Module 1
Triangle Trigonometry

What this module is about

This module will guide you to determine the kind of equation you will use to solve the missing parts of a right triangle. This will require the use of trigonometric functions. Here, you will also learn how to solve problems involving right triangles.

What you are expected to learn

This module is designed for you to:

1. determine the equation in solving the missing parts of a right triangle.

2. Apply trigonometric functions to solve right triangle given:

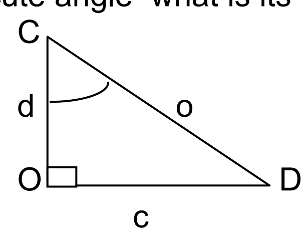
   a. the length of the hypotenuse and length of one leg
   b. the length of the hypotenuse and one of the acute angles
   c. the length of one leg and one of the acute angles
   d. the length of both sides

3. Solve problems involving right triangle.

How much do you know

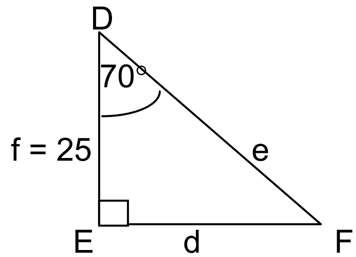
In rt. ΔCOD angled at O, if $\angle$D is an acute angle what is its

1. opposite side
2. adjacent side
3. hypotenuse



Give the equation that can be used to find the required parts of right triangle DEF.

If m $\angle D = 70$ and f = 25, find:

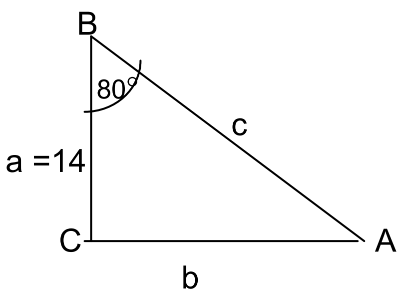


4. d
5. e

Using another acute $\angle F$, if $m\angle F = 60°$ and d = 20 , find

6. e
7. f

Solve right ΔBCA, If $\angle B = 80°$, and a = 14,



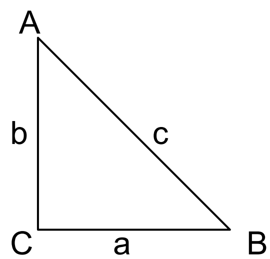
find:

8. b
9. c
10. $\angle A$

Lesson 1

Determine the Equation in Solving the Missing Parts of a Right Triangle

Consider the right triangle ACB below.



The hypotenuse of the triangle is *c*.
The side opposite angle A is *a*.
The side adjacent to angle A is side *b*.

SOH-CAH-TOA is a mnemonic used for remembering the equations:

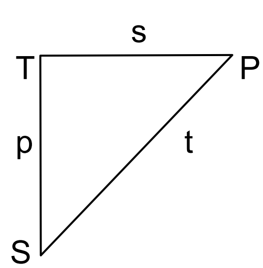
Sin = $\frac{\text{Opposite}}{\text{Hypotenuse}}$

Cos = $\frac{\text{Adjacent}}{\text{Hypotenuse}}$

Tan = $\frac{\text{Opposite}}{\text{Adjacent}}$

Examples:

1. Identify the opposite and adjacent side as well as the hypotenuse of right triangle STP for any acute angle of the right Δ.



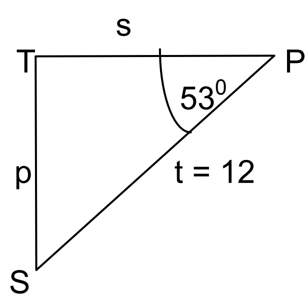
a. Using $\angle$P as the acute angle:

p – is the opposite side
s – is the adjacent side
t – is the hypotenuse of rt. Δ STP

b. Using $\angle$S as the acute angle:

s - is the opposite side
p – is the adjacent side
t – is the hypotenuse

2. Without solving, determine the equation for the missing parts of a rt. Δ.



Given: $\angle P = 53^0$ and t = 12.

a. Solve for s:

Solution: $\angle$P is the acute angle, *t* is the length of the hypotenuse, *s* is the length of the adjacent side of $\angle$P, we can use CAH.

$$\cos P = \frac{s}{t}$$

$$\cos 53^0 = \frac{s}{12}$$

$$s = 12 \cos 53^0$$ the required equation

b. Solve for p:

Solution: $\angle$P is the acute angle , *t* is the hypotenuse, and *p* is the length of the opposite side of $\angle$P, we can use SOH.

$\sin P = \frac{p}{t}$

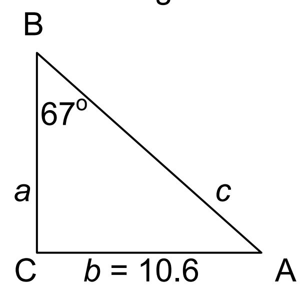
$\sin 53^0 = \frac{p}{12}$

$p = 12 \sin 53^0$ the required equation

3. In the figure, if $m\angle B = 67^0$ and b = 10.6 cm

a. Solve for *a*:

Solution: $\angle B$ is the acute angle, *b* is the opposite side and *a* is the adjacent side of the given acute angle. Use TOA.



$\tan B = \frac{b}{a}$

$\tan 67^0 = \frac{10.6}{a}$

$a \tan 67^0 = 10.6$

$a = \frac{10.6}{\tan 67^o}$ the required equation

b. Solve for *c*:

Solution: $\angle B$ is the acute angle, *b* is the opposite side and *c* is the hypotenuse of the given acute angle, we can use SOH.

$\sin B = \frac{b}{c}$

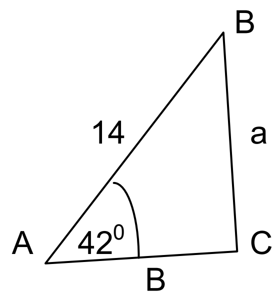
$\sin 67^0 = \frac{10.6}{c}$

$c \sin 67^0 = 10.6$

$c = \frac{10.6}{\sin 67^o}$ the required equation

Try this out

Using the figure below, write the equations that would enable you to solve each problem.



1. If A = 15 and c = 37, find a.
2. If A = 76 and a = 13, find b.
3. If A = 49 13’ and a = 10, find c.
4. If a = 21.2 and A = 71 13’, find b.
5. If a = 13 and B = 16 , find c.
6. If A = 19 07’ and b = 11, find c.
7. If c = 16 and a = 7, find b.
8. If b = 10 and c = 20, find a.
9. If a = 7 and b = 12, find A.
10. If a = 8 and c = 12, find B.

Lesson 2

Solve right triangle given the length of the hypotenuse and length of one leg

To solve right triangle means to find the measures of other angles and sides of a triangle. In order to avoid committing errors, maximize the use of the given values of the parts of the right triangle.

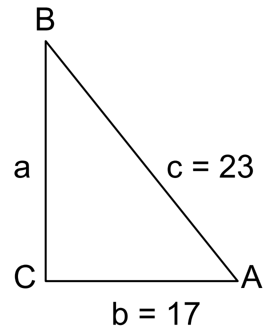
Example:

In right triangle BCA angled at C, if c = 23 and b = 17, find ∠ A, ∠B and a.

(*note: use scientific calculator for values of trigonometric functions)*

Solution:

Sketch the figure:



a. Solve for ∠A :

From the given, b is the adjacent side, c is the hypotenuse of a right ΔBCA, so use CAH.

Cos A = $\frac{b}{c}$

Cos A = $\frac{17}{23}$

Cos A = 0.7391

A = 42^0 20’

b. Solve for ∠B:

To solve for ∠B, make use of the given parts, *b* as the opposite side of ∠B and *c* the hypotenuse. We can use SOH.

Sin B = $\frac{b}{c}$

Sin B = $\frac{17}{23}$

sin B = 0.7391

B = 47°40’

c. Solve for a:

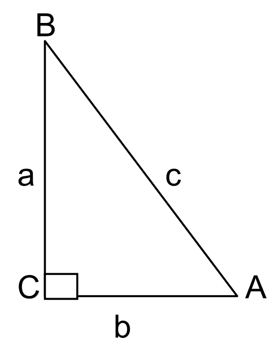
Using the Pythagorean theorem:

$$a^2 + b^2 = c^2$$
$$a^2 + (17)^2 = 23^2$$
$$a^2 + 289 = 529$$
$$a^2 = 529 - 289$$
$$a = \sqrt{240}$$
$$a = 15.49$$

Try this out

In the given figure, solve for each right triangle ACB, given the following:

1. If b = 17 cm, c = 23 cm
   Find : a, $\angle A$, $\angle B$
2. If c = 16 and a = 7
   Find : b, $\angle A$, $\angle B$
3. If b = 10 and c = 20
   Find : a, $\angle A$, $\angle B$
4. If b = 6 and c = 13
   Find : a, $\angle A$, $\angle B$
5. If c = 13 and a = 12
   Find : b, $\angle A$, $\angle B$

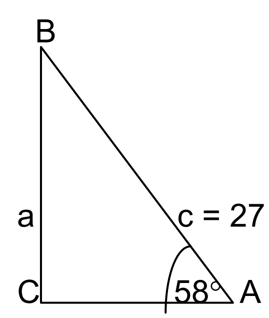


Lesson 3

Solve Right Triangle Given the Length of the Hypotenuse and the Measure of One Acute Angle

In right triangle BCA angled at C if c = 27 and $\angle A = 58^0$, find $\angle B$, b, a.

Solution:



a. Solve for $\angle B$:

In rt. ΔBCA, $\angle B$ and $\angle A$ are complementary angles.

$$\angle B + \angle A = 90^0$$
$$\angle B = 90^0 - 58^0$$
$$\angle B = 32^0$$

b. Solve for b:

Since *b is* the adjacent side of $\angle A$ and *c* is the hypotenuse of rt. ΔBCA , we can use CAH:

$$\cos A = \frac{b}{27}$$
$$\cos 58^0 = \frac{b}{27}$$
$$b = 27 \cos 58^0$$
$$b = 27\ (0.5299)$$
$$b = 14.31$$

c. Solve for a:

Since *a* is the opposite side of $\angle A$ and *c* is the hypotenuse of rt. ΔBCA then, we can use SOH:

$$\sin A = \frac{a}{27}$$
$$\sin 58^0 = \frac{a}{27}$$
$$a = 27 \sin 58^0$$
$$a = 27(0.8480)$$
$$a = 22.9$$

Try this out

Sketch the figure and solve each right Δ ACB angled at C if given the following:

1. If A = 15 and c = 37
   Find : B, a, b

2. If B = 64 and c = 19.2
   Find : A, a, b

3. If A = 15 and c = 25
   Find: B, a, b

4. If A = 45 and c = $7\sqrt{2}$
   Find : B, a, b

5. If B = 55^0 55' and c = 16
   Find : A, a, b

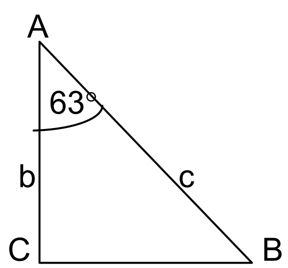
Lesson 4

Solve Right Triangle Given the Length of One Leg and the Measure of One Acute Angle

Example:

In rt. ACB angled at C, if $\angle A = 63^0$ and a = 11 cm, find $\angle B$, b, c.

Solution:



a. Solve for $\angle B$:

In rt. Δ ACB, $\angle B$ and $\angle A$ are complementary angles.

$\angle B + \angle A = 90^0$

$\angle B = 90^0 - 63^0$

$\angle B = 27^0$

b. Solve for b:

Use TOA, *b* is the adjacent side of ∠A and *a* is the opposite side of ∠A.

$$\tan A = \frac{a}{b}$$

$$\tan 63^0 = \frac{11}{b}$$

$$b \tan 63^0 = 11$$

$$b = \frac{11}{\tan 63^o}$$

$$b = \frac{11}{1.9626}$$

$$b = 5.60 \text{ cm}$$

c. Solve for c:

Use SOH. *c* is the hypotenuse of a rt. Δ and *a* is the opposite side of ∠A.

$$\text{Sin } A = \frac{a}{c}$$

$$\text{Sin } 63^0 = \frac{11}{c}$$

$$c \text{ Sin } 63^0 = 11$$

$$c = \frac{11}{\sin 63^o}$$

$$c = \frac{11}{0.8910}$$

$$c = 12.35 \text{ cm}$$

Try this out

Sketch the figure and solve each right Δ ACB angled at C given the following:

1. If A = 76 and a = 13
   Find : B, b, c

2. If A = 22^0 22' and b = 22
   Find : B, a, c

3. If B = 30 and b = 11
   Find : A, a, c

4. If B = 18 and a = $\sqrt{18}$
   Find: A, b, c

5. If A = 77 and b = 42
   Find : B, a, c

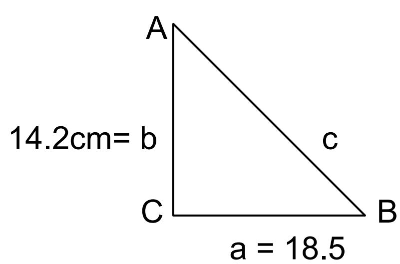
Lesson 5

Solve Right Triangle Given the Length of Both Sides

Example:

In rt. Δ ACB angled at C, a = 18.5 cm and b = 14.2 cm, find c, ∠A, ∠B.

Solution:



a. Solve for c:

   Use Pythagorean theorem:

$$a^2 + b^2 = c^2$$
$$(18.5)^2 + (14.2)^2 = c^2$$
$$342.25 + 201.64 = c^2$$
$$543.89 = c^2$$
$$23.32 = c$$

b. Solve for ∠A:

Use TOA, since *a* and *b* are opposite & adjacent side of ∠A respectively.

$$\text{Tan A} = \frac{a}{b}$$

$$\text{Tan A} = \frac{18.5}{14.2}$$

Tan A = 1.3028

A = 52^0 29'

c. Solve for ∠B:

Use TOA again because **b** opposite side and **a** adjacent side of ∠B.

$$\text{Tan B} = \frac{b}{a}$$

$$\text{Tan B} = \frac{14.2}{18.5}$$

Tan B = 0.7676

B = 37^0 31'

Try this out

Sketch the figure and solve each right triangle given the following:

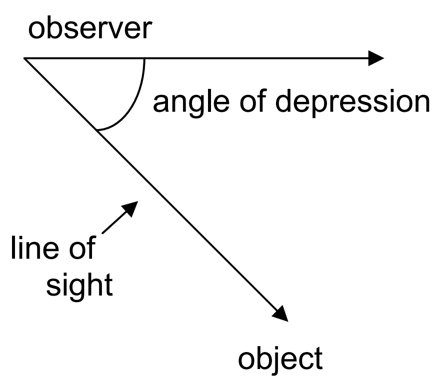
1. If a = 15.8, b = 21
   Find : ∠A, ∠B, c
2. If a = 7 and b = 12
   Find: ∠A, ∠B, c
3. If a = 2 and b = 7
   Find : ∠A, ∠B, c
4. If a = 3 and b = √3
   Find : ∠A, ∠B, c
5. If a = 250 and b = 250
   Find : ∠A, ∠B, c

Lesson 6

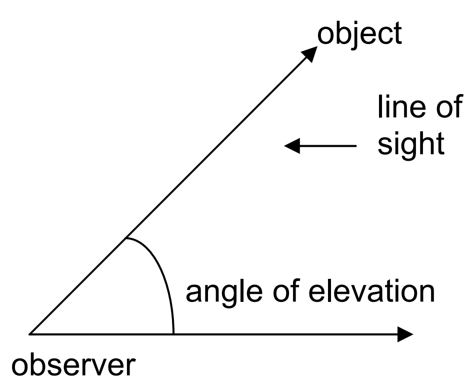
Problem Solving

Solving problems involving right triangles require knowledge of some terms of importance in a particular field. For instance, in surveying the term line of sight, angle of elevation, and angle of depression are frequently used. So we start with familiarizing in these terms.

Line of sight – is an imaginary line that connects the eye of an observer to the object being observed. If the observer is in a higher elevation than the object of observation, the acute angle measured from the eye level of the observer to his line of sight is called the *angle of depression*.

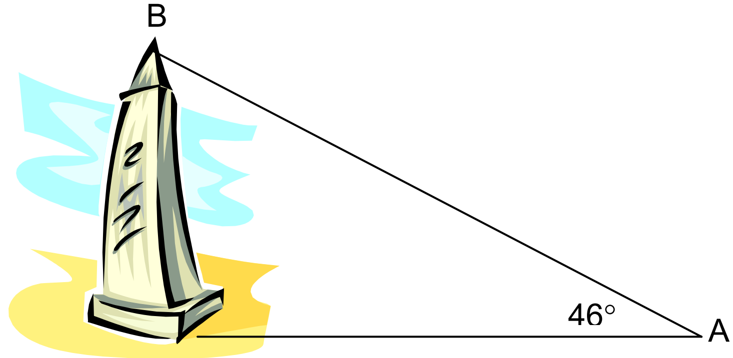


On the other hand, if the situation is reversed, that is, the observer is at the lower elevation than the object being observed, the acute angle made by the line of sight and the eye level of the observer is called the *angle of elevation*.



Examples:

1. Two hikers are 400 meters from the base of the radio tower. The measurement of the angle of elevation to the top of the tower is 46°. How high is the tower?



Solution:

Use the mnemonic TOA. *x* is the opposite side and *b* is adjacent side of ∠A.

$$\text{Tan } 46° = \frac{x}{400}$$

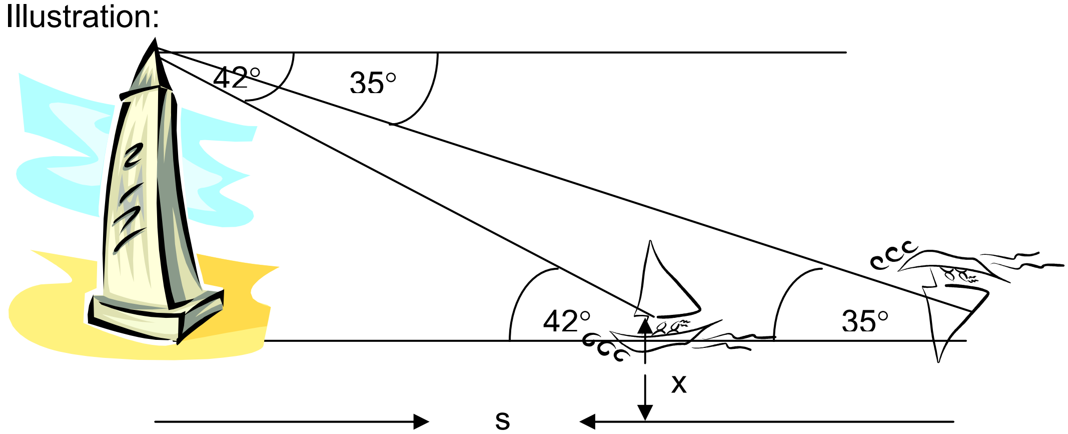
x = 400 tan46°

x = 400(1.0355)

x = 414.2 m

2. An observer on a lighthouse 160 ft. above sea level saw two vessels moving directly towards the lighthouse. He observed that the angle of depression are 42° and 35°. Find the distance between the two vessels, assuming that they are coming from the same side of the tower.

Illustration:



Solution:

Solve for the distance of each boat.

a. For the further boat

$$\text{Tan } 35° = \frac{160}{s}$$

$$s \tan 35° = 160$$

$$s = \frac{160}{\tan 35^o} \longrightarrow \text{equation 1}$$

b. For the nearer boat

$$\tan 42° = \frac{160}{s - x}$$

$$(s - x)\tan 42° = 160$$

$$s - x = \frac{160}{\tan 42^o}$$

$$s = x + \frac{160}{\tan 42^o} \longrightarrow \text{equation 2}$$

Equate: equation 1 and equation 2

$$\frac{160}{\tan 35^o} = x + \frac{160}{\tan 42^o}$$

$$\frac{160}{\tan 35^o} - \frac{160}{\tan 42^o} = x$$

$$228.50 - 177.70 = x$$

$$50.81 \text{ ft.} = x$$

Try this out

Solve the following problems. Sketch the figure.

1. If a 150 ft church tower cast a shadow 210 ft. long. Find the measure of the angle of elevation of the sun.

2. From the top of the control tower 250 m tall, a rock is sighted on the ground below. If the rock Is 170 m from the base of the tower, find the angle of depression of the rock from the top of the control tower.

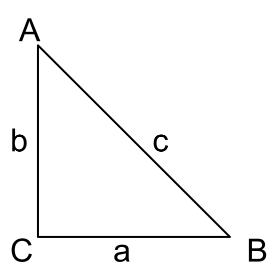
3. From a point on the ground 12 ft. from the base of a flagpole, the angle of elevation of the top of the pole measures 53°. How tall is the flagpole?

4. Ricky's kite is flying above a field at the end of 65 m of string. If the angle of elevation to the kite measures 70°, how high is the kite above Ricky's waist?

5. On a hill, inclined at an angle of 19 with the horizontal , stand a mango tree. At a point A 25 meters down the hill from the foot of the tree, the angle of elevation of the top of the mango tree is 45° . Find the height of the mango tree.

Let's summarize

Consider the right triangle ACB below.



The hypotenuse of the triangle is *c*.
The side opposite angle A is *a*.
The side adjacent to angle A is side *b*.

Use SOH-CAH-TOA, a mnemonic for remembering the equations:

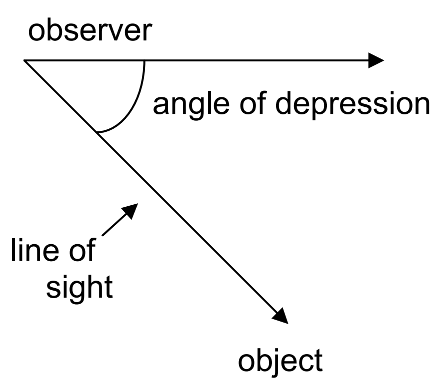
Sin = $\frac{\textbf{O}\text{pposite}}{\textbf{H}\text{ypotenuse}}$

Cos = $\frac{\textbf{A}\text{djacent}}{\textbf{H}\text{ypotenuse}}$

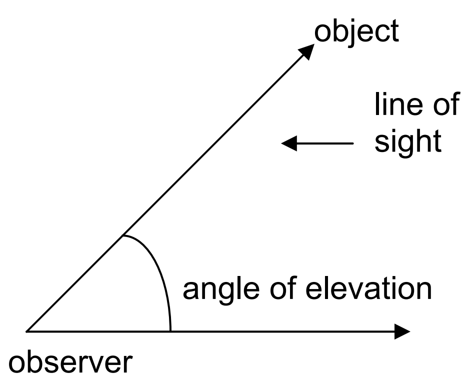
Tan = $\frac{\textbf{O}\text{pposite}}{\textbf{A}\text{djacent}}$

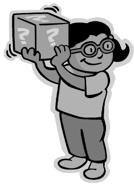
In solving problems, remember this terms:

a. Line of sight – is an imaginary line that connects the eye of an observer to the object being observed. If the observer is in a higher elevation than the object of observation, the acute angle measured from the eye level of the observer to his line of sight is called the *angle of depression*.



B. *Angle of elevation* - the acute angle made by the line of sight and the eye level of the observer is called the.





What have you learned

In rt.ΔXYZ angled at Y, if ∠X is an acute angle what is the

1. opposite side
2. adjacent side
3. hypotenuse

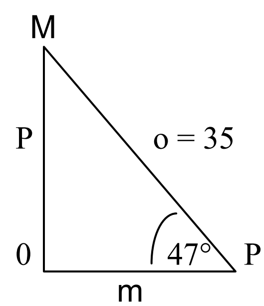
Give the equation that could be used to find the missing parts of a rt. Δ VWM. If m ∠V = 48° and m = 17, find:

4. V
5. w

Using another acute ∠M if m∠M = 23° and w = 13, give the equation to be used to compute:

6. v
7. m

Solve rt. Δ MOP, using the given information;



Find:

8. p
9. m
10. ∠M

Answer Key

How much do you know

1. OC or d
2. OD or c
3. CD or o
4. d =25 tan 70°
5. e = 25 cos 70°
6. e = 20 cos 60°
7. f = 20 tan 60°
8. b = 79.4
9. c = 80.6
10. ∠A = 10°

Try this out

Lesson 1

1. a = 14 sin 15°
2. b = $\dfrac{13}{\tan 76^0}$
3. c = $\dfrac{10}{\sin 49^0 13'}$
4. b = $\dfrac{21.2}{\tan 71^0 13'}$
5. c = $\dfrac{13}{\cos 16^0}$
6. c = $\dfrac{11}{\cos 19^0 07'}$
7. b = $\sqrt{16^2 - 7^2}$
8. a = $\sqrt{20^2 - 10^2}$

9. Tan A = $\frac{7}{12}$

10.Cos B = $\frac{8}{12}$

Lesson 2

1.
a = 15.5
$\angle$A = 42° 20
$\angle$B = 47° 39

2.
b = 14.4
$\angle$A = 25° 57'
$\angle$B = 64° 3'

3.
a = 17.3
$\angle$A = 60°
$\angle$B = 30°

4.
a = 11.5
$\angle$A = 62° 31'
$\angle$B = 27° 29'

5.
b = 5
$\angle$A = 67° 23'
$\angle$B = 22° 37'

Lesson 3

1.
$\angle$B = 75°
a = 9.6
b = 35.7

2.
$\angle$ A = 26°
a = 8.4
b = 17.3

3.

$\angle B = 75°$

$a = 6.5$

$b = 24.1$

4.

$\angle B = 45°$

$a = 7$

$b = 7$

5.

$\angle A = 34° 5'$

$a = 8.9$

$b = 13.3$

Lesson 4

1.

$\angle B = 14°$

$b = 3.2$

$c = 13.4$

2.

$\angle B = 67° 38'$

$a = 9.1$

$c = 23.8$

3.

$\angle A = 60°$

$a = 19.1$

$c = 22$

4.

$\angle A = 72°$

$b = 1.4$

$c = 4.5$

5.

$\angle B = 13°$

$a = 181.9$

$c = 186.7$

Lesson 5

1.

$\angle A = 36° 57'$

$\angle B = 53° 3'$

c = 26.3

2.

$\angle A = 30^\circ\ 15'$
$\angle B = 59^\circ\ 45'$
c = 13.9

3.

$\angle A = 15^\circ\ 57'$
$\angle B = 74^\circ\ 3'$
c = 7.3

4.

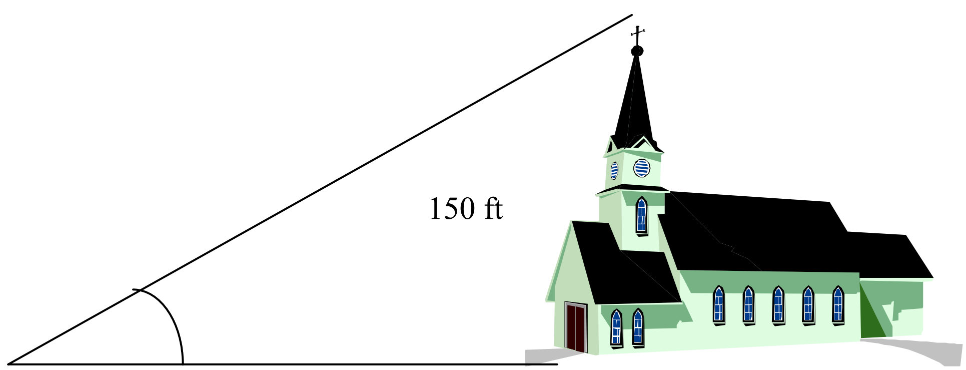
$\angle A = 60^\circ$
$\angle B = 30^\circ$
c = 3.5

5.

$\angle A = 45^\circ$
$\angle B = 45^\circ$
c = 353.6

Lesson 6

1. Figure:



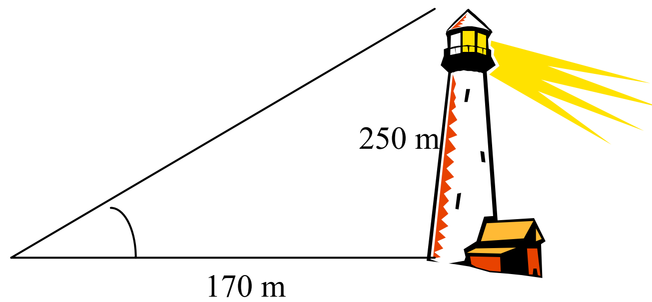
$\tan B = \frac{b}{a}$

$\tan B = \frac{150}{210}$

tan B = 0.7143

B = $35^0\ 32'$

2. Figure:



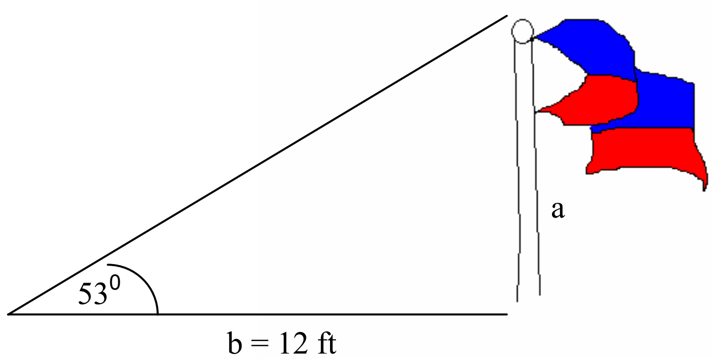
$\tan A = \frac{a}{b}$

$\tan A = \frac{250}{170}$

$\tan A = 1.4706$

$A = 55^0\ 48'$

1. Figure:



$\tan A = \frac{a}{b}$

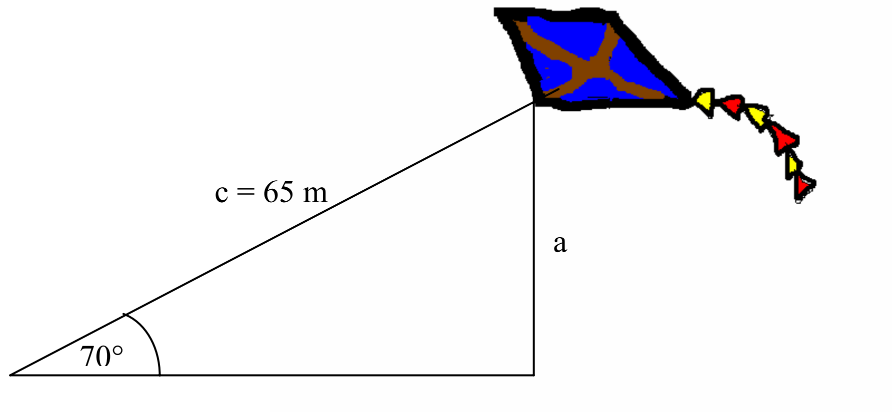
$\tan 53^o = \frac{a}{12}$

$a = 12 \tan 53^o$

$a = 12\ (1.3270)$

$a = 15.92$ ft height of the flagpole

4. Figure:



$$\text{Sin A} = \frac{a}{c}$$

$$\text{Sin } 70° = \frac{a}{65}$$

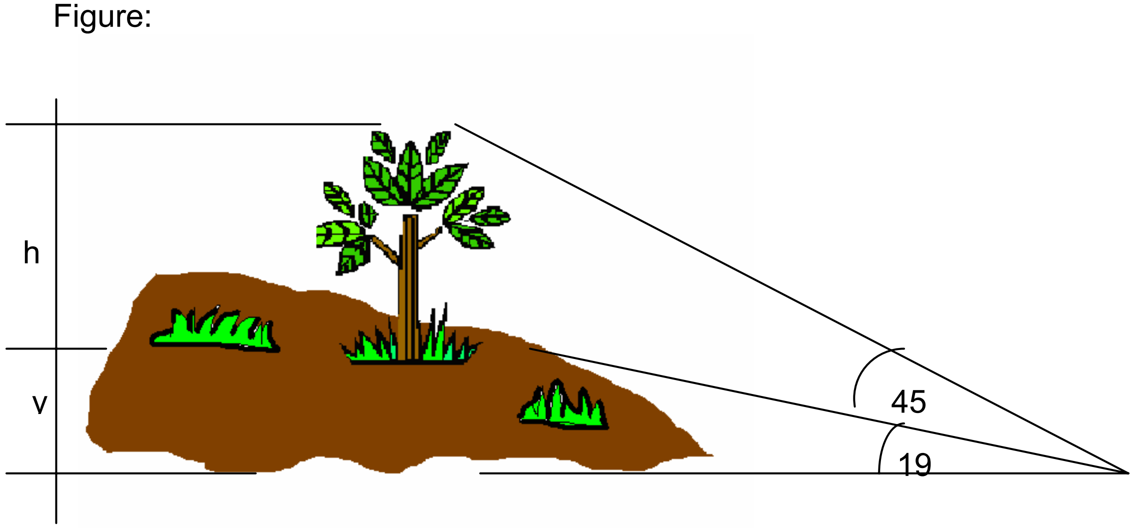
a = 65 sin 70°

a = 65 (0.9397)

a = 61.08 height of the kite

5.

Figure:



First solve for y and x:

Solve for y:

$$\sin 19° = \frac{y}{25}$$

y = 25 sin 19°

$y = 25 (0.3256)$

$y = 8.14$

Solve for x:

$\cos 19° = \frac{x}{25}$

$x = 25 \cos 19°$

$x = 25 (0.9455)$

$x = 23.6$

Solve for h:

$\tan 45° = \frac{h + y}{x}$

$h + y = x \tan 45°$

$h = x \tan 45° - y$

$h = 23.6(1) - 8.1$

$h = 23.6 - 8.1$

$h = 15.5$ height of the mango tree

What have you learned

1. YZ or x
2. XY or z
3. XZ or y
4. $v = 17 \tan 48°$
5. $w = 17 \cos 48°$
6. $v = 13 \cos 23°$
7. $m = 13 \sin 23°$
8. $p = 25.6$
9. $m = 23.9$
10. $\angle M = 43°$