Module 1
Linear Functions

What this module is about

This module is about a special type of relation called Linear Functions. In your study of functions, you learned about relations of quantities in real life situations. This time you will concentrate your study on first degree functions, how it is represented through equations and graphs.

What you are expected to learn

This module is designed for you to:

1. define the linear function f(x) = mx + b.
2. rewrite the standard form Ax + By = C to the slope – intercept form f(x) = mx + b and vice versa.
3. draw the graph of a linear function given the following :
   - any two points
   - x and y – intercept
   - slope and one point
   - slope and y- intercept

How much do you know

1. Which of the following functions is linear?

   a. f(x) = 3x –7
   b. f(x) = x(2 – x)
   c. f(x) = $\frac{2}{3}$ (6 – 9x)
   d. 5y = $5x^2$ – 5
   e. 8x – 2y = 4

2. The graph of 2x + 3 is a _____.

3. In the equation y = mx + b, *m* represents the _____ of the line.

4. In the equation y = 3x + 2, 2 represents the _____ of the line.

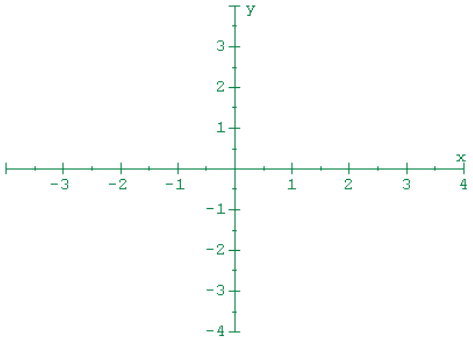
5. Write 9x – 3y + 6 = 0 in the form y = mx + b.

6. The equation y = 3x – 2 if change to Ax + By = C is equal to

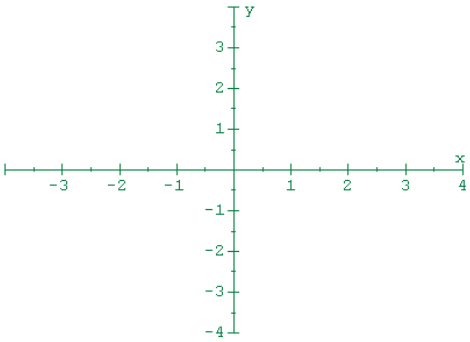
a. 3x – y = 2 b. 3x + y = 2 c. 3x + y = -2 d. 3x - y = - 2

Draw the graph of a linear function given the following:

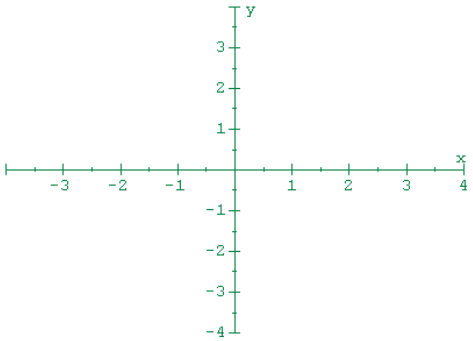
7. Two points: P (0, 0), Q (4, 4)



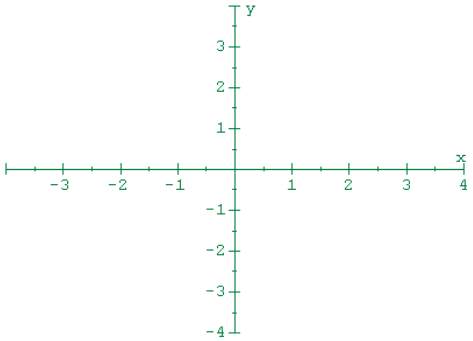
8. Slope (m) = 3; point A (2, 1)



9. x – intercept = 3; y- intercept = -1



10. Slope (m) = 2, y – intercept = -2



What you will do

Lesson 1

Define Linear function f(x) = mx + b

A function is a linear function if and only if its equation can be written in the form $y = mx + b$ or $f(x) = mx + b$, where m is the slope and b is the y-intercept of the line. The graph is a non-vertical straight line.

Examples:

1. Is y + 2x = 2 a linear function?

Solve for y:

y + 2x = 2	The equation is in the form ax + by = c.
y + 2x – 2x = -2x + 2	Add – 2x to both sides of the equation.
y = - 2x + 2	The equation is now in the form y = mx + b
or f(x) = -2x + 2	or f(x) = mx + b.

Since the equation can be written in the form y = mx + b or f(x) = mx + b, where m = -2 and b = 2. Then, y + 2x = 2 is a linear function.

2. Is $x^2 – y + 1 = 0$ a linear function?

Solve for y:

$x^2 – y + 1 = 0$

$-y = - x^2 – 1$

$y = x^2 + 1$

Since you cannot write the equation in the form y = mx + b. Then, $x^2 – y + 1 = 0$ is not a linear function.

3. Is y(x + 6) = x(y + 3) a linear function?

Solve for y:

y(x + 6) = x(y + 3)	Apply distributive property of multiplication.
xy + 6y = xy + 3x	By addition property, add – xy to both sides
6y = 3x	of the equations
$y = \frac{1}{2}x$	

After simplifying, the equation is linear since it can be written in the form y = mx + b, where m = $\frac{1}{2}$ and b = 0.

Try this out

A. Which of the following equation is a linear equation?

1. y = 2x
2. $y = -\frac{1}{5}x + 4$
3. $f(x) = -\frac{5}{2}x$
4. x = -7
5. xy = 10
6. 7y = 0
7. $f(x) = \frac{x}{3} + 6$
8. $f(x) = \frac{2}{7}x + 3$
9. $y = \frac{1+x}{3}$
10. $f(x) = \frac{-2}{3}$

B. Determine whether the relation is linear.

1. x(y + 3) = 0
2. 2y + 2x = 6

3. $5x - 2y + 6 = 0$
4. $2(x-y) = 3(x + y)$
5. $2(x + y) = 5 (y + 1)$

Lesson 2

Given a Linear Function Ax +By = C, Rewrite in the Form f(x) = mx + b, a Slope-Intercept Form and Vice Versa.

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The standard form of linear equation Ax + By = C can be transformed to a linear function f(x) = mx +b called slope - intercept form, where m is the slope and b is the y-intercept and vice versa.

Example 1: Write 3y = -2x – 6 in the form f(x)= mx +b. Give the value of m and b

a. $3y = -2x - 6$

$\frac{3}{3} = -\frac{2}{3}x - \frac{6}{3}$

$y = -\frac{2}{3}x - 2$

or $f(x) = -\frac{2}{3}x - 2$

$m = -\frac{2}{3}, b = -2$

b. $2x - 5y = 10$

$-5y = -2x + 10$

$\frac{-5}{-5}y = \frac{-2}{-5}x + \frac{10}{-5}$

$y = \frac{2}{5}x - 2$

or $f(x) = \frac{2}{5}x - 2$

$m = \frac{2}{5}, b = -2$

Example 2: Write $y = \frac{2}{5}x - 2$ in the form Ax + By = C.

$y = \frac{2}{5}x - 2$

$5[y = \frac{2}{5}x - 2]$ multiply by the LCD

$5y = 2x - 10$

$-2x + 5y = -10$

$-1[-2x + 5y = -10]$ multiply by -1.

$2x - 5y = 10$

Try this out

A. Write the following in the form y = mx + b.

1. 4y + 12x = 20
2. 2 x + 9y – 8 = 10
3. 4y + x + 2 = 0
4. 3x – y = 0
5. 8x – 2y = 6
6. 3x – 4y = 8
7. y + 5x – 3 = 0
8. 2y – 6x + 10 = 0
9. 5x + y = 3
10. –y + 2x – 7 = 0

B. Write the following linear equation in the form Ax + By = C.

1. y = -2x + 3
2. y = 3x – 1
3. y = $\frac{1}{2}$x + 3
4. y = 2x – 2
5. y = 2x + $\frac{1}{2}$
6. y = $\frac{9}{2}$x + $\frac{1}{4}$
7. y = -$\frac{7}{2}$x – 1
8. y = -3x + 5
9. y = $\frac{1}{4}$x – 2
10. y = -$\frac{3}{2}$x – 3

Lesson 3

Draw the Graph of a Linear Function Given Two Points

The graph of a linear function is a straight line. In Geometry, you learned how to graph by connecting points. In this section you will learn how to graph linear functions and determine its slope using the following conditions:

A. Given any two points

Two points determine a straight line, this is a statement in geometry where you can apply to graph linear functions.

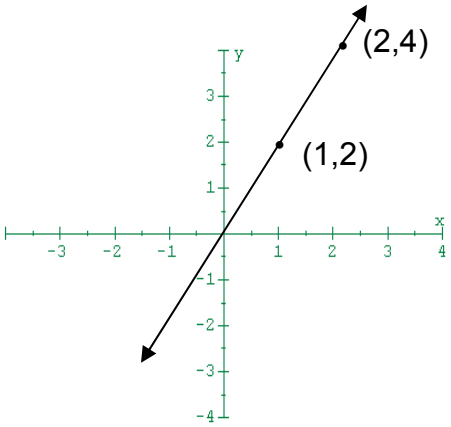
Example:

Draw the graph of a linear function passing through points (1, 2) and (2, 4).

a. First locate the two points

b. Then connect the two points.

The graph of the linear function will look like the figure below.



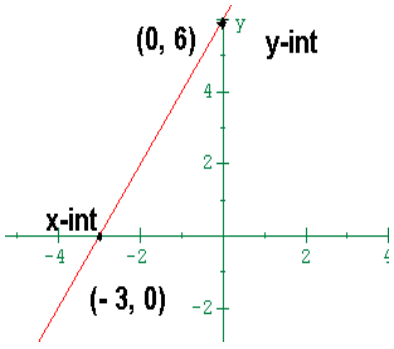
B. Given the x and y – intercepts:

Another way of graphing a linear function is through the points where the graph crosses the x and y axes. This condition also uses two points.

The point at which a line crosses the y-axis has an x coordinate of 0 called *y-intercept*. While, the point at which the line crosses the x-axis has y-coordinate of 0 called *x-intercept*.

Example:

1. x - intercept = -3 ; y - intercept = 6



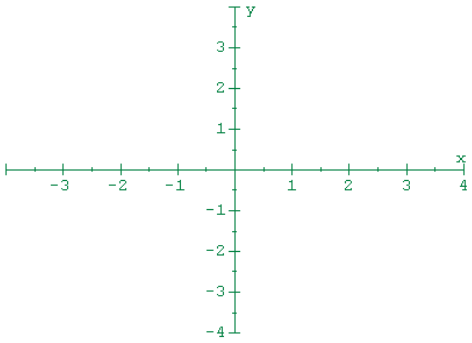
The y – intercept is the y value at point (0, 6). Here the y-intercept is 6

The x – intercept is the x value at point (-3,0). Here the x intercept is -3.

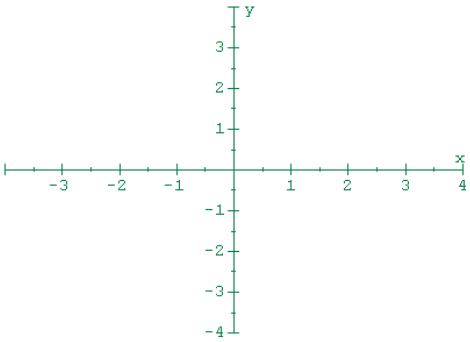
Try this out

A. Draw the graph of the linear function that passes through the given points.

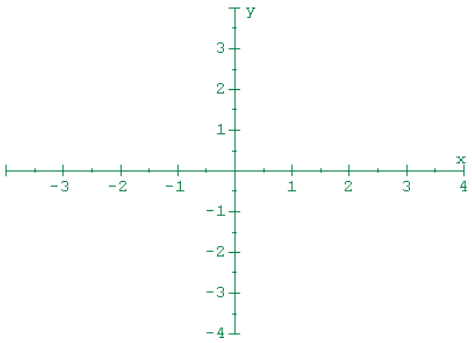
1. (3, -1) and (1, -3)



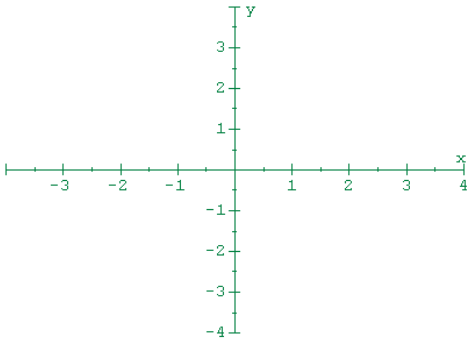
2. (0, -2) and (-2, 1)



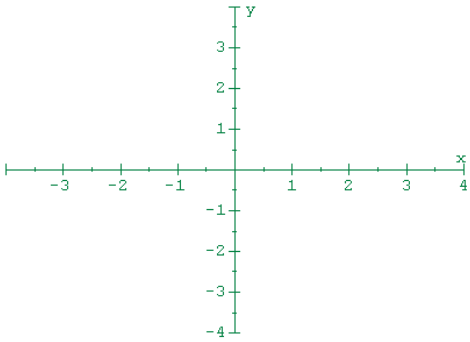
3. (3, 4) and (-4, -3)



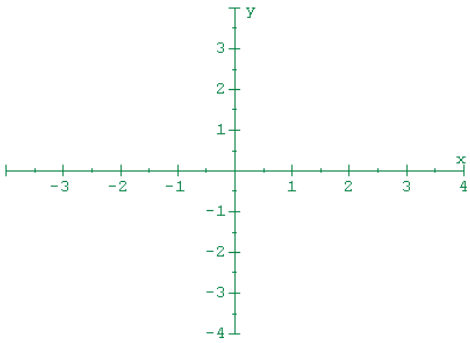
4. (0, 0) and (2, -2)



5. (-1, 2) and (3, -2)

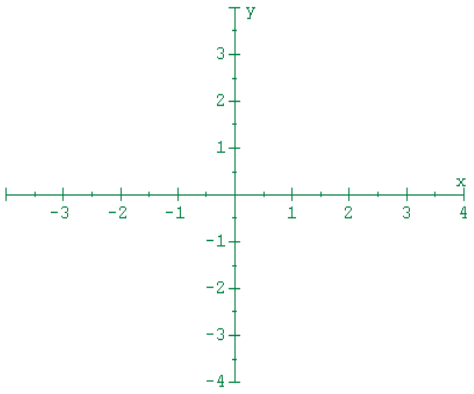


6, (-3 , - 3) and (2, 1)

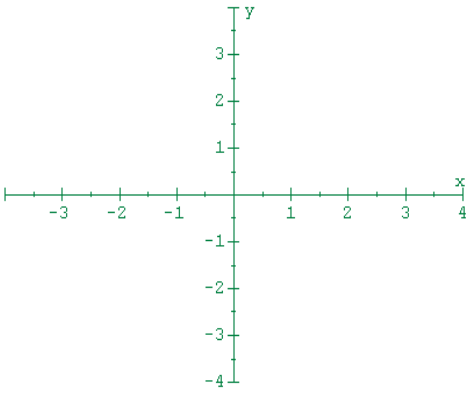


B. Draw the graph of the linear function whose x and y –intercepts are given.

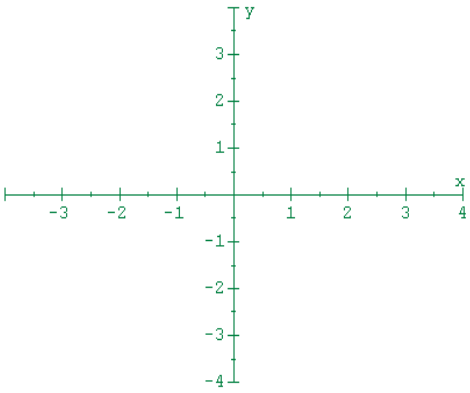
1. x- intercept = 1; y – intercept = 4



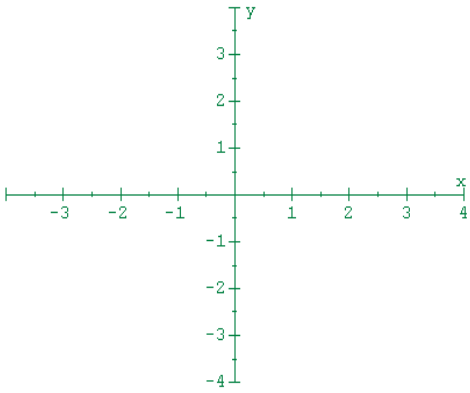
4. x – intercept = - 3; y – intercept = 2



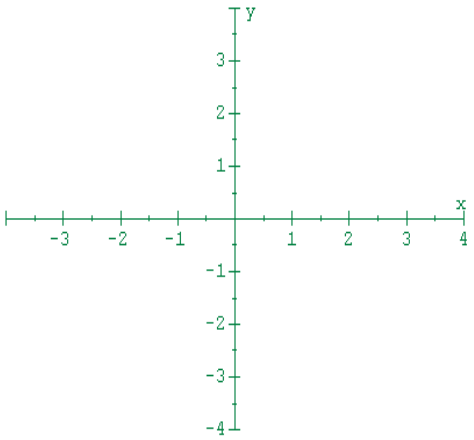
2. x-intercept = -3; y-intercept = -2



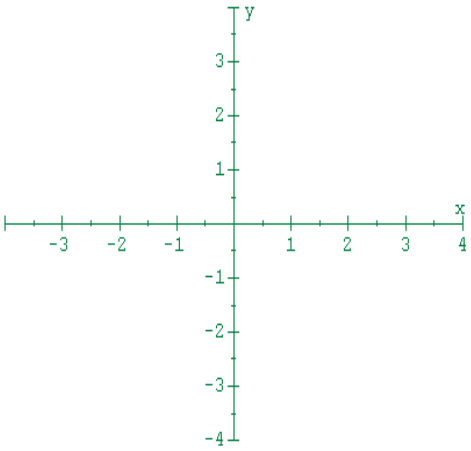
5. x-intercept = 2; y-intercept = -3



3. x-intercept = 2; y-intercept = -2



6. x-intercept = -4; y-intercept = -3



Lesson 4

Draw the Graph of a Linear Function Given the Slope and a Point

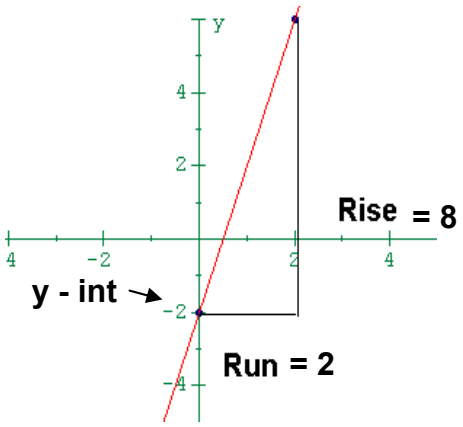
A. Given the slope *m* and y – intercept *b*.

The slope m determines the steepness of a line while the y – intercept is the y value of the point (0, b) where the graph touches the y-axis.

The slope is simply $m = \frac{rise}{run}$

Example 1: In the graph below, the y – intercept is -2 and the slope is $\frac{8}{2}$ or 4.

To graph, start with the y-intercept, and then rise 8 and run 2. These would connect the points (0,-2) and (2, 6)



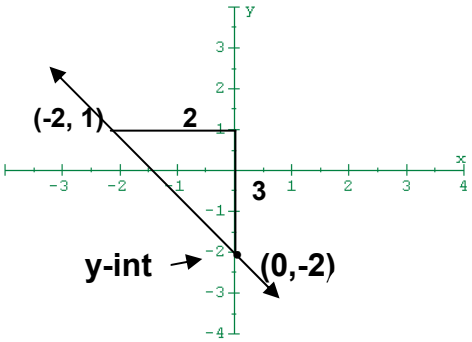
Notice that the slope can be computed using the two points in the formula.

$$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{6 - -2}{2 - 0} = \frac{8}{2} = 4$$

The line rises to the right when the slope is positive.

Example 2: Graph the linear function whose y-intercept = -2 and slope (m)= $\frac{-3}{2}$



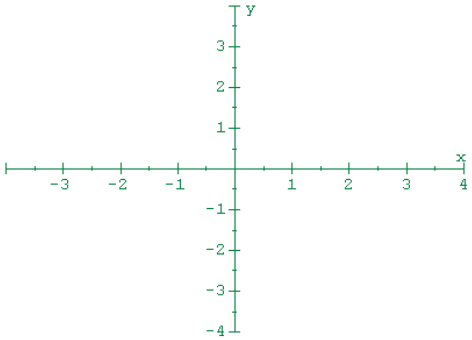
From the y – intercept, at (0, -2) rise 3 and run 2 units to the left. This would connect points (0, -2) and (-2, 1)

This time the direction of the graph goes down to the right because the slope is negative.

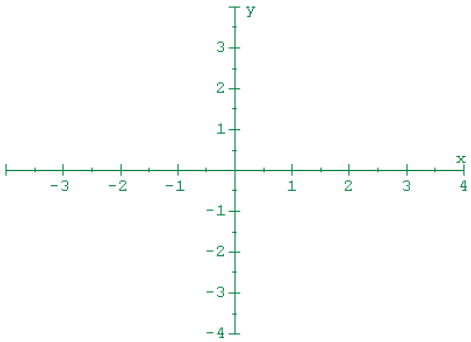
Try this out

A. Draw the graph given the slope and passing through the given point.

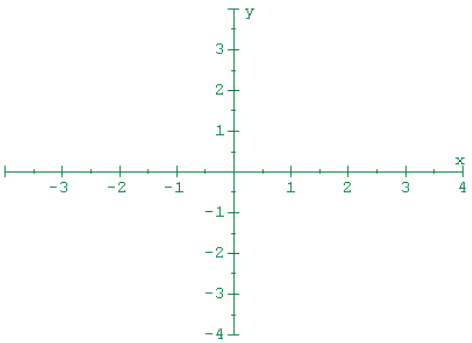
1. $m = \frac{4}{3}$; P(-2, -3)



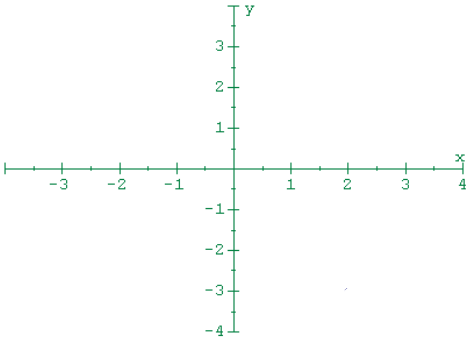
2. m = 3; P(0, 2)



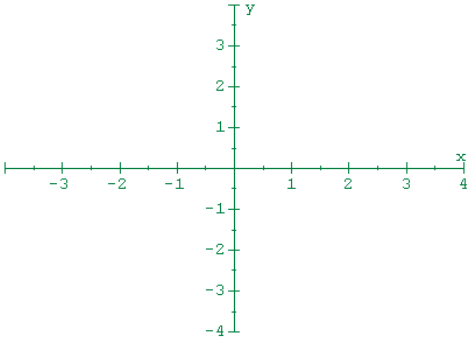
3. m = -3 ; P(-1, 1)



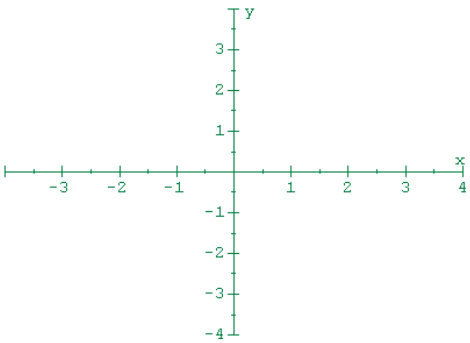
4. m = 2; P(2, 2)



5. $m = \frac{-3}{2}$; P(-2, 3)

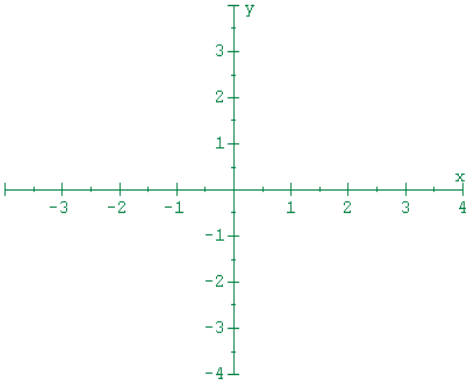


6. $m = \frac{-3}{5}$; P(1, -3)

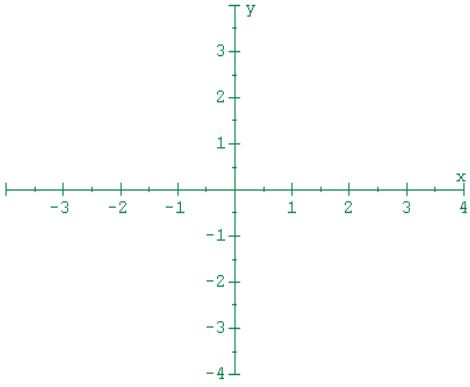


B. Draw the graph with the indicated slope and passing through the given y-intercept.

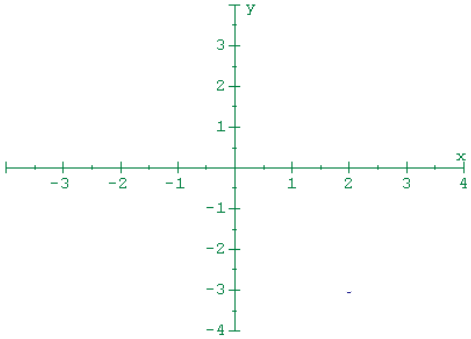
1. m = $\frac{-5}{3}$; y-intercept = -3



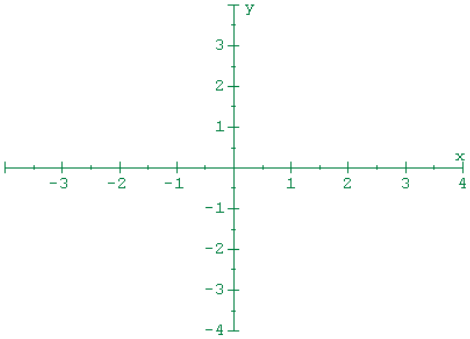
4. m = 2; y-intercept = -3



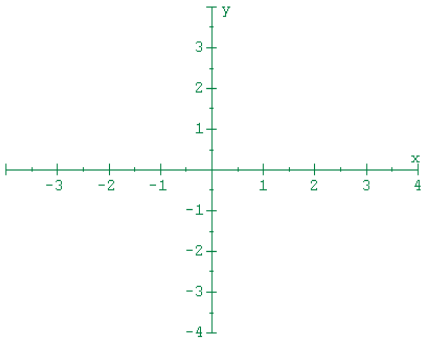
2. m = $\frac{-1}{2}$; y-intercept = 3



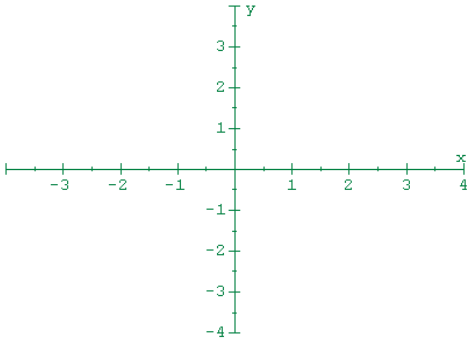
5. m = $\frac{1}{2}$; y-intercept = -2



3. m = $\frac{3}{4}$; y-intercept = 1



6. m = -4; y-intercept = -4



Let's summarize

1. The standard form of linear equation is ax + by = c, where a, b, and c are real numbers, and a and b are not both zero.

2. Linear function can be defined by f(x) = mx + b, where m is the slope of the line and b is the y – intercept.

3. The graph of a linear function is a straight line.

4. The graph can be drawn if the following are given:

   - any two points
   - x and y- intercept
   - slope and any one point
   - slope and y-intercept

What have you learned

A. Which of the following equations/functions is linear?

1. f(x) = -x + 3
2. f(x) = x(x – 3)
3. $y = 2x^2 – 3x + 1$
4. $y = \frac{1}{2}(4 + 6x)$
5. y = - 4x

B. Write each equation in the standard form.

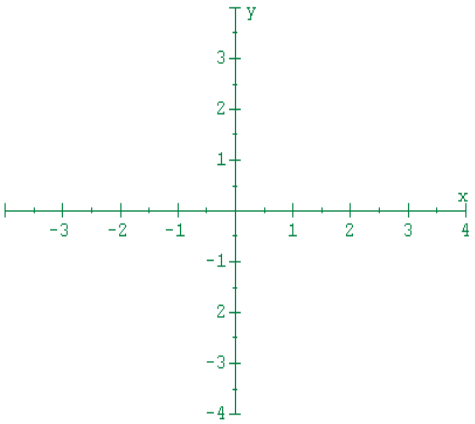
1. $y = \frac{5}{8}x + 1$
2. $x = \frac{1}{3}y - 4$
3. $y = \frac{-2}{3}x + 14$

C. Write each equation in the slope-intercept form.

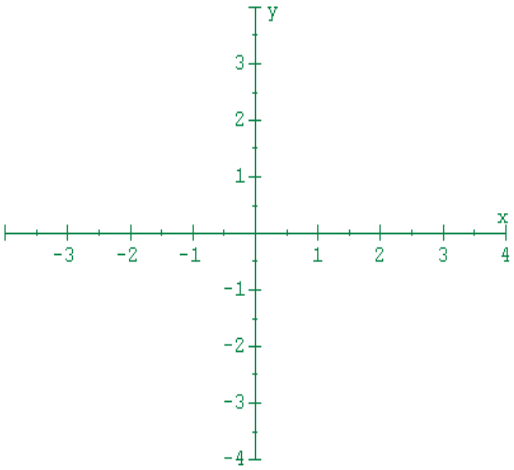
1. $5x + 3y = 9$
2. $3x – 2y = 12$
3. $x + 2y = 7$

D. Draw the graph of linear functions given the following:.

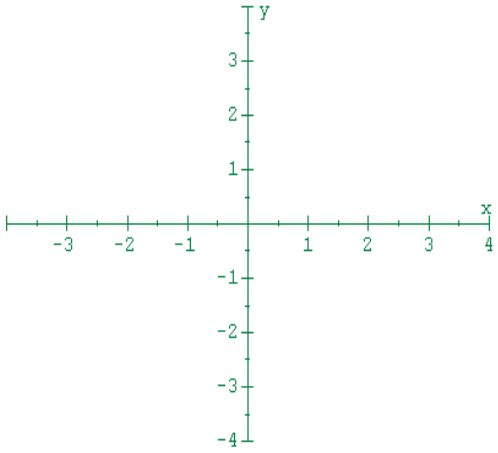
1. Two points (1, 2), (2, 4)



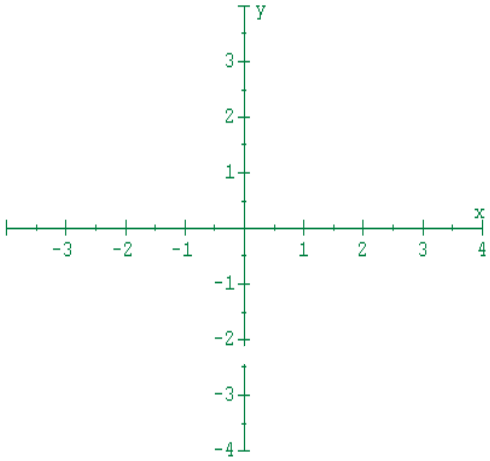
2. slope (m) = -2, passes through (-1, 3)



3. x- intercept = 4, y-intercept = 3



4. slope = $\frac{4}{3}$, y-intercept = -3



Answer Key

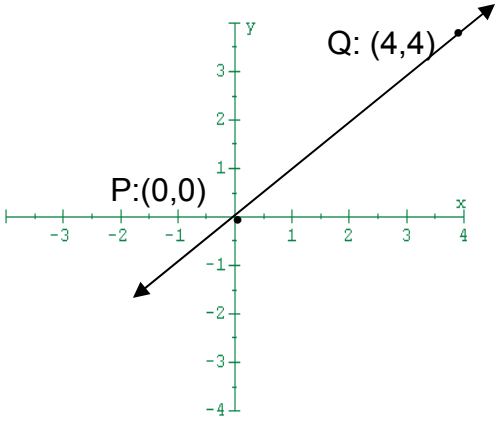
How much do you know

A. 1.

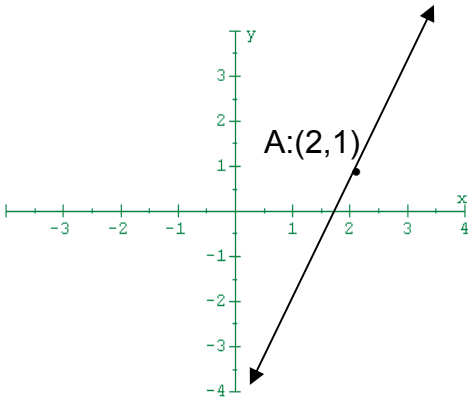
a. Linear
b not linear
c. linear
d. not linear
e. linear

2. straight line
3. slope
4. y-intercept
5. y = 3x + 2
6. a

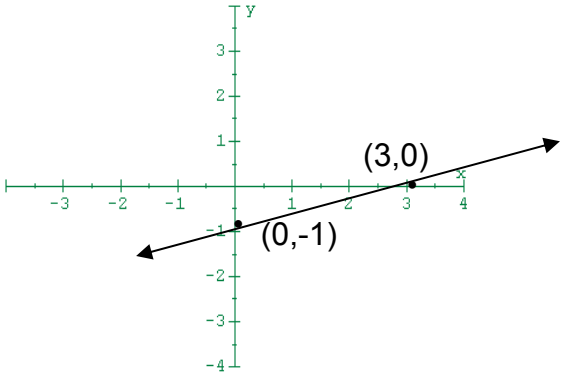
7. Two points: P(0, 0); Q(4, 4)



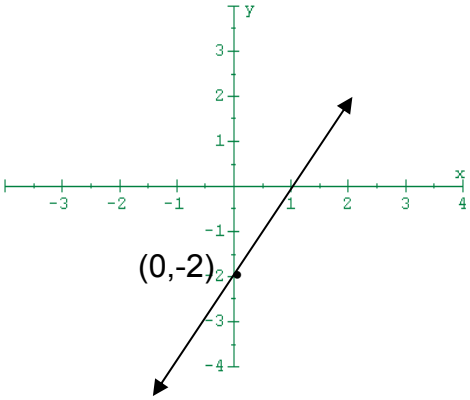
8. Slope (m) = 3; point A (2,1)



9. x- intercept = 3; y –intercept = -1



10. Slope (m) = 2; y-intercept = -2



Try this out

Lesson 1

A.

1. linear
2. linear
3. linear
4. not linear
5. not linear
6. not linear
7. linear
8. linear
9. linear
10. not linear

B.

1. not linear
2. linear
3. linear
4. linear
5. linear

Lesson 2

A. 1. $y = -3x + 5$

2. $y = -\frac{2x}{9} + 2$

3. $y = -\frac{1}{4}x - \frac{1}{2}$

4. $y = 3x$

5. $y = 4x - 3$

6. $y = \frac{3x}{4} - 2$

7. $y = -5x + 3$

8. $y = 3x - 5$

9. $y = -5x + 3$

10. $y = 2x - 7$

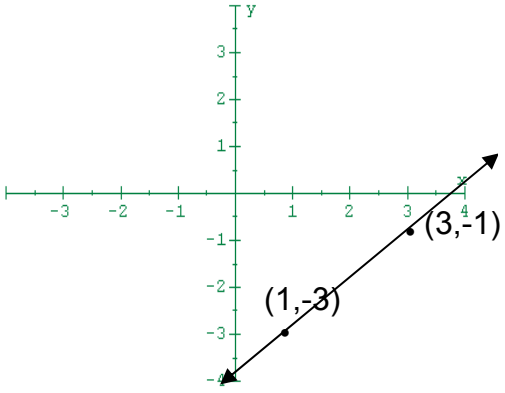
B.

1. $2x + y = 3$
2. $3x - y = 1$
3. $x - 2y = -6$
4. $2x - y = 2$
5. $4x - 2y = -1$
6. $18x - 4y = -1$
7. $7x + 2y = -2$
8. $3x - y = 5$
9. $x - 4y = 8$
10. $3x + 2y = -6$

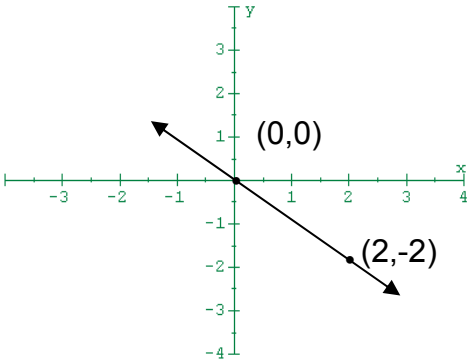
Lesson 3

A

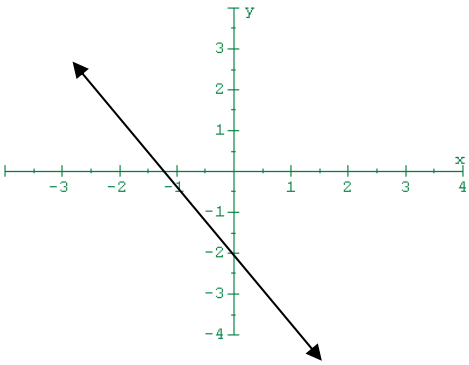
1. (3, -1), (1, -3)



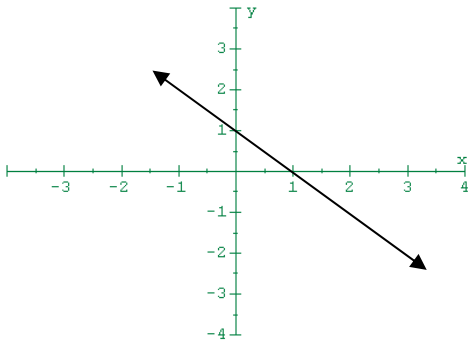
4. (0, 0), (2, -2)



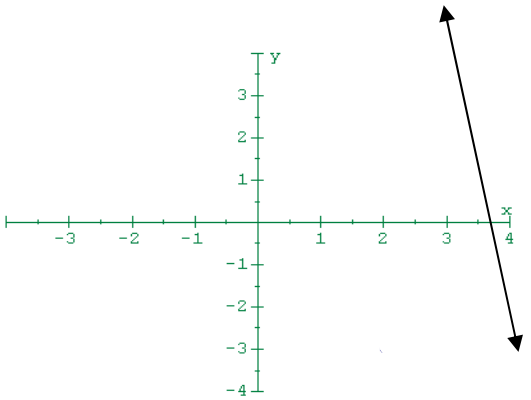
2. (0, -2), (-2,1)



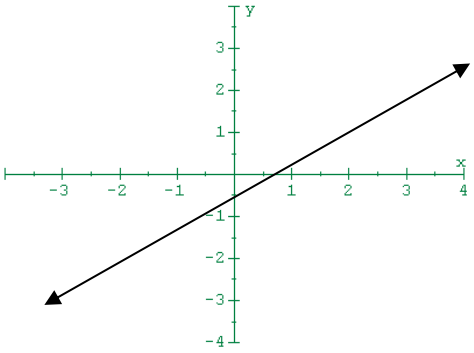
5. (-1, 2), (3, -2)



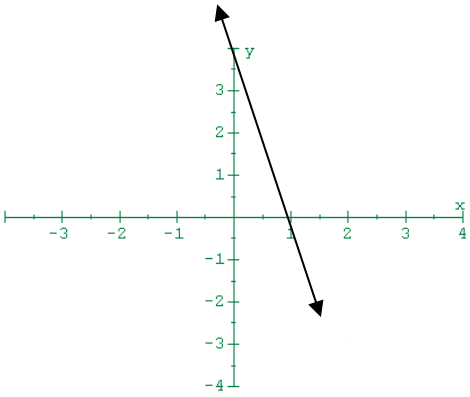
3. (3, 4), (4, -3)



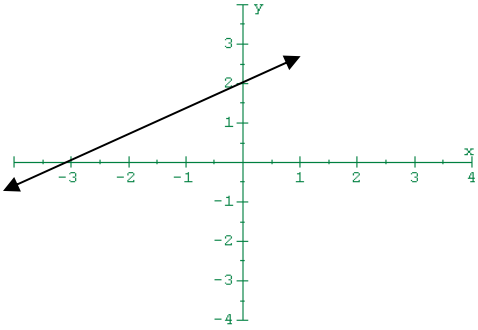
6. (-3, -3), (2, 1)



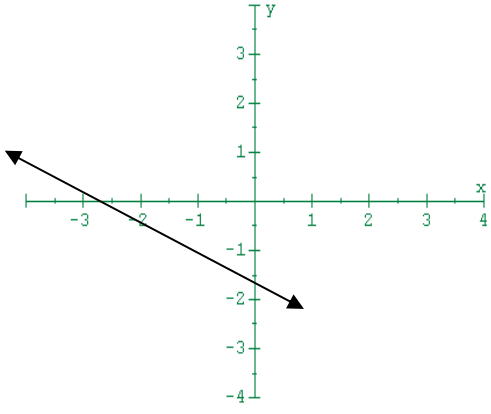
5. x-intercept = 1, y-intercept = 4



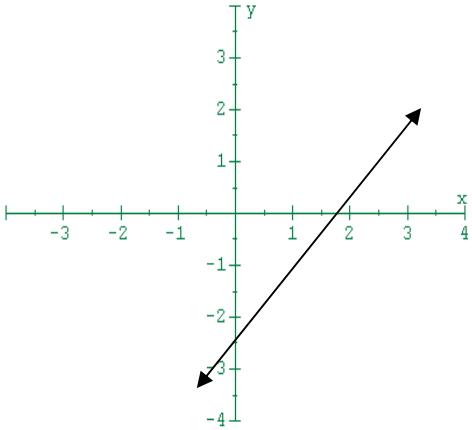
4. x-intercept = -3, y-intercept = 2



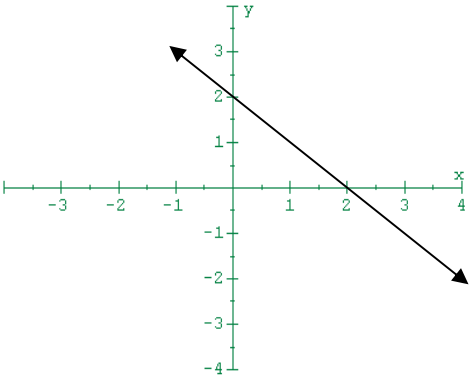
2. x-intercept = -3 ; y-intercept = -2



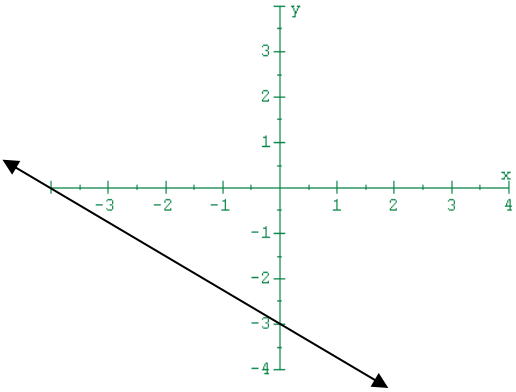
5. x-intercept = 2; y-intercept = -3



3. x-intercept = 2; y-intercept = 2



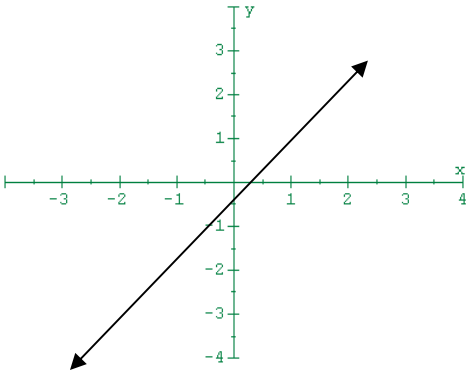
6. x-intercept = -4; y-intercept = -3



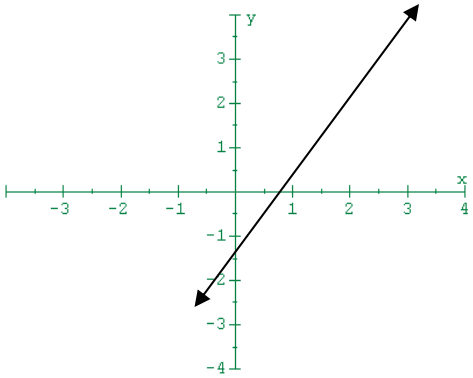
Lesson 4

A.

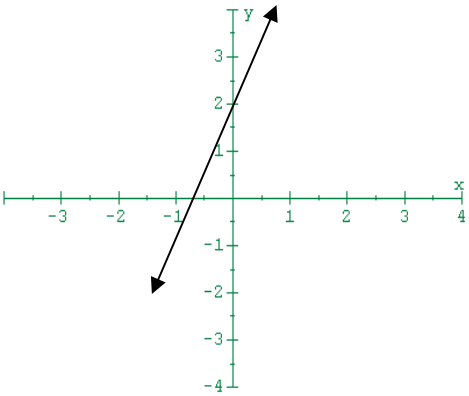
1. m = $\frac{4}{3}$, P(-2, -3)



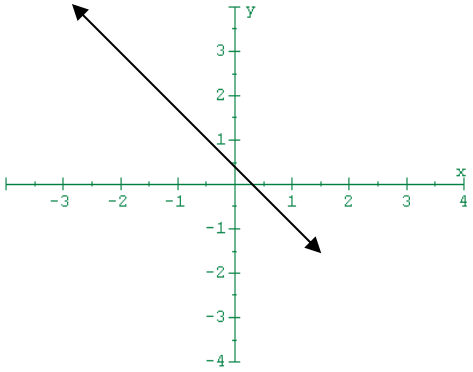
4. m= 2, P(2, 2)



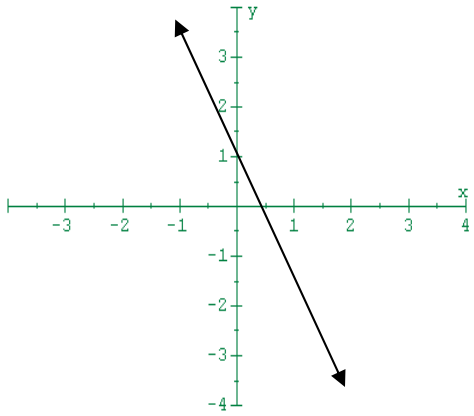
2. m = 3, P(0, 2)



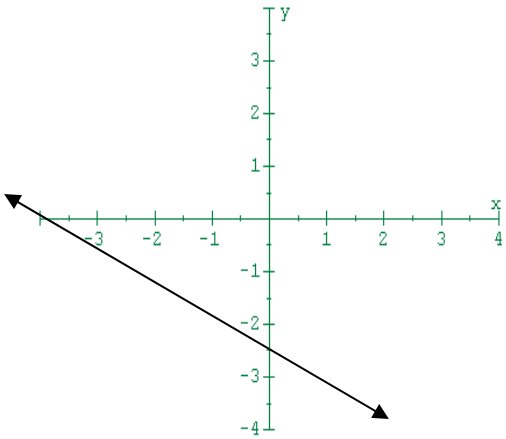
5. m= $\frac{-3}{2}$, P(-2, 3)



3. m= -3; P(1, -1)

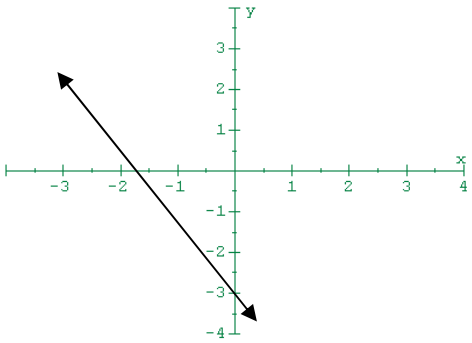


6. m= $\frac{-3}{5}$; P(1, -3)

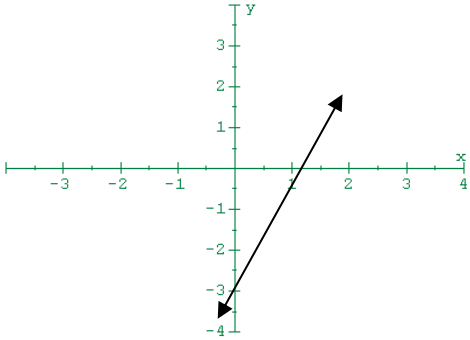


B.

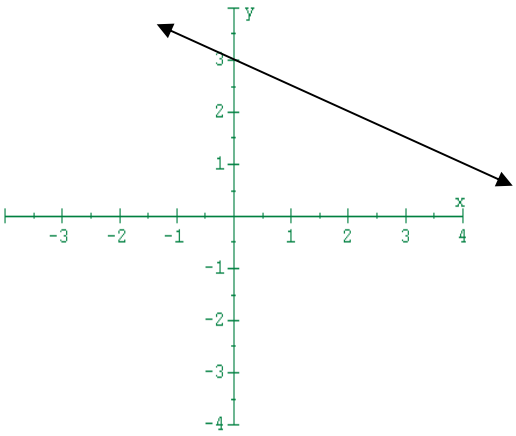
1. m = $\frac{-5}{3}$, y-intercept = -3



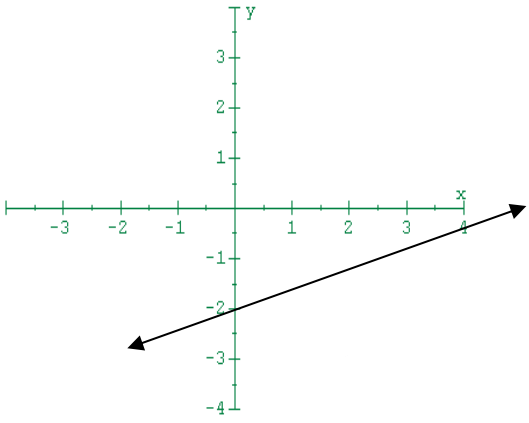
4. m = 2, y-intercept = -3



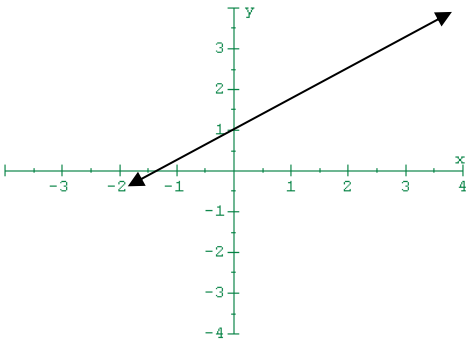
2. m= $\frac{-1}{2}$, y-intercept = 3



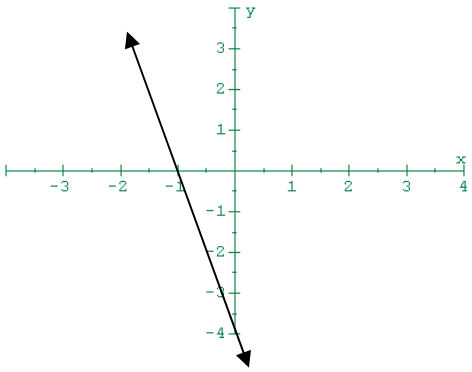
5. m= $\frac{1}{2}$, y-intercept= -2



3. m= 3/4, y-intercept = 1



6. m= -4; y-intercept = -4



What have you learned

A.

1. Linear
2. not linear
3. not linear
4. Linear
5. Linear

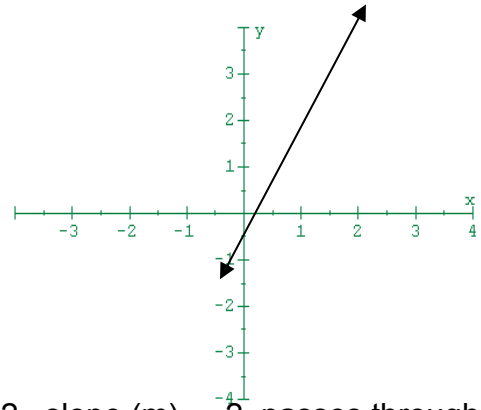
B.

1. 5x – 8y = 8
2. 3x – y = -12
3. 2x + 3y = 42

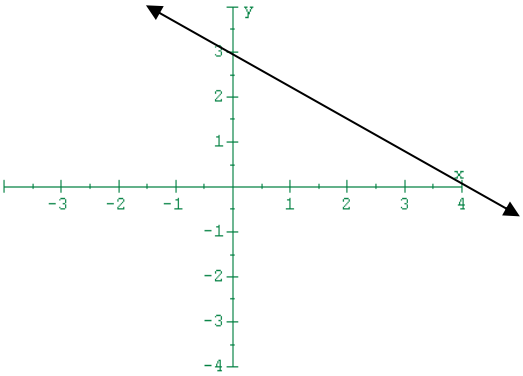
C.

1. $y = \frac{-5}{3}x + 3$
2. $y = \frac{3}{2}x - 6$
3. $y = -\frac{1}{2}x + \frac{7}{2}$

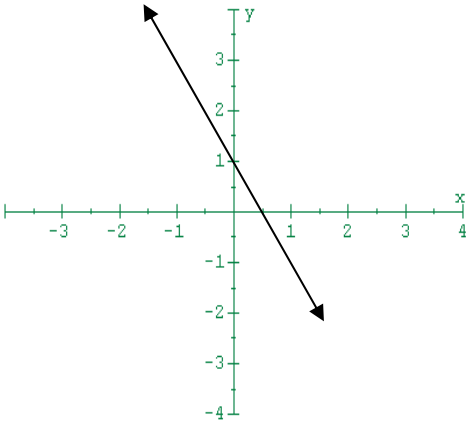
D. 1. (1, 2), (2, 4)



3. x-intercept = 4, y-intercept = 3



2. slope (m) = -2, passes through (-1, 3)



4. slope = $\frac{4}{3}$, y-intercept = -3