

Module 8

Geometry of Shape and Size



What this module is about

This module is about volume of solids. The volume of a solid is the number of cubic units contained in the solid. If measures are given in centimeter, the volume is stated in cubic cm, written as cm^3 .



What you are expected to learn

This module is designed for you to:

1. define volume of solids.
2. find the volume of solids such as:
 - cube
 - prism (rectangular, triangular)
 - pyramid (square, rectangular, triangular)
 - cylinder
 - cone
 - sphere
3. solve problems involving volume of solids.



How much do you know

Find the volume of each solid:

1. a cube with side (s) = 2.4 m
2. a cylinder with $h = 20$ cm, $r = 22$ cm.
3. a rectangular prism with $l = 25$ cm, $w = 17$ cm, $h = 30$ cm

4. a square pyramid with $s = 5$ m, $h = 6$ m.
5. a cone with $r = 2$ cm, $h = 6$ cm.
6. a triangular prism with height 10 cm, base (a right triangle with sides 3, 4 and 5 cm and the right angle between shorter sides).
7. a ball with radius of 17 cm (use $\pi = 3.14$)
8. a triangular pyramid with $b = 4$ cm, $h = 8.2$ cm (altitude of the base), $h = 7$ cm (height of the pyramid).
9. a rectangular pyramid with $l = 6$ cm, $w = 4.3$ cm, $h = 8$ cm (height of the pyramid)
10. a cylindrical tank is 5.3 meters high. If the radius of its base is 2.8 meters, what is its volume?
11. Find the volume of a rectangular prism which is 46 cm long, 37 cm wide and 25 cm high.
12. Find the volume of a pyramid with a square base if the length of the sides of the base is 2.4 m and the height of the triangular face is 3.5 m.
13. cube with edge of $6\frac{2}{3}$ cm.
14. cylinder with radius of base 8.7 cm and height 12 cm.
15. rectangular prism with base 8 m by 10 m by 15 m.



What you will do

Lesson 1

Finding the Volume of a Cube, Prism and Pyramid

One problem with rooms that have high ceilings is that they are hard to heat and cool. The amount of air in a room determines the heating or cooling power needed. To find the amount of air in a room, you need to find the volume of the room.

In finding volume of solids, you have to consider the area of a face and height of the solid. If the base is triangular, you have to make use of the area of a triangle, if rectangular, make use of the area of a rectangle and so on.

The next examples will help you to understand more about volume or the amount of space in three – dimensional figures.

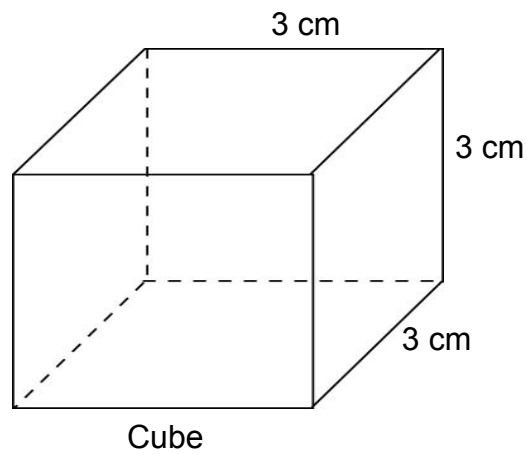
Volume of a cube

The volume V of a cube with edge e is the cube of e . That is,
$$V = e^3.$$

Example:

Find the volume of a cube with edge (e) of 3 cm.

Figure:



Solution:

$$V = e^3$$

Substituting e by 3 cm:

$$V = 3^3$$

$$V = 27 \text{ cm}^3$$

Volume of Prism

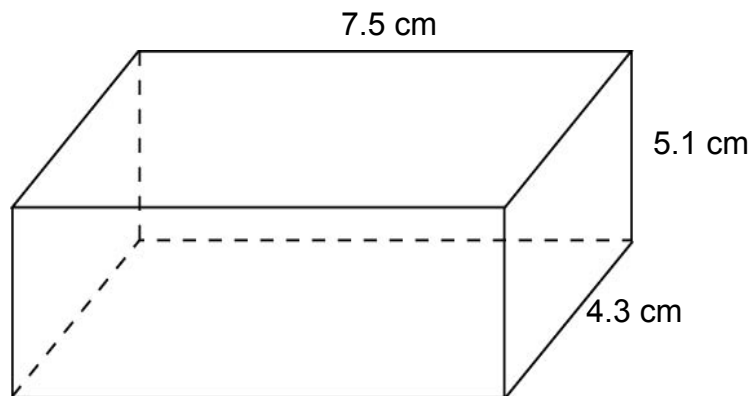
The volume V of a rectangular prism is the product of its altitude h , the length l and the width w of the base. That is,

$$V = lwh.$$

Example:

Find the volume of a rectangular prism whose length is 7.5 cm, width is 4.3 cm and thickness is 5.1 cm.

Figure:



Solution:

$$V = lwh$$

$$V = (7.5 \text{ cm})(4.3 \text{ cm})(5.1 \text{ cm})$$

$$V = 164.475 \text{ cm}^3$$

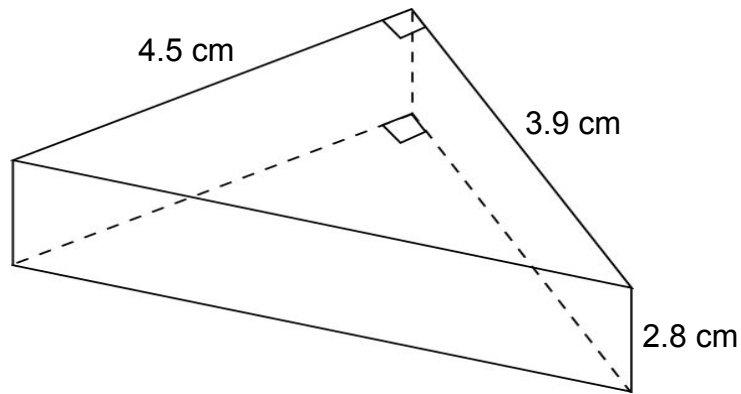
The volume of a prism can also be expressed in terms of area of the base, B .

The volume V of a prism is the product of its altitude h and area B of the base. That is,

$$V = Bh.$$

Example:

Find the volume of a triangular prism whose dimensions is given in the figure below.



Solution:

Let B = area of the triangular base

$$B = \frac{1}{2}bh$$
$$= \frac{1}{2}(4.5 \text{ cm})(3.9 \text{ cm})$$

$$B = 8.775 \text{ cm}^2$$

Finding the volume of the prism:

$$V = Bh$$
$$= 8.775 \text{ cm}^2 (2.8 \text{ cm})$$
$$= 24.57 \text{ cm}^3$$

Volume of Pyramids

Consider a pyramid and a prism having equal altitudes and bases with equal areas. If the pyramid is filled with water or sand and its contents poured into a prism, only a third of the prism will be filled. Thus the volume of a pyramid is $\frac{1}{3}$ the volume of the prism.

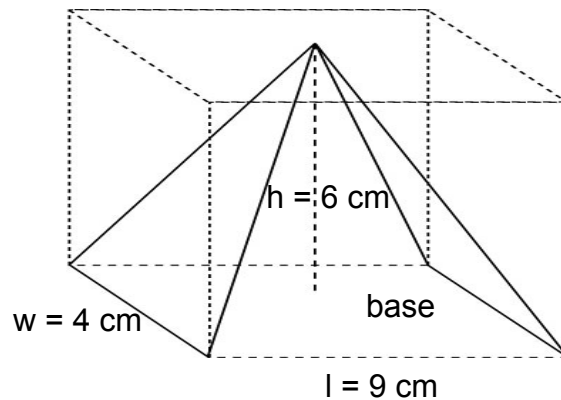
The volume V of a pyramid is one third the product of its altitude h and the area B of its base. That is,

$$V = \frac{1}{3} Bh.$$

Example:

1. Find the volume of the rectangular pyramid with the given dimensions.

Figure:



Solution:

Let B = the area of the rectangular base

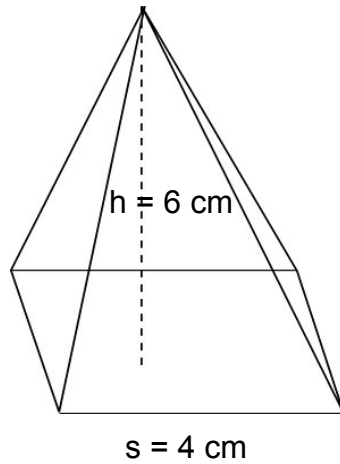
$$\begin{aligned} B &= lw \\ &= (9 \text{ cm})(4 \text{ cm}) \\ &= 36 \text{ cm}^2 \end{aligned}$$

Finding the volume V :

$$\begin{aligned} V &= \frac{1}{3} Bh \\ &= \frac{1}{3} (36 \text{ cm}^2) (6 \text{ cm}) \\ &= 72 \text{ cm}^3 \end{aligned}$$

2. Find the volume of a square pyramid with a side of the base as 4 cm and the height of a pyramid as 6 cm.

Figure:



Solution:

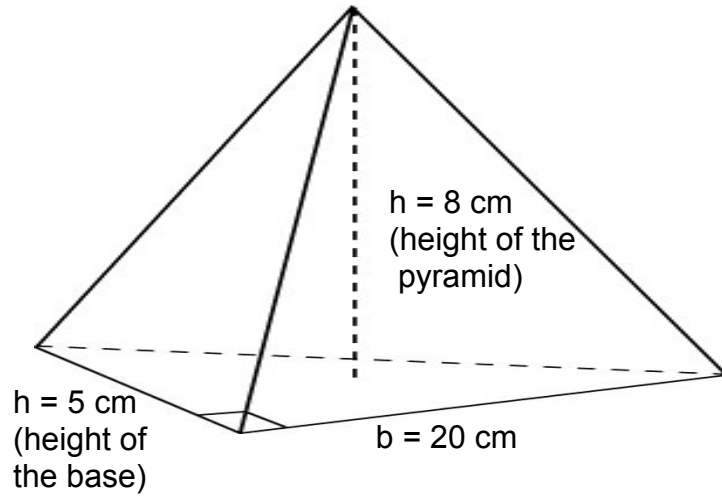
$$\begin{aligned}\text{Let } B &= \text{area of the square base} \\ &= s^2 \\ &= (4 \text{ cm})^2 \\ B &= 16 \text{ cm}^2\end{aligned}$$

Finding the volume of the pyramid:

$$\begin{aligned}V &= \frac{1}{3}Bh \\ &= \frac{1}{3}(16 \text{ cm}^2)(6 \text{ cm}) \\ &= 32 \text{ cm}^3\end{aligned}$$

3. Find the volume of a triangular pyramid with the given dimensions.

Figure:



Solution:

Let B = area of the triangular base

$$\begin{aligned} B &= \frac{1}{2}bh \\ &= \frac{1}{2}(20 \text{ cm})(5 \text{ cm}) \\ B &= 50 \text{ cm}^2 \end{aligned}$$

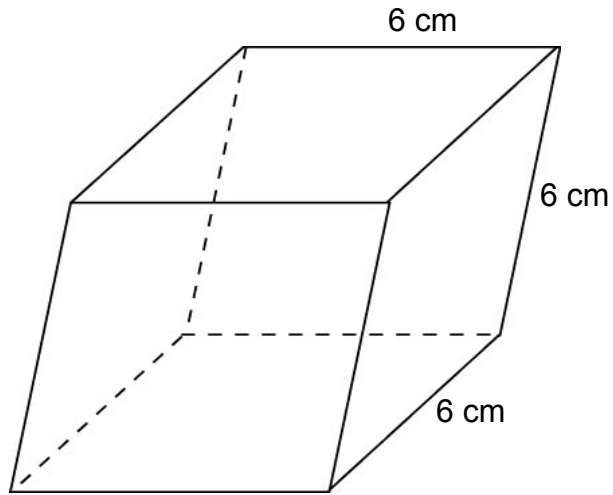
Finding the volume of the pyramid:

$$\begin{aligned} V &= Bh \\ &= (50 \text{ cm}^2)(8 \text{ cm}) \\ V &= 400 \text{ cm}^3 \end{aligned}$$

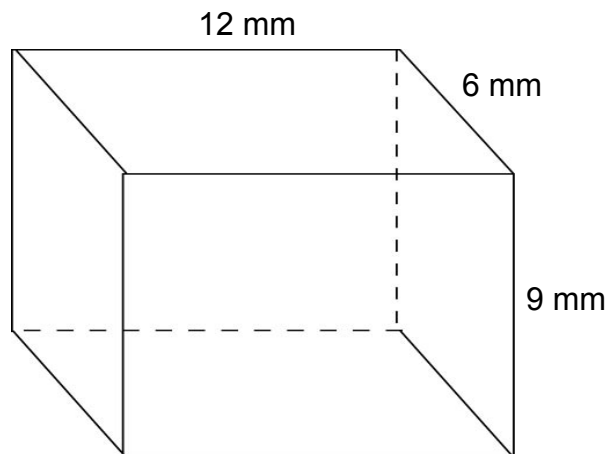
Try this out

Find the volume of each solid:

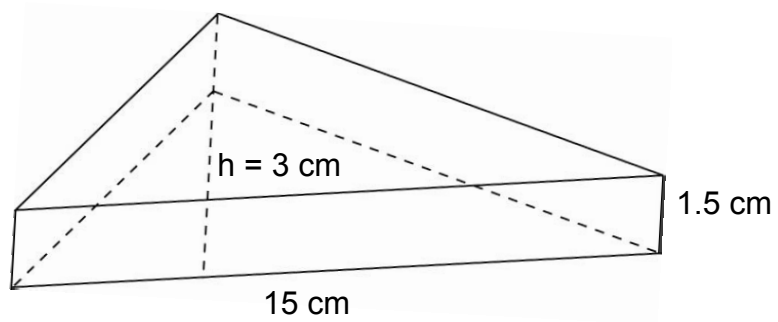
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2.

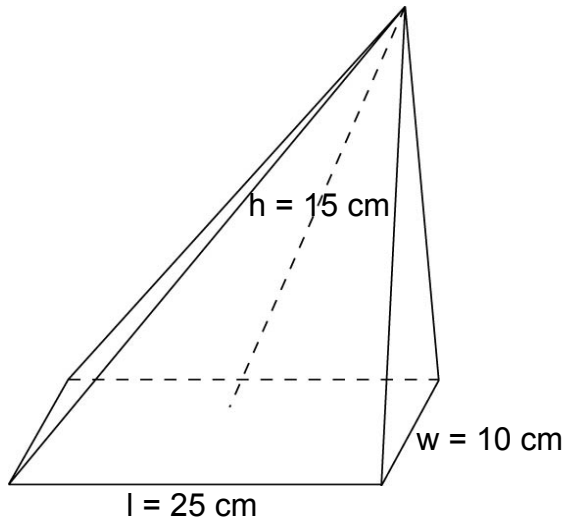


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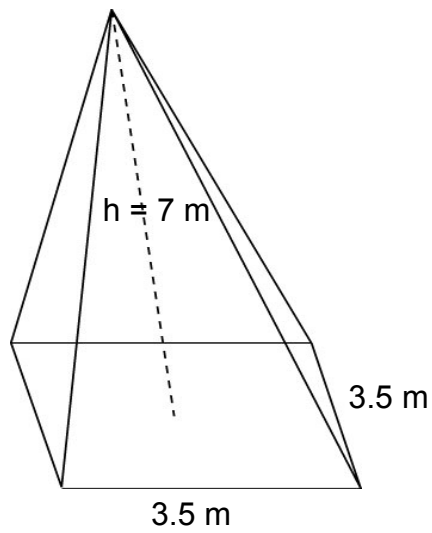


Find the volume of each pyramid:

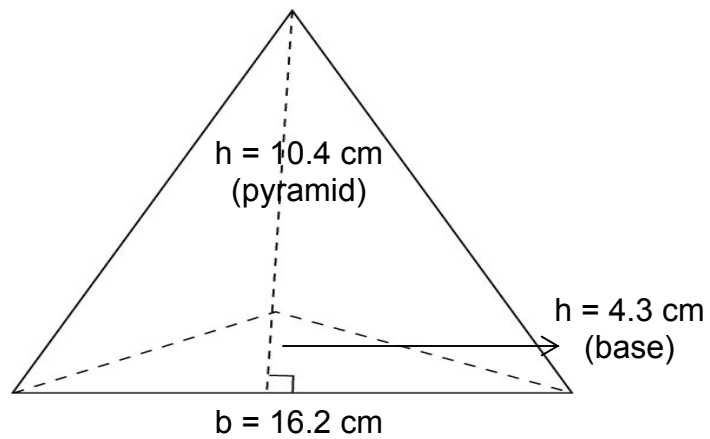
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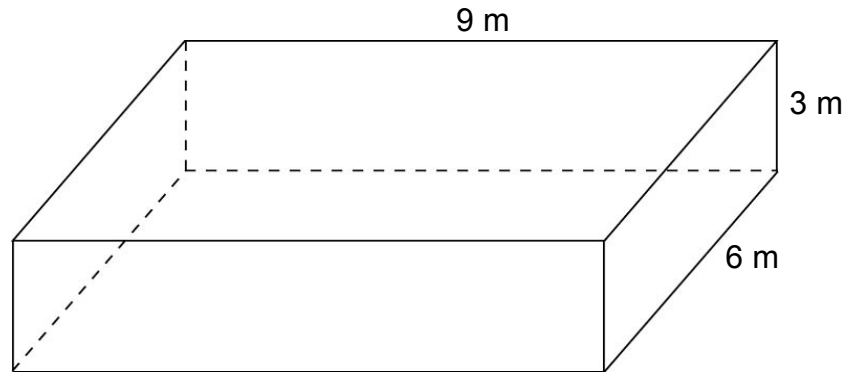
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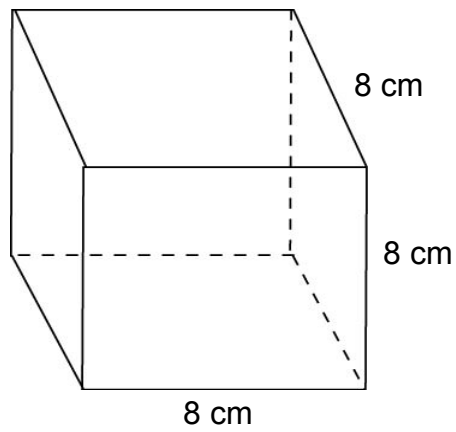
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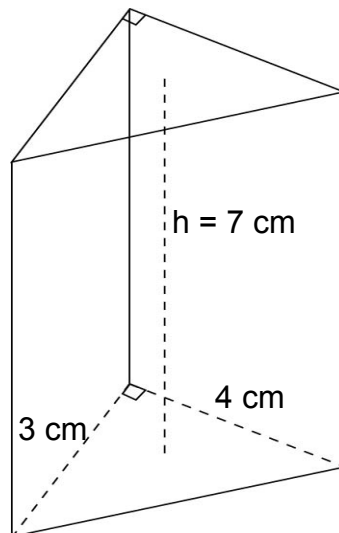
7. What is the volume of a cardboard box that is 9 m long, 6 m wide, and 3 m high?



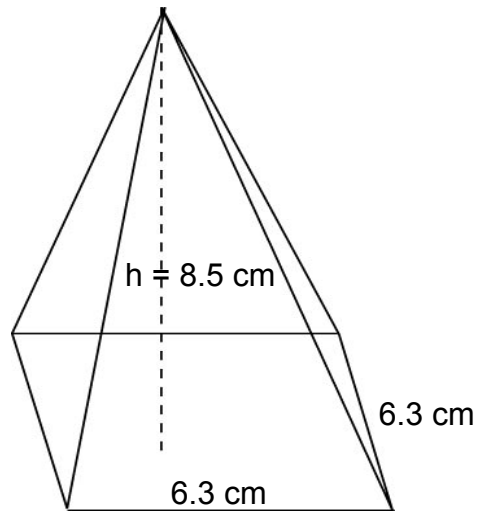
8. Find the volume of a cube with side of 8 cm.



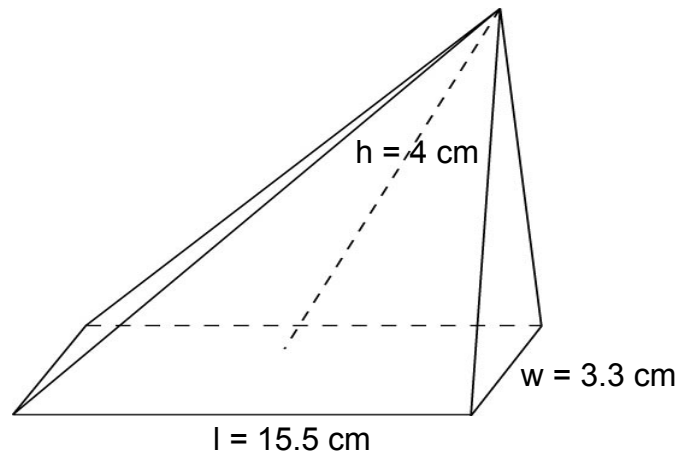
9. Find the volume of a triangular chocolate box.



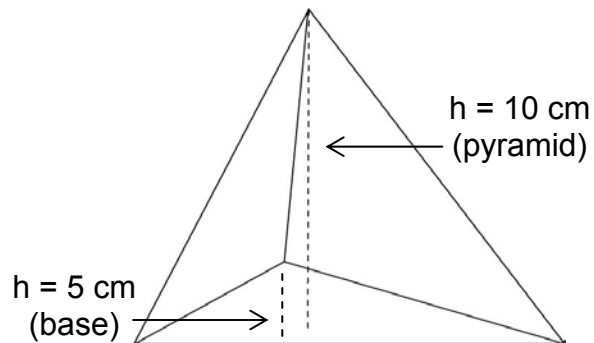
10. Find the volume of a camping tent in a square pyramid shape with a side of the base as 5 cm and the height of a triangle as 7 cm.



11. A pyramid has a rectangular base whose length and width are 15.5 cm and 3.3 cm respectively. The height of the pyramid is 4 cm. Find its volume.



12. Find the volume of a tetra pack juice drink in triangular pyramid shape with the given dimensions.

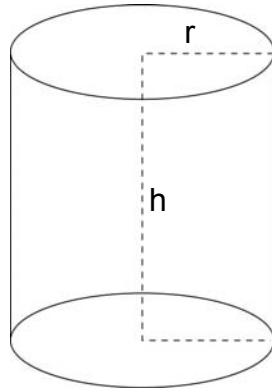


Lesson 2

Finding the Volume of a Cylinder, Cone and Sphere

A cylinder has 2 congruent circular bases. The volume of a cylinder is just like finding the volume of a prism.

Figure:



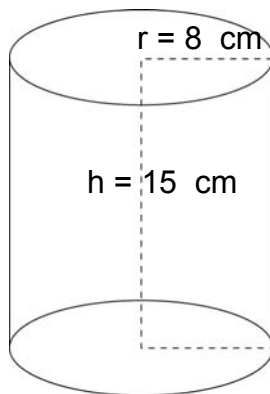
Volume of a Cylinder

The volume V of a circular cylinder is the product of the altitude h and the area B of the base. That is,

$$V = Bh \text{ or } V = \pi r^2 h.$$

Example:

Find the volume of a cylinder which has a radius of 8 cm and a height of 15 cm. (Use 3.14 for π)



Solution:

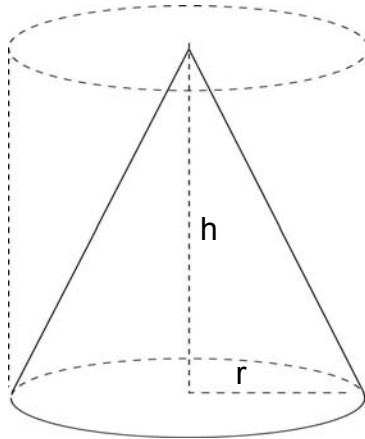
$$\begin{aligned}\text{Let } B &= \text{area of the circular base} \\ &= \pi r^2 \\ &= (3.14)(8 \text{ cm}) \\ B &= 200.96 \text{ cm}^2\end{aligned}$$

Finding the volume of the cylinder:

$$\begin{aligned}V &= Bh \\ &= (200.96 \text{ cm}^2)(15 \text{ cm}) \\ &= 3014.4 \text{ cm}^3\end{aligned}$$

Volume of a Cone

If a cone is filled with water or sand, and then its content is poured into the cylinder (the cone and cylinder have equal areas) only a third of the cylinder will be filled. This shows that the volume of a cone is $\frac{1}{3}$ that of the cylinder.



The volume V of a circular cone is one third the product of the altitude h and the area B of the base. That is,

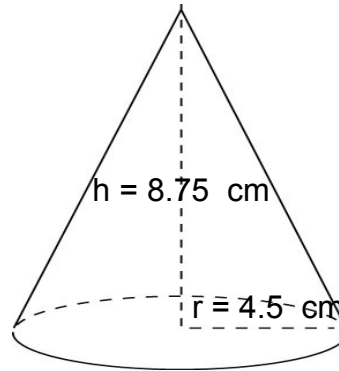
$$V = \frac{1}{3}Bh \text{ or } V = \frac{1}{3}\pi r^2h$$

Example:

Find the volume of a cone if the radius of its base is 4.5 cm and its height is 8.75 cm (Use $\pi = 3.14$)

Solution:

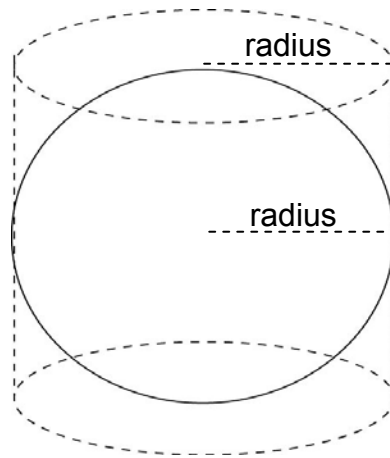
$$\begin{aligned}V &= \frac{1}{3} \pi r^2 h \\&= \frac{1}{3} (3.14)(4.5 \text{ cm})^2 (8.75 \text{ cm}) \\V &= 185.46 \text{ cm}^3\end{aligned}$$



Volume of a Sphere

Fill a cylinder with water. Push the sphere into the cylinder and determine the amount of water displaced. About $\frac{2}{3}$ of the water will be displaced, so the volume of the sphere is $\frac{2}{3}$ that of the cylinder.

Figure:



In the figure, the height of the cylinder is equal to the diameter of the sphere, the volume of the cylinder will now be equal to $2\pi r^3$. Since the volume of the sphere is $\frac{2}{3}$ that of the cylinder and the height of the cylinder = $2r$, then

$$V = \frac{2}{3} (2\pi r^3) = \frac{4}{3} \pi r^3.$$

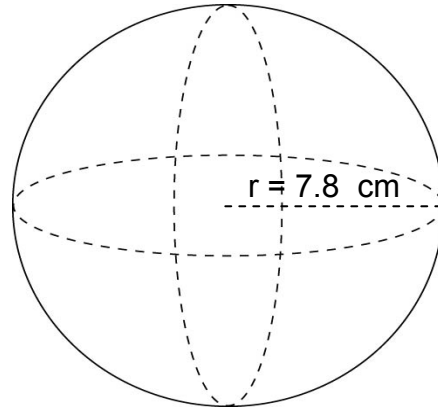
The volume V of a sphere = $\frac{4}{3} \pi r^3$

Example:

What is the volume of a ball with radius equal to 7.8 cm?

Solution:

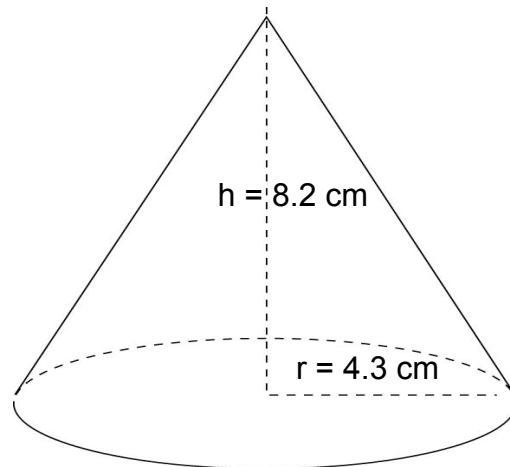
$$\begin{aligned} V &= \frac{4}{3} \pi r^3 \\ &= \frac{4}{3} (3.14)(7.8 \text{ cm})^3 \\ &= 1,986.79 \text{ cm}^3 \end{aligned}$$



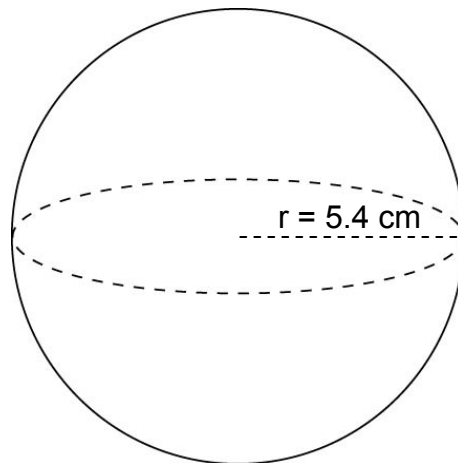
Try this out

Find the volume of each solid.

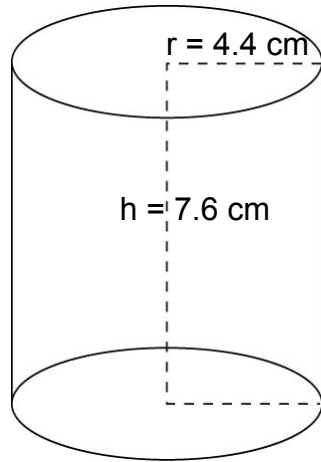
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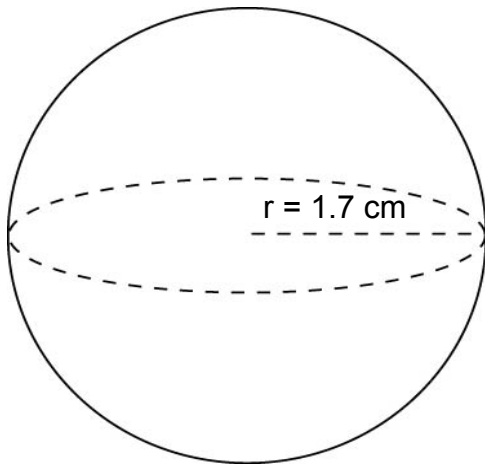
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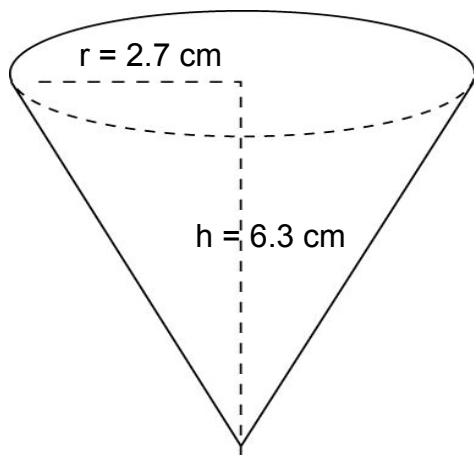
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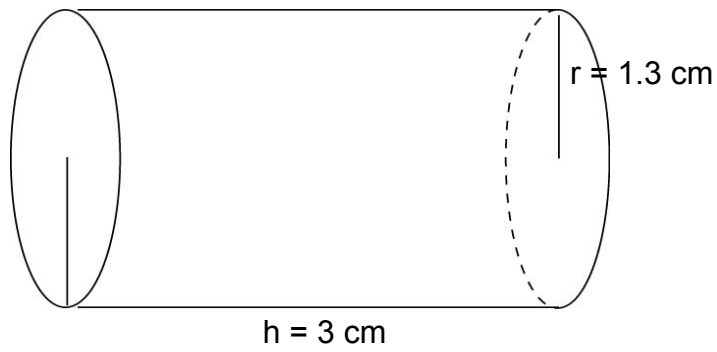
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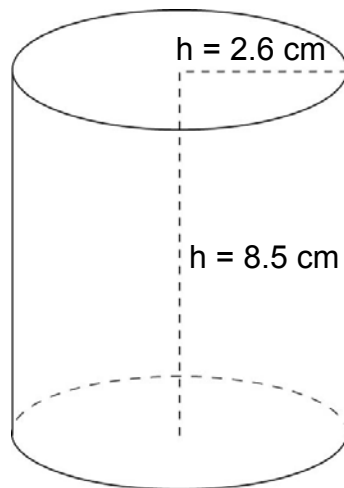
5.



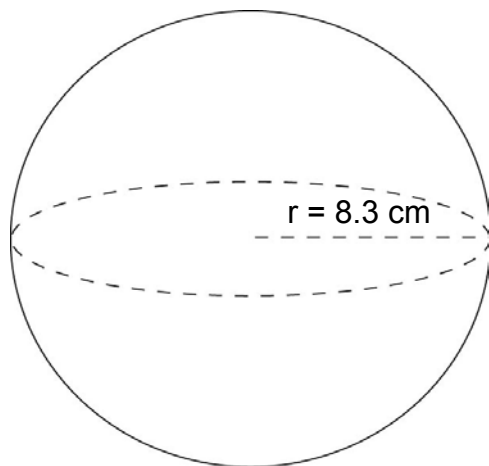
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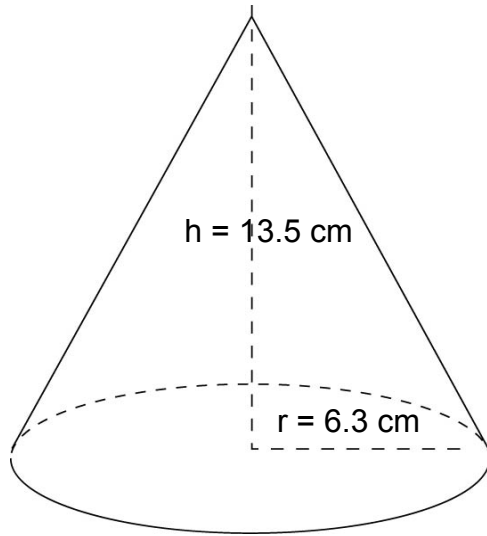
7. A cylindrical water tank is 6.2 meters high. If the radius of its base is 1.8 m, what is its volume.



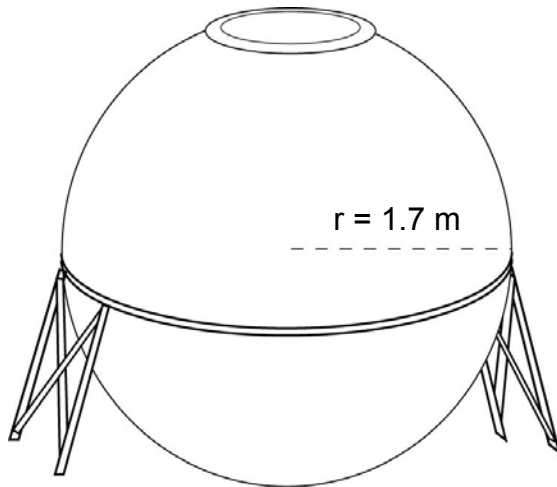
8. The radius of a ball is 5.2 cm. What is its volume?



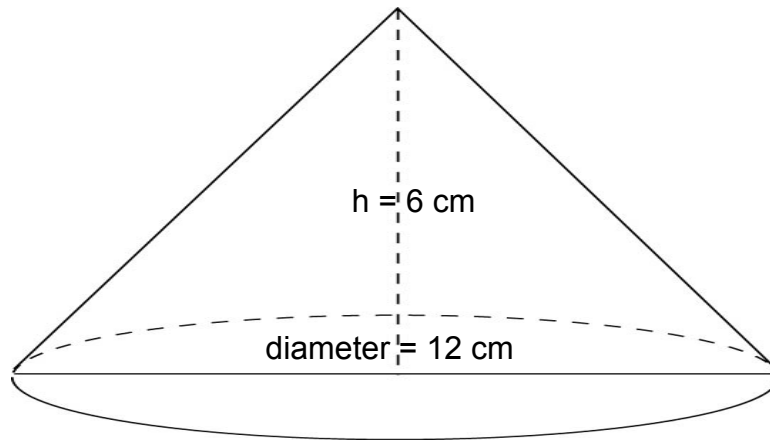
9. Find the volume of a conic solid whose radius is 6.3 cm and its height is 13.5 cm.



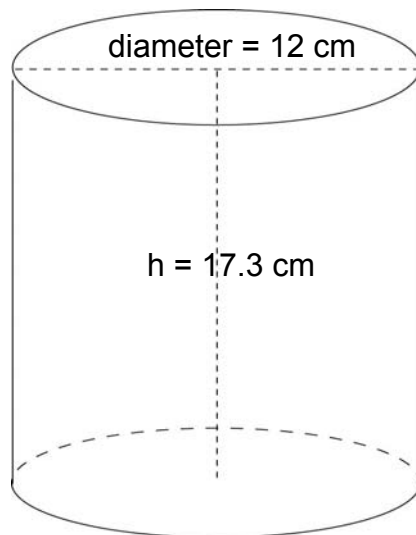
10. Find the volume of a spherical tank whose radius is 1.7 meters.



11. A cone with a diameter of 12 cm and height of 6 cm. Find its volume.



12. A can of milk has a diameter of 12 cm and a height of 17.3 cm. Find its volume.





Let's summarize

The volume of a three dimensional figure is the amount of space it encloses.

The volume V of a cube with edge e is the cube of e . That is,

$$V = e^3.$$

The volume V of a rectangular prism is the product of its altitude h , the length l and the width w of the base. That is,

$$V = lwh.$$

The volume of a prism can be expressed in terms of area of the base, B .

The volume V of a prism is the product of its altitude h and area B of the base. That is,

$$V = Bh.$$

The volume V of a pyramid is one third the product of its altitude h and the area B of its base. That is,

$$V = \frac{1}{3} Bh.$$

The volume V of a circular cylinder is the product of the altitude h and the area B of the base. That is,

$$V = Bh \text{ or } V = \pi r^2 h.$$

The volume V of a circular cone is one third the product of the altitude h and the area B of the base. That is,

$$V = \frac{1}{3} Bh \text{ or } V = \frac{1}{3} \pi r^2 h$$

The volume V of a sphere = $\frac{4}{3} \pi r^3$



What have you learned

Find the volume of each solid:

1. A cube with edge (e) = 6.3 cm.
2. A cylinder with $h = 15$ cm, $r = 7.1$ cm.
3. A rectangular prism with $l = 18$ cm, $w = 7$ cm, $h = 5$ cm.
4. A square pyramid with $s = 8.5$ cm, $h = 6$ cm.
5. A cone with $r = 3.8$ cm, $h = 7.2$ cm.
6. A triangular prism with height 16 cm, base (a right triangle with sides 3, 4 and 5 cm and the right angle between shorter sides).
7. A ball with radius of 13 cm.
8. A triangular pyramid with $b = 5$ cm, $h = 7.2$ cm (altitude of the base),
 $h = 8$ cm (height of the pyramid).
9. A rectangular pyramid with $l = 9$ cm, $w = 6.3$ cm, $h = 8$ cm (height of the pyramid).
10. A cylindrical tank is 5.4 m high. If the radius of its base is 4.9 m, what is its volume?
11. Find the volume of a rectangular prism which is 42 cm long, 38 cm wide and 22 cm high.
12. Find the volume of a pyramid with a square base if the length of the sides of the base is 3.6 m and a height of 1.8 m.
13. Cube with edge 10.5 cm.
14. Cylinder with radius of base 9.7 cm and height of 12 cm.
15. Rectangular prism with base 12 m by 14.6 m and height of 9.1 m.



How much do you know

1. 13.82 m^3
2. $30,395.20 \text{ cm}^3$
3. $12,750 \text{ cm}^3$
4. 50 m^3
5. 25.12 cm^3
6. 60 cm^3
7. $20,569 \text{ cm}^3$
8. 38.27 cm^3
9. 68.8 cm^3
10. 130.47 m^3
11. $42,550 \text{ cm}^3$
12. 6.72 m^3
13. 295.408 cm^3
14. $2,851.99 \text{ cm}^3$
15. $1,200 \text{ m}^3$

Try this out

Lesson 1

1. 216 cm^3
2. 648 mm^3
3. 33.75 cm^3
4. $1,250 \text{ cm}^3$
5. 28.58 m^3
6. 120.74 cm^3
7. 162 m^3
8. 512 cm^3
9. 42 cm^3
10. 112.46 cm^3
11. 68.2 cm^3

12. 125 cm^3

Lesson 2

1. 158.69 cm^3

2. 659.25 cm^3

3. 462.01 cm^3

4. 20.57 cm^3

5. 48.07 cm^3

6. 15.92 m^3

7. 180.42 cm^3

8. $2,393.88 \text{ cm}^3$

9. 560.82 cm^3

10. 20.57 m^3

11. 226.08 cm^3

12. $1,955.59 \text{ cm}^3$

What have you learned

1. 250.047 cm^3

2. $2,374.311 \text{ cm}^3$

3. 630 cm^3

4. 144.5 cm^3

5. 108.82 cm^3

6. 96 cm^3

7. $2,547.168 \text{ cm}^3$

8. 48 cm^3

9. 151.2 cm^3

10. 407.11356 m^3

11. $35,112 \text{ cm}^3$

12. 7.776 m^3

13. $1,157.625 \text{ cm}^3$

14. $3,545.3112 \text{ cm}^3$

15. $1,594.32 \text{ m}^3$