Module 3 Statistics



In the previous modules, you learned about the different measures of central tendencies. Now, you will learn about the measures of variability specifically the range and the standard deviation. You will be presented problems where you will interpret, draw conclusions and make recommendations.

What you are expected to learn

This module is designed for you to:

- 1. give the meaning of measure of variability
- 2. compute the range and standard deviation of the given ungrouped and grouped data
- 3. interpret, draw conclusions and make recommendations from the given statistical data.

How much do you know

A. Write the letter of the correct answer.

- 1. It is the difference between the lowest and the highest value in the distribution.
 - a. mean b. mean deviation c. median d. range
- 2. Which measure of central tendency is generally used in determining the size of the most in demand shoes.
 - a. mean b. median c. mode d. range

3. If the range of a set of scores is 14 and the lowest score is 7, what is the highest score?

a. 21 b. 24 c. 14 d. 7

4. The standard deviation of the scores 5, 4, 3, 6 and 2 is _____.

a. 2 b. 2.5 c. 3 d. 3.5

5. The most important measure of variability is ______.

a. range b. inter quartile range d. mean deviation d. standard deviation

B. Find the range and the standard deviation.

To assure a uniform product, a company measures each extension wire as it comes off the product line. The lengths in centimeters of the first batch of ten wires were: 10, 15, 14, 11, 13, 10, 10, 11, 12 and 13.

- C. On a 20-item quiz, the mean score is 12 and the standard deviation is 4. Find the score that is
 - 1. 5 points above the mean
 - 2. 2 standard deviation above the mean
 - 3. 1.5 standard deviation below the mean
 - 4. Can any of the scores be 20?
- D. Find the mean and standard deviation of the given data.

Х	100 - 110	111 - 121	122 - 132	133 - 143	144 - 154	155 – 165
f	2	1	5	12	9	1

Describe what happen to the mean and standard deviation of a set of numbers under these conditions.

- 1. 5 is added to each member in the set
- 2. 1 is subtracted from each member in the set.



Lesson 1

The Range

The three measures of central tendencies that you have learned in the previous module do not give an adequate description of the data. We need to know how the observations spread out from the average or mean. It is quite possible to have two sets of observations with the same mean or median that differs in the variability of their measurements about the mean.

Consider the following measurements, in liters, for two samples of apple juice in a tetra packed by companies A and B.

Sample A	Sample B
0.97	1.06
1.00	1.01
0.94	0.88
1.03	0.91
1.11	1.14

Both samples have the same mean, 1.00 liters. It is quite obvious that company A packed apple juice with a more uniform content than company B. We say that the variability or the dispersion of the observations from the mean is less for sample A than for sample B. Therefore, in buying apple juice, we would feel more confident that the tetra pack we select will be closer to the advertised mean if we buy from company A.

Statistics other than the mean may provide additional information from the same data. This statistics are the measures of dispersion.

Measures of dispersion or variability refer to the spread of the values about the mean. These are important quantities used by statisticians in evaluation. Smaller dispersion of scores arising from the comparison often indicates more consistency and more reliability.

The Range

The range is the simplest measure of variability. It is the difference between the largest and smallest measurement.

R = H - L

where R = Range, H = Highest measure, L = Lowest Measure

The main disadvantage of the range is that it does not consider every measure in the data.

Examples:

1. The IQs of 5 members of a family are 108, 112, 127, 118 and 113. Find the range.

Solution: The range of the IQs is 127 - 108 = 19.

2. The range of each of the set of scores of the three students is as follows:

Student A	H = 98 L = 92 R = 98 - 92 = 6
Student B	H = 97 L = 90 R = 97 - 90 = 7
Student C	H = 97 L = 90 R = 97 - 90 = 7

Observe that two students are " tie." This indicates that the range is not a reliable measure of dispersion. It is a poor measure of dispersion, particularly if the size of the sample or population is large. It considers only the extreme values and tells us nothing about the distribution of numbers in between.

3. Consider the following two sets of data, both with a range of 12:

Set A	Set B
3	3
4	7
5	7
6	7
8	8
9	8
10	8
12	9
15	15

In set A the mean and median are both 8, but the numbers vary over the entire interval from 3 to 15. In set B the mean and median are also 8, but most of the values are closer to the center of the data. Although the range fails to measure the dispersion between the upper and lower observations, it does have some useful applications. In industry the range for measurements on items coming off an assembly line might be specified in advance. As long as all measurements fall within the specified range, the process is said to be in control.

Disadvantages of the Range:

- 1. It makes use of very little information: that is, it ignores all but two items only.
- 2. It is totally dependent on the two extreme values, so it is greatly affected by any changes in these values.
- 3. It should be used with caution, particularly with data that contain a single extremely large value as this value would have a considerable effect on the range.
- 4. It cannot identify the differences between two sets of data with the same extreme values, example, the two sets of data are

2, 4, 6, 8, 10, 12, 14, 16, 18 and 2, 2, 2, 2, 2, 2, 2, 2, 18

both have the same range 16.

Range of a Frequency Distribution

The range of a frequency distribution is simply the difference between the upper class boundary of the top interval and the lower class boundary of the bottom interval.

Example:

Scores in Second Periodical Test of I - Faith in Mathematics I

Scores	Frequency
46 - 50	1
41 - 45	10
36 - 40	10
31 - 35	16
26 - 30	9
21 - 25	4

Upper Class Boundary = 55.5

Lower Class Boundary = 20.5

Try this out

Answer the following:

A. Compute the range for each set of numbers.

6. (12, 13, 14, 15, 16, 17, 18)
7. (7, 7, 8, 12, 14, 14, 14, 14, 15, 15)
8. (12, 12, 13, 13, 13, 13, 13, 15, 19, 20, 20)
9. (12, 13, 17, 22, 22, 23, 25, 26)
10. (23, 25, 27, 27, 32, 32, 36, 38)

B. Compute the range

1. Two students have the following grades in six math tests.

Pete	Ricky
82	88
98	94
86	89
80	87
100	92
94	90

2.

х	100–110	111–121	122 - 132	133-143	144-154	155–165
f	2	1	5	12	9	1

- C.1. The reaction times for a random sample of 9 subjects to a stimulant were recorded as 2.5, 3.6, 3.1, 4.3, 2.9, 2.3, 2.6, 4.1 and 3.4 seconds. Calculate range.
 - 2. If the range of the set of scores is 29 and the lowest score is 18, what is the highest score?
 - 3. If the range of the set of scores is 14, and the highest score is 31, what is the lowest score?

Lesson 2

The Standard Deviation

The most important measure of dispersion is the standard deviation. Like the mean deviation, standard deviation differentiates sets of scores with equal averages. But the advantage of standard deviation over mean deviation has several applications in inferential statistics

Examples:

1. Compare the standard deviation of the scores of the three students in their Mathematics quizzes.

Student A	97, 92, 96, 95, 90
Student B	94, 94, 92, 94, 96
Students C	95, 94, 93, 96, 92

Solution:

Student A:

Step 1. Compute the mean score.

$$\overline{x} = \frac{97 + 92 + 96 + 95 + 90}{5} = 94$$

Step 2. Find the deviation of each score from the mean.

97 - 94 = 3 92 - 94 = -2 96 - 94 = 2 95 - 94 = 190 - 94 = -4 Step 3. Square each deviation.

$$(3)^2 = 9$$

 $(-2)^2 = 4$
 $(2)^2 = 4$
 $(1)^2 = 1$
 $(-4)^2 = 16$

Step 4. Find the mean of the squared deviations.

$$\frac{9+4+4+1+16}{5} = 6.8$$

Step 5. Get the square root of the mean of the squared deviations.

This is the standard deviation SD.

$$SD = \sqrt{6.8} = 2.6$$

Student B:

Step 1. Compute the mean score.

$$\overline{x} = \frac{94 + 94 + 92 + 94 + 96}{5} = 94$$

Step 2. Find the deviation of each score from the mean.

94 - 94 = 0 94 - 94 = 0 92 - 94 = -2 94 - 94 = 096 - 94 = 2 Step 3. Square each deviation.

 $(0)^{2} = 0$ $(0)^{2} = 0$ $(2)^{2} = 4$ $(0)^{2} = 0$ $(2)^{2} = 4$

Step 4. Find the mean of the squared deviations.

$$\frac{4+4}{5}$$
 = 1.6

Step 5. Get the square root of the mean of the squared deviations.

This is the standard deviation SD.

$$SD = \sqrt{1.6} = 1.3$$

Student C:

Step 1. Compute the mean score.

$$\frac{1}{x} = \frac{95 + 94 + 93 + 96 + 92}{5} = 94$$

Step 2. Find the deviation of each score from the mean.

95 - 94 = 194 - 94 = 093 - 94 = -196 - 94 = 292 - 94 = -2 Step 3. Square each deviation.

$$(1)^{2} = 1$$

 $(0)^{2} = 0$
 $(-1)^{2} = 1$
 $(2)^{2} = 4$
 $(-2)^{2} = 4$

Step 4. Find the mean of the squared deviations.

$$\frac{1+0+1+4+4}{5} = 2$$

Step 5. Get the square root of the mean of the squared deviations.

This is the standard deviation SD.

SD =
$$\sqrt{2}$$
 = 1.4

The steps may be summarized by the formula:

Standard Deviation for Ungrouped Data

$$SD = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

where SD = standard deviation

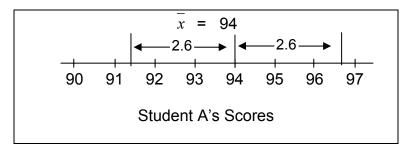
x = individual score

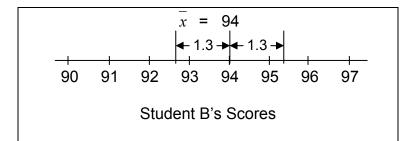
$$\overline{x}$$
 = mean

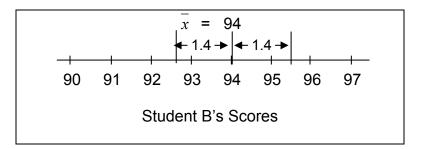
n = number of scores

The result of the computation of the standard deviation of the three students means that student B has more consistent scores.

This can also be illustrated by plotting the scores on the number line.







Graphically, a standard deviation of 2.6 means most of the scores are within 2.6 units from the mean. Standard deviation of 1.3 and 1.4 means that most of the scores are within 1.3 and 1.4 units from the mean. Student B is the most consistent among the three students.

2. Compute the standard deviation of the following set of scores.

39, 10, 24, 16, 19, 26, 29, 30, 5
$$\bar{x} = \frac{5+10+16+19+24+26+29+30+39}{9} = 22$$

Х	x - <i>x</i>	$(x - \bar{x})^2$
5	-17	289
10	-12	144
16	-6	36
19	-3	9
24	2	4
26	4	16
29	7	49
30	8	64
39	17	289

$$SD = \sqrt{\frac{\sum (x - \overline{x})^2}{9}}$$
$$= \sqrt{\frac{900}{9}}$$
$$= \sqrt{100}$$
$$SD = 10$$

The standard deviation is 10. This means that most of the scores are within 10 units from the mean or the values are scattered widely about the mean.

Standard Deviation for Grouped Data

For large quantities, the standard deviation is computed using frequency distribution with columns for the midpoint value, the product of the frequency and midpoint value for each interval, the deviation and its square and finally the product of the frequency and the squared deviation.

To find the standard deviation of a grouped data, use the formula

$$SD = \sqrt{\frac{\sum \left(x - \overline{x}\right)^2 f}{n - 1}}$$

In calculating the standard deviation, the steps to follow are:

1. Prepare a frequency distribution with appropriate class intervals and write the corresponding frequency (f).

- 2. Get the midpoint (x) of each class interval in column 2.
- 3. Multiply frequency (f) and midpoint (x) of each class interval to get fx.
- 4. Add fx of each interval to get $\sum fx$.

5. Compute the mean using
$$\bar{x} = \frac{\sum fx}{n}$$
.

- 6. Calculate the deviation ($x \overline{x}$) by subtracting the mean from each midpoint.
- 7. Square the deviation of each interval to get $(x \overline{x})^2$.
- 8. Multiply frequency (f) and $(x \overline{x})^2$. Find the sum of each product to get $\sum (x \overline{x})^2 f$.
- 9. Calculate the standard deviation using the formula

$$SD = \sqrt{\frac{\sum \left(x - \overline{x}\right)^2 f}{n - 1}}$$

3. The table is the distribution of the number of mistakes 50 students made in factoring 20 quadratic equations. Compute the standard deviation.

Number of	Frequency
Mistakes	
18 – 20	2
15 – 17	5
12 – 14	6
9 – 11	10
6 – 8	15
3 – 5	8
0 – 2	4
Total	50

Solution:

To solve this problem, be sure to follow the different steps.

- 1. Prepare a frequency distribution with appropriate class intervals and write the corresponding frequency (f).
- 2. Get the midpoint (x) of each class interval in column 2.

In the interval 18 - 20, 19 is the midpoint, write this in column 2.

3. Multiply frequency (f) and midpoint (x) of each class interval to get fx.

In the interval 18 - 20, the frequency is 2 and the midpoint is 19, fx = 19 (2) = 38.

Write the product in column 3.

4. Add fx of each interval to get $\sum fx$.

$$\sum fx = 438$$

5. Compute the mean using $\bar{x} = \frac{\sum fx}{n}$

$$\overline{x} = \frac{\sum fx}{n} = \frac{438}{50} = 8.76$$

6. Calculate the deviation ($x - \overline{x}$) by subtracting the mean from each midpoint.

 $x - \overline{x} = 19 - 8.76 = 10.24$ 16 - 8.76 = 7.24

 13 - 8.76 = 4.24 10 - 8.76 = 1.24

 7 - 8.76 = -1.76 4 - 8.76 = -4.76

 1 - 8.76 = -7.76

7. Square the deviation of each interval to get $(x - \overline{x})^2$.

(10.24) ²	=	104.86	(7.24) ² =	= {	52.42
(4.24) ²	=	17.78	(1.24) ²	=	1.54
(-1.76) ²	=	3.1	(-4.76) ²	=	22.66
(-7.76) ²	=	60.22			

8. Multiply frequency (f) and $(x - \overline{x})^2$. Find the sum of each product to get $\sum (x - \overline{x})^2 f$.

(104.86)2 =	209.72	(52.42)5 =	262.1
(17.78)6 =	106.68	(1.54)10 =	15.4
(3.1)15 =	46.5	(22.66)8 =	181.28
(60.22)4 =	240.88		

Number of Mistakes	f	Х	Fx	$(x - \bar{x})$	$(x - \bar{x})^2$	$f(x - \bar{x})^2$
18 – 20	2	19	38	10.24	104.86	209.72
15 – 17	5	16	80	7.24	52.42	262.1
12 – 14	6	13	78	4.24	17.78	106.68
9 – 11	10	10	100	1.24	1.54	15.4
6 – 8	15	7	105	-1.76	3.1	46.5
3 – 5	8	4	32	-4.76	22.66	181.28
0 – 2	4	1	5	-7.76	60.22	240.88
Total	50		438			1062.56

9. Calculate the standard deviation using the formula SD = $\sqrt{\frac{\sum (x - \overline{x})^2 f}{n-1}}$

SD =
$$\sqrt{\frac{\sum (x - \overline{x})^2 f}{n - 1}}$$

= $\sqrt{\frac{(1062.56)}{50 - 1}}$
= $\sqrt{\frac{1062.56}{49}}$
= $\sqrt{21.68}$
SD = 4.66

4. In this example we will use another method of computing the standard deviation of grouped data. Study the two methods and use the one you find easier to use.

Determine the standard deviation of the scores in Mathematics IV Test of IV - Emerald.

Scores	IV – Emerald		
41 – 45	1		
36 - 40	5		
31 – 35	10		
26 – 30	12		
21 – 25	10		
16 – 20	5		
11 – 15	3		
6 – 10	3		
1 – 5	1		

Solution:

$$SD = \frac{\sqrt{\sum f} \left[\sum (fX^2) \right] - \left[\sum (fX) \right]^2}{\sum f \left(\sum f - 1 \right)}$$

where: SD = standard deviation f = frequency

x = midpoint

Scores	F	х	Fx	x ²	fx ²
41 - 45	1	43	43	1849	1849
36 - 40	5	38	190	1444	7220
31 - 35	10	33	330	1089	10890
26 - 30	12	28	336	784	9408
21 - 25	10	23	230	529	5290
16 - 20	5	18	90	324	1620
11 - 15	3	13	39	169	507
6 - 10	3	8	24	64	192
1 - 5	1	3	3	9	9

 $\sum f = 50$

 $\sum fx = 1,285$ $\sum fx^2 = 36,985$

$$SD = \frac{\sqrt{\sum f} \overline{\sum (fX^2)} - \left[\sum (fX)\right]^2}{\sum f (\sum f - 1)}$$
$$= \sqrt{\frac{(50)(36,985) - (1,285)^2}{50(50 - 1)}}$$
$$= \sqrt{\frac{1,819,250 - 1,651,225}{50(49)}}$$

$$= \sqrt{\frac{168,025}{2,450}} \\ = \sqrt{68.58} \\ \text{SD} = 8.28$$

Try this out

A. Compute the standard deviation for each set of numbers.

(12, 13, 14, 15, 16, 17, 18)
 (7, 7, 8, 12, 14, 14, 14, 14, 15, 15)
 (12, 12, 13, 13, 13, 13, 13, 15, 19, 20, 20)
 (12, 13, 17, 22, 22, 23, 25, 26)
 (23, 25, 27, 27, 32, 32, 36, 38)

- B. The reaction times for a random sample of 9 subjects to a stimulant were recorded as 2.5, 3.6, 3.1, 4.3, 2.9, 2.3, 2.6, 4.1 and 3.4 seconds. Calculate range and standard deviation.
- C. The IQ of 100 pupils at a certain high school are given in the following table.

IQ	Frequency
55 - 64	1
65 - 74	3
75 - 84	7
85 - 94	20
95 - 104	32
105 - 114	25
115 - 124	10
125 - 154	1
135 - 144	1

Find the range and the standard deviation.

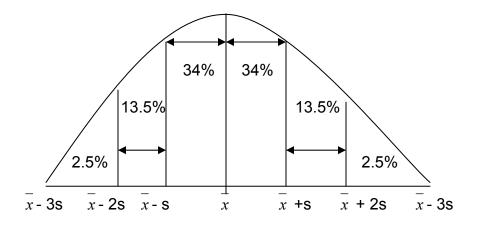
Lesson 3

Characteristics of Data Using the Measures of Variability

The measure of central tendency gives us information about the characteristics of a set of data but this is not enough because it provides only information about the average or centers of a distribution. The measure of variability provides additional information on the variability or the spread of values about the mean. Measure of variability helps visualize the shape of a data set together with its extreme values.

The standard deviation is considered the best indicator of the degree of dispersion among the measures of variability because it represents an average variability of the distribution. Given the set of data, the smaller the range, the smaller the standard deviation, the less spread is the distribution.

if the population being taken into consideration has a normal distribution, then its histogram is a bell shaped. Ina normal distribution, a width of 6 standard deviation covers all the distributions. The percentage that indicates the area that will be within a certain number of standard deviation units away from the mean is shown in the illustration.



When two groups are comparer, the group having a smaller standard deviation is less varied.

Examples:

- 1. In a certain school, 2000 students took the entrance examination for incoming first year. Suppose the results showed that the mean score is 60.5 and the standard deviation is 9.2.
 - a. How many students had score within the range one standard deviation from the mean?
 - b. How many students had scores within the range one standard deviation from the mean?
 - c. if the school will be very selective and will admit only students who got scores one standard deviation above the mean, how many students will be taken in?

Answer:

a. The range of score with one standard deviation from the mean

60.5 - 9.2		to		60.5 + 9.2
= 51.3				= 69.7
	51	to	70	

b. The number of students one standard unit away from mean

34% + 34% = 68% or .68(2000) = 1360

c. The school will only admit (13.5% + 2.5%) = 16%

.16(2000) = 320

2. The scores in Mathematics Quiz of three students.

Student A	97, 92, 96, 95, 90
Student B	94, 94, 92, 94, 96
Students C	95, 94, 93, 96, 92

The computed \overline{x} of the three students is 94 but they differ in standard deviation. The standard deviation of Student A is 2.6, Student B is 1.3 and

Student C is 1.4. Based on the result we can say that Student B has more consistent scores.

3. Consider the scores of two sets of students.

Set A10, 11, 9, 12, 11, 10, 12, 11, 11, 9, 10, 12, 9, 11, 10, 12Set B8, 10, 11, 9, 11, 12, 10, 15, 8, 9, 10, 13, 14, 10, 9, 11

Set A: $\bar{x} = 10.63$ R = 3 Set B: $\bar{x} = 10.63$ R = 7

Set A tells us that this group of students whose scores are very near each other have almost the same abilities and therefore more teachable and would progress at the same rate.

Set B consists of very slow and very fast learners. They are more difficult to manage because different mental abilities.

Try this out

Answer the following.

1. The scores received by Ann and Apple on a ten math quizzes are as follows:

Ann: 4, 5, 3, 2, 2, 5, 5, 3, 5, 0

Apple: 5, 4, 4, 3, 3, 1, 4, 0, 5, 5

- a. Compute the mean and the standard deviation.
- b. Which student had the better grade point average?
- c. Which student was the most consistent.

- 2. The mean IQ score for 2,000 students is 110 with a standard deviation of 12. The scores follow approximately a normal curve
 - a. About how many students have an IQ between 98 and 122?
 - b. About how many students have an IQ below 86?
 - c. About how many students have an IQ below 74 or more than 146?
- 3. In a reading Proficiency Test, the following means and standard deviations were obtained from a sample of 100 students each from the 3rd year and the 4th year.

	Third Year	Fourth Year
Mean	80	81.4
Standard Deviation	2.5	5.5

- a. Which group did better in the test?
- b. Whose scores are more spread? Why?
- 4. Two brands of air conditioning units were compared as to their life span in years and the following data was obtained.

Mean	Brand A 9.2	Brand B 11.5
Standard Deviation	3.3	3.1

Compare the two brands. Decide which brand is better.



The range is the simplest measure of variability. It is the difference between the largest and smallest measurement.

R = H - L

Where,

R = Range, H = Highest measure, L = Lowest Measure

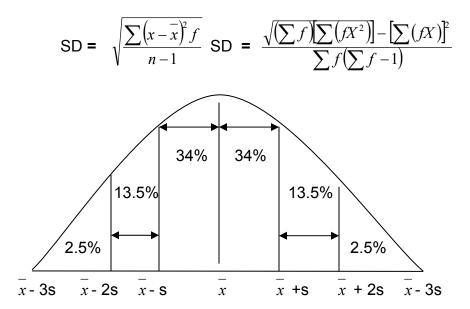
Standard Deviation for Ungrouped Data

$$\mathsf{SD} = \sqrt{\frac{\sum \left(x - \overline{x}\right)^2}{n}}$$

Where, SD = standard deviation

x = individual score $\overline{x} =$ mean n = number of scores

To find the standard deviation of a grouped data, use the formula





Answer the following completely:

- 1. Find the range for each set of data.
 - a. scores on a quiz: 10, 9, 6, 6, 7, 8, 8, 8, 8, 9
 - b. Number of points per game: 16, 18, 10, 20, 15, 7, 16, 24
 - c. Number of VCR's sold per week: 8, 10, 12, 13, 15, 7, 6, 14, 18, 20
- 2. The minimum distance a batter has to hit the ball down the center of the field to get a home run in 8 different stadiums is 410, 420, 406, 400, 440, 421, 402 and 425 ft. Compute the standard deviation.
- 3. Given the set of scores of 2 students received in a series of test

Student A: 60, 55, 40, 48, 52, 36, 52, 50 Student B: 62, 48, 50, 46, 38, 48, 43, 39

- a. What is the mean score of the student?
- b. Compute the range.
- c. Interpret.
- 4. In the entrance examination to an elite school, 9854 took the test. The mean of the applicants was 76.5 with a standard deviation of 8.5. Assume that the results approximated a normal curve.
 - a. How many of the students scored within one standard deviation from the mean?
 - b. If only those applicants who got scores of at least one standard deviation above the mean were accepted and the score of Albert was 87, was he accepted? Why?
 - c. About how many students were accepted in the school?
 - d. If the applicants who got scores 2 standard deviations above the mean were the students given scholarships, about how many students were granted scholarships?



How much do you know

A. 1. d	2. c	3. a	4. d	5. d
B. Range = 5	Stand	dard deviation	= 1.7	
C. 1.17	2.20	3. 18	4. yes	
D. 1. The mean will same.	increase by	5 but the stand	lard deviation	will remain the
2. The mean will same.	decrease by	1 but the stan	dard deviatio	n will remain the
Try this out				
Lesson 1				
A 1. 6	2. 8	3. 8	4. 14	5. 15
6. 6	7. 8	8. 8	9. 14	10. 15
B. 1. Pete = 18	Ricky = 7		2. Range =	66
C. 1. 1.8	2. highest so	core = 47	3. lowest sco	ore = 17
Lesson 2.				
A. 1. Mean = 15	SD = 2		2. Mean =	12.3 SD = 3.11
3. Mean = 14.8	32 SD = 2.	02	4. Mean = 2	20 SD = 5.1
5. Mean = 30	SD = 5			
B. Mean = 2.9	SD = 0.69			
C. Range = 90	Mean = 100	fx = 10000	$fx^2 = 10,1$	80,75 SD = .14

Lesson 3

1.	Ann	Apple	
a. Mean	3.4	3.4	
SD	1.22	1.22	

- b. they have the same point average
- c. both of them were consistent with the results of their test, which means that they are equally good.
- 2. a. 1360 b. 50 students c. 20 students
- 3. a. The 4th year group did better in the test.
 - b. The 4th year group because the standard deviation is higher than the 3rd year group.

What have you learned

- 1. a. 4 b. 17 c. 14
- 2. Mean = 415.5 SD = 11.74
- 3. a. Sudent A = 49.13 Student B = 46.75
 - b. range of Student A = 24 range of Student B = 24
 - c. Both Students A and B were consistent with their scores in the test.
- 4. a. 6701 students
 - b. Pete was accepted
 - c. 246 were granted scholarships
 - d. 1577 students were accepted