Involving Quadratic Function (Maximum or Minimum Point)

Quadratic functions have practical applications. Many applications of quadratic functions are in simple maximization and minimization problems.

The technique for solving such problems is based on the fact that a quadratic function defined by $f(x) = ax^2 + bx + c$ attains its maximum value

(for $a < 0$) or minimum value (for $a > 0$) at $x = h = -\frac{b}{2a}$ and the maximum / minimum value is $k = f(h) = \frac{4ac - b^2}{4a}$.

Examples:

1. The sum of two numbers is 24. Find the maximum product and the value of the two numbers.

Solution:

Let x = the first number

$24 - x$ = the second number

$f(x)$ = the product of the two numbers

$$\begin{aligned}y = f(x) &= x(24 - x) \\ &= 24x - x^2\end{aligned}$$

or $y = -x^2 + 24x$ attains its maximum value at

$$x = \frac{-b}{2a} = \frac{-24}{2(-1)} = 12$$

The maximum product is $12(24 - 12) = 144$

2. A rectangular garden is x meters long and $(18 - x)$ meters wide. If y is the area, find the maximum area that the garden could have. What could be its dimensions?

Solution:

Let x = the length

$18 - x$ = width

$$y(\text{area}) = x(18 - x)$$

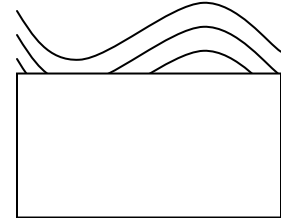
$$= 18x - x^2 \quad \text{or } y = -x^2 + 18x \text{ attains its maximum value}$$

at

$$x = \frac{-b}{2a} = \frac{-18}{-2} = 9$$

The maximum area is $9(18 - 9) = 81$ square meters and the dimensions are length = 9 m and the width = 9 m.

3. A rectangular lot is bounded on the side by a river and on the other three sides by a total of 80 m fencing. Find the possible dimensions of the lot.



Solution:

Step 1. The quantity to be maximized is the area of the lot.

Step 2. Let x = be the width
 y = be the length

If A is the area, then $A = xy$

Step 3. Since the total length of fencing is 80 meters,
 $2x + y = 80$ or $y = 80 - 2x$

Substitute $80 - 2x$ for y in $A = xy$

$$A = x(80 - 2x) = -2x^2 + 80x$$

Step 4. $x = \frac{-b}{2a} = \frac{-80}{2(-2)} = \frac{-80}{-4} = 20$

$$\begin{aligned} \text{Area} &= -2x^2 + 80x \\ &= -2(20)^2 + 80(20) \\ &= -800 + 1600 \\ &= 800 \end{aligned}$$

Then maximum value of A is 800 and occurs when $x = 20$.

Step 5. Since $x = 20$ and $2x + y = 80$, $y = 40$. The dimensions of the largest possible rectangular lot are 40 meters by 20 meters.

4. A company charges P200 for each leather bag on order of 150 or less. The cost of each bag is reduced by P1 for each order in excess of 150. How many bags on order would result in a maximum revenue? what is the maximum revenue?

Solution:

Step 1. The quantity to be maximized is total revenue (R) in pesos from the leather bag to be sold.

Step 2. Let x = be the number of bags ordered
 $x - 150$ = be number of bag ordered in excess of 150

$$\begin{aligned} 200 - 1(x - 150) &= 200 - x + 150 \\ &= 350 - x \text{ is the cost of each bag} \end{aligned}$$

$$R = (\text{cost per bag}) (\text{number of bags sold})$$

$$= (350 - x)x$$

$$= 350x - x^2 \quad \text{or} \quad -x^2 + 350x$$

$$\begin{aligned} \text{Step 3. } x &= \frac{-b}{2a} \\ &= \frac{-350}{2(-1)} \\ &= \frac{-350}{-2} \\ &= 175 \end{aligned}$$

$$R = -x^2 + 350x$$

$$= -(175)^2 + 350(175)$$

$$= 30,625$$

The maximum value of R is 30,625 and it occurs when $x = 175$

- Step 5. A purchase of 175 bags maximizes the company's revenue.
The maximum revenue is P30,625

Try this out

Answer the following problems:

1. What are the dimensions of the largest rectangular field that can be enclosed with 60 meters of wire?
2. The sum of two numbers is 40. If one is x , what is the other? What is the maximum product that the 2 numbers could have?
3. A rectangular lot is bordered on one side by a string and on the other three sides by 600 meters of fencing. Find the dimensions of the lot if its area is a maximum.
4. A theater seats 2000 people and charges P10 for a ticket. At this price, all the tickets can be sold. A survey indicates that if the ticket price is increased by a peso, the number of tickets sold will decrease by 100. What ticket price result in the greatest revenue?



Let's summarize

Steps in Solving word problems involving quadratic equation and function:

1. Identify your unknowns.
2. Write your equation.
3. Operate on the numbers.
4. Convert the equation into the standard form. Remember to change signs.
5. Factor the equation into its two corresponding linear equations. By factoring, you reduce the quadratic equation into two equivalent linear equations.



What have you learned

A. Define a variable and write a quadratic function to describe each of the following.

1. The product of two numbers whose sum is 40.
2. The product of two numbers whose difference is 25.
3. The area of a rectangle whose perimeter is 20 centimeter.
4. The product of two consecutive integers is nine less than the square of the second.
5. Write a quadratic function to describe the area of a circle in terms of its radius.

B. Answer each of the following problems completely.

1. Find two consecutive even integers whose product is 168.
2. The sum of two numbers is 11. The difference of their squares is 11. What are the numbers?
3. The perimeter of a garden is 66 meters and its area is 270 sq. meters. Find the dimensions of the garden.
4. What two integers having a sum equal to 66 will have a maximum product?
5. Two bikers started at the same corner, one going east, the other going north. One biker is traveling at 3 kph faster than the other. After one hour, the two bikers are 15 kilometers apart. Find the rate of each.



Answer Key

How much do you know

A. 1. b

2. d

3. c

4. b

5. d

B. 1. $A = s^2$

2. $P = 2A$

3. $12x + 7 = 3x - 4$

4. $c = 2\pi$

5. $d = rt$

C. 1. 11 and 13 or -13 and -11

2. $l = 16$ cm. $w = 6$ cm

3. 9 and 13

4. 144

Try this out

Lesson 1

1. $x = 3$ and $x + 1 = 4$

2. 14 and 16

3. $n = 9$

4. 5 and 15

5. 5 and 11

6. $\frac{-5}{3}$ and $\frac{3}{5}$

7. 4 and 9

8. -7 and -4 or 4 and 7

9. 7 and 8

10. $n = 29$

Lesson 2

1. width = 4 meters and length = 8 meters

2. length = 20 meters and width = 18 meters

3. base = 28 meters height = 21 meters

4. The length of each side of the pool = 8 meters
area of the Bermuda grass sidewalk = 132 square meters

5. $\frac{1}{2}$ m is the width of the walk

Lesson 3

1. $B = 20\text{kph}$,
 $A = 24\text{ kph}$
2. The speed of Mr Cruz is 40 kph
3. The rate in still water
 $= 4\text{ km/hr}$
4. Rate of mule = 30 kph,
Rate of horse = 60 kph
5. $r = 425\text{ kph}$

Lesson 4

1. Charlie = 5 days,
Cholo = 7 days
2. Luis = 6 hours
Peter = 12 hours
3. Mario = $9\frac{1}{3}$ days
Mang Tano = 7 days
4. son = 10 days
Mr. Cruz = 8 days

Lesson 5

1. width = 15 meters length = 15 meters
2. 20 is the other number; 400 is the maximum product
3. length = 300m width = 150 meters
4. P25 ticket price would result to P22,500 greatest revenue.

What have you learned

- A. 1. $y = x(40 - x)$ 2. $y = x(x - 25)$
3. $A = l(10 - l)$ 4. $P = x(x + 1) = (x + 1)^2 - 9$ 5. $A = \pi r^2$
- B. 1. -14 and -12 or 12 and 14
2. 6 and 5
3. length = 18 meters and width = 15 meters
4. the maximum product is 1089, the numbers are 33 and 33.
5. rate of slower biker = 9 kph
rate of faster biker = 12 kph