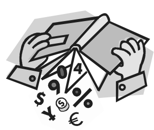
Module 3
Quadratic Functions

What this module is about

This module is about the zeros of quadratic functions and the roots of quadratic equations. As you go over this material, you will be able to determine the zeros of a quadratic function by relating this to the corresponding roots of quadratic equation. This material will likewise develop your skills in finding the roots of quadratic equations using factoring, completing the square and quadratic formula. Moreover, you will be able to use learned concepts and skills in deriving quadratic function given certain conditions.



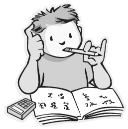
What you are expected to learn

This module is designed for you to:

1. determine the zeros of a quadratic function by relating this to the roots of a quadratic equation.

2. find the roots of quadratic equations by:
   a. factoring
   b. completing the square
   c. quadratic formula

3. derive quadratic functions given:
   a. zeros of the function
   b. table of values
   c. graph



How much do you know

1. What are the zeros of $f(x) = 4x^2 - 64$?

2. Solve the equation $x^2 - 5x + 6 = 0$.

3. Use the quadratic formula in solving $x^2 - x - 3 = 0$.

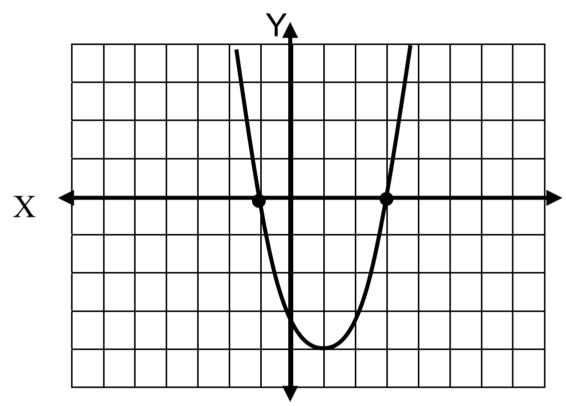
4. Determine the roots of $2x^2 - 3x + 1 = 0$ using the method of completing the square.

5. Find a quadratic function whose zeros are -3 and 2.

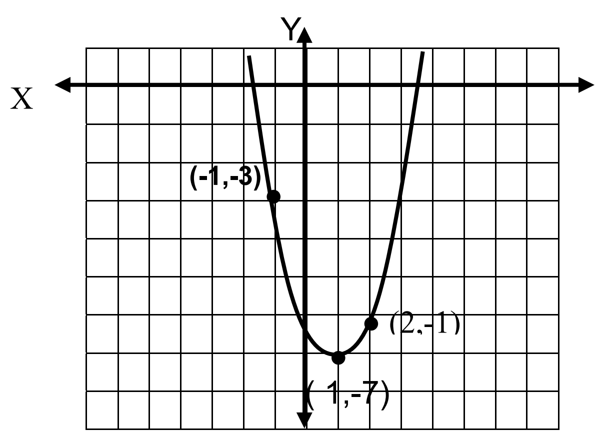
6. Name the quadratic function satisfied by the table below.

x	-2	-1	0	2
f(x)	12	6	2	0

7. Determine the zeros of the quadratic function whose graph is given below.



8. Determine the quadratic function whose graph is given below.



9. What quadratic function has $\sqrt{2}$ and $-\sqrt{2}$ as zeros?

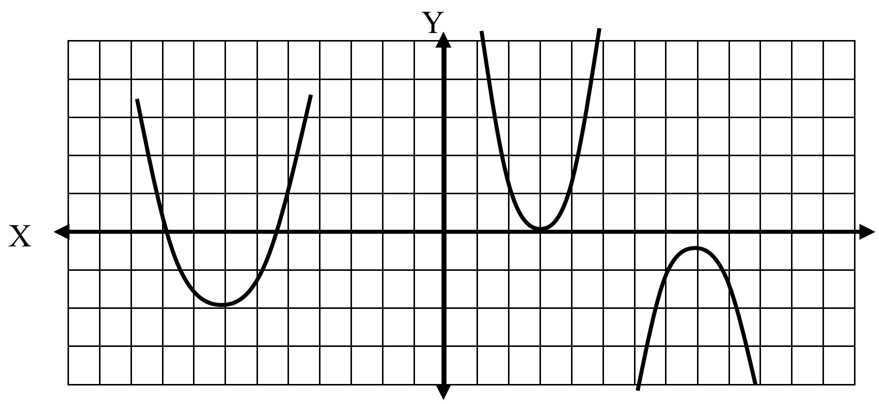
10. What quadratic function has a vertex at (2, 1) and passes through (3, -1)?

What you will do

Lesson 1

Determining the Zeros of Quadratic Function from Its Graph

The parabolic structure of the graph of a quadratic function allows it to intersect the x-axis in different ways. It may cross the x-axis once, twice or none at all as shown below.

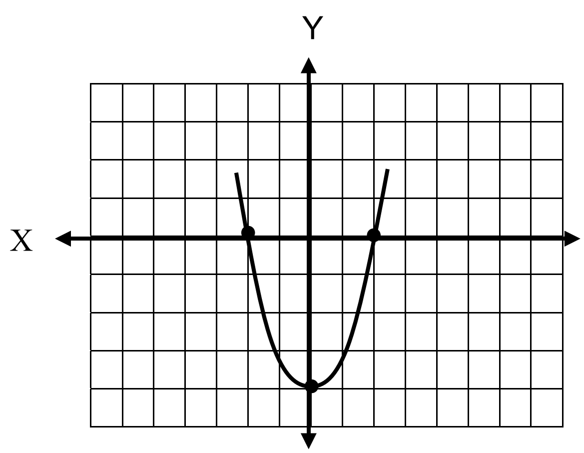


Graphically speaking, the zeros of a quadratic function f(x) are the x-coordinates of the point of intersection of the graph of f(x) and the x-axis, if it does exists. In other word, the zeros of a quadratic function are its x-intercepts.

Examples:

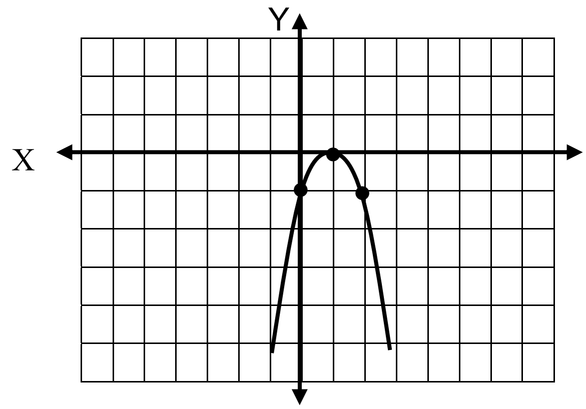
Determine the zeros of the quadratic function whose graph is given below.

1.



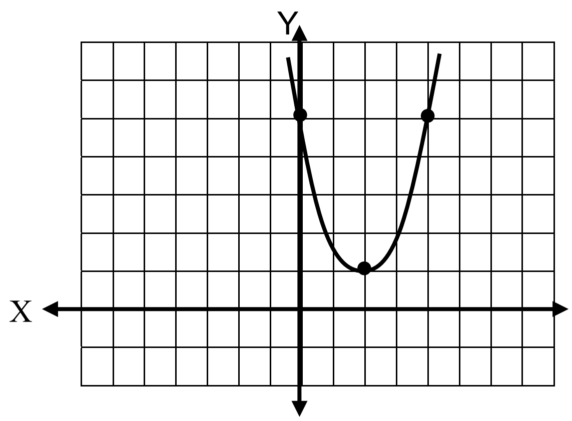
The parabola intersects the x-axis at -2 and 2. Hence, the zeros of the quadratic function represented by the parabola are -2 and 2.

2.



The parabola intersects the x-axis only at 0. Hence, the only zero of the quadratic function represented by the parabola is 0.

3.



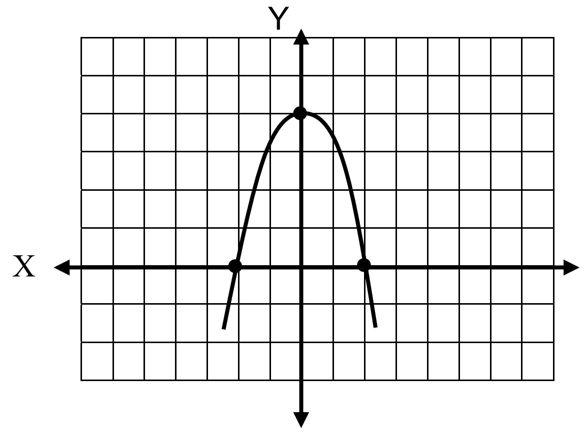
The parabola did not intersect the x-axis. Hence, the quadratic function has no real zero. It is possible that the zeros are imaginary.

Try this out

Set A

Determine the zeros of the quadratic function whose graph is given below.

1.



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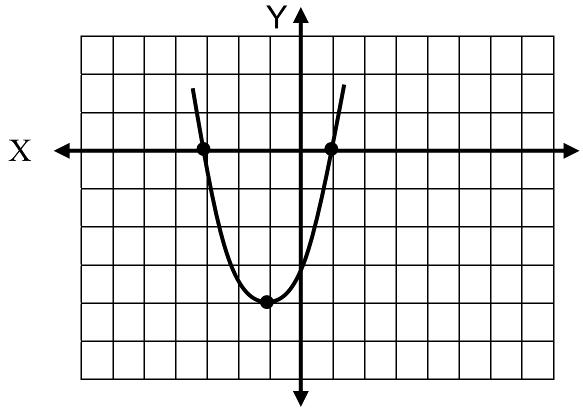

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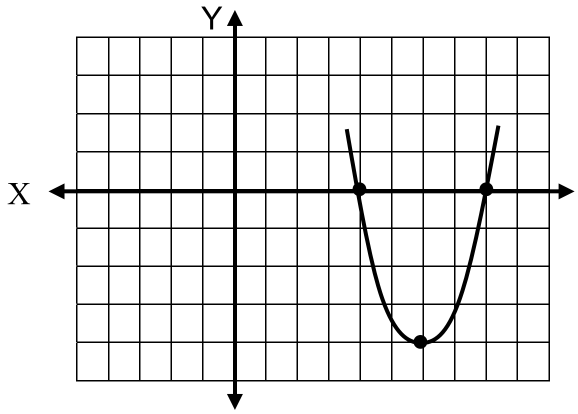

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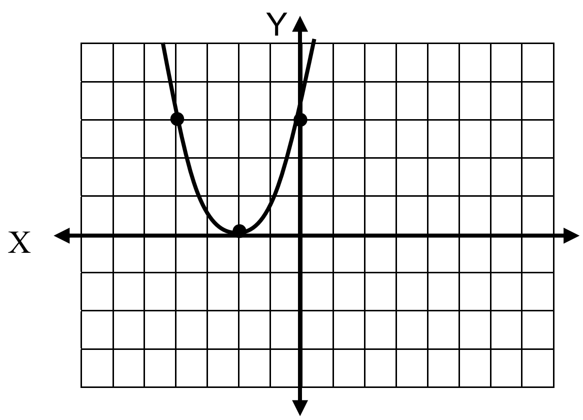

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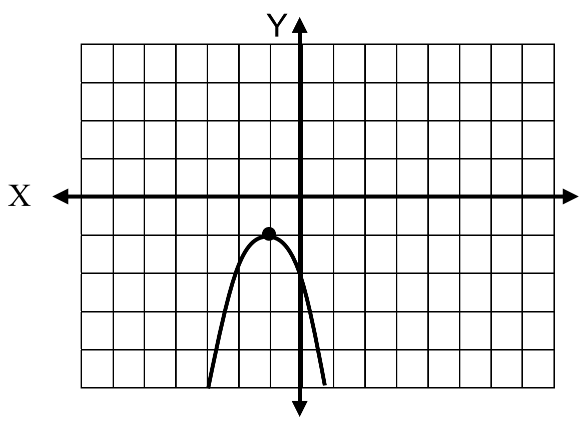
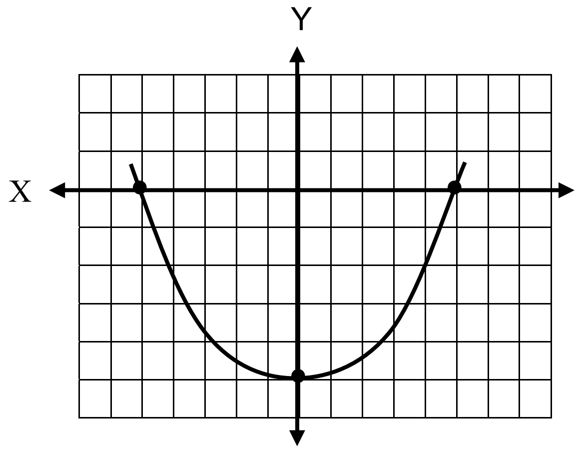


Set B

1.



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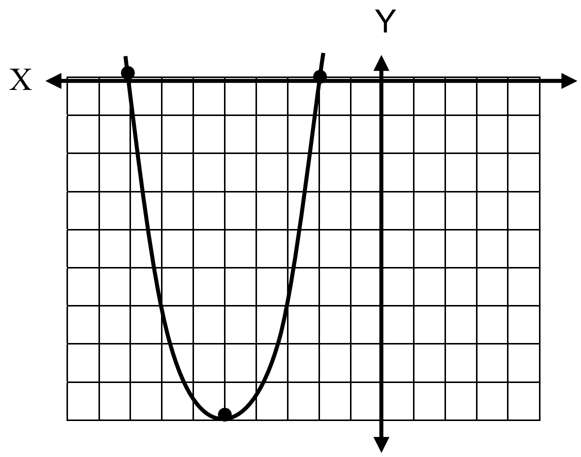

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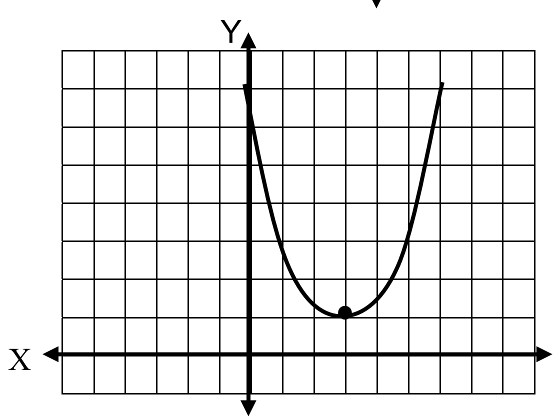

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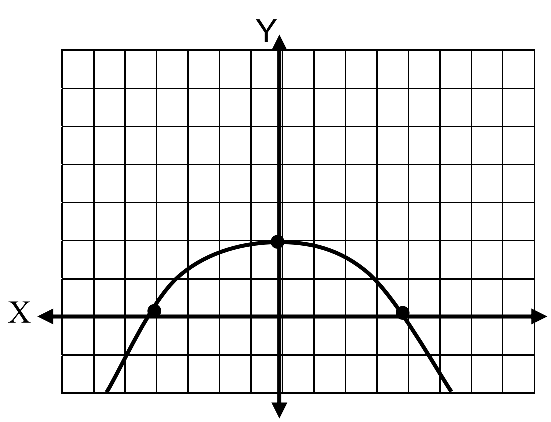

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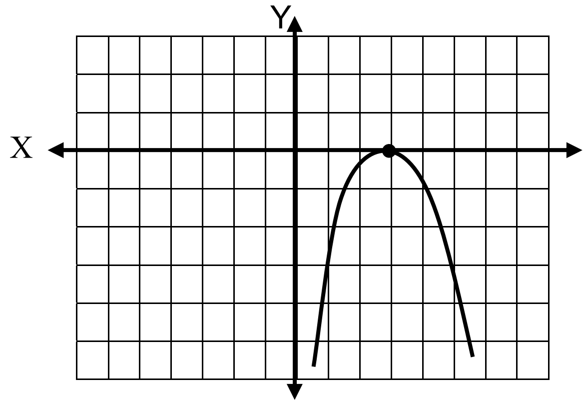
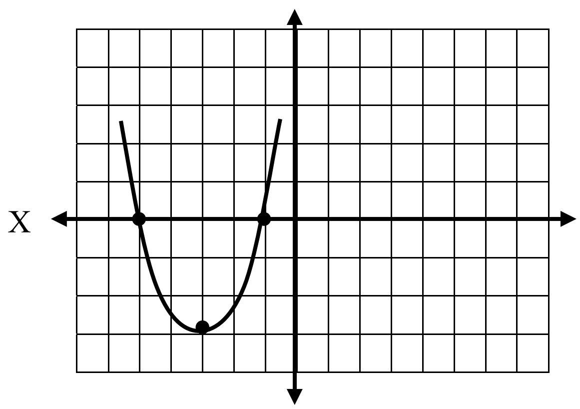


Set C

1.



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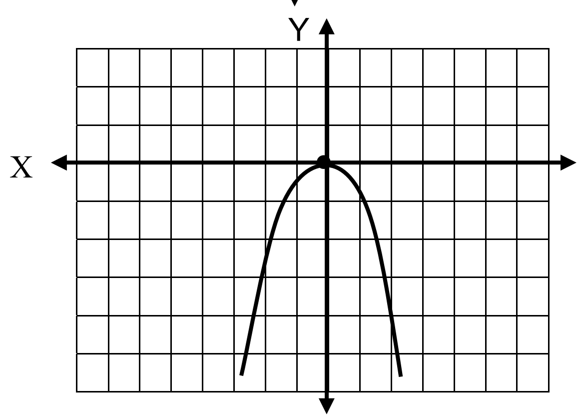

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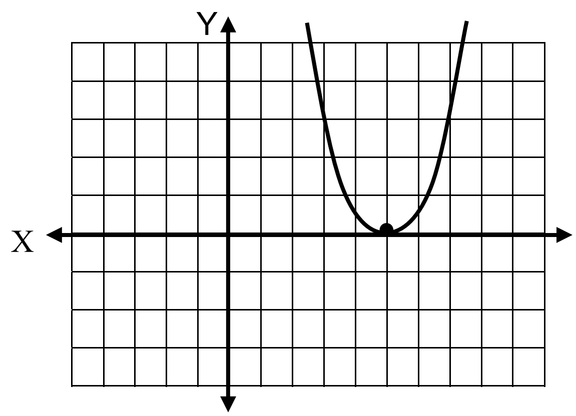

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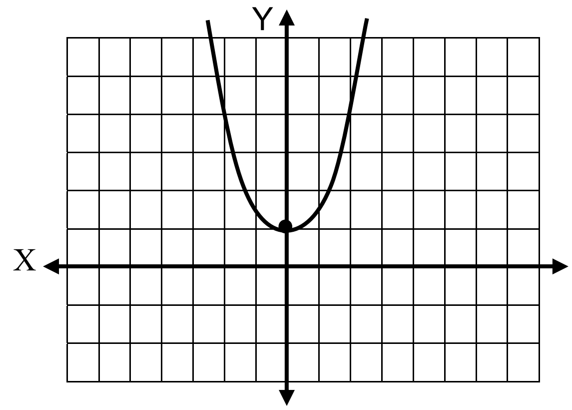

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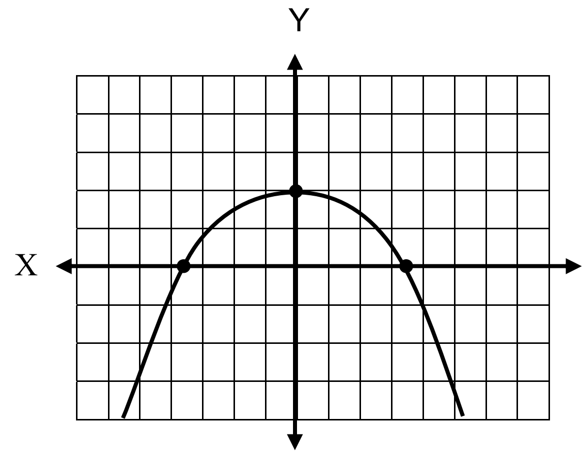


Lesson 2

Determining the Zeros of Quadratic Functions by Factoring

The zeros of a quadratic function of the form $f(x) = ax^2 + bx + c$ are the roots of the quadratic equation $ax^2 + bx + c = 0$.

If the quadratic expression $ax^2 + bx + c$ is factorable, factor it and apply the zero property.

Examples:

Determine the zeros of each quadratic function.

1. $f(x) = 2x^2 + 4x$
   Equate the given function to zero and then solve for x.
   $f(x) = 2x^2 + 4x = 0$
   $2x(x + 2) = 0$ Factor out factor 2x
   $2x = 0$ or $x + 2 = 0$ Apply the Zero property
   $x = 0$ or $x = -2$

Hence, the zeros of $f(x) = 2x^2 + 4x$ are 0 and -2.

2. $g(x) = x^2 – 4$
   Equate the given function to zero and then solve for x.
   $g(x) = x^2 – 4 = 0$
   $(x – 2)(x + 2) = 0$ Factor the difference of two squares
   $x – 2 = 0$ or $x + 2 = 0$ Apply the zero property
   $x = 2$ or $x = -2$

   Hence, the zeros of $g(x) = x^2 – 4$ are 2 and -2.

3. $h(x) = 4x^2 – 4x + 1$
   Equate the given function to zero and then solve for x.
   $h(x) = 4x^2 – 4x + 1 = 0$
   $(2x – 1)^2 = 0$ Factor the Perfect square trinomial
   $2x – 1 = 0$ Apply the zero property
   $x = \frac{1}{2}$

   Hence, the only zero of $h(x) = 4x^2 – 4x + 1$ is $\frac{1}{2}$.

4. $f(x) = x^2 + 2x – 8$
   Equate the given function to zero and then solve for x.
   $f(x) = x^2 + 2x – 8 = 0$
   $(x – 2)(x + 4) = 0$ Factor
   $x – 2 = 0$ or $x + 4 = 0$ Apply the zero property
   $x = 2$ or $x = -4$

   Hence, the zeros of $F(x) = x^2 + 2x – 8$ are 2 and -4.

5. $g(x) = -2x^2 – x + 3$

   Equate the given function to zero and then solve for x.
   $g(x) = -2x^2 – x + 3 = 0$
   $(-2x – 3)(x – 1) = 0$ Factor
   $-2x – 3 = 0$ or $x – 1 = 0$ Apply the zero property
   $x = \frac{3}{2}$ or $x = 1$

   Hence, the zeros of $G(x) = -2x^2 – x + 3$ are $\frac{3}{2}$ and 1.

Try this out

Find the zeros of each quadratic function by factoring.

Set A

1. $f(x) = 5x^2 – 5$
2. $g(x) = 2x^2 – 12$
3. $h(x) = 3x^2 – 27$
4. $f(x) = x^2 + 4x + 3$
5. $g(x) = x^2 + 5x + 6$

Set B

1. $f(x) = 36x^2 – 49$
2. $g(x) = x^2 + 3x + 2$
3. $h(x) = x^2 + 7x + 12$
4. $f(x) = x^2 + 5x – 6$
5. $g(x) = x^2 – x – 12$

Set C

1. $f(x) = x^2 – 5x + 6$
2. $g(x) = x^2 + 5x – 14$
3. $h(x) = x^2 – 4x – 21$
4. $f(x) = 2x^2 + x – 1$
5. $g(x) = 3x^2 + 8x + 4$

Lesson 3

The Imaginary Numbers

It has been mentioned that if a quadratic function has no real zeros, it is possible that the zeros are imaginary.

A non-real number is called imaginary number. The unit imaginary number is defined as follows:

The imaginary number *i* is a number whose square root is -1. In symbols,

$$i = \sqrt{-1} \rightarrow i^2 = -1$$

Examples:

Simplify each of the following:

1. $\sqrt{-25}$

$$\sqrt{-25} = \sqrt{25(-1)} = \left(\sqrt{25}\right)\left(\sqrt{-1}\right) = 5i$$

2. $\sqrt{-12}$

$$\sqrt{-12} = \sqrt{4(3)(-1)} = \left(\sqrt{4}\right)\left(\sqrt{-1}\right)\left(\sqrt{3}\right) = 2i\sqrt{3}$$

3. $\sqrt{-11}$

$$\sqrt{-11} = \sqrt{(-1)(11)} = \left(\sqrt{-1}\right)\left(\sqrt{11}\right) = i\sqrt{11}$$

Try this out

Simplify each of the following:

Set A

1. $\sqrt{-2}$
2. $\sqrt{-4}$
3. $\sqrt{-36}$
4. $\sqrt{-18}$
5. $\sqrt{-20}$

Set B

1. $\sqrt{-81}$
2. $\sqrt{-28}$
3. $\sqrt{\frac{-75}{4}}$
4. $\sqrt{\frac{-27}{16}}$
5. $\sqrt{\frac{-31}{16}}$

Set C

1. $\sqrt{-52}$
2. $\sqrt{-45}$
3. $\sqrt{\frac{-19}{49}}$

4. $\sqrt{\frac{-41}{16}}$
5. $\sqrt{\frac{-162}{121}}$

Lesson 4

Determining the Zeros of Quadratic Functions by Completing the Square

Solving for the zeros of quadratic functions is limited to quadratic expressions that are factorable. However, there is another method that works for quadratic expressions that are factorable or not. This method is called the method of completing the square.

The principle behind the use of completing the square is to produce a perfect square trinomial so that the square root property can be apply.

The following steps are suggested:

1. Equate the given quadratic function to zero.
2. Transpose the constant term.
3. If a $\neq 1$, divide both sides of the equation by a.
4. Add to both sides of the equation the square of half the coefficient of x.
5. Factor the resulting perfect square trinomial.
6. Apply the square root property.
7. Solve for x.

Examples:

Find the zeros of each quadratic function by completing the square.

1. $y = x^2 - 4x - 5$.

$0 = x^2 - 4x - 5$	Equate y to 0
$x^2 - 4x = 5$	Add + 5 to both sides of the equation
$x^2 - 4x + 4 = 5 + 4$	Add the square of one-half of -4 to both sides of the equation.
$(x - 2)^2 = 9$	Factor
$x - 2 = \pm 3$	Take the square root of 9
$x = 2 \pm 3$	Solve for x.

$x = 5$ or $x = -1$

Hence, the zeros of $y = x^2 - 4x - 5$ are 5 and -1.

2. $y = 3x^2 + 4x + 1.$

$0 = 3x^2 + 4x + 1$	Equate y to zero
$3x^2 + 4x = -1$	Add –1 to both sides of the equation
$x^2 + \frac{4}{3}x = \frac{-1}{3}$	Divide both sides by the coefficient 3.
$x^2 + \frac{4}{3}x + \frac{4}{9} = \frac{-1}{3} + \frac{4}{9}$	Add the square of one-half of $\frac{4}{3}$.
$\left(x + \frac{2}{3}\right)^2 = \frac{1}{9}$	Factored
$x + \frac{2}{3} = \pm\frac{1}{3}$	Take the square root of $\frac{1}{9}$
$x = -\frac{2}{3} \pm \frac{1}{3}$	Solve for x.

$x = -\frac{1}{3}$ or $x = -1$

Hence, the zeros of $y = 3x^2 + 4x + 1$ are $-\frac{1}{3}$ and -1.

3. $y = x^2 – 2x – 5$

$0 = x^2 – 2x – 5$	Equate y to 0.
$x^2 – 2x = 5$	Add + 5 to both sides of the equation.
$x^2 – 2x + 1 = 5 + 1$	Add the square of one-half of -2 to both sides of the equation.
$(x – 1)^2 = 6$	Factor
$x – 1 = \pm\sqrt{6}$	Take the square root of 6.
$x = 1 \pm \sqrt{6}$	Solve for x.

$x = 1 + \sqrt{6}$ or $x = 1 - \sqrt{6}$

Hence, the zeros of $y = x^2 – 2x – 5$ are $1 + \sqrt{6}$ and $1 - \sqrt{6}$.

4. $y = 2x^2 – x + 4$

$0 = 2x^2 – x + 4$	Equate y to zero.
$2x^2 – x = -4$	Add – 4 to both sides of the equation.
$x^2 - \frac{1}{2}x = -2$	Divide both sides by the coefficient 2.
$x^2 - \frac{1}{2}x + \frac{1}{16} = -2 + \frac{1}{16}$	Add the square of one-half of $-\frac{1}{2}$ to both sides of the equation.
$\left(x - \frac{1}{4}\right)^2 = -\frac{31}{16}$	Factor
$x - \frac{1}{4} = \pm\frac{i\sqrt{31}}{4}$	Take the square root of $-\frac{31}{16}$.

$$X = \frac{1}{4} \pm \frac{i\sqrt{31}}{4} \qquad \text{(The value of x is solved.)}$$

$$x = \frac{1+i\sqrt{31}}{4} \quad \text{or} \quad x = \frac{1-i\sqrt{31}}{4}$$

Hence, the zeros of $y = 2x^2 - x + 4$ are $\frac{1+i\sqrt{31}}{4}$ and $\frac{1-i\sqrt{31}}{4}$.

Try this out:

Find the zeros of each quadratic function by completing the square.

Set A

1. $f(x) = x^2 - x - 2$
2. $g(x) = x^2 + 6x + 4$
3. $h(x) = x^2 - 7x + 12$
4. $f(x) = 2x^2 - 3x + 1$
5. $g(x) = 3x^2 + 5x - 2$

Set B

1. $f(x) = x^2 - 10x + 21$
2. $g(x) = x^2 + 4x - 1$
3. $h(x) = x^2 - 8x + 3$
4. $y = x^2 + 6x - 5$
5. $y = 2x^2 + 2x + 3$

Set C

1. $y = -x^2 - 3x + 4$
2. $y = x^2 - 2x - 5$
3. $y = x^2 + x - 1$
4. $y = 2x^2 + 2x - 7$
5. $y = 3x^2 + 6x + 1$

Lesson 5

Determining the Zeros of Quadratic Functions by Using the Quadratic Formula

Suppose the general quadratic equation $ax^2 + bx + c = 0$ is solved by the method of completing the square.

$ax^2 + bx + c = 0$

$ax^2 + bx = -c$ — Add –c to both sides of the equation

$x^2 + \frac{b}{a}x = \frac{-c}{a}$ — Divide both sides by the coefficient 2.

$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{-c}{a} + \frac{b^2}{4a^2}$ — Add the square of one-half of $\frac{b}{a}$ to both sides of the equation.

$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$ — Factor

$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$ — Take the square root of the left side.

$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$ — Solve for x.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ — Combine the fractions

The last equation is referred to as the quadratic formula.

Examples:

Find the zeros of each quadratic function by using the quadratic formula.

1. $y = x^2 + 2x – 3$.

First, equate the given quadratic function to 0. Identify the values of a, b, and c. Then, substitute these values to the quadratic formula and solve for x.

a = 1 b = 2 c = -3

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-3)}}{2(1)}$$
$$x = \frac{-2 \pm \sqrt{4 + 12}}{2}$$
$$x = \frac{-2 \pm \sqrt{16}}{2}$$
$$x = \frac{-2 \pm 4}{2}$$

x = 1 or x = -3

Hence, the zeros of $y = x^2 + 2x – 3$ are 1 and -3.

2. $y = 2x^2 - x - 1$.

First, equate the given quadratic function to 0. Identify the values of a, b, and c. Then, substitute these values to the quadratic formula and solve for x.

a = 2 b = -1 c = -1

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-1)}}{2(2)}$

$x = \frac{1 \pm \sqrt{1+8}}{4}$

$x = \frac{1 \pm \sqrt{9}}{4}$

$x = \frac{1 \pm 3}{4}$

$x = 1$ or $x = -\frac{1}{2}$

Hence, the zeros of $y = 2x^2 - x - 1$ are 1 and $-\frac{1}{2}$.

3. $y = x^2 - 5x + 2$.

First, equate the given quadratic function to 0. Identify the values of a, b, and c. Then, substitute these values to the quadratic formula and solve for x.

a = 1 b = -5 c = 2

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(2)}}{2(2)}$

$x = \frac{5 \pm \sqrt{25-8}}{4}$

$x = \frac{5 \pm \sqrt{17}}{4}$

$x = \frac{5+\sqrt{17}}{4}$ 1 or $x = \frac{5-\sqrt{17}}{4}$

Hence, the zeros of $y = x^2 - 5x + 2$ are $\frac{5+\sqrt{17}}{4}$ and $\frac{5-\sqrt{17}}{4}$.

4. $y = 3x^2 - 3x + 2$.

First, equate the given quadratic function to 0. Identify the values of a, b, and c. Then, substitute these values to the quadratic formula and solve for x.

$a = 3 \quad b = -3 \quad c = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(3)(2)}}{2(3)}$$

$$x = \frac{3 \pm \sqrt{9 - 24}}{6}$$

$$x = \frac{3 \pm i\sqrt{15}}{6}$$

$$x = \frac{3 + i\sqrt{15}}{6} 1 \text{ or } x = \frac{3 - i\sqrt{15}}{6}$$

Hence, the zeros of $y = 3x^2 - 3x + 2$ are $\frac{3 + i\sqrt{15}}{6}$ and $\frac{3 - i\sqrt{15}}{6}$.

Try this out

Find the zeros of each quadratic function by using the quadratic formula.

Set A

1. $f(x) = x^2 + 6x + 5$
2. $g(x) = 3x^2 + 6x - 2$
3. $h(x) = 4x^2 - 3x - 2$
4. $F(x) = 2x^2 - x - 3$
5. $G(x) = 4x^2 - 4x - 3$

Set B

1. $f(x) = x^2 + x - 20$
2. $g(x) = x^2 + 4x - 12$
3. $h(x) = 3x^2 + 2x - 1$
4. $y = 2x^2 + x - 5$
5. $y = 3x^2 - x - 2$

Set C

1. $y = -4x^2 + 2x - 3$
2. $y = -4x^2 + 8x + 3$

3. $y = x^2 + x + 3$
4. $y = 2x^2 + 4x - 7$
5. $y = 3x^2 + 6x - 2$

Lesson 6

Deriving a Quadratic Function, given the Zeros of the Function

Recall that the zeros of a quadratic function are also the roots of the corresponding quadratic equation. If the zeros of a quadratic function y are x_1 and x_2, then $x - x_1$ and $x - x_2$ are factors of the quadratic expression. Thus, the corresponding quadratic equation is

$$(x - x_1)(x - x_2) = 0$$

and the quadratic function is

$$y = (x - x_1)(x - x_2)$$

Examples:

Derive the quadratic function, given the zeros of the function:

1. -2, 3

Let y be the quadratic function. If -2 and 3 are the zeros of y, then $x - (-2)$ and $x - 3$ are factors of y.

Hence, $y = [x - (-2)](x - 3)$
$y = (x + 2)(x - 3)$
$y = x^2 - x - 6$

2. $-\frac{1}{2}, 1$

Let f(x) be the quadratic function. If $-\frac{1}{2}$ and 1 are the zeros of f(x), then $x - \left(-\frac{1}{2}\right)$ and $x - 1$ are factors of f(x). The corresponding quadratic equation is

$$\left[x - \left(-\frac{1}{2}\right)\right](x - 1) = 0$$

$$\left(x + \frac{1}{2}\right)(x - 1) = 0$$

$$(2x + 1)(x - 1) = 0$$

$2x^2 - x - 1 = 0.$

Hence, the quadratic function is $f(x) = 2x^2 - x - 1$.

3. $\pm\sqrt{2}$

Let g(x) be the quadratic function. If $\pm\sqrt{2}$ are the zeros of g(x), then $x - \sqrt{2}$ and $x - \left(-\sqrt{2}\right)$ are factors of g(x). The corresponding quadratic equation is

$$\left(x-\sqrt{2}\right)\left[x-\left(-\sqrt{2}\right)\right] = 0$$

$$\left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right) = 0$$

$$x^2 - \left(\sqrt{2}\right)^2 = 0$$
$$x^2 - 2 = 0$$

Hence, the quadratic function is $g(x) = x^2 - 2$.

4. $1 \pm i$

Let h(x) be the quadratic function. If $1 \pm i$ are the zeros of g(x), then $x - \left(1+i\right)$ and $x - \left(1-i\right)$ are factors of h(x). The corresponding quadratic equation is

$$[x - \left(1+i\right)][x - \left(1-i\right)] = 0$$
$$(x - 1 + i)(x - 1 - i) = 0$$
$$[(x - 1) + i][(x - 1) + i] = 0$$
$$(x - 1)^2 - (i)^2 = 0$$
$$x^2 - 2x + 1 - (-1) = 0$$
$$x^2 - 2x + 1 + 1 = 0$$
$$x^2 - 2x + 2 = 0$$

Hence, the quadratic function is $h(x) = x^2 - 2x + 2$.

Try this out

Derive the quadratic function, given the zeros of the function:

Set A

1. 1, 3
2. -3, 4
3. 4, 6

4. -3, 7
5. $-\frac{3}{2}$, 2

Set B

1. $\frac{2}{3}$, 3
2. $-\frac{1}{3}$, 2
3. $\frac{3}{2}$, $-\frac{1}{2}$
4. $\frac{2}{3}$, $\frac{3}{2}$
5. $\frac{3}{4}$, $-\frac{3}{4}$

Set C

1. -8, 5
2. $-\frac{1}{2}$, 4
3. 2, -2
4. 3, 2
5. $-\frac{1}{2} \pm i$

Lesson 7

Deriving a Quadratic Function, Given the Table of Values

If the table of values representing the quadratic function $y = ax^2 + bx + c$ is given, then these values should satisfy $y = ax^2 + bx + c$.

Examples:

Derive a quadratic function, given the table of values

1.

x	-3	0	2
y	-3	0	4

Since (-3, -3), (0, 0), and (2, 4) are points on the graph of y, then they satisfy $y = ax^2 + bx + c$. That is, Using (-3, -3)

$$\begin{array}{llll} y = ax^2 + bx + c & & & \\ -3 = a(-3)^2 + b(-3) + c & \Leftrightarrow & -3 = 9a - 3b + c & \text{Eq. 1} \\ 0 = a(0)^2 + b(0) + c & \Leftrightarrow & 0 = c & \text{Eq. 2} \\ 4 = a(2)^2 + b(2) + c & \Leftrightarrow & 4 = 4a + 2b + c & \text{Eq. 3} \end{array}$$

Substituting Eq. 2 to Eq. 1 and Eq. 3,

$$\begin{array}{llll} -3 = 9a - 3b + 0 & \Leftrightarrow & 3a - b = -1 & \text{Eq. 4} \\ 4 = 4a + 2b + 0 & \Leftrightarrow & 2a + b = 2 & \text{Eq. 5} \end{array}$$

Solving Eq. 4 and Eq. 5 by elimination,

$$\begin{array}{l} 3a - b = -1 \\ \underline{2a + b = 2} \\ 5a \quad\; = 1 \\ \quad a = \frac{1}{5} \end{array}$$

Substituting $a = \frac{1}{5}$ to Eq. 5,

$$2\left(\frac{1}{5}\right) + b = 2$$

$$b = \frac{8}{5}$$

Hence, the quadratic function is $y = \frac{1}{5}x^2 + \frac{8}{5}x$

2.

x	0	1	2
y	-3	-1	-3

Since (0, -3), (1, -1) and (2, -3) are points on the graph of y, then they satisfy $y = ax^2 + bx + c$. That is,

Using (0, -3)

$$\begin{array}{llll} y = ax^2 + bx + c & & & \\ -3 = a(0)^2 + b(0) + c & \Leftrightarrow & -3 = c & \text{Eq. 1} \\ -1 = a(1)^2 + b(1) + c & \Leftrightarrow & -1 = a + b + c & \text{Eq. 2} \\ -3 = a(2)^2 + b(2) + c & \Leftrightarrow & -3 = 4a + 2b + c & \text{Eq. 3} \end{array}$$

Substituting Eq. 1 to Eq. 2 and Eq. 3,

$$-1 = a + b - 3 \quad \Leftrightarrow \quad a + b = 2 \quad \text{Eq. 4}$$
$$-3 = 4a + 2b - 3 \quad \Leftrightarrow \quad 4a + 2b = 0 \quad \text{Eq. 5}$$

Solving Eq. 4 and Eq. 5 by substitution,

$$a = -b + 2$$
$$4(-b + 2) + 2b = 0$$
$$-4b + 8 + 2b = 0$$
$$-2b = -8$$
$$b = 4$$

Substituting b = 4 to Eq. 4,

$$a + 4 = 2$$
$$a = -2$$

Hence, the quadratic function is $y = -2x^2 + 4x - 3$

Try this out

Derive the quadratic function, given the zeros of the function.

Set A

1.

x	-2	0	2
y	0	2	12

2.

x	-2	0	1
y	-12	-16	-15

3.

x	-5	-2	0
y	0	0	10

4.

x	-1	0	2
y	-4	-1	-1

5.

x	-1	0	1
y	1	0	3

Set B

1.

x	1	0	6
y	-8	36	0

2.

x	1	2	3
y	-10	-6	0

3.

x	-1	0	1
y	4	3	4

4.

x	0	1	2
y	0	3	10

5.

x	-2	-1	1
y	15	5	3

Set C

1.

x	-1	1	2
y	0	4	9

2.

x	0	1	2
y	-3	-5	-11

3.

x	-1	0	2
y	4	0	10

4.

x	-1	0	1
y	0	-1	-4

5.

x	-1	0	1
y	2	-2	0

Lesson 8

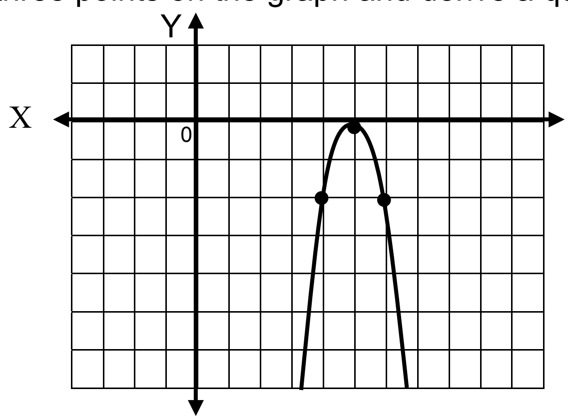
Deriving a Quadratic Function, Given the Graph

If the graph representing the quadratic function $y = ax^2 + bx + c$ is given, identify at least three points on the graph. Use these points to solve the quadratic function.

Examples:

Identify the three points on the graph and derive a quadratic function.

1.



Three of the points on the graph are (4, -2), (5, 0) and (6, -2). They satisfy $y = ax^2 + bx + c$. That is,

Using (4, -2)

$$\begin{array}{lcll} y = ax^2 + bx + c & & & \\ -2 = a(4)^2 + b(4) + c & \Leftrightarrow & -2 = 16a + 4b + c & \text{Eq. 1} \\ 0 = a(5)^2 + b(5) + c & \Leftrightarrow & 0 = 25a + 5b + c & \text{Eq. 2} \\ -2 = a(6)^2 + b(6) + c & \Leftrightarrow & -2 = 36a + 6b + c & \text{Eq. 3} \end{array}$$

Eliminating c in Eq. 1 to Eq. 2 by subtraction,

$$\begin{array}{l} -2 = 16a + 4b + c \\ \underline{\ \ 0 = 25a + 5b + c} \\ -2 = -9a - b \\ \ \ b = -9a + 2 \quad \text{Eq. 4} \end{array}$$

Eliminating c in Eq. 2 to Eq. 3 by subtraction,

$$\begin{array}{l} \ \ 0 = 25a + 5b + c \\ \underline{-2 = 36a + 6b + c} \\ \ \ 2 = -11a - b \\ \ \ b = -11a - 2 \qquad \text{Eq. 5} \end{array}$$

Solving Eq. 4 and Eq. 5 simultaneously,

$$-9a + 2 = -11a - 2$$
$$2a = -4$$
$$a = -2$$

Substituting a = -2 in Eq. 4,

$$b = -9(-2) + 2$$
$$b = 20$$

Substituting a = -2 and b = 20 in Eq. 1

$$-2 = 16(-2) + 4(20) + c$$
$$c = -50$$

The value of a = -2, b = 20 and c = -50

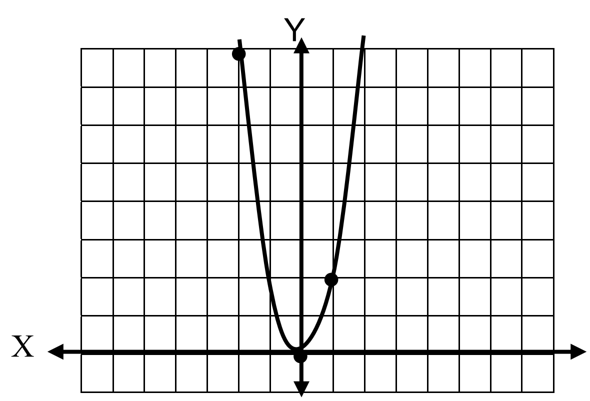
Hence, the quadratic function is $y = -2x^2 + 20x - 50$

Try this out

Identify the coordinates of the given points in the graph and derive a quadratic function.

Set A

1.



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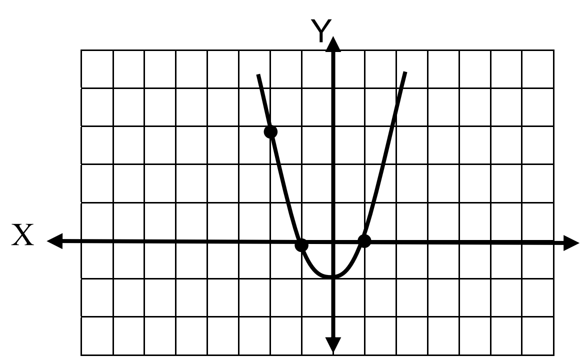

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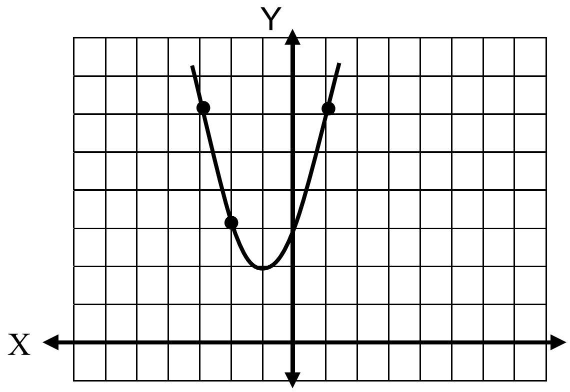

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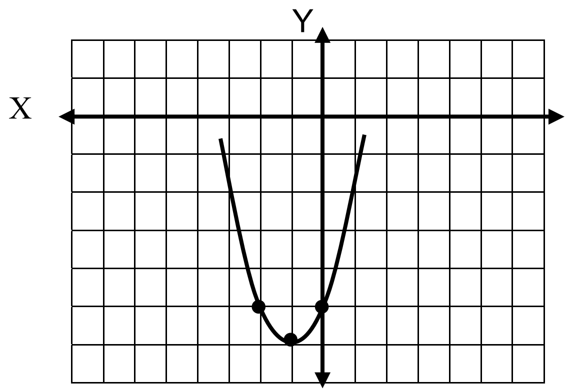

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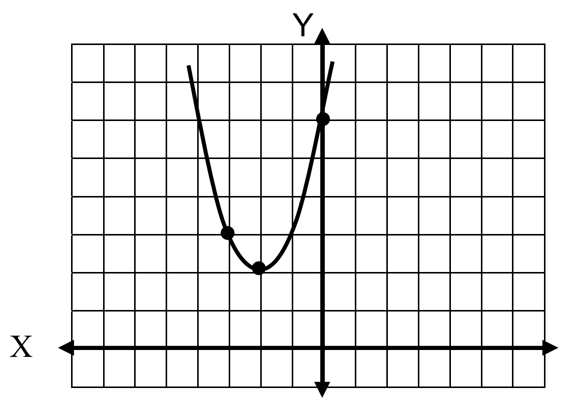
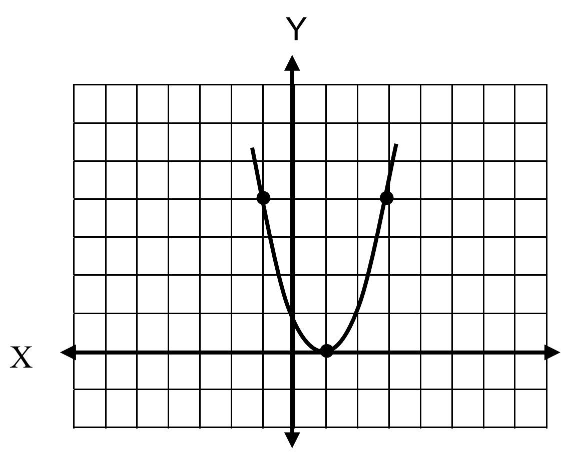
5.

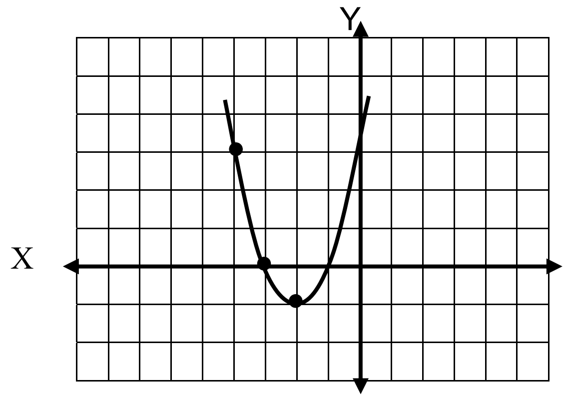


Set B

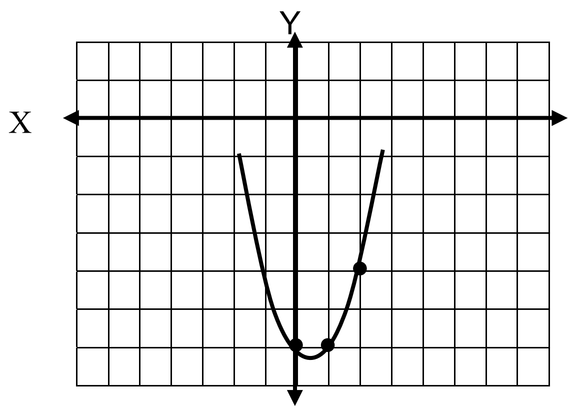
1.



2.



3.



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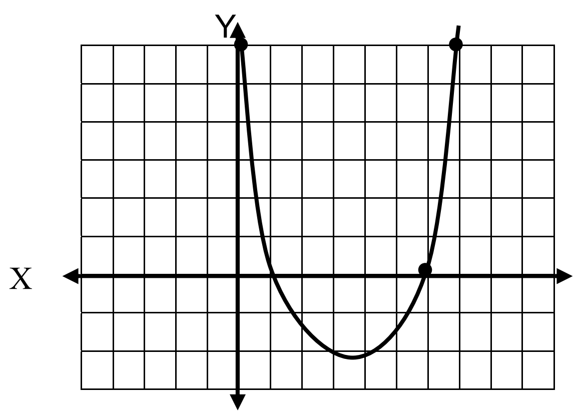
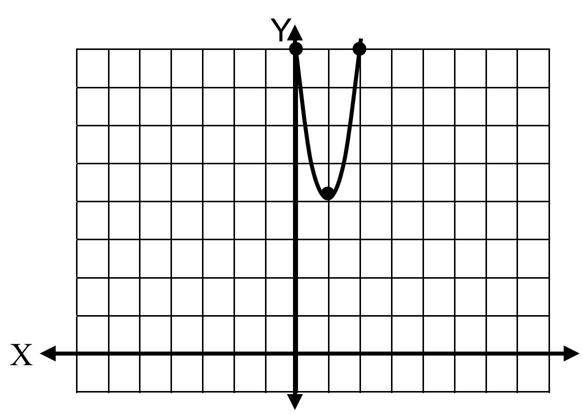
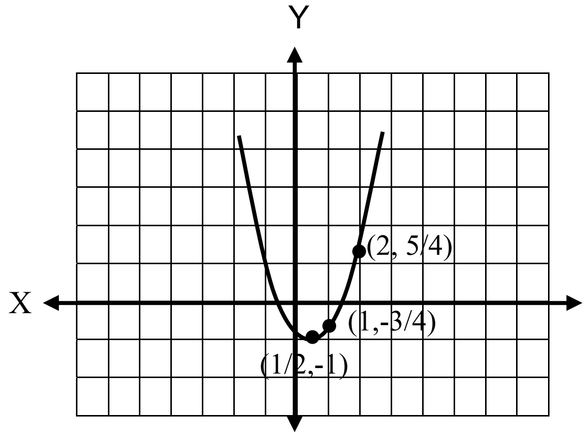


5.

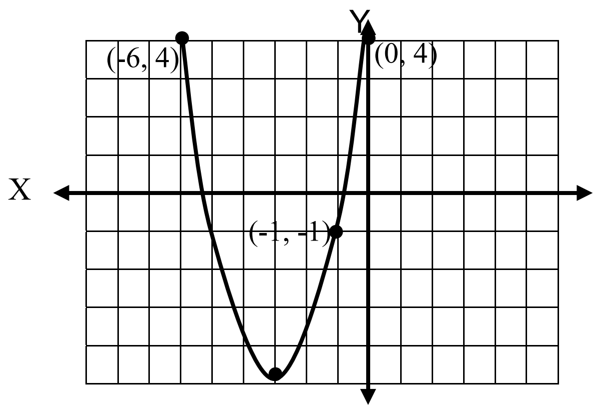


Set C: Given the coordinates, solve for the quadratic functions.

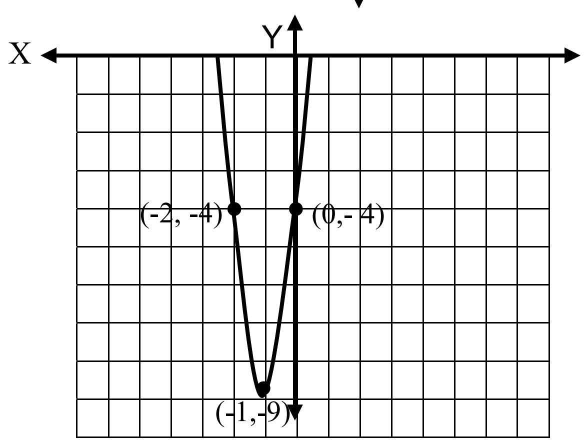
1.



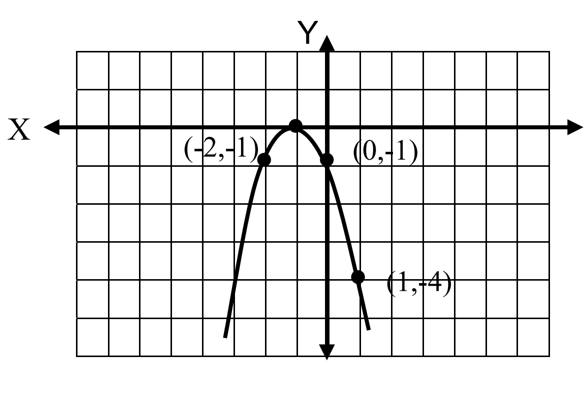
2.



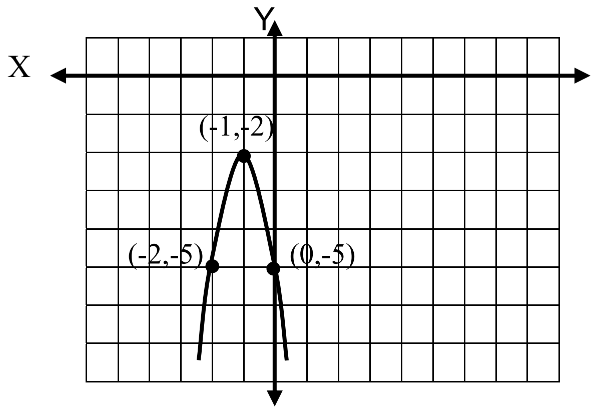
3.



4.



5.



Let's summarize

1. The parabolic structure of the graph of a quadratic function allows it to intersect the x-axis once, twice or none at all.
2. If the quadratic expression $ax^2 + bx + c$ is factorable, its zeros may be found by factoring it and apply the zero property.
3. A non-real number is called imaginary number. The unit imaginary number *i* is a number whose square root is -1. In symbols,

$$i = \sqrt{-1} \rightarrow i^2 = -1$$

4. In solving quadratic equations that are not factorable, the method called completing the square may be used. The principle behind the use of completing the square is to produce a perfect square trinomial so that the square root property can be applied. The following steps are suggested:

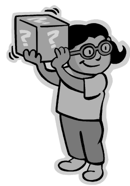
   a. Equate the given quadratic function to zero.
   b. Transpose the constant term.
   c. If $a \neq 1$, divide both sides of the equation by a.
   d. Add to both sides of the equation the square of one-half the coefficient of x.
   e. Factor the resulting perfect square trinomial.
   f. Apply the square root property.
   g. Solve for x.

5. The quadratic formula which can be used to solve any quadratic equation is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

6. If the zeros of a quadratic function y are x_1 and x_2, then the quadratic function is $y = (x - x_1)(x - x_2)$

7. If the table of values representing the quadratic function $y = ax^2 + bx + c$ is given, then those values should satisfy $y = ax^2 + bx + c$.
8. If the graph representing the quadratic function $y = ax^2 + bx + c$ is given, identify at least three point on the graph, and use these point to form the quadratic function.

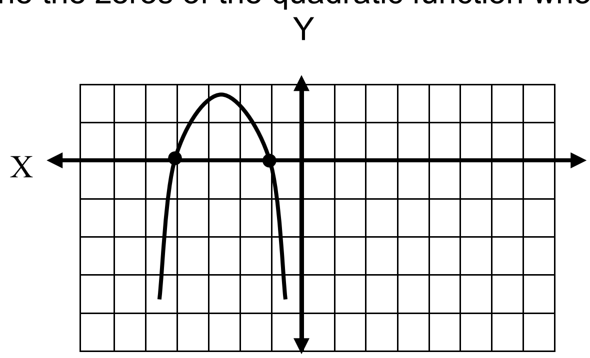


What have you learned

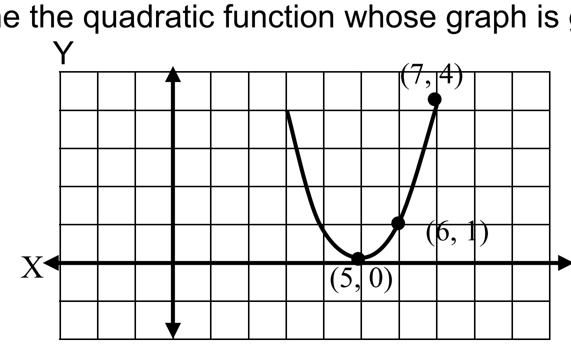
1. What are the zeros of $f(x) = 2x^2 – 32$?
2. Solve the equation $2x^2 – 5x + 2 = 0$.
3. Use the quadratic formula in solving $x^2 – 4x + 3 = 0$.
4. Determine the roots of $8x^2 – 10x + 3 = 0$ using the method of completing the square.
5. Find a quadratic function whose zeros are -2 and 5.
6. Name the quadratic function satisfied by the table below.

x	-2	-1	0	2
f(x)	-4	-5	-4	4

7. Determine the zeros of the quadratic function whose graph is given below.



8. Determine the quadratic function whose graph is given below.



9. What quadratic function has $3\sqrt{2}$ and $-3\sqrt{2}$ as zeros?
10. What quadratic function has a vertex at (3,1) and passes through (4,-3)?

Answer Key

How much do you know

1. x = 4 or x = -4
2. x = 2 or x = 3
3. x = $\frac{-2-\sqrt{13}}{2}$ or x = $\frac{-2+\sqrt{13}}{2}$
4. x = 1 or x = $\frac{1}{2}$
5. $f(x) = x^2 + x - 6$
6. $f(x) = x^2 - 3x + 2$
7. x = -1 or x = 3
8. $f(x) = x^2 - x - 6$
9. $f(x) = x^2 - 2$
10. $f(x) = -2x^2 + 8x - 7$

Try this out

Lesson 1
Set A

1. -2 and 2.
2. -3 and 1
3. 4 and 8
4. -2
5. No zero

Set B

1. -5 and 5
2. -8 and -2
3. No zero
4. -4 and 4
5. 3

Set C

1. -5 and -1
2. 0
3. 5
4. No zero
5. -3.5 and 3.5

Lesson 2
Set A

1. x = -1 or x = 1
2. x = $-\sqrt{6}$ or x = $\sqrt{6}$
3. x = -3 or x = 3
4. x = -3 or x = -1
5. x = -3 or x = -2

Set B

1. x = $-\frac{7}{6}$ or x = $\frac{7}{6}$
2. x = -2 or x = -1
3. x = -3 or x = -4
4. x = -6 or x = 1
5. x = 4 or x = -3

Set C

1. x = 2 or x = 3
2. x = -7 or x = 2
3. x = -3 or x = 7
4. x = -1 or x = $\frac{1}{2}$
5. x = -2 or x = $-\frac{2}{3}$

Lesson 3
Set A

1. $i\sqrt{2}$
2. 2i
3. 6i
4. $3i\sqrt{2}$
5. $2i\sqrt{5}$

Set B

1. 9i
2. $2i\sqrt{7}$
3. $\frac{5}{4}i\sqrt{3}$
4. $\frac{3}{4}i\sqrt{3}$

5. $\frac{1}{4}i\sqrt{31}$

Set C

1. $2i\sqrt{13}$
2. $3i\sqrt{5}$
3. $\frac{1}{7}i\sqrt{19}$
4. $\frac{1}{4}i\sqrt{41}$
5. $\frac{9}{11}i\sqrt{2}$

Lesson 4
Set A

1. x = -1 or x = 2
2. x = $-3+\sqrt{5}$ or x = $-3-\sqrt{5}$
3. x = 3 or x = 4
4. x = $\frac{1}{2}$ or x = 1
5. x = -2 or x = $\frac{1}{3}$

Set B

1. x = 3 or x = 7
2. x = $-2+\sqrt{5}$ or x = $-2-\sqrt{5}$
3. x = $4+\sqrt{12}$ or x = $4-\sqrt{12}$
4. x = $-3+\sqrt{14}$ or x = $-3-\sqrt{14}$
5. x = $-\frac{1}{2}+\frac{1}{4}i\sqrt{5}$ or x = $-\frac{1}{2}-\frac{1}{4}i\sqrt{5}$

Set C

1. x = -4 or x = 1
2. x = $1+\sqrt{6}$ or x = $1-\sqrt{6}$
3. x = $-\frac{1}{2}+\frac{\sqrt{5}}{2}$ or x = $-\frac{1}{2}-\frac{\sqrt{5}}{2}$
4. x = $-\frac{1}{2}+\frac{\sqrt{15}}{2}$ or x = $-\frac{1}{2}-\frac{\sqrt{15}}{2}$

5. x = $-1+\frac{\sqrt{6}}{3}$ or x = $-1-\frac{\sqrt{6}}{3}$

Lesson 5
Set A

1. x = -5 or x = -1
2. x = $-1+\frac{\sqrt{15}}{3}$ or x = $-1-\frac{\sqrt{15}}{3}$
3. x = $\frac{3}{8}+\frac{\sqrt{41}}{8}$ or x = $\frac{3}{8}-\frac{\sqrt{41}}{8}$
4. x = -1 or x = $\frac{3}{2}$
5. x = $-\frac{1}{2}$ or x = $\frac{3}{2}$

Set B

1. x = -4 or x = 5
2. x = -6 or x = 2
3. x = $\frac{1}{3}$ or x = -1
4. x = $-\frac{5}{2}$ or x = 2
5. x = $-\frac{2}{3}$ or x = 1

Set C

1. x = $\frac{1}{4}+\frac{i\sqrt{11}}{4}$ or x = $\frac{1}{4}-\frac{i\sqrt{11}}{4}$
2. x = $1+\frac{\sqrt{7}}{2}$ or x = $1-\frac{\sqrt{7}}{2}$
3. x = $-\frac{1}{2}+\frac{i\sqrt{11}}{2}$ or x = $-\frac{1}{2}-\frac{i\sqrt{11}}{2}$
4. x = $-1+\frac{3\sqrt{2}}{2}$ or x = $-1-\frac{3\sqrt{2}}{2}$
5. x = $-1+\frac{\sqrt{15}}{3}$ or x = $-1-\frac{\sqrt{15}}{3}$

Lesson 6
Set A

1. f(x) = $x^2 - 4x + 3$

2. $f(x) = x^2 – x – 12$
3. $f(x) = x^2 – 10x + 24$
4. $f(x) = x^2 – 4x – 21$
5. $f(x) = 2x^2 – x – 6$

Set B

1. $f(x) = 3x^2 – 11x + 6$
2. $f(x) = 3x^2 – 5x – 2$
3. $f(x) = 4x^2 – 4x – 3$
4. $f(x) = 6x^2 – 13x + 6$
5. $f(x) = 16x^2 – 9$

Set C

1. $f(x) = x^2 + 3x – 40$
2. $f(x) = 2x^2 – 7x – 4$
3. $f(x) = x^2 - 4$
4. $f(x) = x^2 – 5x + 6$
5. $f(x) = x^2 + x - \frac{5}{4}$

Lesson 7

Set A

1. $f(x) = x^2 + 3x + 2$
2. $f(x) = x^2 – 16$
3. $f(x) = x^2 + 7x + 10$
4. $f(x) = -x^2 + 2x – 1$
5. $f(x) = 2x^2 + x$

Set B

1. $f(x) = x^2 – 12x + 36$
2. $f(x) = x^2 + x – 12$
3. $f(x) = x^2 + 3$
4. $f(x) = 2x^2 + x$
5. $f(x) = 3x^2 – x + 1$

Set C

1. $f(x) = x^2 + 2x + 1$
2. $f(x) = -2x^2 – 3$
3. $f(x) = 3x^2 – x$
4. $f(x) = -x^2 – 2x – 1$

5. $f(x) = 3x^2 - x - 2$

Lesson 8
Set A

1. $f(x) = 2x^2$
2. $f(x) = x^2 - 1$
3. $f(x) = x^2 + 2x + 3$
4. $f(x) = x^2 + 2x - 5$
5. $f(x) = x^2 + 4x + 6$

Set B

1. $f(x) = x^2 - 2x + 1$
2. $f(x) = x^2 + 4x + 3$
3. $f(x) = x^2 - x - 6$
4. $f(x) = x^2 - 7x + 6$
5. $f(x) = 3x^2 - 6x + 7$

Set C

1. $f(x) = x^2 - x - \frac{3}{4}$
2. $f(x) = x^2 + 6x - 40$
3. $f(x) = 5x^2 + 10x - 4$
4. $f(x) = -x^2 - 2x - 1$
5. $f(x) = -3x^2 - 6x - 5$

What have you learned

1. $x = -4$ or $x = 4$
2. $x = \frac{1}{2}$ or $x = 2$
3. $x = 1$ or $x = 3$
4. $x = -\frac{1}{4}$ or $x = \frac{3}{2}$
5. $f(x) = x^2 - 3x - 10$
6. $f(x) = x^2 + 2x - 4$
7. -4 and -1
8. $f(x) = x^2 - 10x + 25$
9. $f(x) = x^2 - 18$
10. $f(x) = -4x^2 + 24x - 35$