

Module 7

Geometry of Shape and Size



What this module is about

This module is about surface area of solids. As you go over the exercises, you will develop skills in solving surface area of different solid figures. Treat the lesson with fun and take time to go back if you think you are at a loss.



What you are expected to learn

This module is designed for you to

1. define surface area of solids.
2. find the surface area of solids such as
 - cube
 - prism (rectangular, triangular)
 - pyramid (square, rectangular, triangular)
 - cylinder
 - cone
 - sphere
3. solve problems involving surface area of solids.



How much do you know

Find the surface area of each solid.

1. A cube with side (s) = 2.2 cm.
2. A cylinder with $h = 15$ cm, $r = 3.2$ cm.
3. A rectangular prism with $l = 12$ cm, $w = 7$ cm, $h = 6$ cm.
4. A square pyramid with $s = 4.2$ cm, $h = 7$ cm (slant height).

5. A cone with $r = 5$ cm , $s = 12$ cm (slant height).
6. A triangular prism with height 14 cm, base (a right triangle with sides 6, 8 and 10 cm and the right angle between shorter sides).
7. A ball with radius of 6 cm (Use $\pi = 3.14$).
8. A triangular pyramid with $b = 4$ cm, $h = 8.2$ cm (altitude of the base), $s = 7$ cm (slant height).
9. A rectangular pyramid with $l = 5$ cm, $w = 3.3$ cm, $h = 9$ cm (slant height).
10. A cylindrical tank is 2.6 meters high. If the radius of its base is 0.92 meters, what is its surface area?
11. find the surface area of a rectangular prism which is 45 cm long, 36 cm wide and 24 cm high.
12. Find the surface area of a pyramid with a square base if the length of the sides of the base is 1.4 m and the height of the triangular face is 1.9 m.
13. Cube with edge $4 \frac{2}{3}$ cm
14. Cylinder with radius of base 6.7 cm and height 14 cm
15. Rectangular prism with base 9m by 10m by 12m



What you will do

Lesson 1

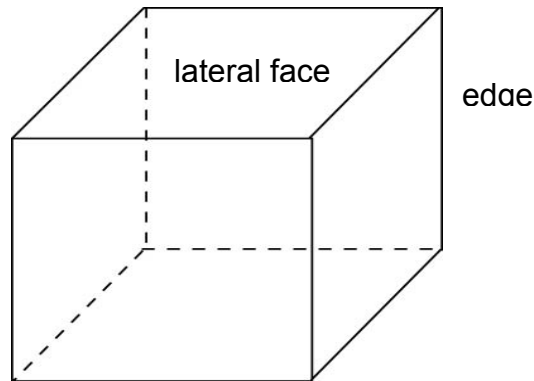
Finding the Surface Area of a Cube, Prism and Pyramid

The surface area of a three-dimensional figure is the total area of its exterior surface. For three-dimensional figures having bases, the surface area is the lateral surface area plus the area of the bases.

Examples :

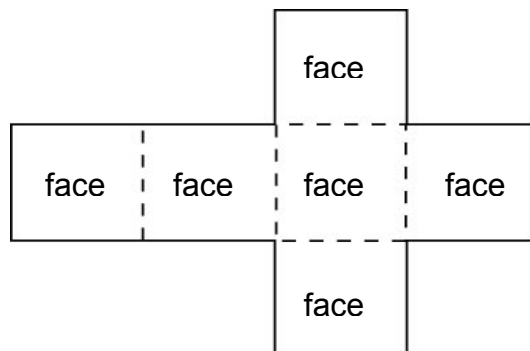
1. Find the surface area of a cube with side of 5 cm.

Figure :



Cube

If you open this up,



This solid is composed of 6 squares or 6 faces. Since the area of a square is the square of its side or s^2 , then the surface area of a cube is $6s^2$.

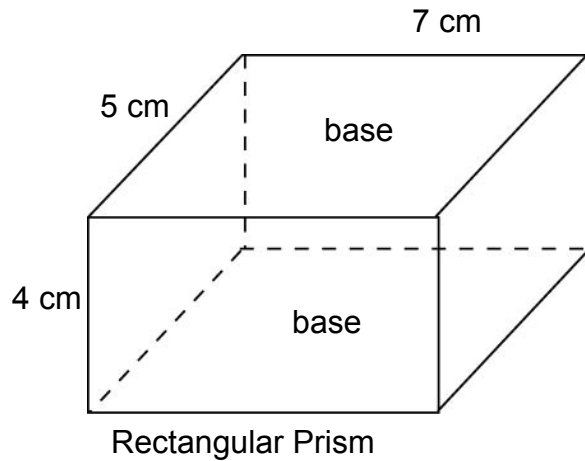
$$\text{SA of a Cube} = 6s^2$$

Substituting s by 5 cm:

$$\begin{aligned} \text{SA} &= 6(5^2) \\ &= 6(25) \\ \text{SA} &= 150 \text{ cm}^2 \end{aligned}$$

2. Find the surface area of a rectangular prism whose length is 7 cm, width is 5 cm and thickness is 4 cm.

Figure :



Rectangular prism, add the areas of its flat surfaces.

Area of top and bottom rectangles (bases) plus area of left and right rectangles and area of back and front rectangles (lateral areas).

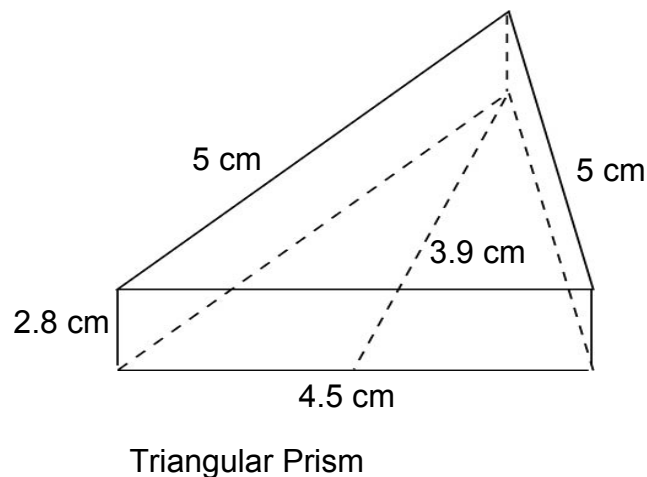
$$\text{SA of rectangular prism} = 2B + LA$$

Solution:

$$\begin{aligned} \text{SA} &= 2(7\text{cm} \times 5\text{ cm}) + 2(4\text{ cm} \times 5\text{ cm}) + 2(7\text{cm} \times 4\text{ cm}) \\ &= 70\text{ cm}^2 + 40\text{ cm}^2 + 56\text{ cm}^2 \\ \text{SA} &= 166\text{ cm}^2 \end{aligned}$$

3. Find the surface area of a triangular prism.

Figure :



SA of a triangular prism = $2B + LA$

Solution:

$$SA = 2 \left(\frac{1}{2} bh \right) + LA$$

$$SA = bh + LA$$

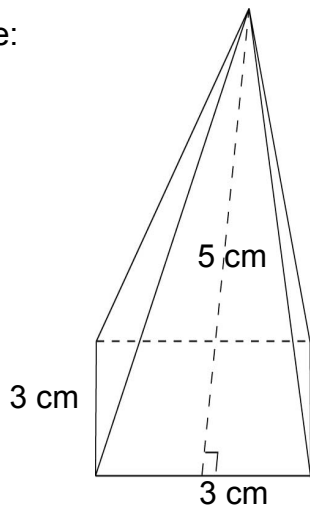
$$= (4.5\text{cm} \times 3.9\text{cm}) + 2(5\text{cm} \times 2.8\text{cm}) + (4.5\text{cm} \times 2.8\text{cm})$$

$$= 17.55\text{ cm}^2 + 28\text{ cm}^2 + 12.6\text{ cm}^2$$

$$SA = 58.15\text{ cm}^2$$

4. Find the surface area of a square pyramid with a side of the base as 3 cm and the height of a triangle as 5 cm.

Figure:



To find the surface area of a square pyramid, add the area of the square base and the areas of the four face triangles.

Solution:

$$SA \text{ of Square Pyramid} = B + 4 \left(\frac{1}{2} bh \right)$$

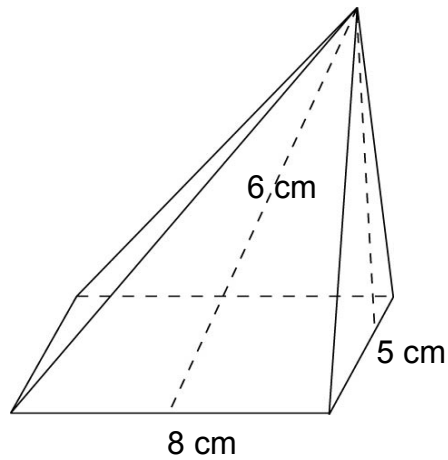
$$SA = 3^2 + 4 \left(\frac{1}{2} \times 3 \times 5 \right)$$

$$= 9 + 30$$

$$SA = 39\text{ cm}^2$$

5. Find the surface area of the rectangular pyramid with the given dimensions.

Figure :



Rectangular Pyramid

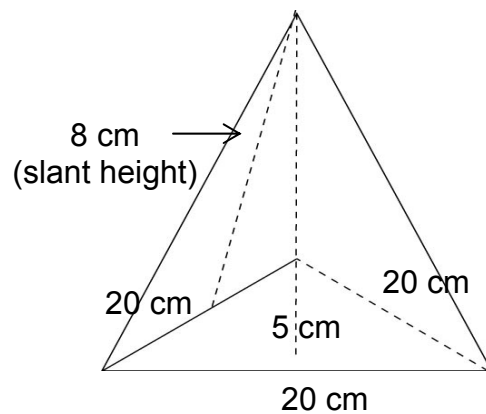
$$\text{SA of Rectangular Pyramid} = B + 2 A_1 + 2 A_2$$

Solution:

$$\begin{aligned} \text{SA} &= bh + 2 \left(\frac{1}{2} b_1 h_1 \right) + 2 \left(\frac{1}{2} b_2 h_2 \right) \\ &= bh + (b_1 h_1) + (b_2 h_2) \\ &= (8 \text{ cm} \times 5 \text{ cm}) + (8 \text{ cm} \times 6 \text{ cm}) + (5 \text{ cm} \times 6 \text{ cm}) \\ &= 40 \text{ cm}^2 + 48 \text{ cm}^2 + 30 \text{ cm}^2 \\ \text{SA} &= 118 \text{ cm}^2 \end{aligned}$$

6. Find the surface area of a triangular pyramid with the given dimensions.

Figure :



Triangular Pyramid

SA of Triangular Pyramid = Area of the base + Area of the 3 triangular faces

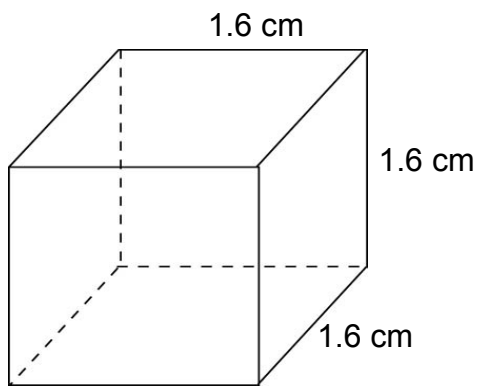
Solution:

$$\begin{aligned} SA &= \left(\frac{1}{2} \times 20 \text{ cm} \times 5 \text{ cm}\right) + (10 \text{ cm} \times 8 \text{ cm}) + \left(\frac{1}{2} \times 20 \text{ cm} \times 8 \text{ cm}\right) \\ &= 50 \text{ cm}^2 + 80 \text{ cm}^2 + 80 \text{ cm}^2 \\ SA &= 210 \text{ cm}^2 \end{aligned}$$

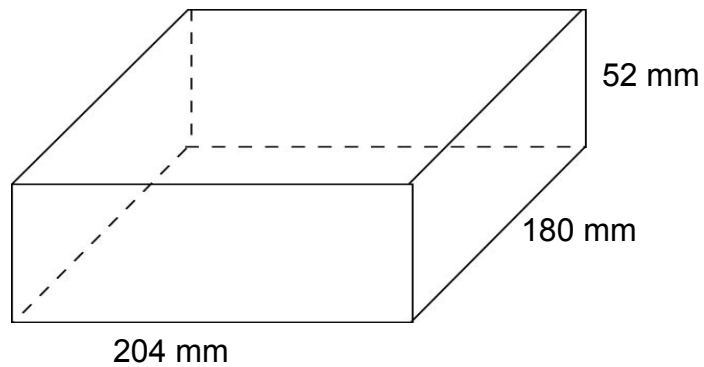
Try this out

Find the surface area of each solid.

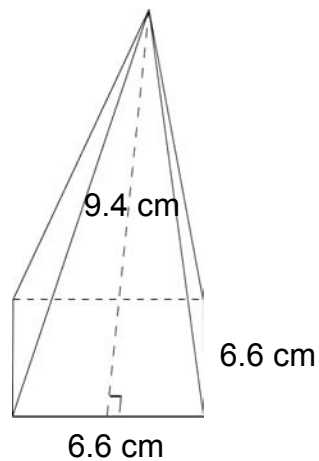
1.



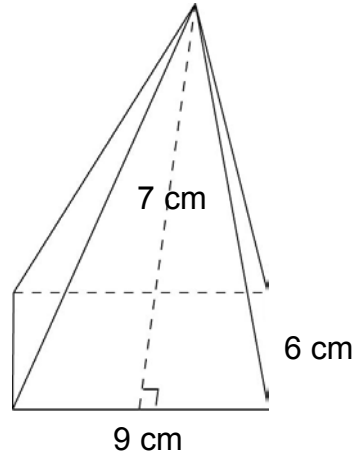
2.



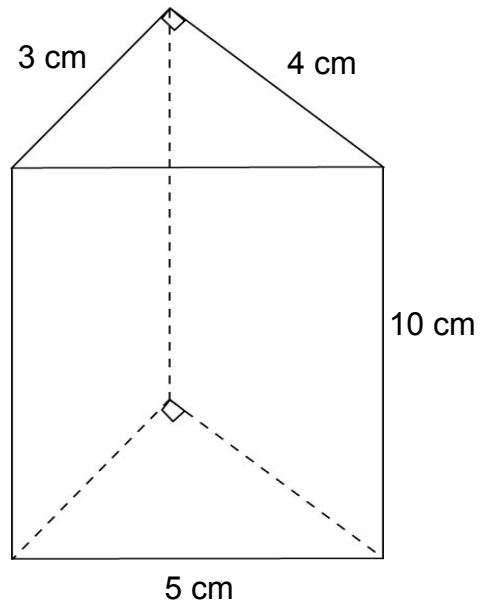
3.



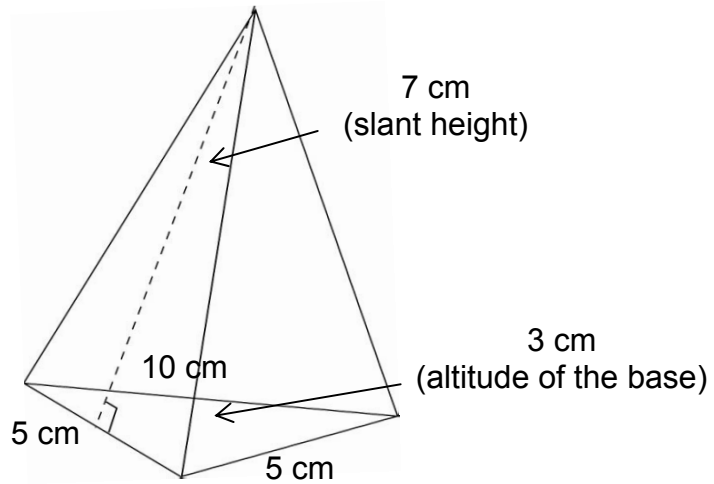
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5.

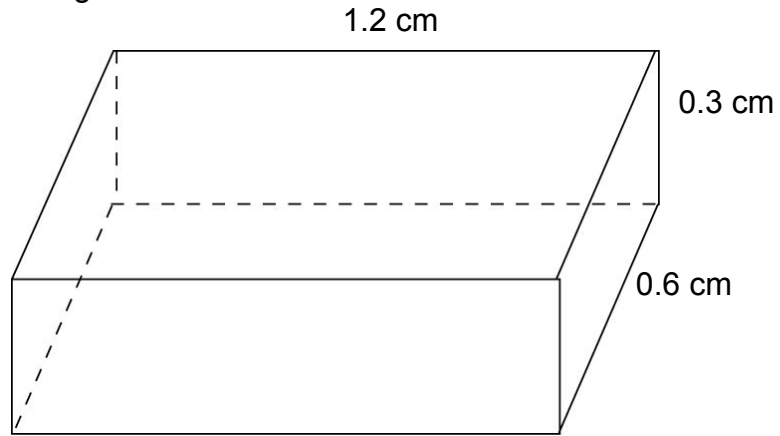


6.



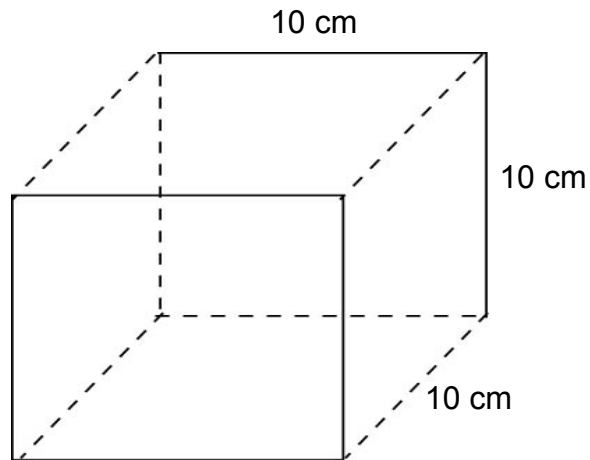
7. What is the total surface area of a cardboard box that is 1.2 m long, 0.6 m wide, and 0.3 m high?

Figure:



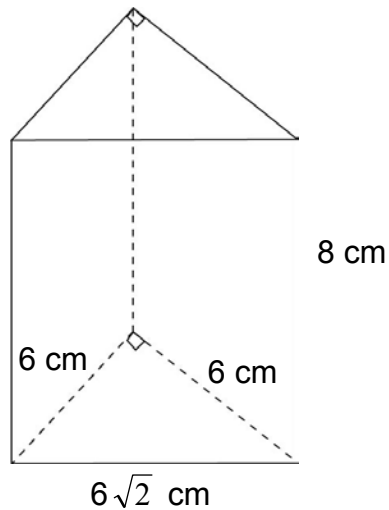
8. Find the surface area of a cube whose side measures 10 cm.

Figure:



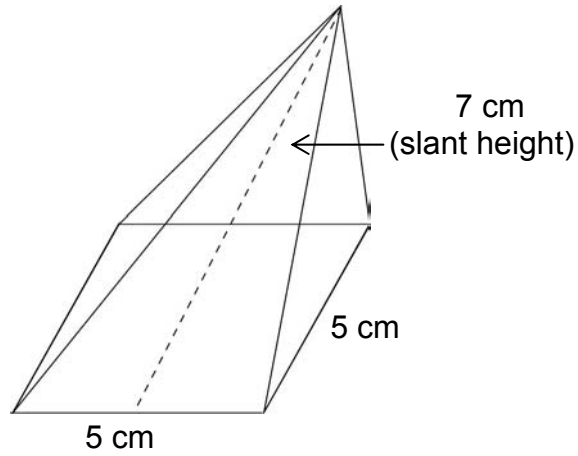
9. Find the surface area of a triangular chocolate box .

Figure:



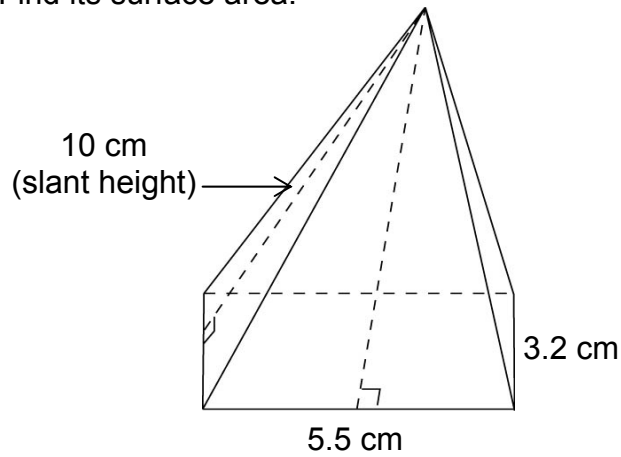
10. Find the surface area of a camping tent in a square pyramid shape with a side of the base as 5 cm and the height of a triangle as 7 cm.

Figure :



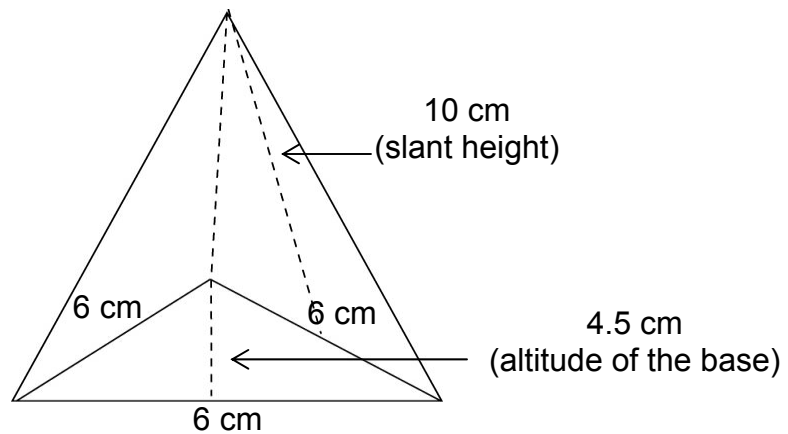
11. A pyramid has a rectangular base whose length and width are 5.5 cm and 3.2 cm respectively. Find its surface area.

Figure:



12. Find the surface area of a tetra pack juice drink in triangular pyramid shape with the given dimensions.

Figure:

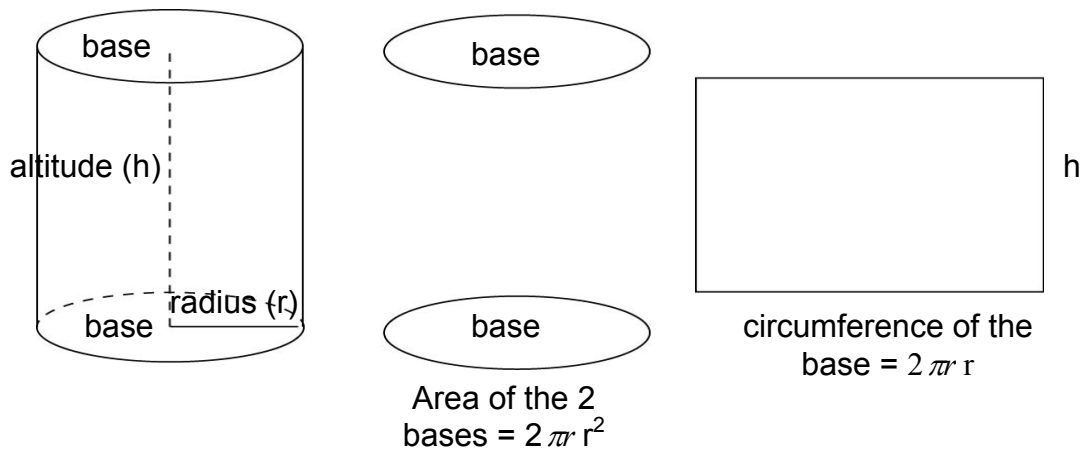


Lesson 2

Finding the Surface Area of Cylinder, Cone and Sphere

Surface Area of a Cylinder

Figure:



To find the surface area of a cylinder, add the areas of the circular bases and the area of the rectangular region which is the body of the cylinder.

SA = Area of 2 Circular Bases + Area of a rectangle

$$SA = 2\pi r^2 + 2\pi rh$$

Example:

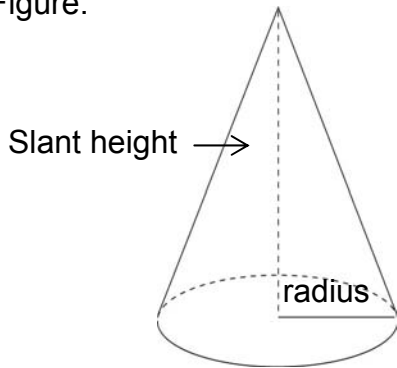
Find the surface area of a cylinder which has a radius of 5 cm and a body length of 20 cm. (Use 3.14 for π)

Solution:

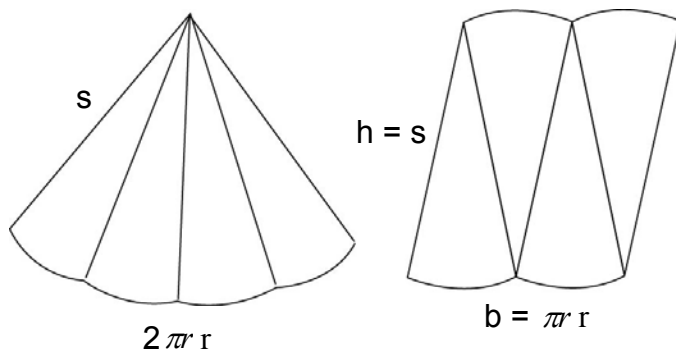
$$\begin{aligned} SA &= 2\pi r^2 + 2\pi rh \\ &= 2(3.14)(5)^2 + 2(3.14)(5)(20) \\ &= (6.28)(25) + (6.28)(100) \\ &= 157 + 628 \\ SA &= 785 \text{ cm}^2 \end{aligned}$$

Surface Area of a Cone

Figure:



Consider an ice-cream cone with its curved surface opened out to resemble a fan.



The fan-shaped surface can be cut into smaller pieces and rearranged to resemble a parallelogram with base $b = \pi r$ and height h equal to side s .

SA = Area of the circular base + Area of the region which resembles a parallelogram

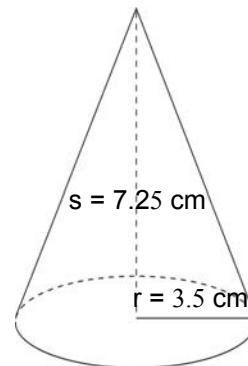
$$SA = \pi r^2 + \pi r s$$

Example:

Find the surface area of a cone if the radius of its base is 3.5 cm and its slant height is 7.25 cm. (Use $\pi = 3.14$)

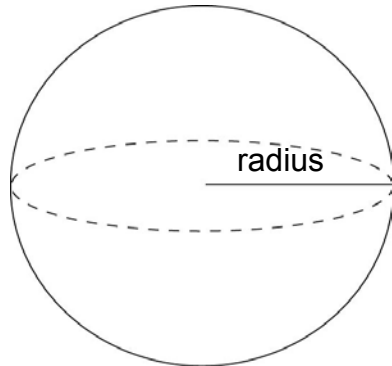
Solution :

$$\begin{aligned} SA &= \pi r^2 + \pi r s \\ &= (3.14)(3.5)^2 + (3.14)(3.5)(7.25) \\ &= (3.14)(12.25) + (10.99)(7.25) \\ &= 38.465 + 79.6775 \\ SA &= 118.1425 \text{ cm}^2 \end{aligned}$$



Surface Area of a Sphere

Figure:



A sphere is a solid where every point is equally distant from its center. This distance is the length of the radius of

SA = area of 4 circles

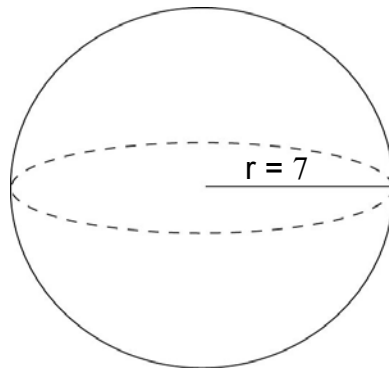
$$SA = 4\pi r^2$$

Example:

What is the surface area of a ball with radius equal to 7 cm? (Use $\pi = 3.14$)

Solution :

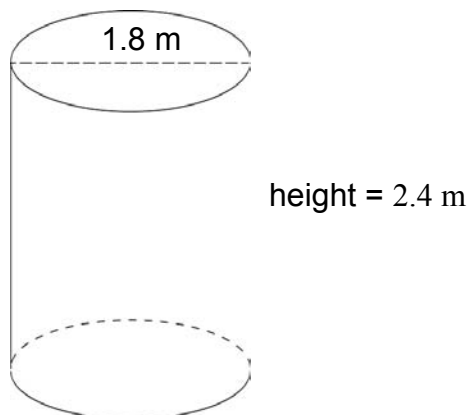
$$\begin{aligned} SA &= 4\pi r^2 \\ &= 4(3.14)(7)^2 \\ &= (12.56)(49) \\ SA &= 615.44 \text{ cm}^2 \end{aligned}$$



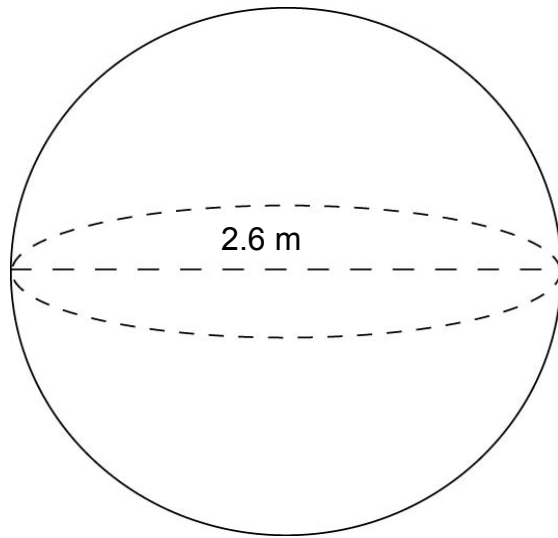
Try this out

Find the surface area of each solid.

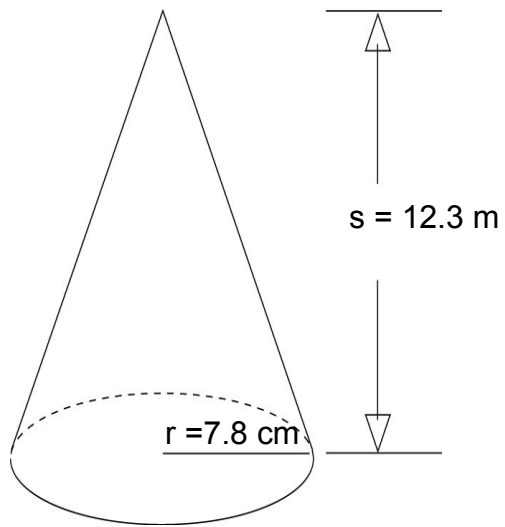
1.



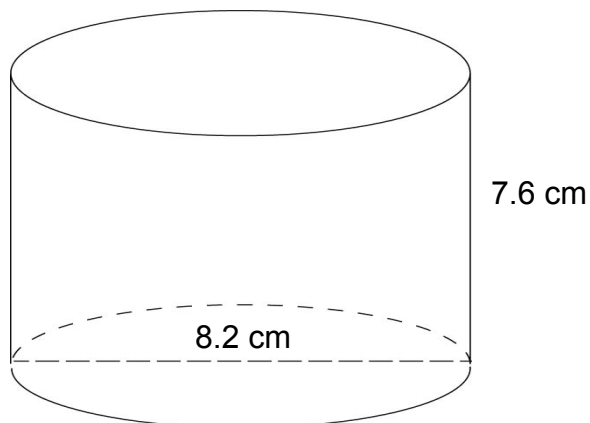
2.



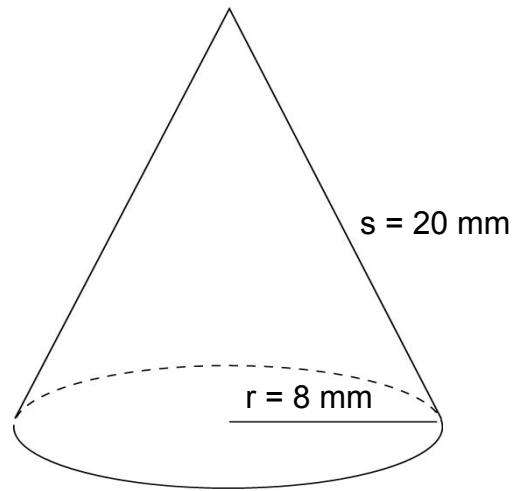
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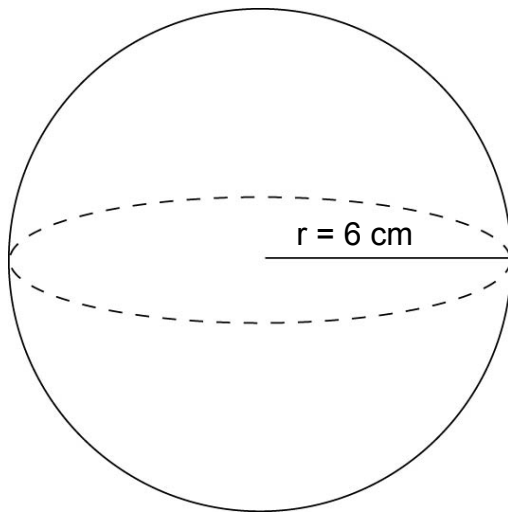
4.



5.

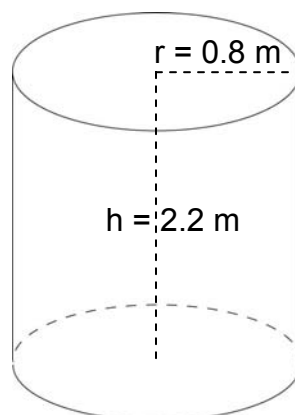


6.



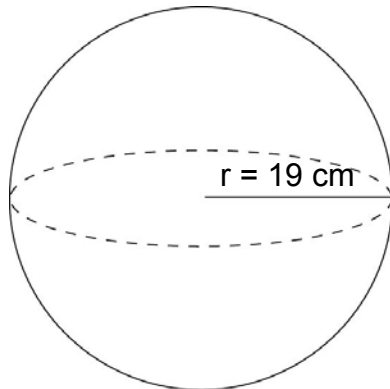
7. A cylindrical water tank is 2.2 meters high. If the radius of its base is 0.8 meter, what is its surface area?

Figure:



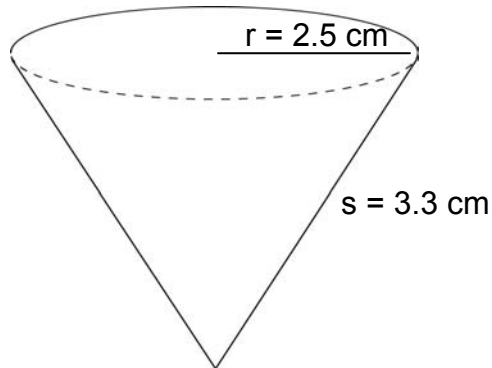
8. The radius of a ball is 19 cm. What is its surface area?

Figure:



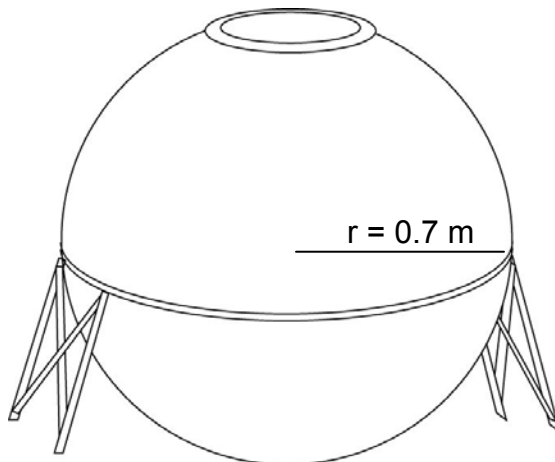
9. Find the surface area of a conic solid whose radius is 2.5cm and its height is 3.3 cm.

Figure:



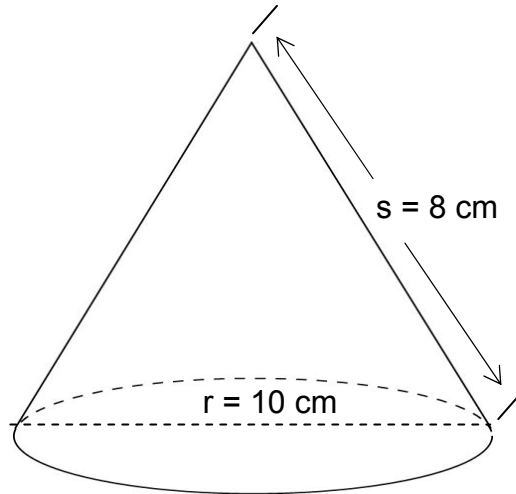
10. Find the surface area of a spherical tank whose radius is 0.7meter.

Figure:



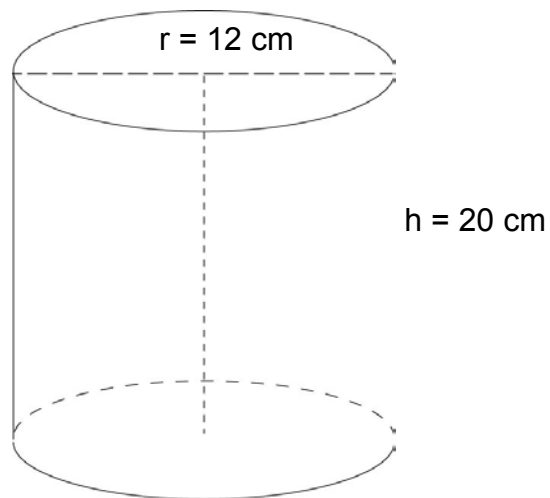
11. A cone with a diameter of 10 cm and height of 8 cm. Find its surface area.

Figure :



12. A can whose height is 20 cm and 12 cm is the diameter. Find its surface area.

Figure:





Let's summarize

The surface area of a three-dimensional figure is the total area of its exterior surface. For three-dimensional figures having bases, the surface area is the lateral surface area plus the area of the bases.

- Surface Area of a Cube = $6s^2$
- Surface Area of Rectangular Prism = $2B + LA$
- Surface Area of Rectangular Prism = $2B + LA$
- Surface Area of Square Pyramid = $B + 4 \left(\frac{1}{2} bh \right)$
- Surface Area of Rectangular Pyramid = $B + 2 A_1 + 2 A_2$
- Surface Area of Triangular Pyramid = Area of the base + Area of the 3 triangular faces
- Surface Area of a Cylinder = Area of 2 Circular Bases + Area of a rectangle

$$SA = 2\pi r^2 + 2\pi rh$$

- Surface Area of a Cone = Area of the circular base + Area of the region which resembles a parallelogram

$$SA = \pi r^2 + \pi rs$$

- Surface Area of a Sphere = area of 4 circles

$$SA = 4\pi r^2$$



What have you learned

Find the surface area of each solid.

1. A cube with side (s) = 4.3 cm
2. A cylinder with h = 13 cm, r = 4.1 cm
3. A rectangular prism with l = 15 cm, w = 8 cm, h = 7 cm
4. A square pyramid with s = 5.4 cm, h = 9 cm (slant height).
5. A cone with r = 4cm , s = 9 cm (slant height).
6. A triangular prism with height 16 cm, base (a right triangle with sides 7 cm, 8 cm and 10 cm and the right angle between shorter sides).
7. A ball with radius of 12 cm. (Use $\pi = 3.14$)
8. A triangular pyramid with b = 5 cm, h = 7.2 cm (altitude of the base), s = 8 cm (slant height)
9. A rectangular pyramid with l = 6 cm, w = 6.3 cm, h = 8 cm (slant height)
10. A cylindrical tank is 3.6 meters high. If the radius of its base is 1.9 meters, what is its surface area?
11. Find the surface area of a rectangular prism which is 40 cm long, 35 cm wide and 23 cm high.
12. Find the surface area of a pyramid with a square base if the length of the sides of the base is 1.6 m and the height of the triangular face is 2.8 m.
13. Cube with edge $5\frac{1}{2}$ cm.
14. Cylinder with radius of base 8.7 cm and height 13 cm.
15. Rectangular prism with base 10m by 12m by 14m.



Answer Key

How much do you know

1. 29.04 cm^2
2. 365.74742 cm^2
3. 396 cm^2
4. 135.24 cm^2
5. 266.9 cm^2
6. 384 cm^2
7. 452.16 cm^2
8. 100.4 cm^2
9. 91.2 cm^2
10. 20.337152 cm^2
11. $7,128 \text{ cm}^2$
12. 12.6 m^2
13. $130 \frac{2}{3} \text{ cm}^2$
14. 870.9732 cm^2
15. 636 m^2

Try this out

Lesson 1

1. 15.36 cm^2
2. $113,376 \text{ mm}^2$
3. 167.64 cm^2

4. 159 cm^2
5. 132 cm^2
6. 85 cm^2
7. 2.52 m^2
8. 600 cm^2
9. 181.88 cm^2
10. 95 cm^2
11. 104.6 cm^2
12. 78.3 cm^2

Lesson 2

1. 18.6516 m^2
2. 21.2264 m^2
3. 492.2892 cm^2
4. 301.3576 cm^2
5. 703.36 mm^2
6. 452.16 cm^2
7. 15.072 m^2
8. $4,534.16 \text{ cm}^2$
9. 45.53 cm^2
10. 6.1544 m^2
11. 204.1 cm^2
12. 979.68 cm^2

What have you learned

1. 110.94 cm^2
2. 131.3148 cm^2
3. 562 cm^2
4. 101.16 cm^2
5. 163.24 cm^2
6. 228 cm^2
7. $1,808.64 \text{ cm}^2$
8. 138 cm^2
9. 136.2 cm^2
10. 65.626 m^2
11. $6,250 \text{ cm}^2$
12. 20.48 m^2
13. 181.5 cm^2
14. $1,185.6012 \text{ cm}^2$
15. 856 m^2