

# Module Functions



## *What this module is about*

This module is about functions. As you go over the discussion and exercises, you will learn about relations and functions, and how to distinguish one over the other. Enjoy learning about functions and do not hesitate to go back if you think you are at a loss.



## *What you are expected to learn*

This module is designed for you to:

1. define a function
2. differentiate a function from a mere relation
  - real life situations
  - set of ordered pairs
  - graph of a given set of ordered pairs
  - vertical line test
  - given equation
3. illustrate the meaning of functional notation  $f(x)$
4. determine the value of  $f(x)$  given a value for  $x$

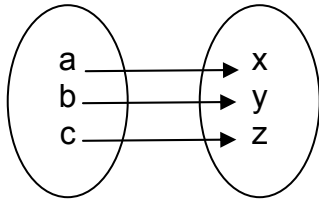


## *How much do you know*

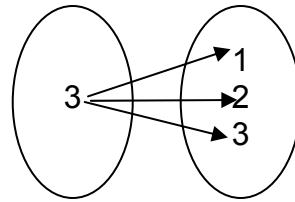
1. Which of the following sets of ordered pairs is a function?
  - a.  $\{(0, 1), (0, -1), (1, 2), (1, -2), (2, 3), (2, -3)\}$
  - b.  $\{(2, 4), (-2, 4), (1, 1), (-1, 1), (3, 9), (-3, 9)\}$
  - c.  $\{(4, 2), (4, -2), (1, 1), (-1, 1), (9, 3), (-9, 3)\}$
  - d.  $\{(1, 1), (1, -1), (1, 2), (1, -2), (1, 3), (1, -3)\}$

1. Which of the following correspondences shows a function?

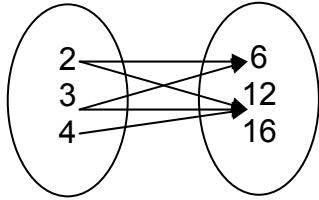
a.



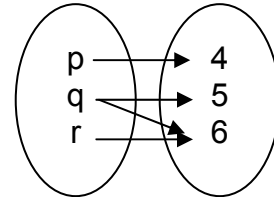
c.



b.

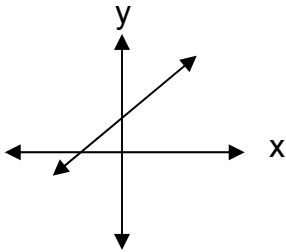


d.

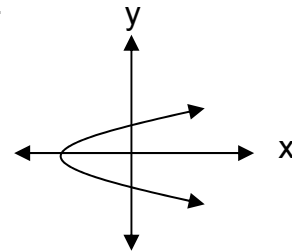


3. Which of the following graphs is **not** a function?

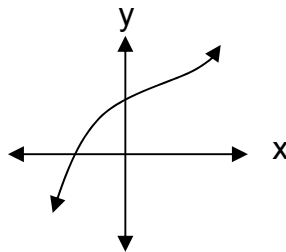
a.



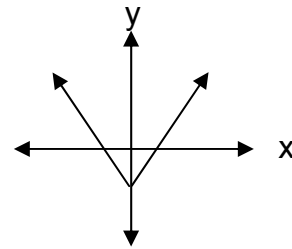
c.



b.



d.



4. Where is the point (-4, 5) located?

- a. Quadrant I
- b. Quadrant II

- c. Quadrant III
- d. Quadrant IV

5. What are the coordinates of the point located 3 units to the left of the y-axis and 4 units above the x-axis?

- a. (-3, 4)
- b. (4, -3)
- c. (3, -4)
- d. (-4, 3)

6. Which of the following equation(s) is/are function(s)?

- i.  $y = 3x + 2$
- ii.  $x^2 + y^2 = 36$
- iii.  $y = \sqrt{3x + 5}$

- a. i only
- b. i and ii
- c. i and iii
- d. ii and iii



Figure 2 shows that each element in the first set is paired with a unique element in the second set; -2 is paired to -2, -3 is paired to -3, and -4 is paired to -4. It is called **one-to-one relation**. The relation shows that elements of both sets are equal.  $B = A$ .

Figure 3 shows that some elements in the first set are paired with the same element in the second set. The pairing is called **many-to-one relation**. The relation shows that the elements of Set B are equal to 5 more than the square of Set A or  $B = A^2 + 5$ .

Figures 2 and 3 above are special kind of relations called **functions**.

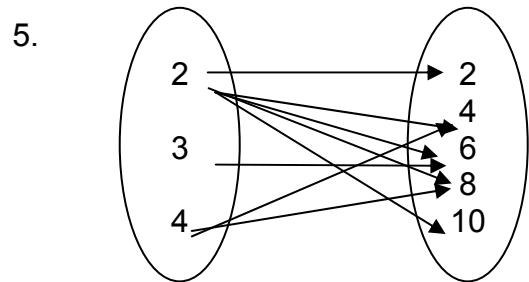
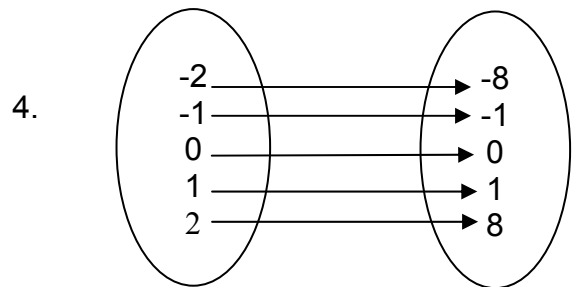
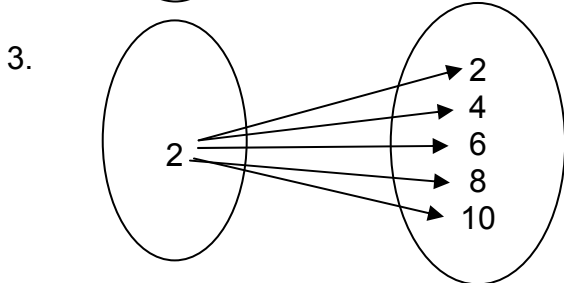
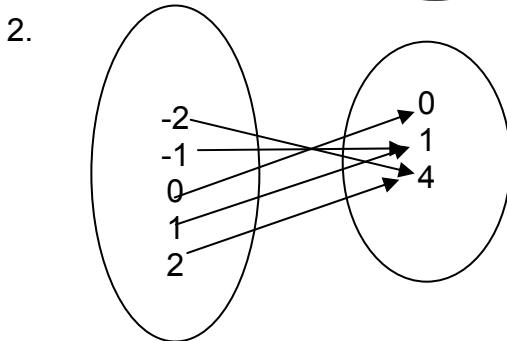
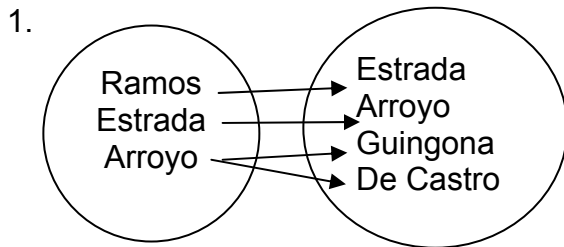
A **function** is a well-defined relation where no two pairs have the same first element.

The main characteristics of a function described from set A to set B can be seen in the pairing in figures 2 and 3.

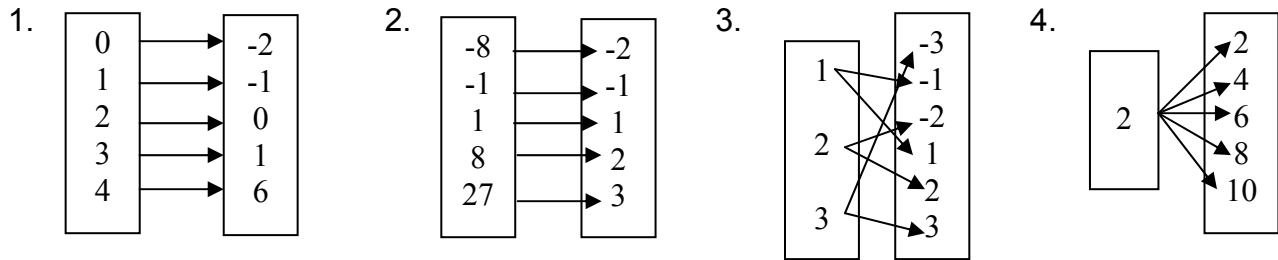
1. Each element in set A is paired with each element in set B.
2. Some elements in set B are not paired with an element in set A.
3. Two or more elements in A may be paired with the same element in B.

Try this out

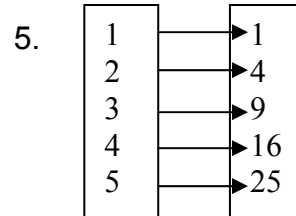
A. Tell whether the correspondence shown in each diagram is a function or not. Explain.



B. Name 5 pairs associated with the following relations. Identify whether the elements show function or not. Justify your answer.



1. \_\_\_\_\_ is 2 more than \_\_\_\_\_.
2. \_\_\_\_\_ is a cube of \_\_\_\_\_.
3. \_\_\_\_\_ is an absolute value of \_\_\_\_\_.
4. \_\_\_\_\_ is a factor of \_\_\_\_\_.
5. \_\_\_\_\_ is a square root of \_\_\_\_\_.



## Lesson 2

### Domain and Range of a Function

The set of all first elements in a relation is called **domain** while the set of all the second elements is called **range**. Since a function is a special relation then it follows that the set of all first elements in a function is also called a **domain** and the set of all second elements is also called a **range**.

**Examples:**

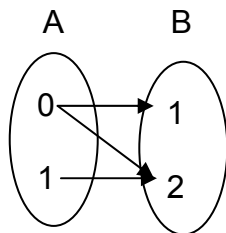


Figure 1

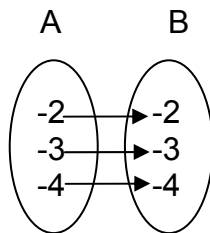


Figure 2

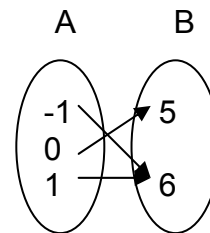


Figure 3

In figure 1,  
Domain = {0, 1} and Range = {1, 2}.

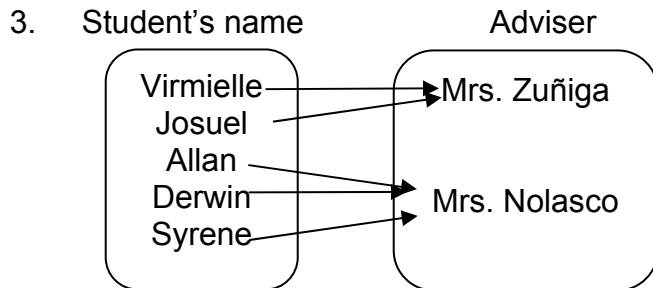
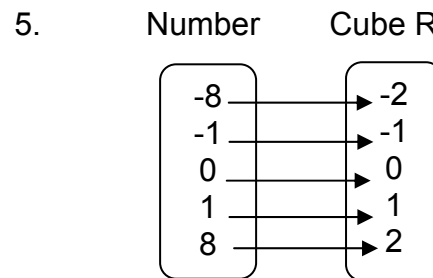
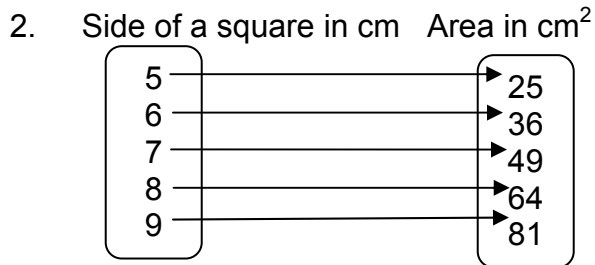
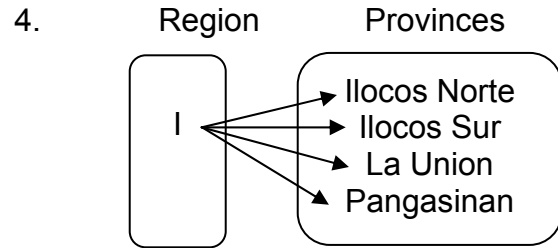
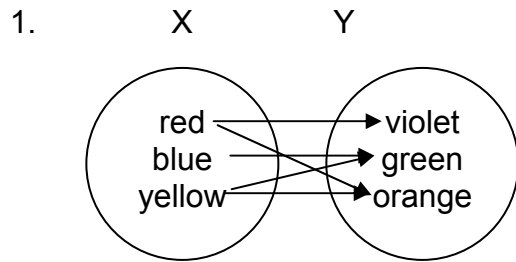
In figure 2,

Domain =  $\{-2, -3, -4\}$  and Range =  $\{-2, -3, -4\}$ .

In figure 3,  
Domain =  $\{-1, 0, 1\}$  and Range =  $\{5, 6\}$ .

Try this out

Identify the domain and range of the following relations. Tell also whether the relations are functions or not.



### Lesson 3

## Identifying Functions in Real Life

There are many situations around you that show functional relationships between variables or quantities so that one variable or quantity depends upon the other.

**Examples:**

1. The perimeter of a square depends upon the length of its side.

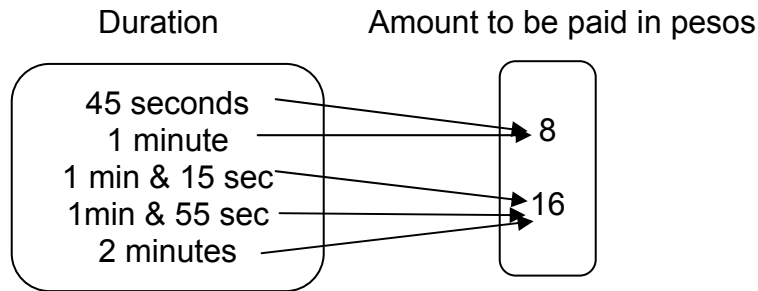
This shows that the perimeter of a square is a function of its length. Recall that the formula for the perimeter of a square is  $P = 4s$  where  $P$  = perimeter and  $s$  = measure of the side of the square. You can assign a specific value of  $s$  for the side and using the formula you will obtain a unique value for the perimeter.

$$\begin{array}{ll} s = 3 \text{ cm} & P = 4(3 \text{ cm}) = 12 \text{ cm} \\ s = 2 \text{ m} & P = 4(2 \text{ m}) = 8 \text{ m} \end{array}$$

The example shows a one-to-one correspondence between the side and the perimeter of a square.

2. The amount you will pay for a long distance call will depend upon the duration of the call.

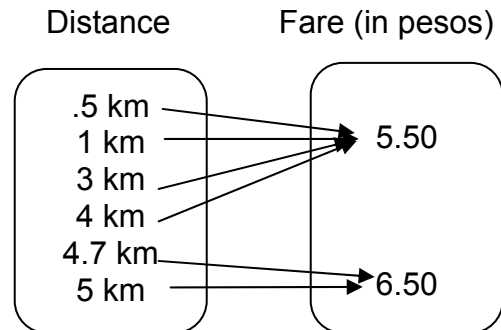
This situation shows that the amount paid for a long distance call is a function of the duration of the call. The computation usually is based on the number of minutes or a fraction of it so that if your call lasted for less than a minute, the amount that you will pay will be for one minute; or if your call is more than 2 minutes but less than 3 minutes, the amount you will pay will be for 3 minutes.



The illustration shows a many-to-one correspondence, hence it is a function.

3. The jeepney fare that a passenger will pay depends upon the distance he or she travels.

This shows that jeepney fare is a function of the distance a passenger travels. At present, the LTRFB of the LTO issued that the computation for jeepney fare is  $\text{₱}5.50$  for the first 4 kilometers and an additional  $\text{₱}1.00$  for every additional kilometer or a fraction of it. This can be shown in the diagram that follows.



The diagram shows that the situation shows a many-to-one correspondence. Hence, the situation is a function.

From the examples given, observe that situations in real life that show *one-to-one* or *many-to-one pairings* are functions.

Hence, you can say that functions in real life are situations that show relationships between variables or quantities in such a way that one variable will depend upon the other.

### Try this out

The situations below show relationship between two quantities. Identify the sets of related quantities and tell whether the correspondence between the variables is one-to-one or many-to-one.

1. The volume of prism is related to the measure of its width, length and depth.
2. The distance traveled by a car is related to the speed and the time it has traveled.
3. The total cost of rice bought is related to the number of kilograms of rice bought.
4. The score obtained on a test is related to the length of time spent in studying a subject.
5. The area of a triangle is related the measure of its base and altitude.
6. The pressure of a given mass of gas at a constant temperature is related to its volume.
7. The total cost of rambutan bought is related to the price per kilogram.
8. The distance traversed by a freely falling body is related to the square of its time of fall.
9. The apparent size of an object is related to its distance from the observer.
10. The intensity of light is related to the square of the distance from the source.

## Lesson 4

### The Representing Functions by Ordered Pairs

In Lesson 1 to 3, you have seen that relations and functions were described by means of arrow diagrams. The arrow diagrams illustrate the pairing or mapping of elements from one set to another set.

Functions can also be described by means of *ordered pairs*.

An ***ordered pair*** is a pair of numbers, usually denoted by  $(x, y)$ , where the order of elements is important;  $x$  is called the *first element or component* and  $y$  is called the *second element or component*. Ordered pairs can also be shown by tables.



A set of ordered pairs represents a function if no two ordered pairs have the same first elements or component.

**Example:** Consider the following sets of ordered pairs:

$$A = \{(-2, -1), (-1, 0), (0, 1), (1, 2), (2, 3)\}$$

$$B = \{(4, 2), (4, -2), (9, 3), (9, -3), (16, 4), (16, -4)\}$$

$$C = \{(-2, 7), (-1, 7), (0, 7), (1, 7), (2, 7)\}$$

Set A is a set of ordered pairs where the second element  $y$  is one more than the first element  $x$ . Observe that the set of ordered pairs shows a one-to-one correspondence or mapping. Hence, the set of ordered pairs illustrates a function.

Set B is a set of ordered pairs where a first element is paired to two different second elements. Notice that the pairing is one-to-many. Hence, the set of ordered pairs does not represent a function.

Set C is a set of ordered pairs where two different first elements are paired to the same second element. The set of ordered pairs shows many-to-one correspondence. Hence, the set of ordered pairs represents a function.

Each of the sets of ordered pairs in the given examples can also be presented using a table (also called a table of ordered pairs).

Set A:

x	-2	-1	0	1	2
y	-1	0	1	2	3

Set B:

x	4	4	9	9	16	16
y	2	-2	3	-3	4	-4

Set C:

x	-2	-1	0	1	2
y	7	7	7	7	7

## Try This Out

A. Determine which of the following sets/tables of ordered pairs represent a function.

- $\{(-2, 3), (-1, 4), (0, 5), (1, 6), (2, 7)\}$
- $\{(-2, 10), (-1, -5), (0, -6), (1, -5), (2, 10), (3, 75)\}$
- $\{(2, 3), (2, 1), (17, 4), (17, 0), (82, 5), (82, -1)\}$
- $\{(-4, 4), (-3, 7), (-2, 10), (-1, 13), (0, 16)\}$
- $\{(-3, \frac{3}{4}), (-2, \frac{3}{2}), (-1, 4), (0, 3), (1, 5)\}$

6.

x	-3	-2	-1	0	1	2	3
y	-9	-9	-9	-9	-9	-9	-9

7.

x	-2	-1	0	1	2	3
y	5	6	7	8	9	10

8.

x	-2	-1	0	1	2	3
y	-9	-2	-1	0	7	26

9.

x	-2	-2	-1	-1	0	0
y	-1	1	-3	3	-5	5

10.

x	3	3	3	3	3
y	-5	-4	-3	-2	-1

B. Give the domain and range of the set/table of ordered pairs in A.

## Lesson 5

### Graphs of Relations and Functions

In Lesson 4, functions were represented by ordered pairs. These ordered pairs, in turn, correspond to points on the Cartesian Plane.

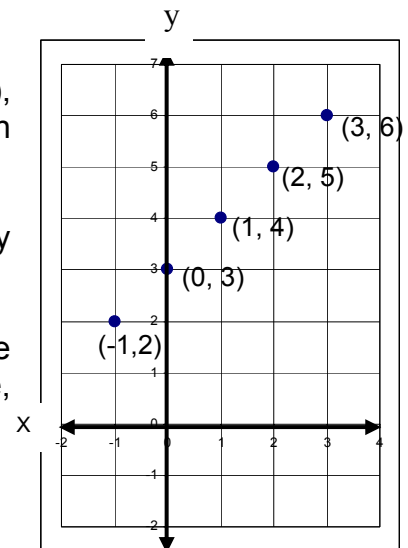
The graph of a function is the set of all points on the coordinate plane corresponding to the ordered pairs in the function. If the domain and range of the function is the set of real numbers, the points are connected producing a continuous graph. If the domain and the range are not real numbers, then the graph will be discrete; the graph is a series of unconnected points.

#### Example 1:

Draw the graph of the set of ordered pairs  $\{(-1, 2), (0, 3), (1, 4), (2, 5), (3, 6)\}$  and determine whether the graph represents a function or not.

The graph of the given set of ordered pairs is obtained by plotting the set of ordered pairs on the Cartesian Plane.

The graph is shown at the right. The domain and the range of the function belong to the set of integers. Therefore, the graph of the function is discrete (unconnected points).



The graph of the set of ordered pairs represents a function since no two ordered pairs have the same first element.

**Example 2:**

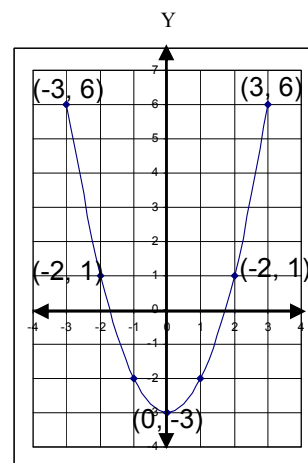
Draw the graph of the relation where both domain and range are real numbers and are represented by the following table of ordered pairs.

x	-3	-2	-1	0	1	2	3
y	6	1	-2	-3	-2	1	6

To sketch the graph of the relation, plot the points in the given table of ordered pairs on the Cartesian Plane. Connect the points since the domain and the range belong to the set of real numbers.

The graph is shown at the right. It is a symmetrical curve which opens upward. It has a turning point at the point (0,3).

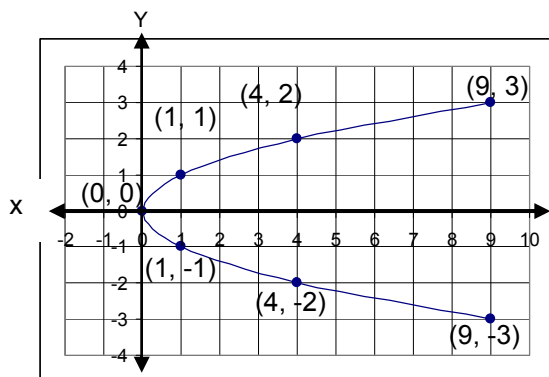
The graph of the relation is a function since there are no ordered pairs having the same first element.



**Example 3:**

Sketch the graph of the relation whose domain and range are both real numbers and is represented by the following table of ordered pairs. Tell whether the graph represents a function or not.

X	9	9	4	4	1	1	0
Y	3	-3	2	-2	1	-1	0

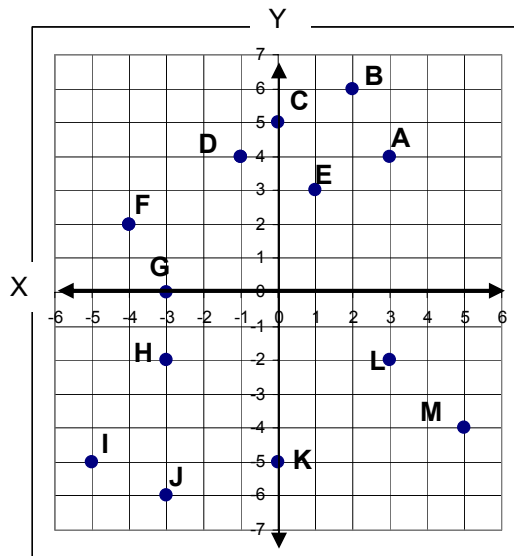


Just like in example 2, plot the given ordered pairs then connect the points since both domain and range are elements of the real numbers.

As can be seen from the table, there are ordered pairs having the same first element. Hence, the graph does not represent a function.

## Try This Out

A. Name the coordinates of the points on the Cartesian Plane.



B. Tell the quadrant or axis where the following points are located.

- |             |               |
|-------------|---------------|
| 1. A(2, -4) | 5. E(3, 1)    |
| 2. B(0, -5) | 6. F(-1, -10) |
| 3. C(-2, 7) | 7. G(17, -7)  |
| 4. D(8, 0)  | 8. H(-3, -4)  |

C. Draw the graph of the following sets or tables of ordered pairs. Tell whether the graphs obtained in each represent a function or not. Assume that the domain and the range in each number are real numbers.

- $\{(-3, 5), (-2, 4), (-1, 3), (0, 2), (1, 1), (2, 0)\}$
- $\{(-3, 12), (-2, 7), (-1, 4), (0, 3), (1, 4), (2, 7), (3, 12)\}$
- $\{(-3, 2), (-3, 3), (-3, 4), (-3, -2), (-3, -3)\}$

4.

x	-3	-2	-1	0	1	3	4	5
y	-5	-4	-3	-2	-1	0	1	2

5.

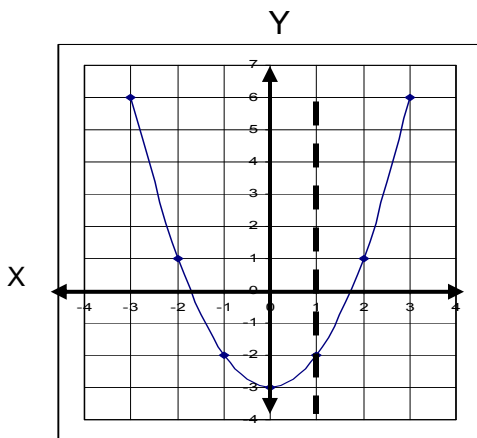
x	-2	-1	0	1	2
y	-8	-1	0	1	8

## Lesson 6

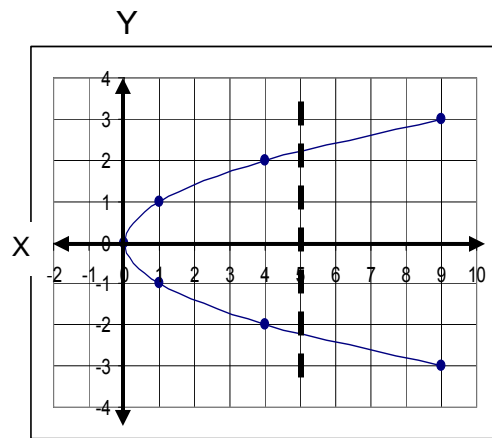
### The Vertical Line Test

Drawing a vertical line through the graph is called the **vertical line test**. This test shows that if a vertical line drawn through the graph intersects the graph of a relation in exactly **one point**, then the relation is a function.

**Examples:**



Function



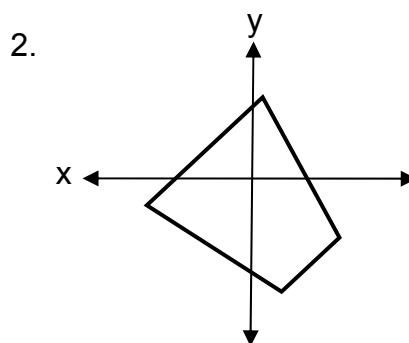
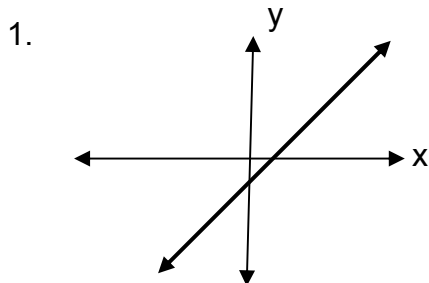
Not Function

In the first figure, the vertical line passes the graph at only one point. Hence, it is a function.

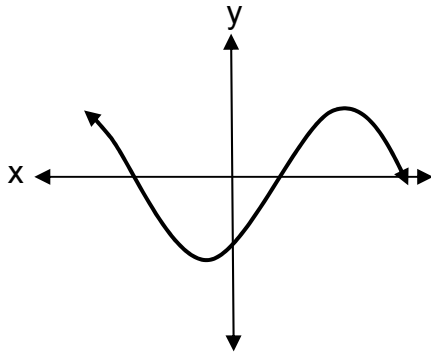
In the second figure, the vertical line passes the graph at two points. Hence, it is not a function.

Try this out

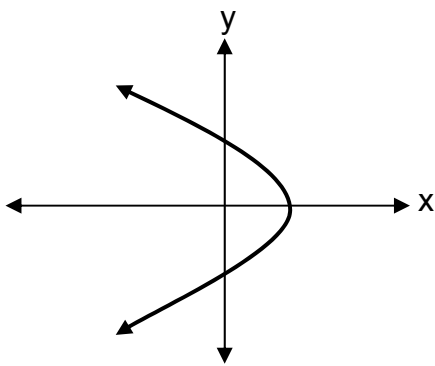
Determine whether the graph represents a function. Justify your answer.



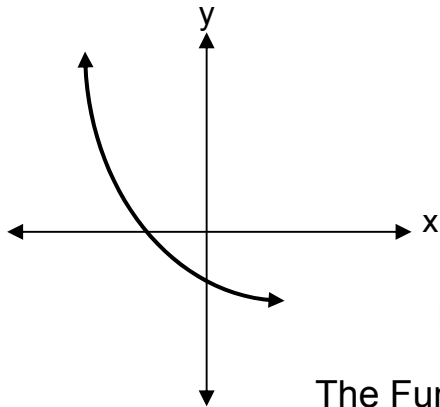
3.



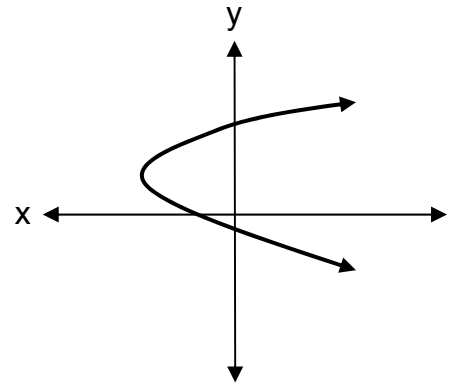
4.



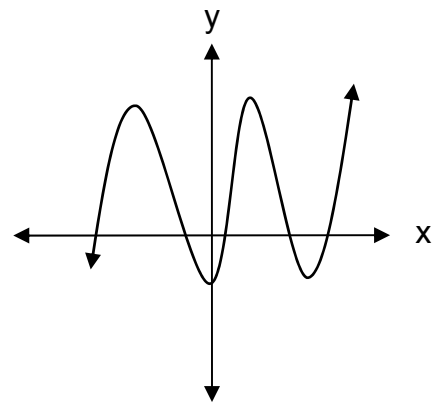
5.



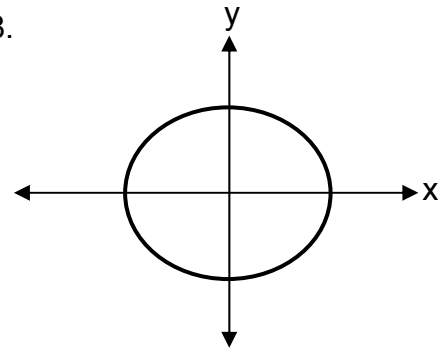
6.



7.



8.



### Lesson 7

### The Functional Notation

To specify that  $y$  is the unique element related to  $x$  by the function  $f$ , the notation

$$y = f(x)$$

read as “*y is a function of x*” is used. This shows that the value of  $y$  depends on the value of  $x$  and that  $f$  is the rule. The rule is usually an equation which assigns to each  $x$  which is an element of the domain a unique value of  $y$  which is an element of the range.  $x$  is also called as the independent variable and  $y$  the dependent variable.

### Examples:

1. The area and perimeter of a square depends on the measure of its side, According to the rule,  $P = 4s$  and  $A = s^2$  where  $P$  and  $A$  stand for perimeter and area of a square, respectively.

In functional notation, we say that  $P(s) = 4s$  and  $A(s) = s^2$ . Hence,

if  $s = 3$  cm, then

$$\begin{aligned} P(3 \text{ cm}) &= 4(3 \text{ cm}) = 12 \text{ cm} \\ &\text{and} \\ A(3 \text{ cm}) &= (3 \text{ cm})^2 = 9 \text{ cm}^2 . \end{aligned}$$

If  $s = 15$  m, then

$$\begin{aligned} P(15 \text{ m}) &= 4(15 \text{ m}) = 60 \text{ m} \\ &\text{and} \\ A(15 \text{ m}) &= (15 \text{ m})^2 = 225 \text{ m}^2 . \end{aligned}$$

It can also be said that in  $y = f(x)$ ,  $x$  is the input and  $y$  is the output.

2. Given  $f(x) = x^2 + 3x - 28$ .

Find: a.  $f(5)$                       b.  $f(-4)$                       c.  $f(a)$

$$\begin{aligned} \text{a. } f(5) &= 5^2 + 3(5) - 28 = 25 + 15 - 28 = 12 \\ \text{b. } f(-4) &= (-4)^2 + 3(-4) - 28 = 16 - 12 - 28 = -24 \\ \text{c. } f(a) &= a^2 + 3(a) - 28 = a^2 + 3a - 28 \end{aligned}$$

3. Given  $f(x) = 3x + 2$  and  $g(x) = x^2$ .

Find: a.  $f(x) + g(x)$                       b.  $g(x) - f(x)$                       c.  $f(x) \bullet g(x)$

d.  $\frac{f(x)}{g(x)}$                       e.  $f(g(x))$                       f.  $g(f(x))$

a.  $f(x) + g(x) = 3x + 2 + x^2 = x^2 + 3x + 2$   
Hence,  $f(x) + g(x) = (f + g)(x)$ .

b.  $g(x) - f(x) = x^2 - (3x + 2) = x^2 - 3x - 2$   
In general,  $f(x) - g(x) = (f - g)(x)$ .

c.  $f(x) \bullet g(x) = (3x + 2) \bullet x^2 = 3x(x^2) + 2(x^2) = 3x^3 + 2x^2$   
Thus,  $f(x) \bullet g(x) = (f \bullet g)(x)$ .

d.  $\frac{f(x)}{g(x)} = \frac{3x+2}{x^2}$   
 Therefore,  $\frac{f(x)}{g(x)} = \frac{f}{g}(x)$ .

e.  $f(g(x)) = f(x^2) = 3(x^2) + 2 = 3x^2 + 2$

f.  $g(f(x)) = g(3x+2) = (3x+2)^2 = 9x^2 + 12x + 4$

Examples e and f above are called composition of functions. In symbols,

$$(f \circ g)(x) = f(g(x)) \text{ and } (g \circ f)(x) = g(f(x)).$$

### Try This Out

1. Let  $f(x) = 4x - 2$

Find: a)  $f(-2)$                       b)  $f(4)$                       c)  $f(4) - f(-2)$                       d)  $f(-2) + f(4)$

2. Let  $g(x) = 2x^2 + 5x - 3$

Find: a)  $f(3)$                       b)  $f(\sqrt{5})$                       c)  $f(3) + f(\sqrt{5})$                       d)  $\frac{f(\sqrt{5})}{f(3)}$

3. Let  $f(x) = 5 - x^2$  and  $g(x) = 2x - 5$

Find: a)  $f(3)$                       b)  $g(-5)$                       c)  $g(f(3))$                       d)  $f(g(-5))$   
 e)  $f(g(x))$                       f)  $g(f(x))$

4. Let  $f(x) = \sqrt{x} - 4$

Find: a)  $f(9)$                       b)  $f(4)$                       c)  $f(9) + f(4)$                       d)  $f(9) - f(4)$   
 e) all values of  $x$  for which  $f(x) = 0$

5. Let  $h(x) = x^3 - 3x^2 + x - 3$

Find: a)  $h(2)$                       b)  $h(-3)$                       c)  $h(h(-2))$                       e) all values of  $x$  for which  $h(x) = 0$

## Lesson 8

### Equations in Two Variables that are Functions

Functions can also be represented using equations in two variables aside from arrow diagrams, sets of ordered pairs, and graphs which you have learned earlier.



To determine whether a given equation represents a function or not, solve for  $y$  in terms of  $x$ . If  $x$  is replaced by a value and a unique value of  $y$  is obtained, then the equation is a function. Otherwise, the equation represents a mere relation.

**Examples:**

1. Determine whether the equation  $x - 4y = 3$  represents a function or not.

Solution:

Solve for  $y$  in terms of  $x$ .

$$\begin{aligned}x - 4y &= 3 \\-4y &= -x + 3 \\y &= \frac{-x + 3}{-4} \text{ or} \\y &= \frac{x - 3}{4}\end{aligned}$$

If  $x$  is replaced by a value, a unique value for  $y$  is obtained. Hence,  $y$  is a function of  $x$  and so  $x - 4y = 3$  represents a function.

2. Determine if the equation  $x^2 + y^2 = 49$  represents a function or not. If it not, what conditions should be set for it to become a function?

Solution:

Solve for  $y$  in terms of  $x$ .

$$\begin{aligned}x^2 + y^2 &= 49 \\y^2 &= 49 - x^2 \\y &= \pm \sqrt{49 - x^2}\end{aligned}$$

Observe that if a value of  $x$  is substituted, two values of  $y$  will be obtained as indicated by the  $\pm$  signs. Therefore, the equation is not a function.

Since the equation is not a function, conditions must be set in order to have a function. You use either

$$y = \sqrt{49 - x^2} \text{ or } y = -\sqrt{49 - x^2} .$$

To determine whether a given equation represents a function or not, solve for  $y$  in terms of  $x$ . If  $x$  is replaced by a value and a unique value of  $y$  is obtained, then the equation is a function.

Try This Out

A. Which of the equations below are functions?

1.  $3x + 4y = 10$

2.  $x^2 + y = 7$

3.  $y^2 - x = 12$

4.  $x^3 - y^3 = 8$

5.  $xy^2 = 20$

6.  $|x| = y$

7.  $y = x^5 - 3$

8.  $x + y^4 = 8$

9.  $x^2 - y^2 = 16$

10.  $y = 3^x$

B. What conditions should be set in order that equations which are not functions in A will become functions?



*Let's summarize*

1. A *relation* is an association or pairing of some kind between two sets of quantities or information.
2. A *one-to-many relation* is a relation where one element from the first set is associated with more than one element in the second set.
3. A *one-to-one relation* is a relation where each element in the first set is paired with a unique element in the second set.
4. A *many-to-one relation* is a relation where the elements in the first set are paired with one element in the second set.
5. *Domain* is the set of all first elements in a relation.
6. *Range* is the set of all the second elements in a relation.
7. A *function* is a well defined *relation* where no two pairs have the same first element.
8. A set of ordered pairs represents a function if no two ordered pairs have the same first elements or component.
9. The graph of a function is the set of all points on the coordinate plane corresponding to the ordered pairs in the function. If the domain and range of the function is the set of real numbers, the points are connected hence producing a continuous graph. If the domain and the range are not real numbers, then the graph will be discrete, that is, the graph is a series of unconnected points.

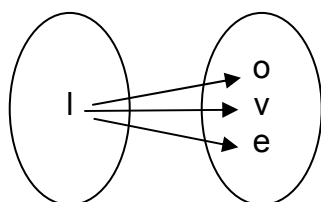
10. The vertical line test is a test that shows whether a graph represents a function or not. If a vertical line is drawn through a graph and it passes through the graph at exactly one point, then the graph shows a function. Otherwise, the graph shows a mere relation.
11.  $y = f(x)$  read as “ $y$  is a function of  $x$ ” shows that the value of  $y$  depends on the value of  $x$  and that  $f$  is the rule, usually an equation, which assigns to each element  $x$  which is an element of the domain a unique value of  $y$  which is an element of the range.
12. In  $y = f(x)$ ,  $x$  is called the independent variable while  $y$  is called the dependent variable.



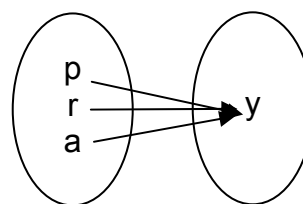
## What have you learned

1. Which of the correspondences below show(s) a function?

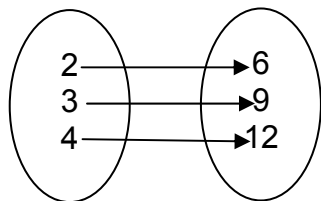
a.



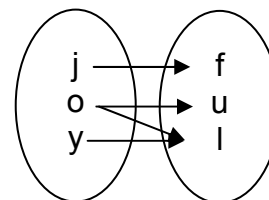
c.



b.



d.

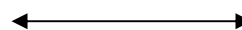
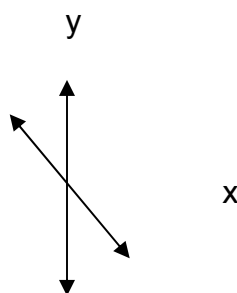


2. Which of the following sets of ordered pairs is **not** a function?

- a.  $\{(1, 0), (-1, 0), (2, 1), (-2, 1), (3, 2), (-3, 2)\}$   
 b.  $\{(4, 2), (4, -2), (1, 1), (1, -1), (9, 3), (9, -3)\}$   
 c.  $\{(-8, -2), (8, 2), (-1, -1), (1, 1), (27, 3), (-27, -3)\}$   
 d.  $\{(1, 1), (-1, ), (2, 1), (-2, 1), (3, 1), (-3, 1)\}$

3. Which of the following graphs is a function?

a.



b. y



a. 78

b. 46

c. -52

d. -8



## Answer Key

How much do you know

- |      |      |      |      |       |
|------|------|------|------|-------|
| 1. b | 3. c | 5. a | 7. a | 9. d  |
| 2. a | 4. b | 6. d | 8. d | 10. b |

Try this out

### Lesson 1

A.

- |                                |                                |
|--------------------------------|--------------------------------|
| 1. not a function; one-to-many | 4. function; one-to-one        |
| 2. function; many-to-one       | 5. not a function; one-to-many |
| 3. not a function; one-to-many |                                |

B. Possible answers

- \_\_\_\_\_ is 2 more than \_\_\_\_\_; function; correspondence is one-to-one  
2 is 2 more than 0                      0 is 2 more than -2                      4 is 2 more than 6  
1 is 2 more than -1                      3 is 2 more than 1
- \_\_\_\_\_ is a cube of \_\_\_\_\_ function; correspondence is one-to-one  
-8 is a cube of -2                      1 is a cube of 1                      27 is a cube of 3  
-1 is a cube of -1                      8 is a cube of 2
- \_\_\_\_\_ is an absolute value of \_\_\_\_\_; not a function; correspondence is one-to-many  
1 is an absolute value of -1                      2 is an absolute value of 2  
1 is an absolute value of 1                      3 is an absolute value of -3  
2 is an absolute value of -2
- \_\_\_\_\_ is a factor of \_\_\_\_\_; not a function; correspondence is one-to-many  
2 is a factor of 2                      2 is a factor of 6                      2 is a factor of 10  
2 is a factor of 4                      2 is a factor of 8
- \_\_\_\_\_ is a square root of \_\_\_\_\_; function; correspondence is one-to-one  
1 is a square root of 1                      4 is a square root of 16  
2 is a square root of 4                      5 is a square root of 25  
3 is a square root of 9

### Lesson 2

- Domain = {red, yellow, blue}, Range = {violet, green, orange}; not a function
- Domain = {5, 6, 7, 8, 9}, Range = {25, 36, 49, 64, 81}; function
- Domain = {Virmielle, Josuel, Allan, Derwin, Syrene}, Range = {Mrs. Zuñiga, Mrs, Nolasco}, not a function

4. Domain = {1}, Range = {Ilocos Norte, Ilocos Sur, La Union, Pangasinan}; not a function
5. Domain = {-8, -1, 0, 1, 8}, Range = {-2, -1, 0, 1, 2, 3}; function

### Lesson 3

1. Volume, dimension of a prism; one-to-one
2. distance, speed and time; one-to-one
3. total cost, number of kilograms; one-to-one
4. score, time; many-to-one
5. Area, height and altitude; one-to-one
6. Pressure, temperature; one to one
7. Total amount, amount per kg; one-to-one
8. distance, time; one to one
9. size, distance; one-to-one
10. intensity, distance; one-to-one

### Lesson 4

#### A

- |                   |             |                    |
|-------------------|-------------|--------------------|
| 1. function       | 5. function | 9. not a function  |
| 2. function       | 6. function | 10. not a function |
| 3. not a function | 7. function |                    |
| 4. function       | 8. function |                    |

#### B

1. Domain = {-2, -1, 0, 1, 2}, Range = {3, 4, 5, 6, 7}
2. Domain = {-2, -1, 0, 1, 2, 3}, Range = {-6, -5, 10, 75}
3. Domain = {2, 17, 82}, Range = {-1, 0, 1, 3, 4, 5}
4. Domain = {-4, -3, -2, -1, 0}, Range = {4, 7, 10, 13, 16}
5. Domain = {-3, -2, -1, 0, 1}, Range =  $\{3\frac{1}{4}, 3\frac{1}{2}, 4, 3, 5\}$
6. Domain = {-3, -2, -1, 0, 1, 2, 3}, Range = {-9}
7. Domain = {-2, -1, 0, 1, 2, 3}, Range = {5, 6, 7, 8, 9, 10}
8. Domain = {-2, -1, 0, 1, 2, 3}, Range = {-9, -2, -1, 0, 7, 5}
9. Domain = {-2, -1, 0}, Range = {-5, -3, -1, 1, 3, 10}
10. Domain = {3}, Range = {-5, -4, -3, -2, -1}

### Lesson 5

#### A.

- |            |             |             |            |
|------------|-------------|-------------|------------|
| A. (3, 4)  | E. (1, 2)   | I. (-5, -5) | M. (5, -4) |
| B. (2, 6)  | F. (-4, 2)  | J. (-3, -6) |            |
| C. (0, 5)  | G. (-3, 0)  | K. (0, -5)  |            |
| D. (-1, 4) | H. (-3, -2) | L. (3, -2)  |            |

B.

- 1. IV
- 2. y-axis

- 3. II
- 4. x-axis

- 5. I
- 6. III

- 7. IV
- 8. III

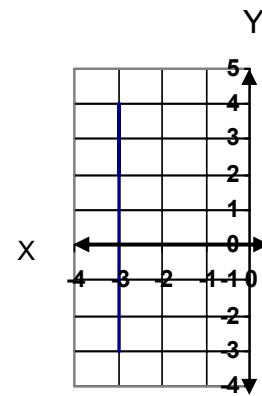
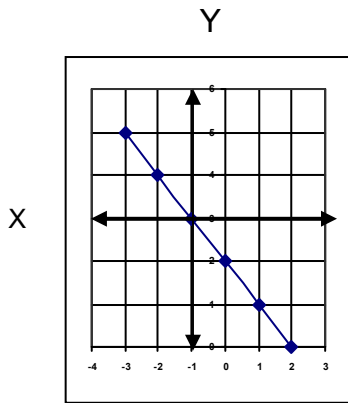
C.

- 1. function
- 2. function

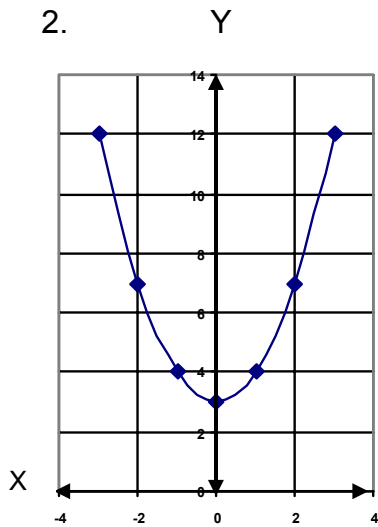
- 3. not a function
- 4. function

- 5. function

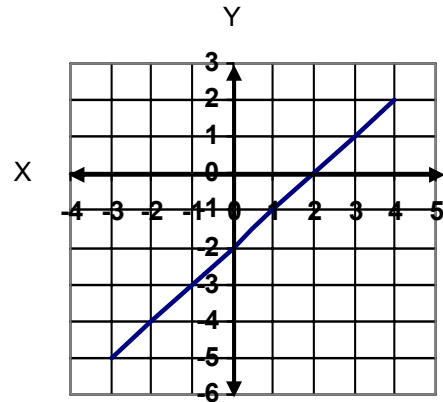
1.



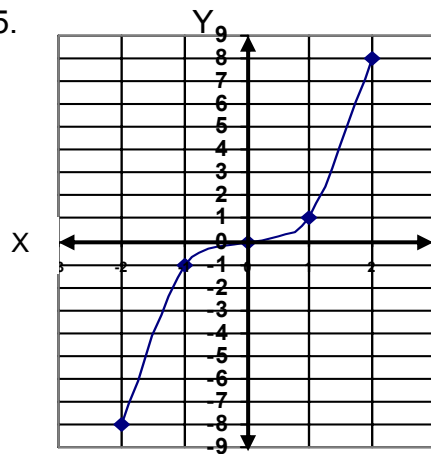
2.



4.



5.





## Lesson 6

1. function
2. not function
3. function

4. not function
5. function
6. not function

7. function
8. not function

## Lesson 7

1. a. -10  
b. 14  
c. 24  
d. 4

3. a. -4  
b. -15  
c. -13  
d. -220  
e.  $-4x^2 + 16x - 11$   
f.  $-2x^2 + 10x - 5$

- d. 1  
e. 16

2. a. 30  
b.  $7 + 5\sqrt{5}$   
c.  $37 + 5\sqrt{5}$   
d.  $\frac{7 + 5\sqrt{5}}{30}$

4. a. -1  
b. -2  
c. -3

5. a. -5  
b. -60  
c. -58  
d.  $-3, \pm i$

## Lesson 8

A. 1, 2, 4, 6, 7, 10

B. 3.  $y = \sqrt{x+12}$  or  $y = -\sqrt{x+12}$

5.  $y = \frac{\sqrt{20x}}{x}$  or  $y = -\frac{\sqrt{20x}}{x}$

8.  $y = \sqrt[4]{8-x}$  or  $y = -\sqrt[4]{8-x}$

9.  $y = \sqrt{16-x^2}$  or  $y = -\sqrt{16-x^2}$

## What have you learned

1. b, c
2. b
3. a
4. d
5. d
6. c
7. a
8. d
9. b
10. b