Module
Triangle Trigonometry

What this module is about

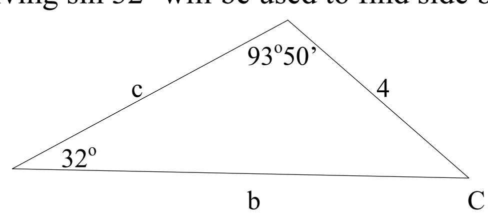
This module is about law of sines. As you go over this material, you will develop the skills in deriving the law of sines. Moreover, you are also expected to learn how to solve different problems involving the law of sines.

What you are expected to learn

This module is designed for you to demonstrate ability to apply the law of sines to solve problems involving triangles.

How much do you know?

1. Find the measure of $\angle B$ in $\triangle ABC$ if $m\angle A = 8^o$ and $m\angle C = 97^o$.
2. Find the measure of $\angle A$ in $\triangle ABC$ if $m\angle B = 18^o14'$ and $m\angle C = 81^o41'$.
3. Two of the measures of the angles of $\triangle ABC$ are $A = 8^o3'$ and $B = 59^o6'$. Which is the longest side?
4. What equation involving sin 32^o will be used to find side b?



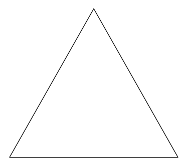
5. Given a = 62.5 cm, $m\angle A = 62^o20'$ and $m\angle C = 42^o10'$. Solve for side b.
6. Solve for the perimeter of $\triangle ABC$ if c = 25 cm, $m\angle A = 35^o14'$ and $m\angle B = 68^o$.
7. Solve $\triangle ABC$ if a = 38.12 cm, $m\angle A = 46^o32'$ and $m\angle C = 79^o17'$.
8. Solve $\triangle ABC$ if b = 67.25 mm, c = 56.92 mm and $m\angle B = 65^o16'$.
9. From the top of a building 300 m high, the angles of depression of two street signs are 17.5^o and 33.2^o. If the street signs are due south of the observation point, find the distance between them.
10. The angles of elevation of the top of a tower are 35^o from point A and 51^o from another point B which is 35 m from the base of the tower. If the base of the tower and the points of observation are on the same level, how far are they from each other?

What you will do

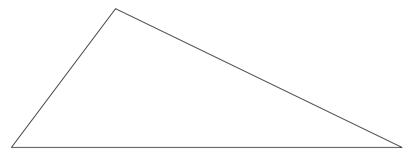
Lesson 1

Oblique Triangles

An oblique triangle is a triangle which does not contain a right angle. It contains either three acute angles (acute triangle) or two acute angles and one obtuse angle (obtuse triangle).



Acute triangle



Obtuse triangle

If two of the angles of an oblique triangle are known, the third angle can be computed. Recall that the sum of the interior angles in a triangle is 180^o.

Examples:

Given two of the angles of $\triangle ABC$, solve for the measure of the third angle.

1. $A = 30^o$, $B = 45^o$.

   $A + B + C = 180^o$.
   $30^o + 45^o + C = 180^o$.
   $C = 180^o – (30^o + 45^o)$
   $C = 180^o – 75^o$
   $C = 105^o$

2. $B = 69.25^o$, $C = 114.5^o$.

   $A + B + C = 180^o$.
   $A + 69.25^o + 114.5^o = 180^o$.
   $A = 180^o – (69.25^o + 114.5^o)$
   $A = 180^o – 183.75^o$
   $A = 3.75^o$

3. $A = 56^o38'$, $C = 64^o49'$.

   $A + B + C = 180^o$.
   $56^o38' + B + 64^o49'^o = 180^o$.
   $B = 180^o – (56^o38' + 64^o49')$
   $B = 180^o – 121^o27'$
   $B = 58^o33'$

Try this out!

Given two of the angles of $\triangle ABC$, solve for the measure of the third angle.

Set A

1. $A = 60^o$, $B = 45^o$
2. $B = 76^o$, $C = 64^o$
3. $A = 69.25^o$, $C = 14.5^o$

4. $B = 19.15^{\circ}$, $A = 107.05^{\circ}$
5. $C = 85^{\circ}38'$, $A = 74^{\circ}23'$

Set B

1. $A = 57^{\circ}$, $B = 118^{\circ}$
2. $B = 46^{\circ}$, $C = 69^{\circ}$
3. $A = 51.63^{\circ}$, $C = 70.47^{\circ}$
4. $B = 37.3^{\circ}$, $A = 77.24^{\circ}$
5. $C = 58^{\circ}6'$, $A = 47^{\circ}24'$

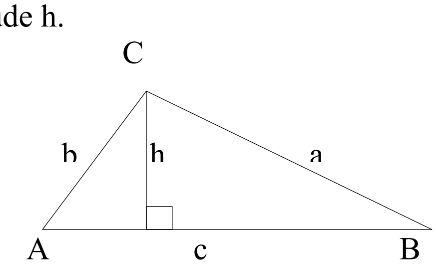
Set C

1. $A = 58^{\circ}$, $B = 77^{\circ}$
2. $B = 64^{\circ}$, $C = 64^{\circ}$
3. $A = 18.82^{\circ}$, $C = 109.3^{\circ}$
4. $B = 56.85^{\circ}$, $A = 37.18^{\circ}$
5. $C = 27^{\circ}34'$, $A = 72^{\circ}43'$

Lesson 2

Deriving the Law of Sines;
Solving Oblique Triangles Involving Two Angles and a Side Opposite One of Them

Consider $\triangle ABC$ with altitude h.



The area of this triangle is

$$\text{Area} = \frac{1}{2}ch \qquad (1)$$

From the right triangle

$$\sin A = \frac{h}{b} \qquad (2)$$

Solving h in (2) gives

$$H = b\sin A \qquad (3)$$

Substituting (3) in (1) gives

$$\text{Area} = \frac{1}{2}c(b\sin A) \qquad (4)$$

Similarly, two other formulas for the area of $\triangle ABC$ can be derived, namely:

$$\text{Area} = \frac{1}{2}a(c\sin B) \qquad (5)$$

and

$$\text{Area} = \frac{1}{2}b(a\sin C) \qquad (6)$$

Since (4), (5), and (6) represent the area of one and the same triangle, then

$$\frac{1}{2}c(b\sin A) = \frac{1}{2}a(c\sin B) = \frac{1}{2}b(a\sin C)$$

or equivalently

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

This is the Law of Sines which states that "In any triangle ABC, with a, b, and c as its sides, and A, B and C as its angles, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$."

Note that this is also equivalent to

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

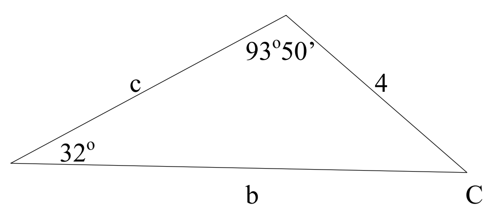
The Law of Sines is used to solve different cases of oblique triangles. One case of oblique triangle that can be solved using the law of sines involves two angles and a side opposite one of them.

Examples:

Solve each triangle ABC described below.

1. $a = 4$, $A = 32^o$, $B = 93^o50'$

   The given parts of the triangle involve two angles A and B, and side a which is opposite A. The given parts and the unknown parts are shown in the figure below.



To solve for C:

$$A + B + C = 180^o$$
$$32^o + 93^o50' + C = 180$$
$$C = 180^o - (32^o + 93^o50')$$
$$C = 180^o - 125^o50'$$
$$C = 54^o10'$$

To solve for b:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$b = \frac{a\sin B}{\sin A}$$
$$b = \frac{4(\sin 93°50')}{\sin 32°}$$

$$b = \frac{4(0.99776)}{0.52992}$$

$$b = 7.53$$

To solve for c:

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$c = \frac{a \sin C}{\sin A}$$

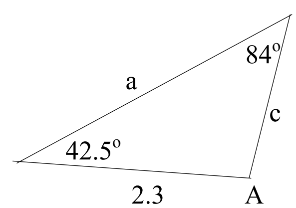
$$c = \frac{4(\sin 54°10')}{\sin 32°}$$

$$b = \frac{4(0.81072)}{0.52992}$$

$$b = 6.12$$

2. $b = 2.3$, $C = 42.5^o$, $B = 84^o$

The given parts of the triangle involve two angles B and C, and side b which is opposite B. The given parts and the unknown parts are shown in the figure below.



To solve for A:

$$A + B + C = 180^o$$

$$A + 84^o + 42.5^o = 180^o$$

$$A = 180^o - (84^o + 42.5^o)$$

$$A = 180^o - 126.5^o$$

$$A = 53.5^o \text{ or } 53^o30'$$

To solve for c:

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

$$c = \frac{b \sin C}{\sin B}$$

$$c = \frac{2.3(\sin 42.5°)}{\sin 84°}$$

$$c = \frac{2.3(0.67559)}{0.99452}$$

$$c = 1.6$$

To solve for a:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$a = \frac{b \sin A}{\sin B}$$

$$a = \frac{2.3(\sin 53°30')}{\sin 84°}$$

$$a = \frac{2.3(0.80386)}{0.99452}$$

$$a = 1.86$$

Try this out!

Solve each triangle ABC described below.

Set A

1. $a = 12$, $A = 60^o$, $B = 45^o$
2. $b = 20$, $B = 32^o$, $C = 24^o$
3. $c = 42.5$, $A = 35.2^o$, $C = 29^o$
4. $a = 9$, $A = 42.4^o$, $C = 37.8^o$
5. $b = 75$, $B = 30^o45'$, $C = 32^o30'$

Set B

1. $a = 10$, $A = 61^o$, $B = 49^o$
2. $c = 80$, $C = 56^o$, $B = 68^o$
3. $b = 38$, $B = 47^o$, $A = 30^o$
4. $c = 8$, $C = 77.2^o$, $A = 88^o$
5. $b = 8$, $B = 67^o15'$, $A = 33^o5'$

Set C

1. $A = 65^o50'$, $B = 14^o$, $b = 16$
2. $A = 69^o15'$, $C = 28^o40'$, $a = 80$
3. $B = 47.5^o$, $A = 80^o$, $a = 15.1$
4. $B = 63^o36'$, $C = 89^o$, $b = 8$
5. $C = 57.25^o$, $A = 33.5^o$, $c = 23.5$

Lesson 3

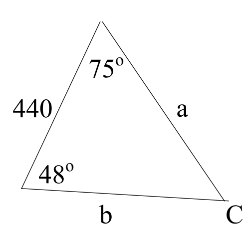
Solving Oblique Triangles Involving Two Angles and the Included Side

Another case of oblique triangle that can be solved using the law of sines involves two angles and the included side.

Examples:

1. $c = 440$, $A = 48^o$, $B = 75^o$

   The given parts of the triangle involve two angles and the included side. This case can also be solved using the law of sines. The given parts and the unknown parts are shown in the figure below.



To solve for C:

$$A + B + C = 180^o$$
$$48^o + 75^o + C = 180$$
$$C = 180^o – (48^o + 75^o)$$
$$C = 180^o – 123^o$$
$$C = 57^o$$

To solve for a:

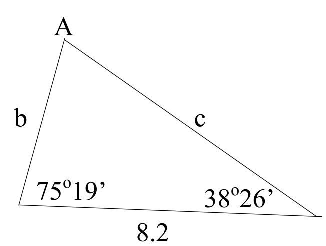
$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$a = \frac{c \sin A}{\sin C}$$
$$a = \frac{440(\sin 48°)}{\sin 57°}$$
$$a = \frac{440(0.74315)}{0.83867}$$
$$a = 389.87$$

To solve for b:

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$b = \frac{c \sin B}{\sin C}$$
$$b = \frac{440(\sin 48°)}{\sin 57°}$$
$$b = \frac{440(0.74314)}{0.96593}$$
$$b = 338.51$$

2. $a = 8.2$, $B = 38^o26'$, $C = 75^o19'$

    The given parts of the triangle involve two angles and the included side. This case can also be solved using the law of sines. The given parts and the unknown parts are shown in the figure below.



To solve for A:

$$A + B + C = 180^o$$
$$A + 38^o26' + 75^o19' = 180$$
$$A = 180^o – (38^o26' + 75^o19')$$
$$C = 180^o – 113^o45'$$
$$C = 66^o15'$$

To solve for b:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$b = \frac{a \sin B}{\sin A}$$

$$b = \frac{8.2(\sin 38°26')}{\sin 75°19'}$$

$$b = \frac{8.2(0.62160)}{0.96734}$$

$$b = 5.27$$

To solve for c:

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$c = \frac{a \sin C}{\sin A}$$

$$c = \frac{8.2(\sin 66°15')}{\sin 75°19'}$$

$$c = \frac{8.2(0.91531)}{0.96734}$$

$$c = 7.76$$

Try this out!

Solve each triangle ABC described below.

Set A

1. a = 12, A = 60^o, B = 45^o
2. b = 20, A = 32^o, C = 24^o
3. c = 42.5, A = 35.2^o, B = 79^o
4. a = 9, A = 42.4^o, B = 37.8^o
5. b = 0.751, A = $100^o30'$, C = $32^o45'$

Set B

1. a = 248, A = 51^o, B = 49^o
2. c = 8.5, A = 56^o, B = 68^o
3. b = 38, A = 47.2^o, C = 50.5^o
4. a = 8, B = $77^o26'$, C = $28^o40'$
5. c = 1.2, A = 67.3^o, B = 33.9^o

Set C

1. A = $65^o50'$, B = 14^o, c = 16
2. B = 69.2^o, C = 28.8^o a = 0.8
3. A = $15^o28'$, C = 77^o, b = 100
4. a = 1.8, B = $36^o36'$, C = 98^o
5. c = 23.5, B = 57.25^o, A = $33^o33'$

Lesson 4

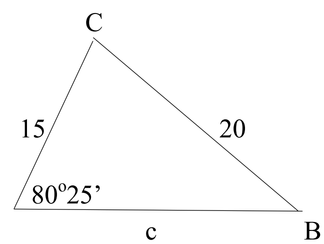
Solving Oblique Triangles Involving Two Sides and an Angle Opposite One of Them

Another case of oblique triangle that can be solved using the law of sines involves two sides and an angle opposite one of them.

Examples:

1. b = 15, a = 20, A = $80^o25'$

   The given parts and the unknown parts are shown in the figure below.



To solve for B:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\sin B = \frac{b \sin A}{a}$$

$$\sin B = \frac{15(\sin 80°25')}{20}$$

$$\sin B = \frac{15(0.98604)}{20}$$

$$\sin B = 0.73953$$

$$B = \text{Arcsin}\ (0.73953)$$

$$B = 47^o41'29''$$

To solve for C:

$$A + B + C = 180^o$$

$$80^o25' + 47^o41' + C = 180$$

$$C = 180^o - (80^o25' + 47^o41')$$

$$C = 180^o - 128^o6'$$

$$C = 51^o54'$$

To solve for c:

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$c = \frac{a \sin C}{\sin A}$$

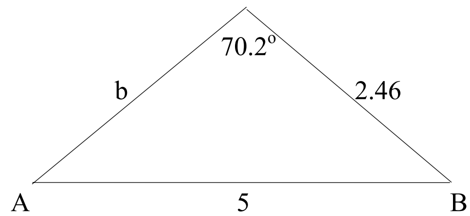
$$c = \frac{20(\sin 51°54')}{\sin 80°25'}$$

$$b = \frac{20(0.78694)}{0.98604}$$

$$b = 15.96$$

2. a = 2.46, c = 5, C = 70.2°

The given parts and the unknown parts are shown in the figure below.



To solve for A:

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\sin A = \frac{a \sin C}{c}$$

$$\sin A = \frac{2.46(\sin 70.2°)}{5}$$

$$\sin A = \frac{2.46(0.94088)}{5}$$

$$\sin A = 0.46291$$

$$A = \text{Arcsin}\ (0.46291)$$

$$A = 27.58^o$$

To solve for B:

$$A + B + C = 180^o$$

$$27.58^o + B + 70.2^o = 180$$

$$B = 180^o – (27.58^o + 70.2^o)$$

$$B = 180^o – 97.78^o$$

$$B = 82.22^o$$

To solve for b:

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$b = \frac{c \sin B}{\sin C}$$

$$b = \frac{5(\sin 82.22°)}{\sin 70.2°}$$

$$b = \frac{5(0.99080)}{0.94088}$$

$$b = 5.27$$

Try this out!

Solve each triangle ABC described below.

Set A

1. $a = 12$, $b = 18$, $B = 45^o$
2. $b = 20$, $c = 32$, $B = 24^o$
3. $a = 2.5$, $c = 5.2$, $C = 79^o$
4. $a = 9$, $b = 14.4$, $B = 37.8^o$
5. $b = 0.751$, $c = 1$, $C = 32^o45'$

Set B

1. $a = 140$, $b = 150$, $B = 49^o$
2. $a = 8.5$, $c = 5.6$, $A = 68^o$
3. $a = 47.2$, $b = 38$, $A = 50.5^o$
4. $a = 0.8$, $c = 0.72$, $C = 28^o40'$
5. $c = 1.2$, $b = 6.3$, $B = 131.5^o$

Set C

1. $A = 65^o50'$, $a = 15$, $c = 16$
2. $B = 95.2^o$, $b = 2.8$, $c = 0.8$
3. $C = 15^o28'$, $c = 77$, $b = 100$
4. $B = 36^o36'$, $a = 1.8$, $b = 2.98$
5. $A = 33^o33'$, $a = 33.5$, $b = 57.25$

Lesson 5

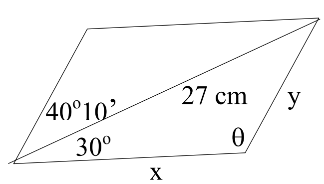
Solving Problems Involving Oblique Triangles Using The Law of Sines

The following examples show how the law of sines is used to solve word problems involving oblique triangles.

Examples:

Solve each problem. Show a complete solution.

1. The longer diagonal of a parallelogram is 27 cm. Find the perimeter of the parallelogram if the angles between the sides and the diagonal are 30^o and $40^o10'$.



The perimeter of any parallelogram is twice the sum of the lengths of the two consecutive sides.

Let x and y be the consecutive sides as shown in the figure. The angle opposite x is $40^o10'$. Hence, the angle θ between x and y is

$$30^o + 40^o10' + \theta = 180^o$$

$$\theta = 180^o - (30^o + 40^o10')$$

$\theta = 180^o - 70^o10'$
$\theta = 109^o50'$

To solve for x:

$$\frac{x}{\sin 40°10'} = \frac{27}{\sin 109°50'}$$
$$x = \frac{27 \sin 40°10'}{\sin 109°50'}$$
$$x = \frac{27(0.64501)}{0.94068}$$
$$x = 18.51 \text{ cm}$$

To solve for y:

$$\frac{y}{\sin 30°} = \frac{27}{\sin 109°50'}$$
$$y = \frac{27 \sin 30°}{\sin 109°50'}$$
$$y = \frac{27(0.5)}{0.94068}$$
$$x = 14.35 \text{ cm}$$

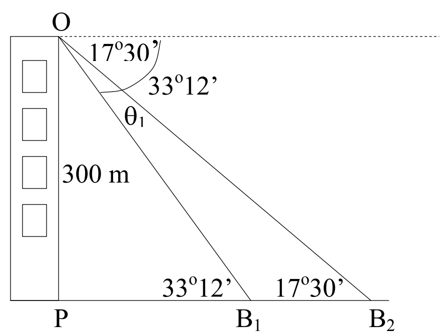
The perimeter P of the parallelogram is

$P = 2(x + y)$
$P = 2(18.51 + 14.35)$
$P = 2(32.86)$
$P = 65.72$ cm

2. From the top of a 300 m building, the angle of depression of two traffic aides on the street are $17^o30'$ and $33^o12'$, respectively. If they are due south of the observation point, find the distance between them.

Let B_1B_2 be the distance between the two traffic aides.

First, find θ_1:



$\theta_1 = 33^o12' - 17^o30'$
$\theta_1 = 15^o42'$

Then, find OB_1:

$$\cos 33°12' = \frac{300}{OB_1}$$
$$OB_1 = \frac{300}{\cos 33°12'}$$

$$OB_1 = \frac{300}{0.83676}$$

$$OB_1 = 358.53$$

To solve for B_1B_2:

$$\frac{B_1B_2}{\sin 15°42'} = \frac{OB_1}{\sin 17°30'}$$

$$B_1B_2 = \frac{OB_1(\sin 15°42')}{\sin 17°30'}$$

$$B_1B_2 = \frac{358.53(0.27060)}{0.30071}$$

$$B_1B_2 = 322.63 \text{ m}$$

Try this out!

Solve each problem. Show a complete solution.

Set A

1. The shorter diagonal of a parallelogram is 5.2 m. Find the perimeter of the parallelogram if the angles between the sides and the diagonal are 40^o and $30^o10'$.
2. From the top of a 150 m lighthouse, the angles of depression of two boats on the shore are 20^o and 50^o, respectively. If they are due north of the observation point, find the distance between them.
3. Two policemen 122 meters apart are looking at a woman on top of a tower. One cop is on the east side and the other on the west side. If the angles of elevation of the woman from the cops are 42.5^o and 64.8^o, how far is she from the two cops?
4. The angle between Rizal St. and Bonifacio St. is 27^o and intersect at P. Jose and Andres leaves P at the same time. Jose jogs at 10 kph on Rizal St. If he is 3 km from Andres after 30 minutes, how fast is Andres running along Bonifacio St?
5. The angle of elevation of the top of a tower is $40^o30'$ from point X and 55^o from another point Y. Point Y is 30 meters from the base of the tower. If the base of the tower and points X and Y are on the same level, find the approximate distance from X and Y.

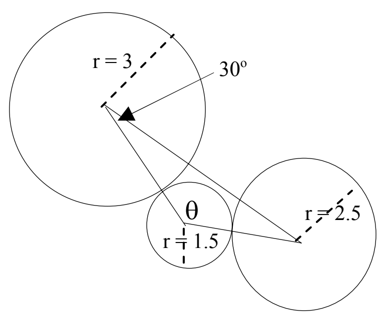
Set B

1. The vertex of an isosceles triangle is 40^o. If the base of the triangle is 18 cm. find its perimeter.
2. An 80-meter building is on top of a hill. From the top of the building, a nipa hut is sighted with an angle of depression of 54^o. From the foot of the building the same nipa hut was sighted with an angle of depression of 45^o. Find the height of the hill and the horizontal distance from the building to the nipa hut.
3. The lengths of the diagonals of a parallelogram are 32.5 cm and 45.2 cm. The angle between the longer side and the longer diagonal is 35^o. Find the length of the longer side of the parallelogram.

4. A diagonal of a parallelogram is 35 cm long and forms angles of 34^o and 43^o with the sides. Find the perimeter and area of the parallelogram.
5. A wooden post is inclined 85^o with the ground. It was broken by a strong typhoon. If a broken part makes an angle of 25^o with the ground, and the topmost part of the post is 40 ft from its base, how long was the post?

Set C

1. The measures of two of the angles of a triangle are 50^o and 55^o. If its longest side measures 17 cm, find the perimeter of the triangle.
2. Bryan's (B) and Carl's (C) houses are along the riverbank. On the opposite side is Angel's (A) house which is 275 m away from Carl's. The angles CAB and ACB are measured and are found to be 125^o and 49^o, respectively. Find the distance between Angel's and Bryan's houses.
3. A park is in the shape of an obtuse triangle. One angle is 45^o and the opposite side is 280-m long and the other angle is 40^o. Find the perimeter of the park.
4. Three circles are arranged as shown in the figure. Their centers are joined to form a triangle. Find angle θ.



5. Two forces are acting on each other at point A at an angle 50^o. One force has a magnitude of 60 lbs. If the resultant force is 75 lbs find the magnitude of the second force.

Let's summarize

1. An oblique triangle is a triangle which does not contain a right angle. It contains either three acute angles (acute triangle) or two acute angles and one obtuse angle (obtuse triangle).
2. If two of the angles of an oblique triangle are known, the third angle can be computed. The sum of the interior angles in a triangle is 180^o.
3. In any triangle ABC, with a, b, and c as its sides, and A, B and C as its angles,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

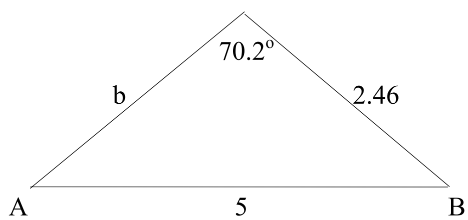
This is also equivalent to

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

4. The Law of Sines is used to solve different cases of oblique triangles. This law is used to solve oblique triangles that involve
   a. two angles and a side opposite one of them.
   b. two angles and the included side
   c. two sides and the angle opposite one of them

What have you learned?

1. Find the measure of ∠B in △ABC if m∠A = 8^o and m∠C = 97^o.
2. Find the measure of ∠A in △ABC if m∠B = $18^o14'$ and m∠C = $81^o41'$.
3. The measures of the side of △ABC are c = 83 cm and b = 59 cm and a = 45 cm. Which is the largest angle?
4. What equation that involves sin 70.2^o will be used to find angle A?



5. Given a = 62.5 cm, m∠A = $112^o20'$ and m∠C = $42^o10'$. Solve for side b.
6. Solve for the perimeter of △ABC if c = 25 cm, m∠A = $35^o14'$ and m∠B = 68^o.
7. Solve △ABC if b = 38.12 cm, m∠B = $46^o32'$ and m∠A = $79^o17'$.
8. Solve △ABC if b = 67.25 mm, c = 56.92 mm and m∠B = $65^o16'$.
9. From the top of a building 300 m high, the angles of depression of two street signs are 17.5^o and 33.2^o. If the street signs are due south of the observation point, find the distance between them.
10. A park is in the shape of an acute triangle. One angle is 55^o and the opposite side is 300-m long and the other angle is 60^o. Find the perimeter of the park.

Answer Key

How much do you know?

1. 75^o
2. $80^o5'$
3. c
4. $\frac{b}{\sin 93°50'} = \frac{4}{\sin 32°}$
5. 68.32 cm
6. 14.82 cm

7. B = $54^o11'$, b = 42.59 cm, c = 51.61 cm
8. C = $50^o14'30''$, A = $64^o29'30''$, a = 66.83 mm
9. 155.45 m
10. 26.73 m

Lesson 1

Set A

1. 75^o
2. 40^o
3. 96.25^o
4. 53.8^o
5. $19^o59'$

Set B

1. 5^o
2. 65^o
3. 57.9^o
4. 65.46^o
5. $74^o30'$

Set C

1. $45\,^o$
2. $52\,^o$
3. 51.88^o
4. 85.97^o
5. $79^o43'$

Lesson 2

Set A

1. C = 75^o, b = 9.80, c = 13.38
2. A = 124^o, a = 31.29, c = 15.35
3. B = 115.8^o, a = 50.53, b = 78.92
4. B = 99.8^o, b = 13.15, c = 8.18
5. A = $116^o45'$, a = 130.99, c = 78.81

Set B

1. C = 70^o, b = 8.63, c = 10.74
2. A = 56^o, a = 80, b = 89.47
3. C = 103^o, a = 25.98, c = 50.63
4. B = 14.8^o, a = 8.20, b = 2.10
5. C = $79^o40'$, a = 4.74, c = 8.53

Set C

1. C = $100^o10'$, a = 60.34, c = 65.10
2. B = $82^o5'$, b = 84.73, c = 41.04
3. C = 52.5^o, b = 11.30, c = 12.16

4. $A = 27^o24'$, a = 4.11, c = 8.93
5. $B = 89.25^o$, a = 15.42, b = 27.94

Lesson 3
Set A
1. $C = 75^o$, b = 9.80, c = 13.38
2. $B = 123^o$, a = 12.64, c = 9.70
3. $C = 65.8^o$, a = 26.84, b = 45.74
4. $C = 99.8^o$, b = 8.18, c = 13.15
5. $B = 46^o45'$, a = 1.014, c = 0.558

Set B
1. $C = 80^o$, b = 240.84, c = 314.27
2. $C = 56^o$, a = 8.50, b = 9.51
3. $B = 82.3^o$, a = 28.14, c = 29.59
4. $A = 73^o54'$, b = 8.13, c = 3.99
5. $C = 79^o33'$, a = 1.13, b = 0.68

Set C
1. $C = 100^o$, a = 14.83, b = 14.83
2. $A = 82^o$, b = 0.76, c = 0.39
3. $B = 87^o32'$, a = 26.69, c = 97.53
4. $A = 45^o24'$, b = 1.51, c = 2.50
5. $C = 89^o12'$, a = 13.00, b = 19.77

Lesson 4
Set A
1. $A = 28^o7'32''$, $C = 106^o52'28''$, c = 10.83
2. $A = 115^o23'48''$, $C = 40^o36'12''$, a = 44.42
3. $A = 28^o9'36''$, $B = 72^o50'24''$, b = 5.06
4. $A = 22^o31'56''$, $C = 119^o40'4''$, c = 20.41
5. $A = 123^o16'45''$, $B = 23^o58'15''$, a = 1.55

Set B
1. $A = 44^o46'51''$, $C = 86^o13'9''$, c = 198.32
2. $B = 74^o20'56''$, $C = 37^o39'4''$, b = 8.83
3. $B = 38^o24'20''$, $C = 91^o5'40''$, c = 61.16
4. $A = 32^o12'34''$, $B = 119^o7'26''$, b = 1.31
5. $A = 40.3^o$, C = 8.2, a = 5.44

Set C
1. $B = 37^o27'57''$, $C = 76^o42'3''$, b = 10.00
2. $A = 71.21^o$, $C = 16.59^o$, a = 0.76
3. $A = 144^o16'12''$, $B = 20^o15'48''$, a = 168.61
4. $A = 21^o6'31''$, $C = 122^o17'29''$, c = 4.23
5. $B = 70^o49'4''$, $C = 75^o37'56''$, c = 58.72

Lesson 5

Set A

1. The perimeter is 12.66 m.
2. The distance between them is approximately 286.26 m.
3. The distances of the woman from the two policemen are approximately 86.33 m and 115.62 m.
4. Andres ran approximately at the rate of 3.21 kph.
5. The distance between X and Y is approximately 20.16 m.

Set B

1. The perimeter of the isosceles triangle is approximately 70.62 cm.
2. The horizontal distance of the building to the nipa hut is 212.55 m.
3. The longer side is 28.31 cm.
4. The perimeter of the parallelogram is 89.18 cm and its area is 1,962.21 sq.cm.
5. The wooden post is 60.40 ft long.

Set C

1. The perimeter is 44.90 cm.
2. The distance between Angel's and Bryan's houses is 1,985.54 m.
3. The perimeter of the park is 929 m.
4. The angle θ is $115^o46' 12''$
5. The second force is approximately 97.83 lbs.

How much have you learned?

1. 75^o
2. $80^o5'$
3. Angle C
4. $\frac{\sin A}{2.46} = \frac{\sin 70.2°}{5}$
5. 29.20 cm
6. 63.63 cm
7. $C = 54^o11'$, a = 51.61 cm, c = 42.59 cm
8. $C = 50^o14'30''$, $A = 64^o29'30''$, a = 66.83 mm
9. The distance between the signs is 493.03 m.
10. The perimeter is 949.09 m