Module 1
Quadratic Functions

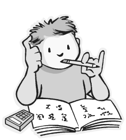
What this module is about

This module is about identifying quadratic functions, rewriting quadratic functions in general form and standard form, and the properties of its graph. As you go over the discussion and exercises, you will understand more about this function, and how to differentiate it from a linear function which you have learned in the previous modules. Enjoy learning and do not hesitate to go back if you think you are at a loss.

What you are expected to learn

This module is designed for you to:

1. identify quadratic functions $f(x) = ax^2 + bx + c$
2. rewrite a quadratic function $f(x) = ax^2 + bx + c$ in the form $f(x) = a(x - h)^2 + k$ and vice versa
3. given a quadratic function, determine
   - highest or lowest point (vertex)
   - axis of symmetry
   - direction or opening of the graph



How much do you know

1. Tell whether the function is quadratic or linear.
   a. $y = 2x + 3$
   b. $y = 3x^2 + 5x - 6$
   c. $f(x) = -2(x + 1)^2 - 5$
   d. $f(x) = 7(4x + 5)$
   e. $f(x) = 9 - 2x$

2. Which of the following table of ordered pairs represents a quadratic function?

a.

x	-2	-1	0	1	2
y	1	-2	-3	-2	1

b.

x	0	1	2	3	4
y	2	5	8	11	14

c.

x	-2	-1	0	1	2
y	8	1	0	-1	-8

d.

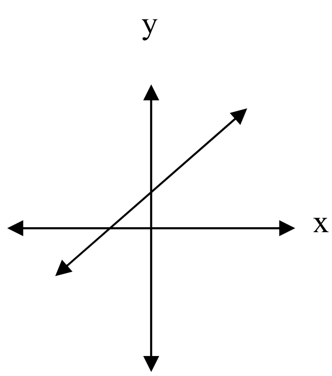
x	-1	0	1	2	3
y	10	7	4	1	-2

3. What is $f(x) = (x + 1)^2 – 3$ in general form?
   a. $f(x) = x^2 + 2x – 4$
   b. $f(x) = x^2 + 2x – 2$
   c. $f(x) = x^2 + 2x + 2$
   d. $f(x) = x^2 + 2x + 4$

4. How is $f(x) = x^2 – 6x + 14$ written in standard form?
   a. $f(x) = (x + 3)^2 + 2$
   b. $f(x) = (x – 3)^2 – 5$
   c. $f(x) = (x – 6)^2 + 5$
   d. $f(x) = (x – 3)^2 + 5$

5. Which of the following is the graph of a quadratic function?

a.



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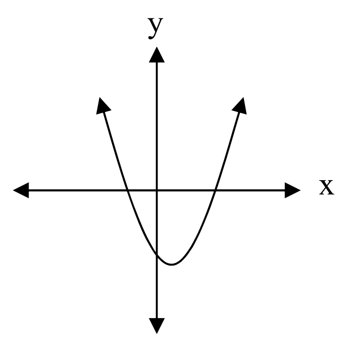

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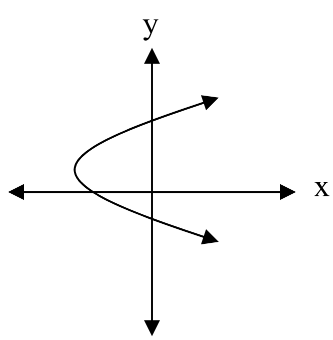

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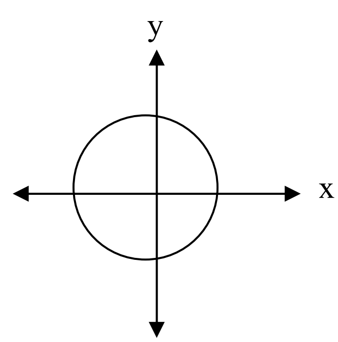


6. What do you call the graph of a quadratic function?
   a. Parabola
   b. Line
   c. Circle
   d. Curve

7. Which of the following quadratic functions will open upward?
    a. $y = 2 – 3x – 5x^2$
    b. $y = -(x + 4)^2$
    c. $y = -(3x^2 + 5x – 1)$
    d. $y = 4x^2 – 12x + 9$

8. Determine the vertex of the quadratic function $f(x) = 3x^2 – 6x + 5$.
    a. (1, 2)
    b. (-1, 2)
    c. (1, -2)
    d. (-1, -2)

9. Which of the following is the axis of symmetry of $y = -2x^2 + 12x – 23$?
    a. x = -3
    b. x = -5
    c. x = 3
    d. x = 5

10. What is the maximum value of $f(x) = -5(x + 1)^2 + 4$?
    a. y = -5
    b. y = 4
    c. y = -1
    d. y = -4

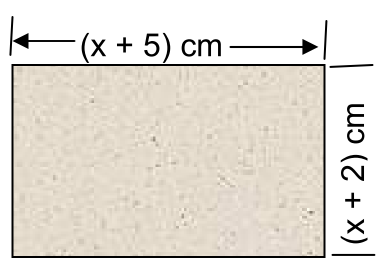
What you will do

Lesson 1

Identifying Quadratic Functions

Consider a rectangle whose width is (x + 2) cm and whose length is (x + 5) cm. How do you find the area of this rectangle?

The situation above is illustrated in the figure that follows.



You recall that the formula for the area of a rectangle is *A = lw* where *l* is the length and *w* is the width. Thus, the area of the given rectangle is –

$$A = [(x + 5)\text{ cm}]\,[(x + 2)\text{ cm}]$$
$$A = (x + 5)(x + 2)\text{ cm}^2$$

Multiplying the binomial then simplifying, the area of the rectangle is -

$$A = (x^2 + 5x + 2x + 10)\text{ cm}^2$$
$$\mathbf{A = (x^2 + 7x + 10)\text{ cm}^2}$$

If x = 3, the area of the rectangle can be obtained by substituting in the equation above.

$$A = 3^2 + 7(3) + 10\text{ cm}^2$$
$$A = 9 + 21 + 10\text{ cm}^2$$
$$A = 40\text{ cm}^2$$

Notice that in the given example, the area of the rectangle is a function of its dimension. Thus, the area of the rectangle can also be written in functional notation as

$$\mathbf{Area = f(x) = (x^2 + 7x + 10)\text{ cm}^2.}$$

Observe that the highest exponent is 2. Hence, the degree of $f(x) = x^2 + 7x + 10$ is 2 which is called a ***quadratic function.***

The following are examples of quadratic functions.

1. $f(x) = x^2 + 7$
2. $f(x) = 6x^2 – 4x + 3$
3. $y = 9 + 2x – x^2$
4. $y = x^2 – 6x - 16$
5. $f(t) = t\left(\frac{t-1}{2}\right)$
6. $y = (x + 7)^2 - 9$

Why do you think the examples above are called quadratic functions?

The following are not quadratic functions.

1. $y = 5x + 8$
2. $f(x) = 5^x + 2$
3. $f(x) = x^3 – 27$
4. $y = \sqrt{x} - 2x^x + 3$
5. $y = x(x^2 + 7x - 1)$

Why do you think the examples above are not quadratic functions?

Now that you know how to identify a quadratic function given an equation, how will you identify a quadratic function from a given set of ordered pairs or a table of values?

Example 1:

Consider the ordered pairs of values for the quadratic function $f(x) = x^2$ for the integers $-3 \le x \le 3$.

$$\{(-3, 9), (-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4), (3, 9)\}$$

The ordered pairs of values above can also be presented using a table of ordered pairs as shown below.
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x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9

Observe the characteristics of a quadratic function. when ordered pairs are given.

1 1 1 1 1 1 Differences in x

x	-3	-2	-1	0	1	2	3
Y = f(x)	9	4	1	0	1	4	9

-5 -3 -1 1 3 5 Differences in y

Notice that the differences in x are equal while the differences in y are not. Let us call the differences in y obtained above as first differences in y.

Look what happens when second differences in y is obtained.

1 1 1 1 1 1 *Differences in x*

x	-3	-2	-1	0	1	2	3
y = f(x)	9	4	1	0	1	4	9

-5 -3 -1 1 3 5 *First differences in y*

2 2 2 2 2 *Second differences in y*

Observe that the second differences in y are equal. Hence, for the quadratic function, $f(x) = x^2$, *equal differences in x produce equal second differences in y.* The method presented above is called the *equal differences method.*

Example 2:

Consider the table of values for the quadratic function $f(x) = 5 – 2x^2$.

x	-3	-1	1	3	5	7	9
y = f(x)	-13	3	3	-13	-45	-93	-157

Solution:

Verify if equal differences in x will produce equal second differences in y.

2 2 2 2 2 2 *Differences in x*

x	-3	-1	1	3	5	7	9
y = f(x)	-13	3	3	-13	-45	-93	-157

16 0 -16 -32 -48 -64 *First differences in y*

-16 -16 -16 -16 -16 *Second differences in y*

Observe that like in $f(x) = x^2$, the table of values for $f(x) = 5 – 2x^2$ showed that equal differences in x produced equal **second differences in y = f(x**). This is true for all quadratic functions.

Example 3:

Determine if the ordered pairs of numbers given in the table below represents a quadratic function of not.

x	-6	-4	-2	0	2	4	6	8
y	-7	-3	1	5	9	13	17	21

Solution:

Apply the same method as seen in examples 1 and 2 and see if equal differences in x will also produce equal second differences in y.

2 2 2 2 2 2 2

x	-6	-4	-2	0	2	4	6	8
y	-7	-3	1	5	9	13	17	21

4 4 4 4 4 4 4

Notice that the differences in x produced equal **first differences in y**. Thus, the ordered pairs of numbers in the given table **does not represent a quadratic function**.

Try this out

A. Tell whether the following functions are quadratic functions or not. Explain.

1. $f(x) = x^2 - 9$
2. $f(x) = 3x + 15$
3. $f(x) = 24 + 5x - x^2$
4. $f(x) = 27 – 4x^2$
5. $f(x) = 2(x-6)^2 + 1$
6. $f(x) = (3x + 2)(x - 5)^2$
7. $f(x) = 5x^2 + x – 2$
8. $f(x) = \frac{4-x}{5}$
9. $f(x) = -\frac{5}{6}x^2$
10. $f(x)=\frac{7}{2}x + \frac{3}{4}$

B. Using the equal differences method, determine which of the following ordered pairs represent a quadratic function. Justify your answer.

1. {(-1, 11), (0, 6), (1, 3), (2, 2), (3, 3), (4, 6), (5, 11)}
2. {(-3, -35), (-2, -16), (-1, -9), (0, 0), (1, -7), (2, 0), (3, 19)
3. {(1, 5), (3, 13), (5, 29), (7, 53), (9, 85)}
4. {(-2, -13), (-1, -6), (0, -5), (1, -4), (2, 3)}
5. {(-5, 40), (-4, 28), (-3, 18), (-2, 10), (-1, 4), (0, 0)}
6.

x	-5	-2	1	4	7
y	64	91	100	91	64

7.

x	-3	-2	-1	0	1
y	56	37	30	29	28

8.

x	-4	-2	0	2	4
y	39	24	9	-6	-21

9.

x	-3	-1	1	3	5
y	25	10	-5	-20	-35

10.

x	-10	-5	0	5	10
y	-20	-85	-100	-85	-20

C. The sum of two numbers is 12.

1. If one number is represented by x, what is the other number?
2. If f(x) represents their product, express f(x) in terms of x.
3. Give a table of values for this relation for the integers $0 \le x \le 12$.
4. Determine whether the table of values represents a quadratic function or not.
5. Determine the product of each pair of numbers.
6. Which pair of numbers in (a) gives the greatest product?
7. Which pair of numbers in (a) gives the least product?

Lesson 2

Rewriting Quadratic Functions from $f(x) = ax^2 + bx + c$ to $f(x) = a(x-h)^2 + k$ and vice versa

In lesson 1, you learned that $f(x) = ax^2 + bx + c$ is the standard form of a quadratic function. This function can be written in an equivalent form using the process of *completing the square.* Study the steps as shown below.

$f(x) = ax^2 + bx + c$	Standard form of a quadratic function
$f(x) = a(x^2 + \frac{b}{a}x) + c$	Factor out a from x^2 and x terms
$f(x) = a\left[x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right] + c - a\left(\frac{b}{2a}\right)^2$	Complete the square by adding and subtracting $a\left(\frac{b}{2a}\right)^2$
$f(x) = a\left[x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right] + c - a\left(\frac{b^2}{4a^2}\right)$	Expand the terms added and subtracted in the previous step
$f(x) = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right)^2 + c - \frac{b^2}{4a}$	Simplify $a\left(\frac{b^2}{4a^2}\right)$ to $\frac{b^2}{4a}$
$f(x) = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$	Factor the trinomial inside the bracket and simplify the last two terms

From the result let $\frac{b}{2a} = -h$ and $\frac{4ac-b^2}{4a} = k$. Substituting this to the equation above will result to $f(x) = a(x - h)^2 + k$. Hence, $f(x) = ax^2 + bx + c$ is equivalent to $f(x) = a(x - h)^2 + k$.

Examples:

Rewrite the following quadratic functions in the form $f(x) = a(x- h)^2 + k$.

1. $f(x) = x^2 - 2x - 15$
2. $f(x) = 2x^2 - 7$
3. $y = 4 + x – 3x^2$
4. $y = 4x^2 + 5x$

Solution 1: Using completing the square.

1. $f(x) = x^2 - 2x – 15$

$f(x) = (x^2 - 2x) – 15$	Factor out 15 in the x terms.
$f(x) = (x^2 - 2x + 1) - 15 – 1$	Complete the square inside the parenthesis by adding and subtracting 1.
$f(x) = (x - 1)^2 – 16$	Factor the trinomial inside the parenthesis and simplify the last 2 terms

2. $f(x) = 2x^2 – 7$

$f(x) = 2x^2 - 0x – 7$	Write function in the form $f(x) = ax^2 + bx + c$.
$f(x) = 2\left(x^2 - \frac{0}{2}x\right) - 7$	Factor out 2 in the x terms.
$f(x) = 2\left(x^2 - \frac{0}{2}x + 0\right) - 7 - 0$	Complete the square by adding and subtracting 0.
$f(x) = 2(x - 0)^2 – 7$	Factor the trinomial inside the parenthesis and combine the last two terms.

3. $y = 4 + x - 3x^2$

$y = -3x^2 + x + 4$	Rewrite the equation in the form $y = ax^2 + bx + c$.
$y = -3\left(x^2 - \frac{1}{3}x\right) + 4$	Factor out -3 in the x terms
$y = -3\left[x^2 - \frac{1}{3}x + \left(-\frac{1}{6}\right)^2\right] + 4 - (-3)\left(-\frac{1}{6}\right)^2$	Complete the square by adding and subtracting $-3\left(-\frac{1}{6}\right)^2$
$y = -3\left(x^2 - \frac{1}{3}x + \frac{1}{36}\right) + 4 - (-3)\left(\frac{1}{36}\right)$	Square the added number.
$y = -3\left(x - \frac{1}{6}\right)^2 + 4 - \left(-\frac{1}{12}\right)$	Factor the trinomial inside the parenthesis and reduce the fraction to the lowest term.
$f(x) = -3\left(x - \frac{1}{6}\right)^2 + \frac{49}{12}$	Add the last two terms

4. $y = 4x^2 + 5x$ — Given

$y = 4\left(x^2 + \frac{5}{4}x\right)$ — Factor out 4 in the x terms

$y = 4\left[x^2 + \frac{5}{4}x + \left(\frac{5}{8}\right)^2\right] - 4\left(\frac{5}{8}\right)^2$ — Complete the square by adding and subtracting $4\left(-\frac{5}{8}\right)^2$

$y = 4\left(x^2 + \frac{5}{4}x + \frac{25}{64}\right) - 4\left(\frac{25}{64}\right)$ — Square the added number

$y = 4\left(x + \frac{5}{8}\right)^2 - \frac{25}{16}$ — Factor the trinomial inside the parenthesis and reduce the fraction to the lowest term

Solution 2:

Using the formula in solving the values of h and k:

In the relation $-h = \frac{b}{2a}$, the value of h can be obtained using the multiplication property of equality so that $h = \frac{-b}{2a}$.

1. $f(x) = x^2 - 2x - 15$.

   Substitute the values a = 1, b = -2, and c = -15 in the formula.

$$h = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1$$

$$k = \frac{4ac - b^2}{4a} = \frac{4(1)(-15) - (-2)^2}{4(1)} = \frac{-60 - 4}{4} = \frac{-64}{4} = -16$$

   Substituting the values of h and k to $f(x) = a(x - h)^2 + k$.

   Thus, $f(x) = x^2 - 2x - 15$ is equivalent to $f(x) = (x - 1)^2 - 16$.

2. $f(x) = 2x^2 - 7$.

   Substitute the values a = 2, b = 0 and c = -7 in the formula.

$$h = \frac{-b}{2a} = \frac{-0}{2(2)} = \frac{0}{4} = 0$$

$$k = \frac{4ac - b^2}{4a} = \frac{4(2)(-7) - 0^2}{4(2)} = \frac{-56 - 0}{8} = \frac{-56}{8} = -7$$

Substitute the values of h and k to $f(x) = a(x - h)^2 + k$.

Therefore, $f(x) = 2x^2 - 7$ is equivalent to $f(x) = 2(x - 0)^2 - 7$.

3. $y = 4 + x - 3x^2$. Here, a = -3, b= 1, c = 4

$$h = \frac{-b}{2a} = \frac{-(1)}{2(-3)} = \frac{-1}{-6} = \frac{1}{6}$$

$$k = \frac{4(-3)(4)-(1)^2}{4(-3)} = \frac{-48-1}{-12} = \frac{-49}{-12} = \frac{49}{12}$$

Thus, $y = -3\left(x - \frac{1}{6}\right)^2 + \frac{49}{12}$.

4. $y = 4x^2 + 5x$. Here a = 4, b = 5 and c = 0.

$$h = \frac{-b}{2a} = \frac{-5}{2(4)} = \frac{-5}{8}$$

$$k = \frac{4(4)(0) - (5)^2}{4(4)} = \frac{0 - 25}{16} = \frac{-25}{16}$$

Hence, $y = 4\left[x - \left(-\frac{5}{8}\right)\right]^2 + \left(-\frac{25}{16}\right)$ or $y = 4\left(x + \frac{5}{8}\right)^2 - \frac{25}{16}$

Observe that the two solutions resulted to the same answer. Thus, a quadratic function in the form $f(x) = ax^2 + bx + c$ can be transformed in the form $f(x) = a(x - h)^2 + k$ by completing the square or the relation $h = \frac{-b}{2a}$ and $k = \frac{4ac - b^2}{4a}$.

Now, how will you transform a quadratic function in the form $f(x) = a(x - h)^2 + k$ to the standard form $f(x) = ax^2 + bx + c$?

To do this, simply follow the given steps.

1. Expand the square of the binomial indicated in the function.
2. Multiply the result by the value of *a*.
3. Combine the similar terms.

Now, study the examples below.

Examples:

Transform the following equation to standard form.

1. $f(x) = (x – 3)^2 – 7$
2. $f(x) = -2[x – (-5)]^2 + 50$
3. $y = 5(x + 4)^2 – 3$
4. $y = -\frac{3}{4}(x-2)^2 + 9$

Solutions:

1. $f(x) = (x – 3)^2 + 7$
   $f(x) = x^2 – 6x + 9 + 7$ — Square the binomial
   $f(x) = x^2 – 6x + 16$ — Combine the similar terms

2. $f(x) = -2[x – (-5)]^2 + 50$
   $f(x) = -2(x + 5)^2 + 50$ — Simplify the term inside the parenthesis
   $f(x) = -2(x^2 + 10x + 25) + 50$ — Square the binomial
   $f(x) = -2x^2 - 20x -50 + 50$ — Multiply the result by -2
   $f(x) = -x^2 - 20x$ — Combine the similar terms

3. $y = 5(x + 4)^2 – 3$
   $y = 5(x^2 + 8x + 16) – 3$ — Square the binomial
   $y = 5x^2 + 40x + 80 – 3$ — Multiply the result by 5
   $y = 5x^2 + 40x + 77$ — Combine the similar terms

4. $y = -\frac{3}{4}(x-2)^2 + 9$
   $y = -\frac{3}{4}(x^2 - 4x + 4) + 9$ — Square the binomial
   $y = -\frac{3}{4}x^2 + 3x - 3 + 9$ — Multiply the result by $-\frac{3}{4}$
   $y = -\frac{3}{4}x^2 + 3x + 6$ — Combine the similar terms

Try this out

A. Rewrite the following quadratic functions to $f(x) = a(x – h)^2 + k$.

1. $f(x) = 2x^2 - 12x + 33$
2. $f(x) = x^2 + 8x$
3. $f(x) = 5x^2 - 6$
4. $f(x) = 3x^2 + 24x + 43$
5. $f(x) = 0.5x^2 + 11$
6. $f(x) = 1 + 16x - 8x^2$

7. $f(x) = 12 - x^2$

8. $f(x) = \frac{3}{2}x^2 + 15$

9. $f(x) = \frac{5}{3} - 7x^2$

10. $f(x) = \frac{4x^2 - 5x + 7}{2}$

B. Transform the following quadratic function to $f(x) = ax^2 + bx + c$.

1. $(x) = 3(x - 2)^2 + 5$
2. $f(x) = -7(x + 1)^2 - 3$
3. $f(x) = 5(x - 9)^2 + 1$
4. $f(x) = -5(x - 3)^2 + 2$
5. $f(x) = (x + 5)^2 - 12$
6. $f(x) = -4(x - 1)^2 + 14$
7. $f(x) = 4(x - \frac{1}{2})^2 - 2$
8. $f(x) = \frac{2}{3}(x - 5)^2$
9. $f(x) = -\frac{4}{7}(x - 1)^2 + 3$
10. $f(x) = \frac{6}{7}(x - 7)^2 - 1$

Lesson 3

Properties of the Graph of a Quadratic Function

The graph of a quadratic function is called a **parabola**. It is the set of all points on the Cartesian Coordinate Plane that satisfies the function defined by $f(x) = ax^2 + bx + c$ or the vertex form $f(x) = a(x - h)^2 + k$ where (h, k) is the vertex.
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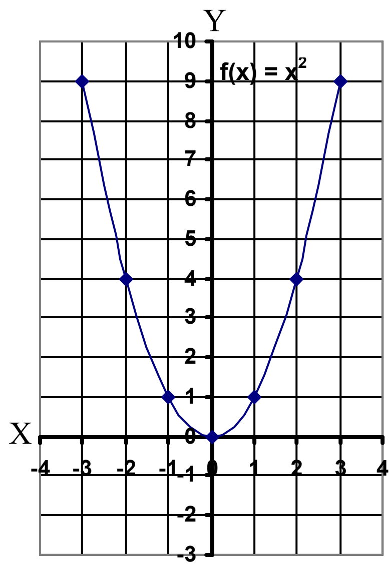
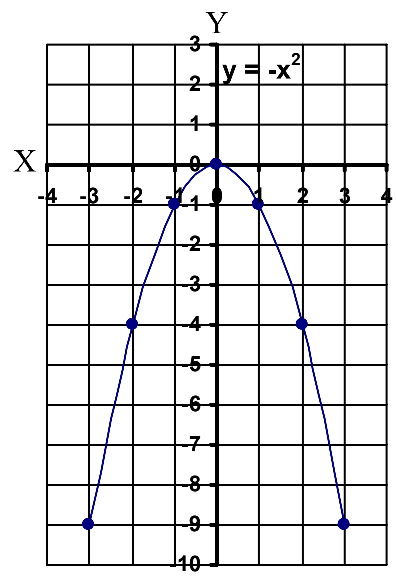
Look at the two graphs. What do you notice about the parabolas? In what way are they similar? In what way are they different?





The following properties of the parabolas should be observed.

1. The graph of $y = x^2$ opens upward while the graph of $y = -x^2$ opens downward.

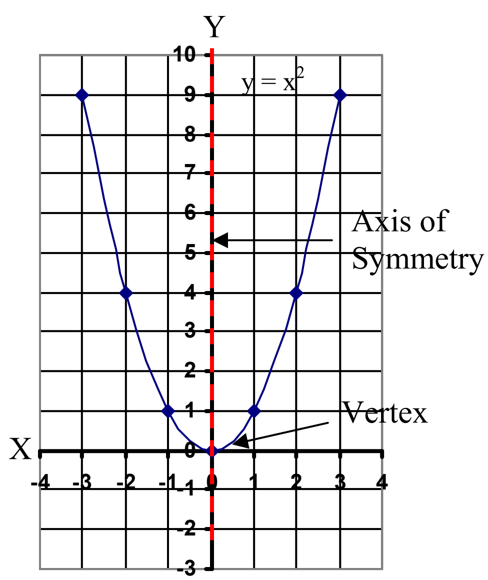
   The direction of opening is indicated by the sign of *a* in the equation. Note that in $y = x^2$, *a* is positive or $a > 0$ while in $y = -x^2$, *a* is negative or $a < 0$.

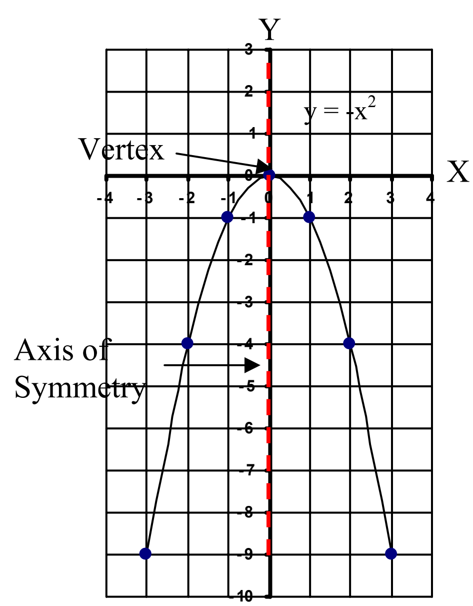
2. The two graphs have turning points. The turning point is called a **vertex**. The vertex maybe the *minimum point* or the *maximum point* of the parabola depending on the direction of opening of the graph.

   The vertex of $y = x^2$ is at **(0, 0)** also denoted by V(0, 0). It is the lowest or minimum point on the graph. It is the minimum point if the parabola opens upward

   The vertex of $y = -x^2$ is also at **V(0,0)** but it is the highest or maximum point on the graph. It is the maximum point if the parabola opens downward.

3. Drawing a vertical line through the vertices of each graph divide both graphs into two congruent or symmetrical parts such that one part is a mirror image of the other. We call this line **axis of symmetry.** Thus, the axis of symmetry of both graphs is the y-axis or the line x = 0.





Example 2:

Draw the graph of $f(x) = 2(x - 1)^2 - 3$ and give the properties the function.

Solution:

Step 1: Construct a table of values for x and f(x). For this particular example, let us use for x the values { -1, 0, 1, 2, 3}.

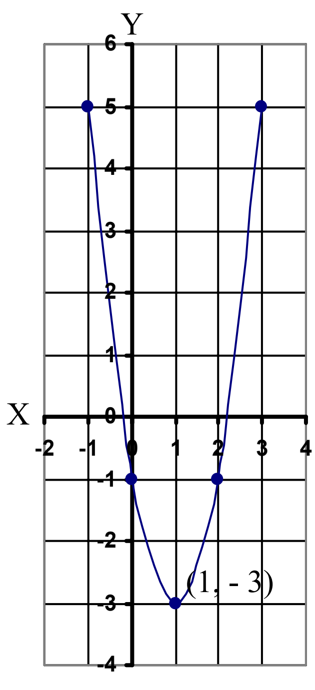
Substitute these values in $f(x) = 2(x - 1)^2 - 3$.

$f(-1) = 2(-1 - 1)^2 - 3 = 2(-2)^2 - 3 = 2(4) - 7 = 8 - 3 = 5$
$f(0) = 2(0 - 1)^2 - 3 = 2(-1)^2 - 3 = 2(1) - 3 = 2 - 3 = -1$
$f(1) = 2(1 - 1)^2 - 3 = 2(0)^2 - 3 = 2(0) - 3 = 0 - 3 = -3$
$f(2) = 2(2 - 1)^2 - 3 = 2(1)^2 - 3 = 2(1) - 3 = 2 - 3 = -1$
$f(3) = 2(3 - 1)^2 - 3 = 2(2)^2 - 3 = 2(4) - 3 = 8 - 3 = 5$

The results in a table.

x	-1	0	1	2	3
$f(x) = 2(x - 1)^2 - 3$	5	-1	-3	-1	5

Step 2: Plot and connect the points on the Cartesian Plane.



The following properties of the quadratic function can be observed from the parabola.

The functions is $f(x) = 2(x - 1)^2 - 3$ in the form $f(x) = a(x - h)^2 + k$

Here, a = 2, h = 1 and k = -3.

1. The parabola opens upward because a > 0.
2. The vertex (h, k) is (1, -3), the lowest point of the graph.
3. The axis of symmetry is x = 1, the value of h in the vertex.
4. The minimum value is y = -3 which is the value of k in the vertex.

Hence, for any quadratic function, its vertex is at the point V(h, k) where $h = \frac{-b}{2a}$ and $k = \frac{4ac - b^2}{4a}$. You have learned this in the previous lesson.

Example 3*:*

Determine the direction of opening, vertex, axis of symmetry, and minimum or maximum point of the quadratic function defined by $y = -2x^2 + 12x - 5$.

Solution: In the given equation, $a = -2$, $b = 12$, and $c = -5$.

a. Opening: downward since *a* is negative or *a* < 0.
b. Vertex:

Since the function is not in the vertex form $f(x) = a(x - h)^2 + k$, we cannot easily determine the vertex (h, k).

Use the formula for h and k:

$$h = \frac{-b}{2a} = \frac{-12}{2(-2)} = \frac{-12}{-4} = 3$$

$$k = \frac{4ac-b^2}{4a} = \frac{4(-2)(-5)-12^2}{4(-2)} = \frac{40-144}{-8} = \frac{-104}{-8} = 13$$

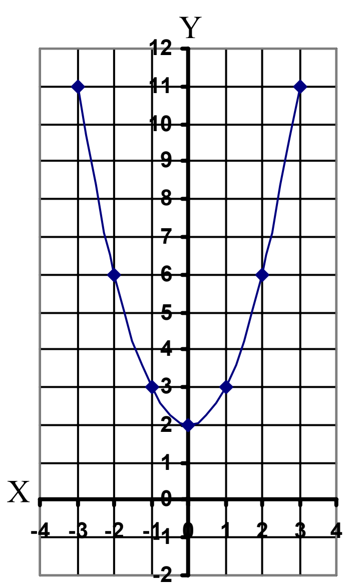
Hence, the vertex is (3, 13);

c. The axis of symmetry is x = 2
d. The maximum value is y = 13 since the parabola opens downward.

Try this out

A. Give the sign of the leading coefficient, the coordinates of the vertex, the axis of symmetry, and the highest/lowest value of the quadratic function represented by the given parabolas.

1.



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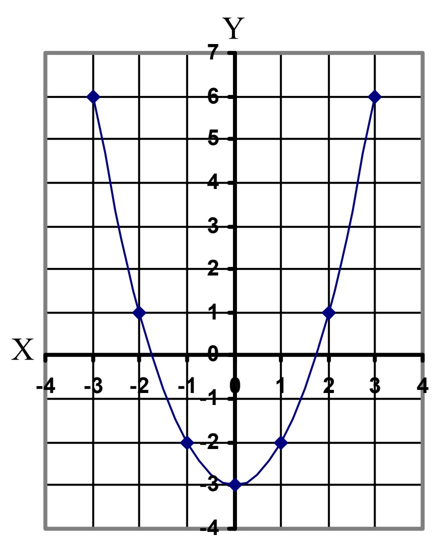

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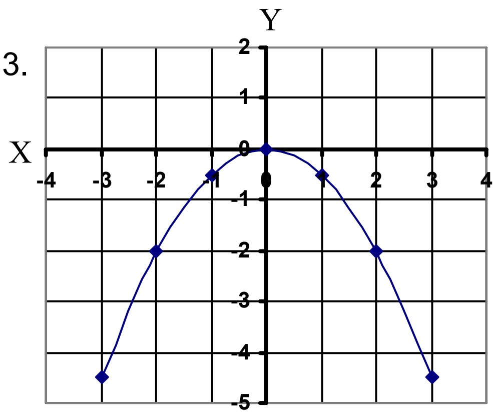

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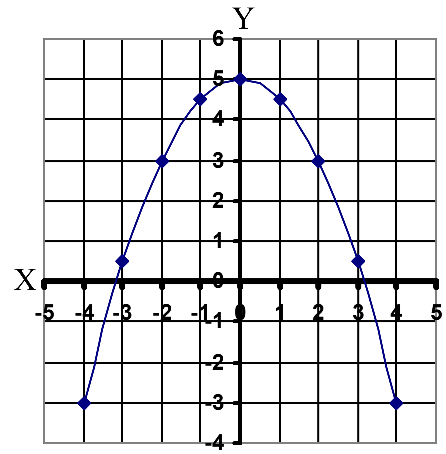

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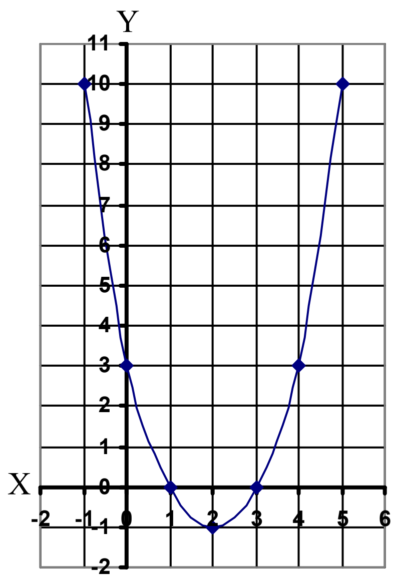

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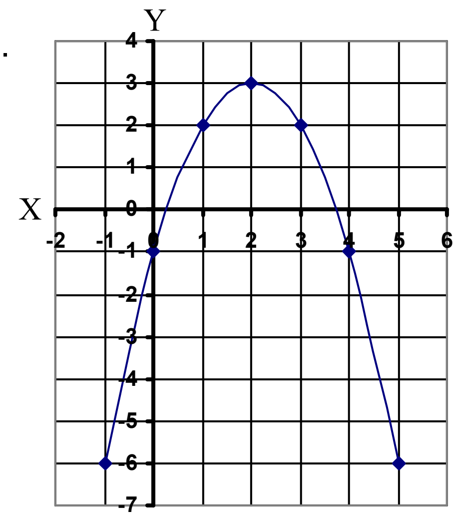

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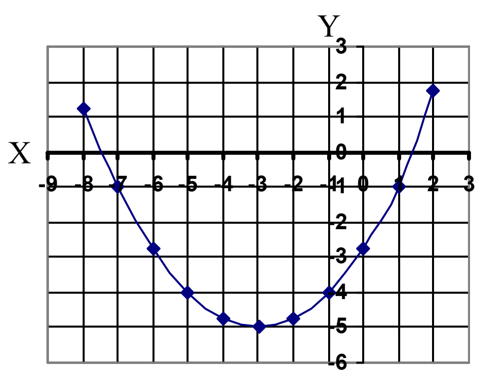

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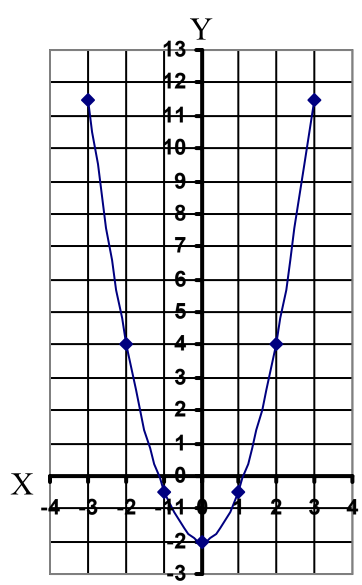

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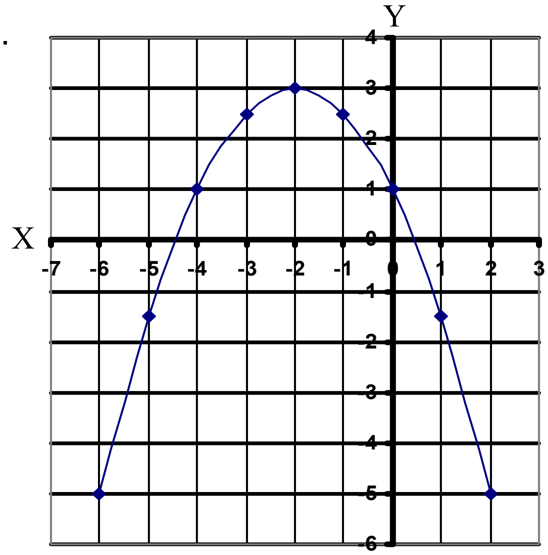

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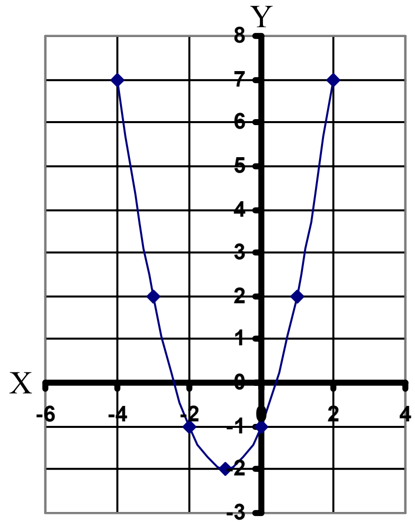


B. Determine the direction of opening of the parabola, the vertex, the axis of symmetry and minimum or maximum value of the following quadratic functions.

1. $f(x) = (x - 3)^2$
2. $f(x) = -(x + 2)^2 - 1$
3. $f(x) = - x^2 + 9$
4. $f(x) = \frac{3}{2}(x - 4)^2 + 7$
5. $f(x) = -4(x - 6)^2 + 5$
6. $f(x) = x^2 + 3x – 4$
7. $f(x) = 2x^2 + 4x - 7$
8. $f(x) = x^2 - x - 20$
9. $f(x) = 5 + 3x – x^2$
10. $f(x) = 2 - 5x - 3x^2$

Let's Summarize

1. A quadratic function is a second degree function in the form $f(x) = ax^2 + bx + c$ where $a \neq 0$.

2. Quadratic functions can be written in two forms- the standard form $f(x) = ax^2 + bx + c$ or its equivalent form $f(x) = a(x – h)^2 + k$.

3. To rewrite a quadratic function from the form $f(x) = ax^2 + bx + c$ to the form $f(x) = a(x – h)^2 + k$, use completing the square; or determine the values of a, b, and c then solve for h and k. Substitute the obtained values in $f(x) = a(x – h)^2 + k$. To find the values of h and k, use the relationships,

$$h = \frac{-b}{2a} \text{ and } k = \frac{4ac - b^2}{4a}$$

4. To rewrite a quadratic function from the form $f(x) = a(x-h)^2 + k$ to the form $f(x) = ax^2 + bx + c$, expand the square of the binomial, multiply by a and add k, then simplify by combining similar terms.

5. The graph of a quadratic function is called a parabola.

6. A parabola may open upward or downward depending upon the sign of a. If $a>0$, the parabola opens upward while if $a<0$, the parabola opens downward.

7. The highest or lowest point of a parabola is called the turning point or vertex. It is denoted by the ordered pair V(h, k) where

$$h = \frac{-b}{2a} \text{ and } k = \frac{4ac - b^2}{4a}$$

   $x = h$ is called the axis of symmetry while $y = k$ is the highest or lowest value of f(x).

8. The axis of symmetry is the line that passes through the vertex and divides the parabola into two equal parts such that one part is the mirror image of the other.

What have you learned

1. Which of the following functions is quadratic?

   a. $y = x + 3$
   b. $y = 5 – 6x^2$
   c. $f(x) = 2^{x+1} - 5$
   d. $f(x) = 7(4x + 5)$

2. Which of the following table of ordered pairs represents a quadratic function?

   a.

x	-2	-1	0	1	2
y	-8	-4	0	4	8

   b.

x	0	1	2	3	4
y	-2	1	4	7	10

   c.

x	-2	-1	0	1	2
y	9	2	1	0	-7

   d.

x	-1	0	1	2	3
y	13	7	5	7	13

3. What is $f(x) = 2(x - 1)^2 + 8$ in general form?

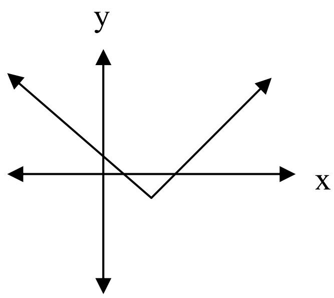
   a. $f(x) = 2x^2 - 2x + 9$
   b. $f(x) = 2x^2 - x + 10$
   c. $f(x) = 2x^2 + 4x + 8$
   d. $f(x) = 2x^2 - 4x + 10$

4. How is $f(x) = x^2 - 10x + 29$ written in standard form?

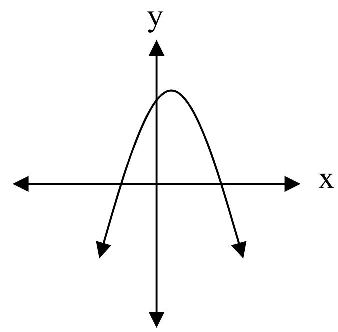
a. $f(x) = (x - 5)^2 + 4$
b. $f(x) = (x + 5)^2 + 4$
c. $f(x) = (x - 5)^2 - 4$
d. $f(x) = (x + 5)^2 - 4$

5. Which of the following is the graph of a quadratic function?

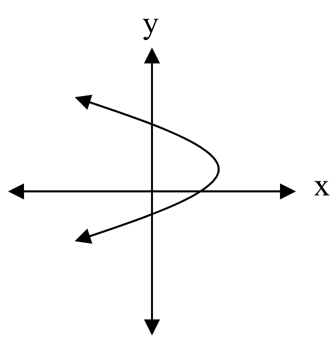
a.



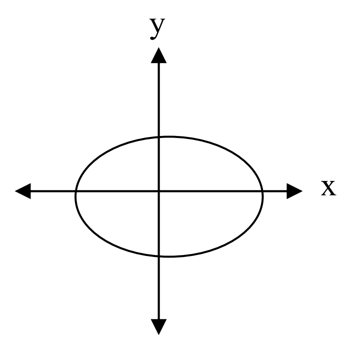
c.



b.



d.



6. What do you call the highest or lowest point in the graph of a quadratic function?

a. Axis of symmetry
b. Slope
c. Vertex
d. Major axis

7. Which of the following quadratic functions will open downward?

a. $f(x) = 2x^2 - 3x - 5$
b. $f(x) = (x - 4)^2$
c. $f(x) = 3 + 5x + 2x^2$
d. $f(x) = 4 - 2x - 9x^2$

8. Determine the vertex of the quadratic function $f(x) = 2x^2 - 12x + 11$.

a. (3, 7)
b. (-3, -7)
c. (3, -7)
d. (-3, -7)

9. Which of the following is the axis of symmetry of $y = -3x^2 - 30x - 75$?

a. $x = 5$
b. $x = -5$
c. $x = 3$
d. $x = -3$

10. What is the minimum value of $f(x) = 2(x + 1)^2 - 7$?

a. $y = -7$
b. $y = 7$
c. $y = 1$
d. $y = 2$

Answer Key

How much do you know

1. a. Linear Function
   b. Quadratic Function
   c. Quadratic Function
   d. Linear Function
   e. Linear Function
2. a
3. b
4. d
5. c
6. a
7. d
8. a
9. c
10. b

Lesson 1

A. 1. Quadratic function since the degree of the function is 2.
   2. Not a quadratic function since the degree of the function is 1,
   3. Quadratic function since the degree of the function is 2.
   4. Quadratic function since the degree of the function is 2.
   5. Quadratic function since the degree of the function is 2.
   6. Not a quadratic function since the degree of the function is 3.
   7. Quadratic function since the degree of the function is 2.
   8. Not a quadratic function since the degree of the function is 1,
   9. Quadratic function since the degree of the function is 2.
   10. Not a quadratic function since the degree of the function is 1,

B. 1. Quadratic function; equal differences in x produced equal second differences in y.
   2. Not a quadratic function; equal differences in x did not produce equal second differences in y.
   3. Quadratic function; equal differences in x produced equal second differences in y.
   4. Not a quadratic function; equal differences in x did not produce equal second differences in y.
   5. Quadratic function; equal differences in x produced equal second differences in y.
   6. Quadratic function; equal differences in x produced equal second differences in y.
   7. Not a quadratic function; equal differences in x did not produce equal second differences in y.
   8. Not a quadratic function; equal differences in x did not produce equal second differences in y.
   9. Not a quadratic function; equal differences in x did not produce equal second differences in y.

10. Quadratic function; equal differences in x produced equal second differences in y.

C.

1. $12 - x$
2. $f(x) = x(12 - x)$ or $f(x) = 12x - x^2$
3.

x	0	1	2	3	4	5	6	7	8	9	10	11	12
y	12	11	10	9	8	7	6	5	4	3	2	1	0

4. No, since equal differences in x did not produce equal second differences in y.
5.

x	0	1	2	3	4	5	6	7	8	9	10	11	12
y	12	11	10	9	8	7	6	5	4	3	2	1	0
product	0	11	20	27	32	35	36	35	32	27	20	11	0

6. (6, 6)
7. (0, 12) and (12, 0)

Lesson 2

A.

1. $f(x) = 2(x - 3)^2 + 5$
2. $f(x) = (x + 4)^2 - 16$
3. $f(x) = 5(x - 0)^2 - 6$ or $f(x) = 5(x + 0)^2 - 6$
4. $f(x) = 3(x + 4)^2 - 5$
5. $f(x) = 0.5(x - 0)^2 + 11$ of $f(x) = 0.5(x + 0)^2 + 11$
6. $f(x) = -8(x - 1)^2 + 9$
7. $f(x) = -(x - 0)^2 + 12$ or $f(x) = -(x + 0)^2 + 12$
8. $f(x) = \frac{3}{2}(x - 0)^2 + 15$ or $f(x) = \frac{3}{2}(x + 0)^2 + 15$
9. $f(x) = -7(x - 0)^2 + \frac{5}{3}$ or $f(x) = -7(x + 0)^2 + \frac{5}{3}$
10. $f(x) = 2\left(x - \frac{5}{8}\right)^2 - \frac{9}{16}$

B.

1. $f(x) = 3x^2 - 12x + 17$
2. $f(x) = -7x^2 - 14x - 10$
3. $f(x) = 5x^2 - 90x + 406$
4. $f(x) = -5x^2 + 30x - 43$
5. $f(x) = x^2 + 10x + 13$
6. $f(x) = 1 + 16x - 8x^2$ or $f(x) = -8x^2 + 16x + 1$
7. $f(x) = 4x^2 - 4x - 1$
8. $f(x) = \frac{2}{3}(x^2 - 10x + 25)$ or $f(x) = \frac{2}{3}x^2 - \frac{20}{3}x + \frac{50}{3}$
9. $f(x) = -\frac{4}{7}x^2 + \frac{8}{7}x - \frac{11}{7}$
10. $f(x) = \frac{6}{7}x^2 - 12x + 41$

Lesson 3

A.

No.	Sign of leading coefficient	Vertex	Axis of Symmetry	Minimum/Maximum value
1	Positive	(0, 2)	x = 0 or y-axis	Minimum value; y = 2
2	Positive	(0, 3)	x = 0 or y - axis	Minimum value; y = 3
3	Negative	(0, 0)	x = 0 or y-axis	Maximum value; y = 0
4	Negative	(0, 5)	x = 0 or y-axis	Maximum value; y = 5
5	Positive	(2, 1)	x = 2	Minimum value; y = 1
6	Negative	(2, 3)	x = 2	Maximum value; y = 3
7	Positive	(-3, -5)	x = -3	Minimum value; y = -5
8	Positive	(0, -2)	x = 0 or y-axis	Minimum value; y = -2
9	Negative	(-2, 3)	x = -2	Maximum value, y = 3
10.	Positive	(-1, -2)	x = -1	Minimum value, y = -2

B.

No.	Direction of Opening	Vertex	Axis of Symmetry	Minimum/Maximum Value
1	Upward	(3, 0)	x = 3	Minimum value, y = 0
2	Downward	(-2, -1)	x = - 2	Maximum value, y = -1
3	Downward	(0, 9)	x = 0 or y-axis	Maximum value, y = 9
4	Upward	(4, 7)	x= 4	Minimum value, y = 7
5	Downward	(6, 5)	x = 6	Maximum value, y = 5
6	Upward	$\left(\frac{3}{2}, -\frac{25}{4}\right)$	x = $\frac{3}{2}$	Minimum value, y = $-\frac{25}{4}$
7	Upward	(-1, -9)	x = -1	Minimum value, y = -9
8	Upward	$\left(\frac{1}{2}, -\frac{81}{4}\right)$	x = $\frac{1}{2}$	Minimum value, y = $-\frac{81}{4}$
9	Downward	$\left(\frac{3}{2}, \frac{29}{4}\right)$	x = $\frac{3}{2}$	Maximum value, y = $\frac{29}{4}$
10	Downward	$\left(-\frac{5}{6}, \frac{49}{12}\right)$	x = $-\frac{5}{6}$	Maximum value, y = $\frac{49}{12}$

What have you learned

1. b
2. d
3. d
4. a
5. c
6. c
7. c
8. d
9. b
10. a