Module 2
Quadratic Functions

What this module is about

This module is about Quadratic Functions. As you go over the exercises, you will develop skills in solving quadratic equations and ability to apply this in solving problems. Treat the lesson with fun and take time to go back if you think you at a loss.

What you are expected to learn

This module is designed for you to:

1. Draw the graph of a quadratic functions using the
    - vertex
    - axis of symmetry
    - direction of the opening of the graph
    - given points
2. Analyze the effects on the graph of the changes in a, h, and k in $f(x) = a(x-h)^2 + k$

How much do you know

A. Tell the direction or the opening of the graph of the following functions.

1. $y = -2x^2 + 5$
2. $y = x^2 - 3$
3. $y = 3x^2 - 9x + 2$

B. Sketch the graph of the following functions.

4. $y = -2x^2 + 4x - 3$
5. $y = x^2 - 4x + 2$

C. Using $f(x) = x^2$ as the reference graph, which graph opens wider/narrower.

6. $y = 3x^2 + 2$

   $y = \frac{1}{3} x^2 - 5$

7. $y = -4x^2 - 4$

   $y = \frac{-1}{4} x^2 + 2$

D. Which graphs translate to the right or to the left of the origin?

8. $f(x) = (x + 3)^2$
9. $f(x) = 3(x - 4)^2$
10. $f(x) = -(x - 2)^2$
11. $y = 2(x + 2)^2$

E. Which graphs translate upward or downward considering $f(x) = x^2$ as the reference graph.

12. $f(x) = x^2 - 4$
13. $f(x) = 3x^2 + 1$
14. $f(x) = -2(x - 3)^2 + 3$
15. $y = (x + 2)^2 - 2$

What you will do

Lesson 1

Graph of Quadratic Function

The graph of a quadratic function is a parabola. You can graph using your previous knowledge about the characteristics of the graph of a quadratic function such as vertex, axis of symmetry and the direction of the opening.

Steps in graphing quadratic functions:

1. find the coordinates of the vertex
2. determine the axis of symmetry

3. determine the direction of the opening of the graph
4. make the table of values (choose symmetric values with respect to the value of h)

Examples: Construct a table of values and graph the following functions:

1. $f(x) = (x + 1)^2 - 2$

Vertex = (-1, -2)
Axis of symmetry: x = -1
Direction of opening: upward

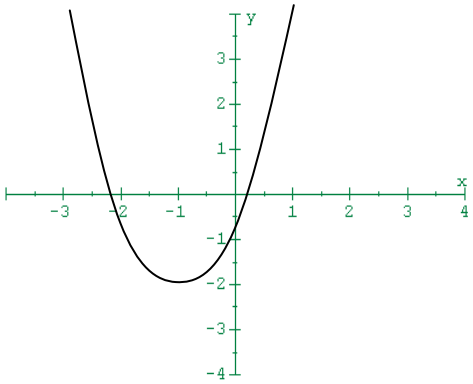


Table of values		
x	$(x + 1)^2 - 2$	f(x)
1	$(1 + 1)^2 - 2$	2
0	$(0 + 1)^2 - 2$	-1
-1	$(-1 + 1)^2 - 2$	-2
-2	$(-2+ 1)^2 - 2$	-1
-3	$(-3 + 1)^2 - 2$	2

2 $f(x) = -(x - 2)^2 + 3$

vertex = (2, 3)
axis of symmetry: x = 2
direction of opening: Downward

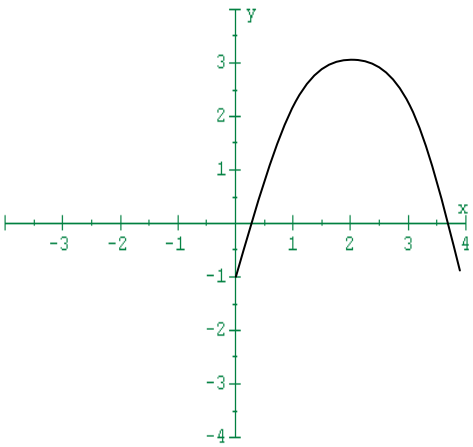


Table of values		
x	$-(x - 2)^2 + 3$	f(x)
4	$-(4 - 2)^2 +3$	-1
3	$-(3 - 2)^2 +3$	-2
2	$-(2 - 2)^2 +3$	3
1	$-(1 - 2)^2 +3$	2
0	$-(0 - 2)^2 +3$	-1

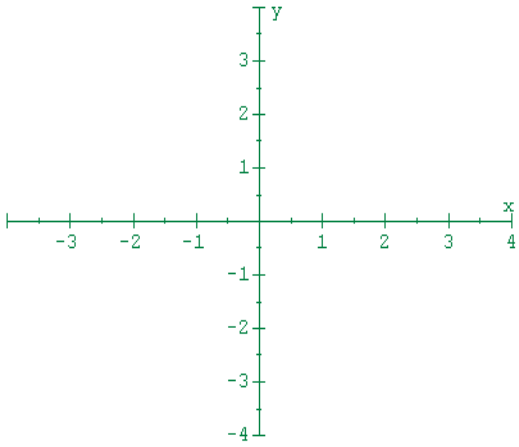
Try this out

Draw the graph of each of the following functions by following the steps mentioned in the given examples.

1. $f(x) = (x + 2)^2 - 3$

Vertex: _____
Axis of symmetry: _____
Direction of opening: _____

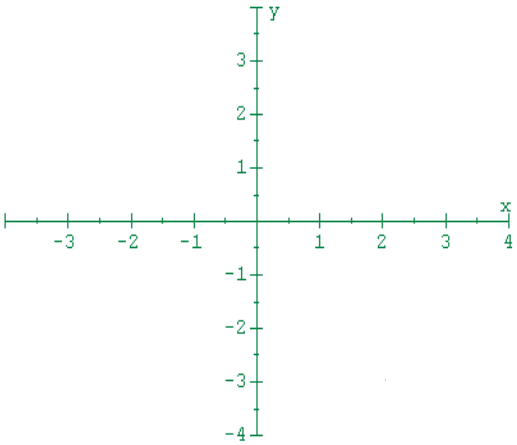
Table of values		
x	$(x + 2)^2 - 3$	f(x)
0		
-1		
-2		
-3		
-4		



2. $f(x) = -(x - 2)^2 + 4$

vertex: _____
Axis of symmetry: _____
Direction of opening: _____

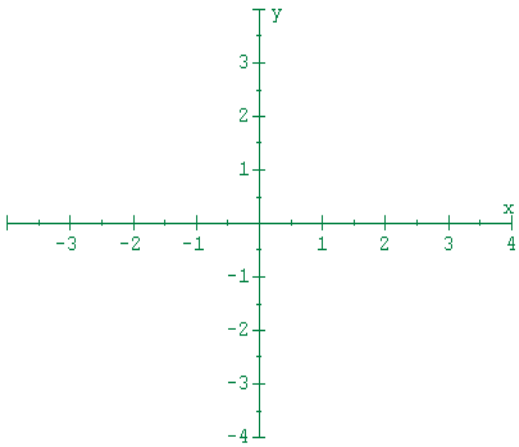
Table of values		
x	$-(x - 2)^2 + 4$	f(x)
3		
2		
1		
0		
-1		



3. $f(x) = 3(x + 1)^2 + 3$

vertex:: _____
Axis of symmetry: _____
Direction of the opening: _____

Table of values		
x	$3(x + 1)^2 + 3$	f(x)
3		
2		
1		
0		
-1		



4. $f(x) = (x - 1)^2 + 3$

vertex: _____
Axis of symmetry: _____
Direction of opening: _____

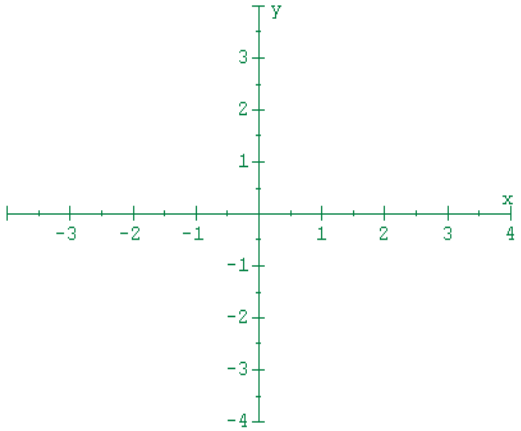


Table of values		
x	$(x - 1)^2 + 3$	f(x)
3		
2		
1		
0		
-1		

5. $f(x) = 1/3(x - 1)^2 + 2$

vertex : _____
Axis of symmetry: _____
Direction of opening: _____

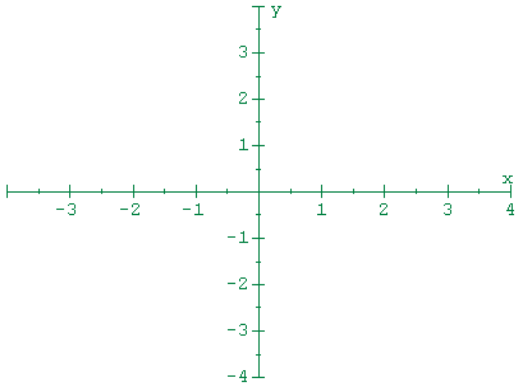


Table of values		
x	$1/3(x - 1)^2 + 2$	f(x)
3		
2		
1		
0		
-1		

Lesson 2

Analyze the Effect of the Changes in ***a*** in the Graph of the Function $f(x) = ax^2$

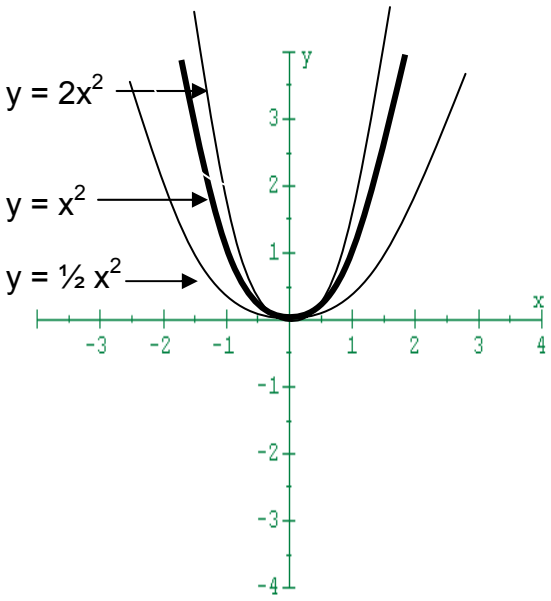
The value of ***a*** in $f(x) = ax^2$ has an effects on the graph of a quadratic function. It determines the width of the parabola with respect to the axis of symmetry. If $0 < a < 1$, the graph is wider and tends to flatten out. If > 1, the graph is narrower and steeper.

Consider the following functions and $f(x) = x^2$ as reference function.

1. $y = 2x^2$

Vertex: (0,0)
Axis of symmetry: x = 0
Opening of graph: Upward

	Table of values	
x	$2x^2$	f(x)
2	$2(2)^2$	8
1	$2(1)^2$	2
0	$2(0)^2$	0
-1	$2(-1)^2$	2
-2	$2(-2)^2$	8



2. $f(x) = 1/2x^2$

vertex: (0,0)
Axis of symmetry: x = 0
Opening of the graph:
Upward

	Table of values	
x	$\frac{1}{2}(x)^2$	f(x)
2	$\frac{1}{2}(2)^2$	2
1	$\frac{1}{2}(1)^2$	½
0	$\frac{1}{2}(0)^2$	0
-1	$\frac{1}{2}(-1)^2$	½
-2	$\frac{1}{2}(-2)^2$	2

You will notice that the graph of $f(x) = 2x^2$ is narrower compared to the graph of $f(x) = x^2$. While, the graph of $f(x) = \frac{1}{2}x^2$ is wider compared to the graph of $f(x) = x^2$.

Try this out

For each set of functions, tell which graph is narrower or wider.

1. $f(x) = 2x^2$

   $f(x) = 3x^2$

2. $f(x) = \frac{-1}{2} x^2$

f(x) = $-2x^2$

3. f(x) = $4x^2$

   f(x) = $\frac{1}{4}x^2$

4. f(x) = $5x^2$

   f(x) = $4x^2$

5. f(x) = $-3x^2$

   f(x) = $\frac{-1}{3}x^2$

6. f(x) = $-x^2$

   f(x) = $-3x^2$

7. f(x) = $2x^2$

   f(x) = $4x^2$

8. f(x) = $-5x^2$

   f(x = $-2x^2$

9. f(x) = $\frac{2}{3}x^2$

   f(x) = $\frac{1}{2}x^2$

10. f(x) = $\frac{-1}{3}x^2$

    f(x) = $\frac{-1}{2}x^2$

Lesson 3

Analyze the Effect on the Changes in *h* in the Graph of the Function $f(x) = a(x-h)^2$

The graph of the function $f(x) = a(x-h)^2$ is the same as the graph of

$f(x) = ax^2$, except that its vertex is translated horizontally to the right of the origin when $h > 0$.

The graph of the function $f(x) = a(x - h)^2$ is the same as the graph of $f(x) = ax^2$ except the vertex is translated to the left of the origin when $h < 0$.

Example:

1. $y = (x + 1)^2$

vertex : (-1, 0)
Axis of symmetry: x = -1
Opening: Upward

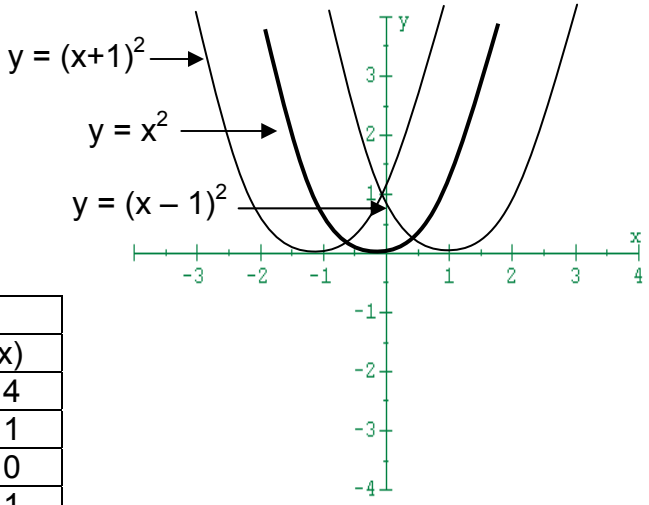


	Table of values	
x	$(x + 1)^2$	f(x)
1	$(1+ 1)^2$	4
0	$(0 + 1)^2$	1
-1	$(-1 + 1)^2$	0
-2	$(-2+ 1)^2$	1
-3	$(-3+ 1)^2$	4

2. $y = (x - 1)^2$

vertex : (1, 0)
Axis of symmetry: x = 1
Opening of the graph: Upward

	Table of values	
x	$(x - 1)^2$	f(x)
-1	$(-1- 1)^2$	4
0	$(0 - 1)^2$	1
1	$(1 - 1)^2$	0
2	$(2 - 1)^2$	1
3	$(3 - 1)^2$	4

The graph of $y = (x + 1)^2$ moves to the left of the origin while the graph of $y = (x – 1)^2$ moves to the right of the origin.

Try this out

Given are the following functions. Tell the directions of each graph, if it translates to the right or to the left of the reference graph $f(x) = x^2$.

1. $f(x) = (x + 4)^2$
2. $f(x) = (x – 4)^2$
3. $f(x) = (x – 2)^2$

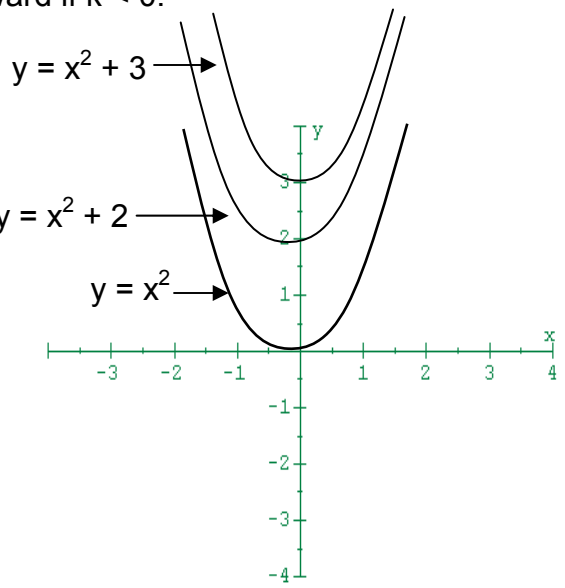
4. $f(x) = (x + 2)^2$
5. $f(x) = (x + 5)^2$
6. $f(x) = 2(x – 5)^2$
7. $f(x) = 3(x – 4)^2$
8. $y = (x - 6)^2$
9. $y = (x + 1)^2$
10. $y = (x - ½)^2$

Lesson 4

Analyze the Effect on the Changes in k in the Graph of the Function $f(x) = ax^2 + k$

Using the graph of $f(x) = x^2$ as the reference, the value of k translates the graph vertically, upward if $k > 0$ or downward if $k < 0$.

Examples:



1. $f(x) = x^2 + 2$

vertex : (0, 2)
axis of symmetry: x= 0
Opening of the graph: Upwards

Table of values		
x	$x^2 + 2$	f(x)
2	$(2)^2 + 2$	6
1	$(1)^2 + 2$	3
0	$(0)^2 + 2$	2
-1	$(-1)^2 + 2$	3
-2	$(-2)^2 + 2$	6

2. $f(x) = x^2 + 3$

vertex : (0, 3)
axis of symmetry: x = 0
opening of the graph: upwards

	Table of values	
x	$x^2 + 3$	f(x)
2	$(2)^2 + 3$	7
1	$(1)^2 + 3$	4
0	$(0)^2 + 3$	3
-1	$(-1)^2 + 3$	4
-2	$(-2)^2 + 3$	7

Try this out

Tell the direction of each graph, if it translate upwards or downwards . Consider the graph of $f(x) = x^2$ as your reference graph.

1. $y = x^2 + 6$
2. $y = x^2 - 4$
3. $y = (x - 3)^2$
4. $y = (x + 2)^2$
5. $y = (x - 2)^2$
6. $y = x^2 - 1$
7. $y = x^2 + 5$
8. $y = x^2 + 8$
9. $y = (x + 3)^2$
10. $y = (x - 1)^2$

Lesson 5

Analyze the Effects of the Changes in *a* in the Graph of the Function $f(x) = a(x-h)^2 + k$.

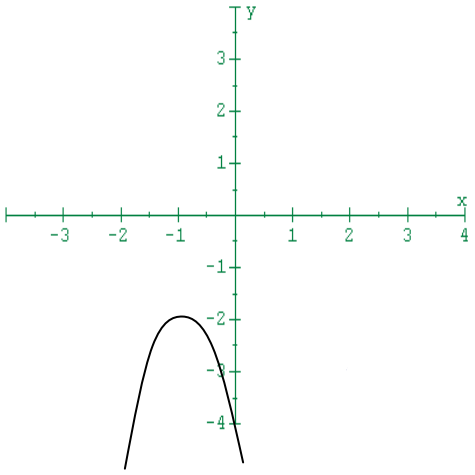
Using your previous knowledge regarding the characteristics and different forms of quadratic functions lets you analyze how the graph of $f(x) = ax^2$ is affected by both h and k. This would require translation of the graph from both the x and y axis.

Example 1

If a = -2, h = -1 and k = -2 the quadratic function becomes

$y = -2(x + 1)^2 - 2.$
Vertex: (-1, -2)
Axis of symmetry: x = -1
Opening of the graph: Downward

	Table of values	
X	$-2(x + 1)^2 - 2$	f(x)
1	$-2(1 + 1)^2 - 2$	-10
0	$-2(0 + 1)^2 - 2$	-4
-1	$-2(-1 + 1)^2 - 2$	-2
-2	$-2(-2 + 1)^2 - 2$	-4
-3	$-2(-3 + 1)^2 - 2$	-10



Example 2:

Write the equation of parabola if the graph of $y = 2x^2$ is shifted 4 units to the left and 1 unit downward.

Answer The equation is $y = 2(x + 4)^2 - 1$.

Try this out

A. Sketch the graph of the following quadratic functions. Complete the table of values (choose points symmetric to the value of h) and other characteristics such as vertex, axis of symmetry and direction of the graph.

1. $y = -(x - 2)^2 + 1$
2. $y = 2(x + 2)^2 - 3$
3. $y = (x - 1)^2 + 2$
4. $y = (x + 1)^2 - 2$
5. $y = (x + 1)^2 - 2$

B. Write the equation for each parabola described.

1. The graph of $y = x^2$ shifted 5 units upward.
2. The graph of $y = x^2$ shifted 3 units downward
3. The graph of $y = 2x^2$ shifted 2 units above the origin
4. The graph of $y = x^2$ shifted 4 units to the right of the origin
5. The graph of $y = 3x^2$ shifted 2 units to the left of the origin
6. The graph of $y = 2x^2$ shifted 3 units to the left and 5 units upward
7. The graph of $y = 3x^2$ shifted 3 units to the right and 2 units downward
8. the graph of $y = x^2$ shifted 3 units to the left and 4 units downward
9. the graph of $y = -2x^2$ shifted 5 units to the right and 3 units upward
10. the graph of $y = -3x^2$ shifted 2 units to the left and 4 units upward

Let's summarize

1. The graph of a quadratic function is called parabola.
2. Graphing quadratic functions there are steps to be followed

   a. Find the coordinates of the vertex
   b. Determine the axis of symmetry
   c. Determine the direction of the opening of the graph
   d. Prepare the table of values (choose the values of x symmetric to the value of h)

3. The graph of the function of the form $f(x) = ax^2$, as **a** increases the graph narrows.
4. The graph of $f(x) = a(x - h)^2$ has the same shape and direction of opening as the graph of $f(x) = ax^2$. But its position is translated h units to the right or left.
5. As the value of k changes, the graph of $f(x) = a(x - h)^2 + k$ is translated k units up or down.

What have you learned

A. Tell the direction or the opening of the graph of the following functions.

1. $y = 2x^2 - 4$
2. $y = x^2 + 3$
3. $y = -x^2 + 2x - 5$

B. Sketch the graph of the following functions.

4. $y = 3x^2 - 9x + 2$
5. $y = -x^2 + 2x - 1$

C. Using $f(x) = x^2$ as the reference graph , which graph is wider or narrower

6. $y = 2x^2 - 2$
   $y = x^2 + 4$
7. $y = -3x^2 + 3$
   $y = \frac{-1}{2} x^2 - 2$

D. Which graphs translates to right or to the left of the origin.

8. $f(x) = (x - 3)^2$
9. $f(x) = 4(x + 2)^2$
10. $f(x) = -2(x - 4)^2$
11. $f(x) = 3(x + 3)^2$

E. Which graph translates upwards or downwards from $f(x) = x^2$.

12. $f(x) = 3x^2 - 4$
13. $f(x) = -2x^2 + 3$
14. $fX) = -3(x - 3)^2 + 5$
15. $f(x) = (x + 4)^2 - 3$

Answer key

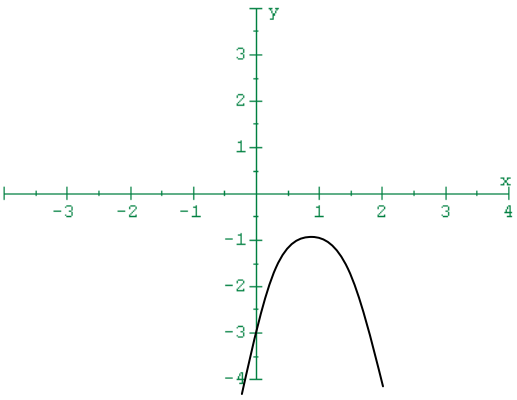
How much do you know

A. 1. Downward

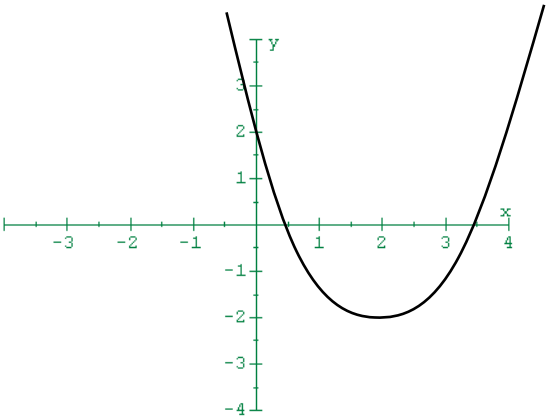
2. Upward

3. Upward

B. 4. $y = -2x^2 + 4x - 3$



5. $y = x^2 - 4x + 2$



C.

6. narrower
   wider

7. narrower
   wider

D.

8. to the left

9. to the right
10. to the right
11. to the left

E.

12. downward
13. upward
14. upward
15. downward

Try this out

Lesson 1

1. $f(x) = (x + 2)^2 - 3$

Vertex: (-2, -3)
Axis of symmetry: x = -2
Direction of opening: upward

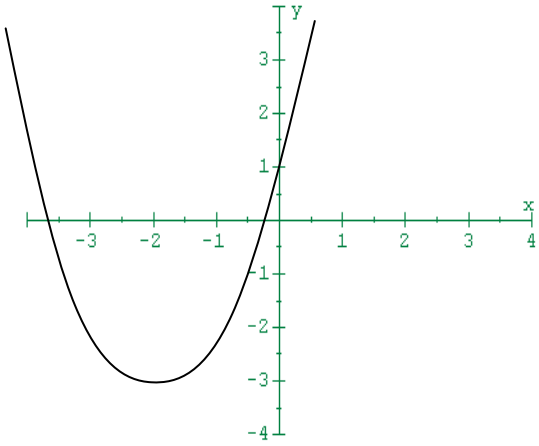


	Table of values	
x	$(x + 2)^2 - 3$	f(x)
0	$(0 + 2)^2 - 3$	1
-1	$(-1 + 2)^2 - 3$	-2
-2	$(-2 + 2)^2 - 3$	-3
-3	$(-3 + 2)^2 - 3$	-2
-4	$(-4 + 2)^2 - 3$	1

2. $f(x) = -(x - 2)^2 + 4$

vertex: (2, 4)
Axis of symmetry: x = 2
Direction of opening: downward

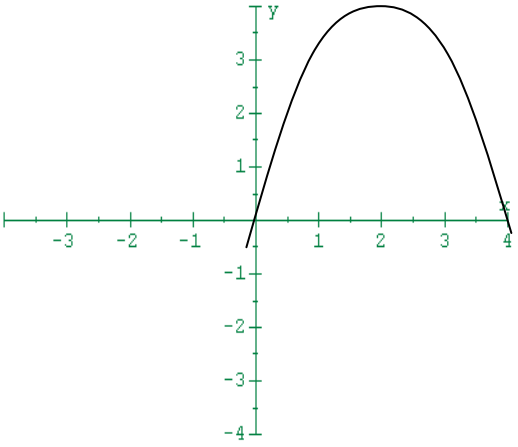
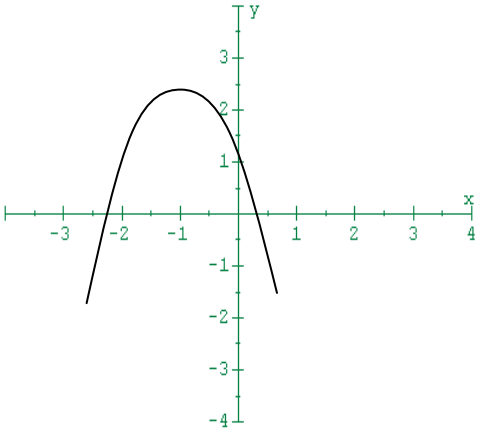


	Table of values	
x	$-(x - 2)^2 + 4$	f(x)
4	$-(4 - 2)^2 + 4$	0
3	$-(3 - 2)^2 + 4$	3
2	$-(2 - 2)^2 + 4$	4
1	$-(1 - 2)^2 + 4$	3
0	$-(0 - 2)^2 + 4$	0

3. $f(x) = -(x + 1)^2 + 3$

vertex: (-1, 3)
Axis of symmetry: x= -1
Direction of the opening: Upward

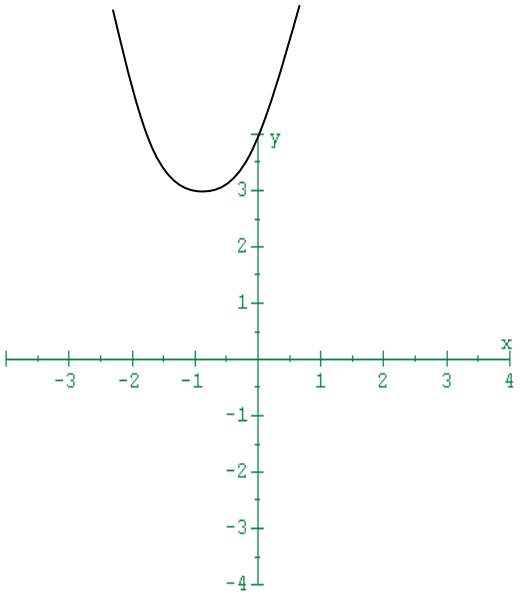
	Table of values	
x	$-(x + 1)^2 + 3$	f(x)
1	$-(1 + 1)^2 + 3$	-1
0	$-(0 + 1)^2 + 3$	2
-1	$-(-1 + 1)^2 + 3$	3
-2	$-(-2 + 1)^2 + 3$	2
-3	$-(-3 + 1)^2 + 3$	-1



4. $f(x) = (x - 1)^2 + 3$

vertex: (1, 3)
Axis of symmetry: x = 1
Direction of opening: Upward

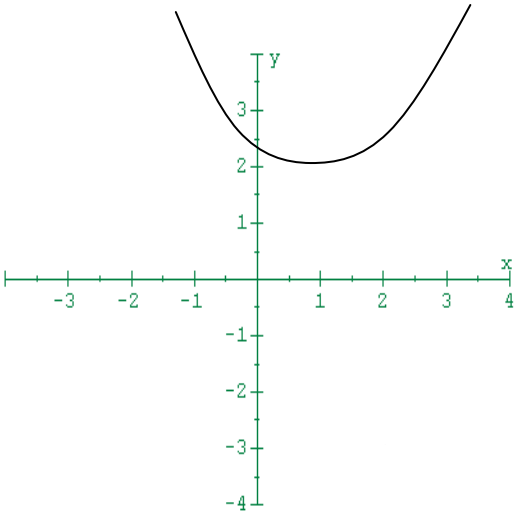
	Table of values	
x	$(x - 1)^2 + 3$	f(x)
3	$(3 - 1)^2 + 3$	7
2	$(2 - 1)^2 + 3$	4
1	$(1 - 1)^2 + 3$	3
0	$(0 - 1)^2 + 3$	4
-1	$(-1 - 1)^2 + 3$	7



5. $f(x) = 1/3(x - 1)^2 + 2$

vertex : (1, 2)
Axis of symmetry: x = 1
Direction of opening: Upward

	Table of values	
x	$1/3(x-1)^2 + 2$	f(x)
3	$1/3(3-1)^2 + 2$	3.3
2	$1/3(2-1)^2 + 2$	2.3
1	$1/3(1-1)^2 + 2$	2
0	$1/3(0 -1)^2 + 2$	2.3
-1	$1/3(-1-1)^2 + 2$	3.3



Lesson 2
For each set of functions, tell which graph is narrower or wider.

1. $f(x) = 2x^2$, wider

   $f(x) = 3x^2$, narrower

2. $f(x) = -\frac{1}{2}x^2$, wider

   $f(x) = -2x^2$, narrower

3. $f(x) = 4x^2$, narrower

   $f(x) = \frac{1}{4}x^2$, wider

4. $f(x) = 5x^2$, narrower

   $f(x) = 4x^2$, wider

5. $f(x) = -3x^2$, narrower

   $f(x) = -\frac{1}{3}x^2$, wider

6. $f(x) = -x^2$, wider

   $f(x) = -3x^2$, narrower

7. $f(x) = 2x^2$, wider

   $f(x) = 4x^2$, narrower

8. $f(x) = -5x^2$, narrower

   f(x = $- 2x^2$, wider

9. $f(x) = \frac{2}{3}x^2$, narrower

   $f(x) = \frac{1}{2}x^2$, wider

10. $f(x) = -\frac{1}{3}x^2$, wider

    $f(x) = -\frac{1}{2}x^2$, narrower

Lesson 3:

1. moves to the left
1. moves to the right
2. moves to the right
4 moves to the left
12. moves to the left
13. moves to the right
14. moves to the right
15. moves to the right
16. moves to the left
17. moves to the right

Lesson 4

A.

1. upwards
2. downwards
3. downwards
4. upwards
5. downwards
6. downwards
7. upwards
8. upwards
9. downwards
10. upwards

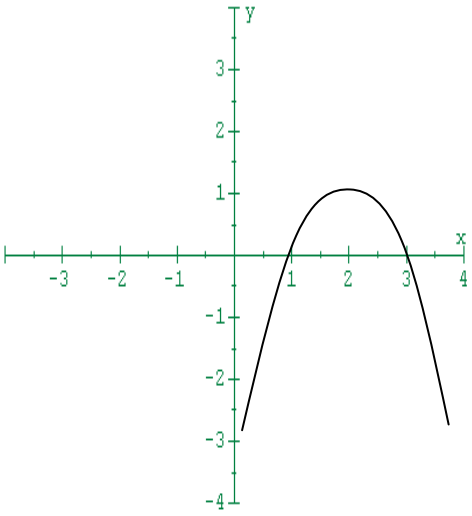
Lesson 5

A.

1. $y = -(x - 2)^2 + 1$

vertex : (2,1)
Axis of symmetry: x + 2
Direction of the graph: Downward

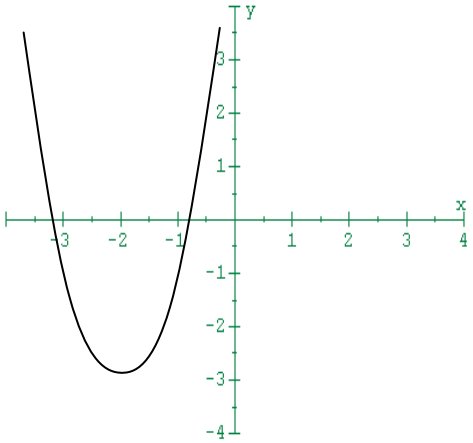
	Table of values	
x	$-(x - 2)^2 + 1$	f(x)
4	$-(4 - 2)^2 + 1$	-3
3	$-(3 - 2)^2 + 1$	0
2	$-(2 - 2)^2 + 1$	1
1	$-(1 - 2)^2 + 1$	0
0	$-(0 - 2)^2 + 1$	-3



2. $y = 2(x + 2)^2 - 3$

vertex: (-2 – 3)
Axis of symmetry: x = -2
Direction of the graph: Upwards

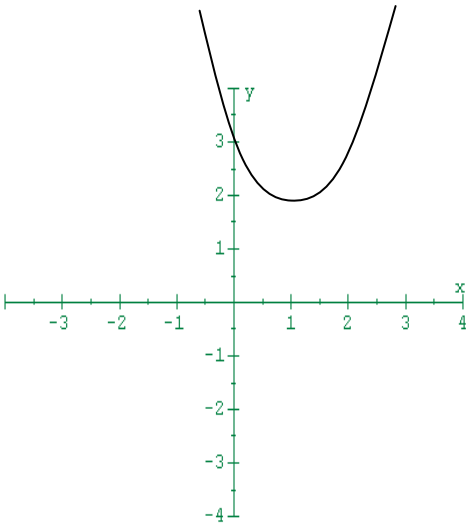
Table of values		
x	$2(x + 2)^2 - 3$	f(x)
0	$2(0 + 2)^2 - 3$	5
-1	$2(-1 + 2)^2 -3$	-1
-2	$2(-2 +2)^2 -3$	-3
-3	$2(-3 + 2)^2 - 3$	-1
-4	$2(-4 + 2)^2 - 3$	5



3. $y = (x – 1)^2 + 2$

Vertex: (1, 2)
Axis of symmetry: x = 1
Direction of the graph: Upwards

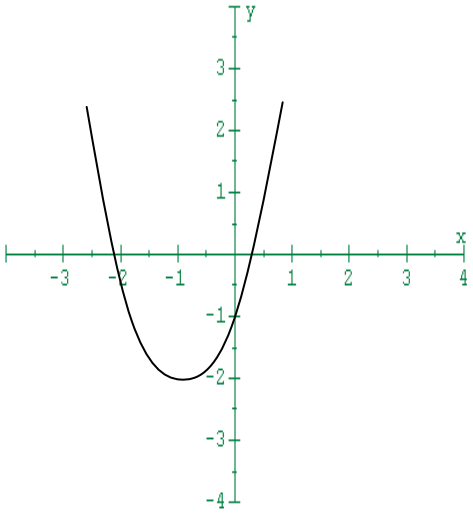
Table of values		
x	$(x – 1)^2 + 2$	x
3	$(3 –1)^2+ 2$	6
2	$(2 – 1)^2 + 2$	3
1	$(1 – 1)^2 + 2$	2
0	$(0 – 1)^2 + 2$	3
-1	$(-1 – 1)^2 + 2$	6



4. $y = (x + 1)^2 – 2$

Vertex: (-1, -2)
Axis of symmetry: x = -1
Direction of the graph: Upwards

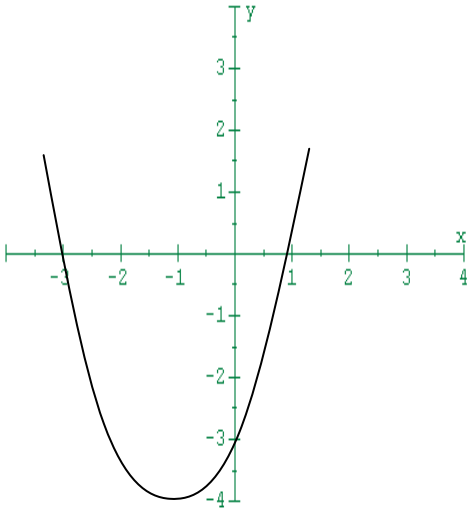
Table of values		
x	$(x + 1)^2 –2$	f(x)
1	$(1 + 1)^2 - 2$	2
0	$(0 + 1)^2 - 2$	-1
-1	$(-1 + 1)^2 - 2$	-2
-2	$(-2 + 1)^2 - 2$	-1
-3	$(-3 + 1)^2 - 2$	2



5. $y = (x + 1) – 4$

Vertex: (-1, -4)
Axis of symmetry: x = -1
Direction of the graph: Upwards

	Table of values	
x	$(x + 1)^2 – 4$	f(x)
1	$(1 + 1)^2 - 4$	0
0	$(0 + 1)^2 - 4$	-3
-1	$(-1 + 1)^2 - 4$	-4
-2	$(-2 + 1)^2 - 4$	-3
-3	$(-3 + 1)^2 - 4$	0



B.

1. $y = x^2 + 5$
2. $y = x^2 -3$
3. $y = 2x^2 + 2$
4. $y = (x - 4)^2$
5. $y = 3(x + 2)^2$
6. $y = 2(x + 3)^2 + 5$
7. $y = 3(x – 3)^2 – 2$
8. $y = (x + 3)^2 – 4$
9. $y = -2(x – 5)^2 + 3$
10. $y = -3(x + 2)^2 + 4$

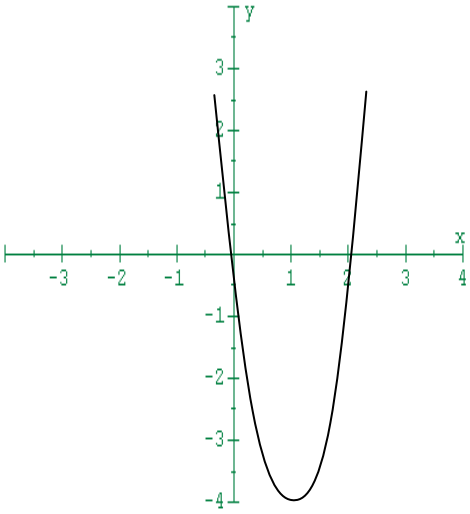
What have you learned

A.

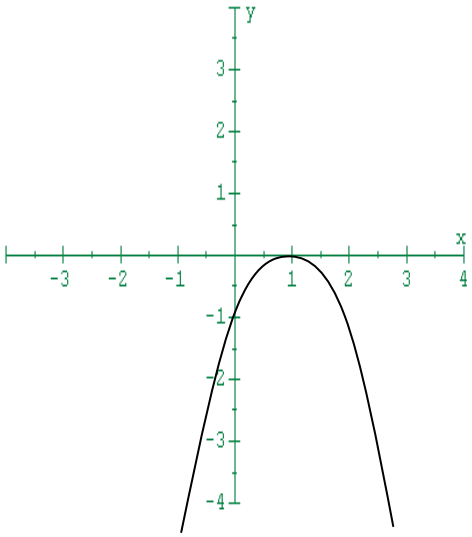
1. Upward
2. Upward
3. Downward

B.

4. $y = 3(x - 1)^2 - 4$



5. $y = -x^2 + 2x - 1$



C.

6. narrower
   wider

7. narrower
   wider

D.

8. to the right
9. to the left
10. to the right
11. to the left

E.

12. downward
13. upward
14. upward
15. downward