Module 6
Geometry of Shape and Size

What this module is about

This module is about areas of plane figures. In this module you will study the areas of squares, rectangles, parallelograms, triangles, trapezoids and circles, and learn to compute them.

What you are expected to learn

This module is designed for you to

1. Apply the formulas for the measurements of the following plane figures

   a. square,
   b. rectangle,
   c. parallelogram,
   d. triangle,
   e. trapezoid, and
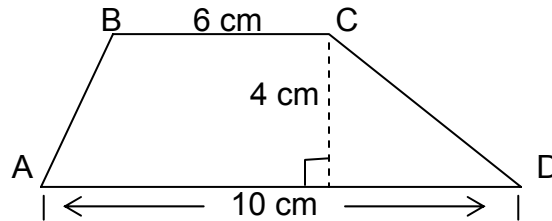   f. circle.

2. Solve problems involving areas of plane figures

How much do you know

1. Find the area of a square whose side is 9 cm.
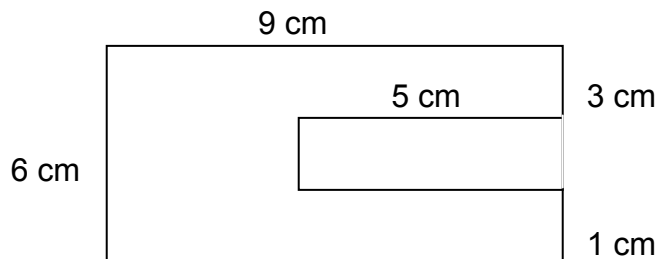
2. Find the area of the triangle below.

8 cm

| ← 6 cm → |
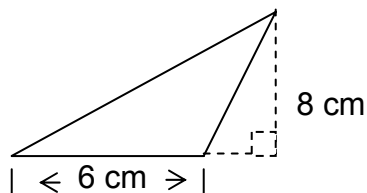
3. Find the area of a rectangle whose length and width are 12 cm and 5 cm respectively.

4. Find the area of the trapezoid ABCD below

5. Find the area of a circle whose radius is 7 mm.

6. Find the base of a triangle if the altitude is 4 cm and the area is 16 cm^2.

7. Find the area of a parallelogram with base 12 cm and height 8 cm.

8. The area of the parallelogram ABCD below is 96 cm^2. Find x.
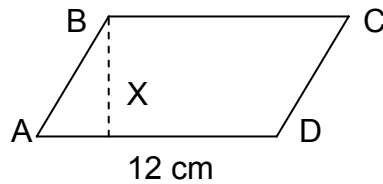
9. Find the area of the figure

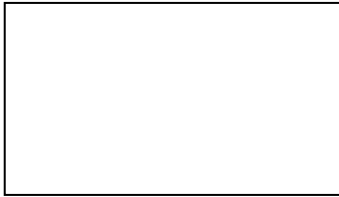10. Beth's garden is 4 meters wide and 6 meters long. Find the area of the garden.

Lesson 1

Areas of Rectangles and Squares

The first figure below is a rectangle and the second is a rectangular region.
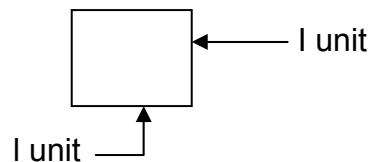
Rectangle   Rectangular region

A rectangular region is a union of a rectangle and its interior. When you are asked to find the area of a rectangle, you are actually asked to determine the area of a rectangular region. The area of a region is the number of square units contained in the region. A square unit is a square with a side 1 unit in length.

I unit ← I unit

I unit →

Example 1

In the rectangle below, each small square is one unit in length. Find the area of the rectangle.
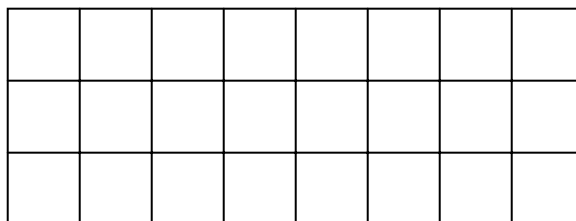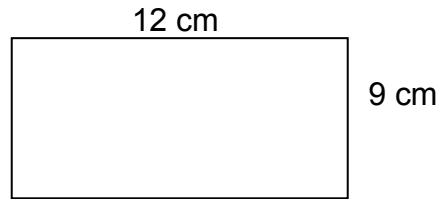
The area can be determined by counting the number of small squares. Since there are 24 small squares, therefore, the area is 24 square units.

The standard units of area are square units, such as square centimeters, square decimeters and square meters.

Example 2

Find the area of the rectangle below.

12 cm

9 cm

Solution:

The length (l) of the rectangle is 12 cm and the width (w) is 9 cm. Substitute these data in the formula.

$A = lw$
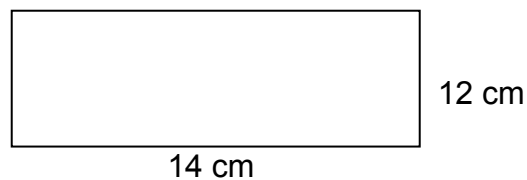$= (12)(9)$
$= 108 \text{ cm}^2$

The area is 108 square centimeters.

Example 3

Find the area of a rectangle whose length and width are 14 cm and 12 cm respectively.

Solution:

Step 1. Draw and label the figure.

12 cm

14 cm

Step 2. Substitute the data in the formula. The figure shows that l = 14 cm and w = 12 cm.

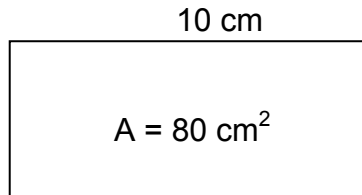$A = lw$
$= (14)(12)$
$= 168 \text{ cm}^2$

The area is 168 square centimeters.

.

Example 4

Find the width of the rectangle with an area of 80 cm^2 and length equal to 10 cm.
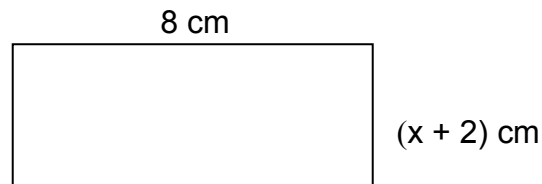
Solution:

Step 1. Draw and label the figure.

10 cm

A = 80 cm^2

Step 2. A = 80 cm^2 , l = 10 cm. Substitute these data in the formula.

A = lw
80 = 10 w
10w = 80
w = 8 cm

The width is 8 cm

Example 5

The area of rectangle EFGH below is 48 cm^2. Find its width.

8 cm

(x + 2) cm

Solution:

In the figure, l= 8 cm and w = (x + 2) cm. Substitute 8 in place of l and (x + 2) in place of w in the formula A = lw. Replace A by 48 since the area of the rectangle is given as 48 cm^2. Then solve the resulting equation for x.

A = lw.
48 = 8 (x + 2)
48 = 8x + 16
48 –16 = 8x
32 = 8x   or 8x = 32
x = 4

Since the width is represented by x + 2, then the width is 4 + 2 or 6 cm

The Area of a Square

When the length and width of a rectangle are equal, the figure is a square. The formula for finding the area of a square is $A = s^2$ where s = length of a side.
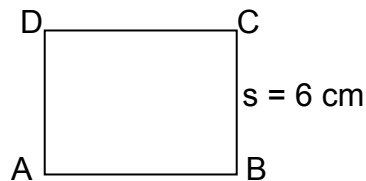
D ___s___ C

S

A |____| B
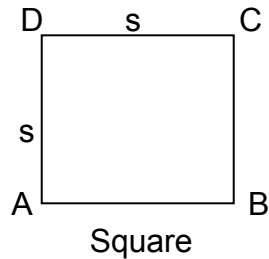
Square

$$AB = BC = CD = DA$$

Example 1

Find the area of the square ABCD.

Solution:

$A = s^2$
$= 6^2$
$= (6)(6)$
$= 36 \text{ cm}^2$

D _____ C

s = 6 cm

A _____ B

The area is 36 square centimeters

Example 2
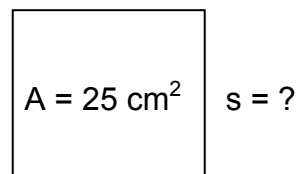
Find the side of a square whose area is 25 cm^2.

Solution:

Step 1. Draw and label the figure.
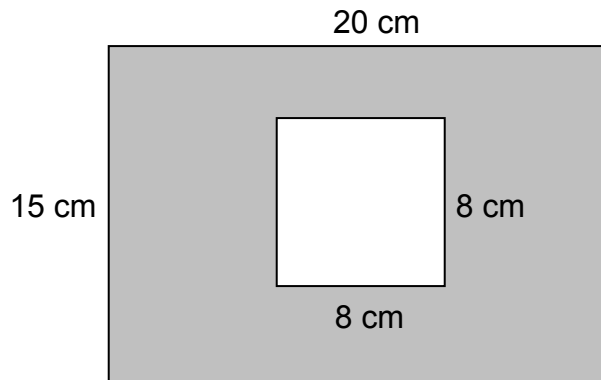
Step 2. Substitute the data in the formula.

A = 25 cm^2 s = ?

$A = s^2$
$25 = s^2$
$s^2 = 25$
$\sqrt{s^2} = \sqrt{25}$
$s = 5 \text{ cm}$

The side is 5 centimeters

Example 3

Find the area of the shaded region.

20 cm

15 cm   8 cm

8 cm

Solution:

Step 1. Find the area of the rectangle

$A = lw$
$= (20)(15)$
$= 300 \text{ cm}^2$

Step 2. Find the area of the square

$A = s^2$
$= 8^2$
$= (8)(8)$
$= 64 \text{ cm}^2$

Step 3. Find the area of the shaded region by subtracting the area of the square from the area of the rectangle.

$A = lw - s^2$
$A = 300 - 64$
$= 236 \text{ cm}^2$

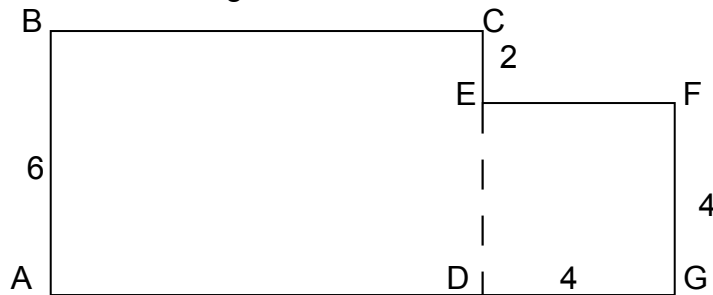The area of the shaded region is 236 square centimeters.

Another important thing you will learn from this lesson is that areas of plane figures can be added as long as the figures do not overlap.

The area of the entire figure in the following illustration is the sum of the areas of the rectangle and the square.
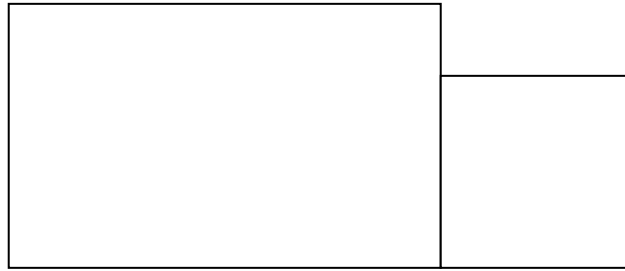
Example:

Find the area of the figure below.

Notice that polygon ABCD is a rectangle and polygon DEFG is a square. The area of the entire figure is the sum of the areas of the rectangle and the square.

Solution:

Step 1. Find the area of the rectangle ABCD
$$A_1 = lw$$
$$= (10)(6)$$
$$= 60$$

Step 2. Find the area of the square DEFG.
$$A_2 = s^2$$
$$= 4^2$$
$$= (4)(4)$$
$$= 16$$

Step 3. Find the area of the entire figure by adding the area of the rectangle to the area of the square.
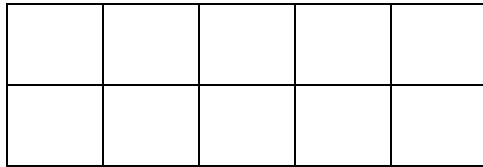$$A_3 = A_1 + A_2$$
$$= 60 + 16$$
$$= 76 \text{ cm}^2$$

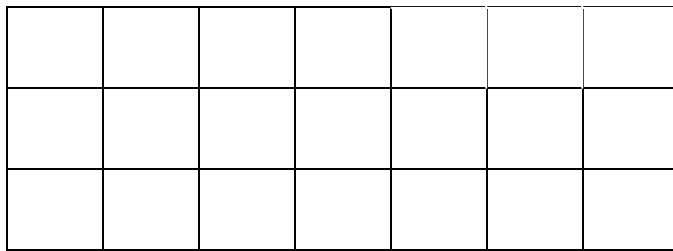The area of the entire figure is 76 square centimeters

Try this out

In exercises 1-4, each plane figure is divided into small squares each with a side 1 unit in length.
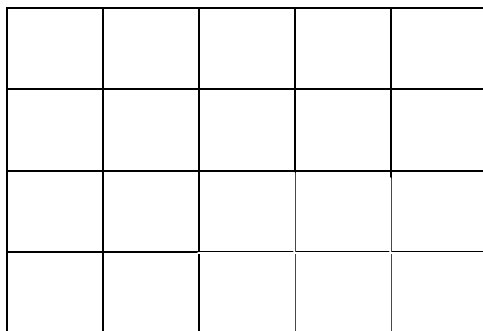
1. Find the area.

A = _____ square units.
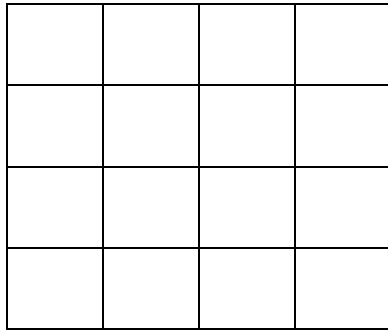
2. Find the area.

A = _____ square units

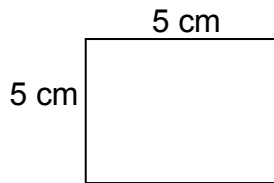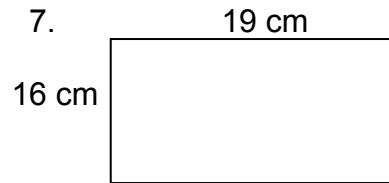3. Find the area.

A = _____ square units

4. Find the area.

A = _____ square units

Find the area of each rectangle or square described below.

5. 5 cm

5 cm

7. 19 cm

16 cm

8 cm

6.

16 cm

5.5 m

8.

5.5 m
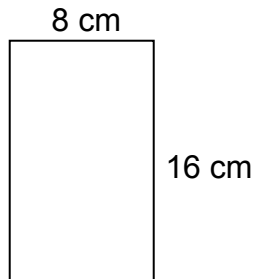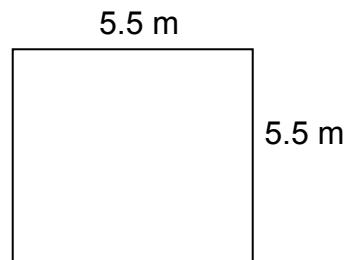
9. A rectangle with a length of 14 meters and a width of 10 meters.

10. A square with a side of 7 mm.

Set B

1. Find the area of a square whose side is 15 cm.
2. Find the area of a rectangle whose length and width are 12 m and 7 m respectively.
3. Find the area of a square whose side is 8.5 m
4. Find the area of a rectangle with length of 8 cm and width of 5 cm.
5. A side of a square is 13 cm. Find its area.
6. The length and width of a rectangle are 18 cm and 10 cm respectively. Find its area.

Find the area of the following figure.

7.
2 cm

3 cm

2 cm

4 cm

8.
3 cm

3 cm

2 cm

5 cm
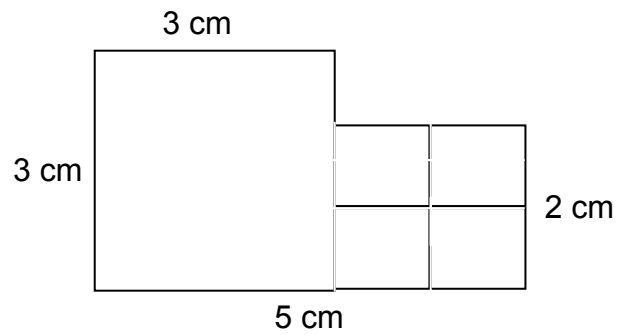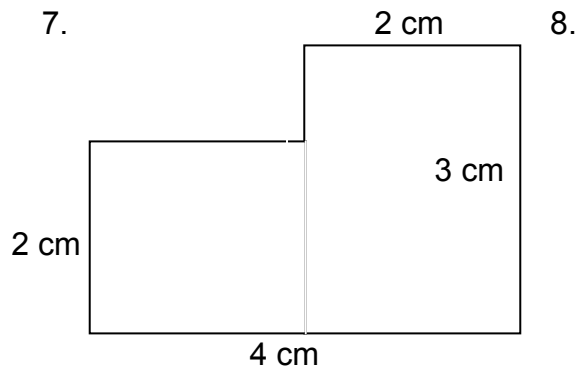
Find the area of the shaded region in the following figures

9.

8

6
6

8

10

12

4

6

10
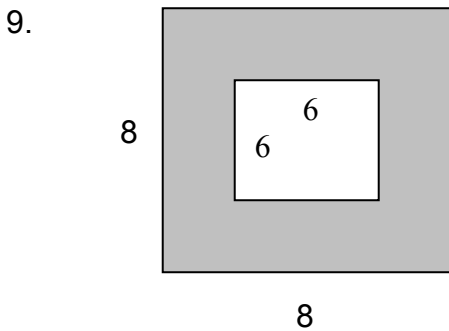
Set C

Find the area of each square

1. side = 4 m

2. side = 4.5 cm

Find the area of each rectangle

3. Length = 11 cm and width = 8 cm

4. length = 9 cm and width = 8 cm

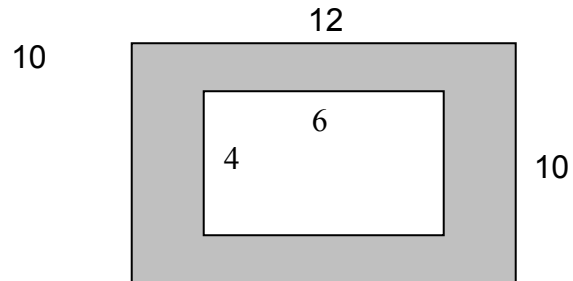Find the length of a side of each square.

5. Area = 36 cm^2

6. Area = 81 c m^2

Find the length of each rectangle

7. Area = 112 c m^2 and width = 8 cm
8. Area = 135 c m^2 and width = 9 cm

Find the width of each rectangle

9. Area = 176 c m^2 and length = 16 cm
10 Area = 216 c m^2 and length = 18 cm

Lesson 2

Areas of Parallelograms and Triangles

The area of a parallelogram is equal to the product of the base times the height. The formula is A = bh
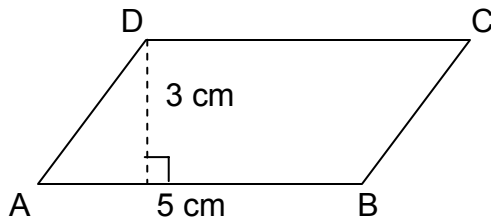
Example 1

Find the area of a parallelogram with a base of 5 cm and a height of 3 cm.

Solution:

Step 1. Draw and label the figure.

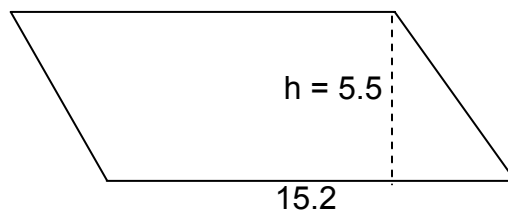The base AB has length of 5 cm and the height or altitude is 3 cm.

Step 2. Substitute the data in the formula

A = bh
= 5(3)
= 15 cm^2

The area is 15 square centimeters

Example 2

Find the area of the parallelogram below

The figure shows that b = 15.s cm and h = 5.5 cm. Substitute these data in the formula.

Solution:

A = bh
  = (15.2)(5.5)
  = 83.6 cm^2

The area is 83.6 square centimeters

Example 3

The area of a parallelogram is 84 m^2. The lengths of a base is 6 m. Find the length of the corresponding altitude.

Solution:

Step 1. Draw and label the figure.

A = 84 m^2

h = ?

6 m
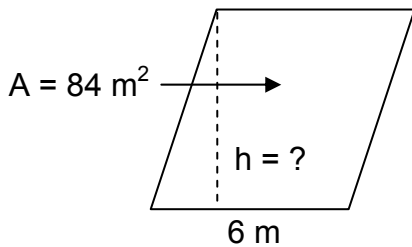
Step 2. Substitute the data in the formula

A = bh
84 = 6h
6h = 84
 h = 84/6
 h = 14 m

The height is 14 m

Example 4

Find the height of a parallelogram that has a base of 14 cm and an area of 126 cm^2

Solution:

Step 1. Draw and label the figure.

A = 126 cm^2

h = ?

14 cm

Step 2. Substitute the data in the formula

$$A = bh$$
$$126 = 14\,h$$
$$14\,h = 126$$
$$h = 126/14$$
$$h = 9\text{ cm}$$

The height is 9 centimeters

The Area of a Triangle

The diagonal separates the parallelogram into two congruent triangles.
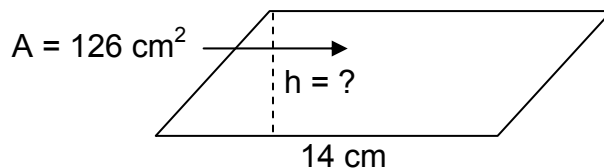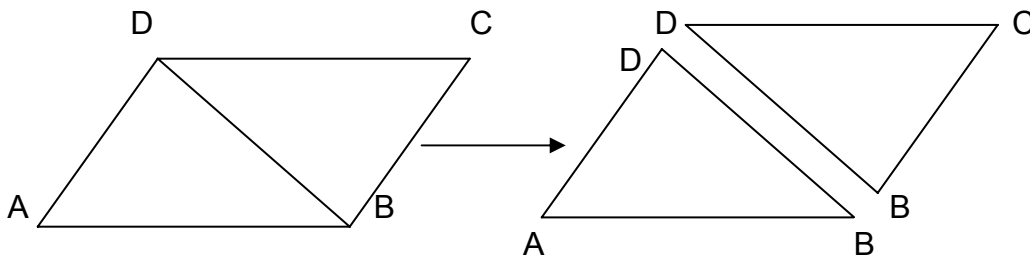

The diagonal BD divides the parallelogram ABCD into two congruent triangles. Two congruent triangles have equal areas. The area of $\triangle ABD$ is equal to the area of $\triangle CDB$. Since the formula for finding the area of a parallelogram is A = bh, therefore, the formula for finding the area of a triangle is A = $\frac{1}{2}$ bh.

Example 1

Find the area of the triangle shown below.
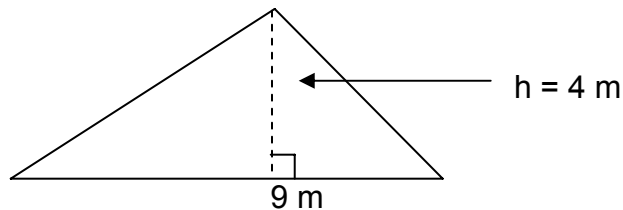
Solution:

$$A \ = \ \frac{1}{2}\,bh$$
$$= \ \frac{1}{2}\,(\,9\,)\,(\,4\,)$$
$$= 18\text{ m}^2$$

The area is 18 square meters

Example 2

Find the area of the triangle below.

12 dm
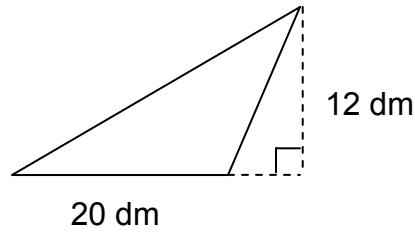
20 dm

Solution

$A = \dfrac{1}{2} bh$

$= \dfrac{1}{2} (20)(12)$

$= 120 \text{ dm}^2$

The area is 120 square decimeters

Example 3

Find the area of the triangle shown below.

5.5 m

7.2 m
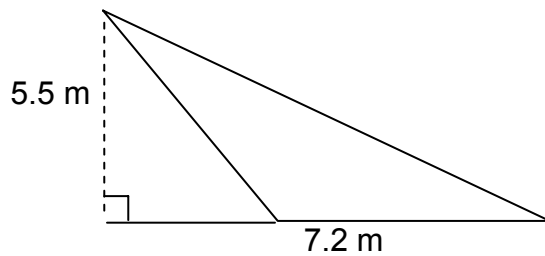
Solution:

$A = \dfrac{1}{2} bh$

$= \dfrac{1}{2} (7.2)(5.5)$

$= 19.8 \text{ m}^2$

The area is 19.8 square meters

Example 4

Find the area the triangle with base equal to 12 cm and altitude equal to 10 cm.

Solution:

Step 1. Draw and label the triangle
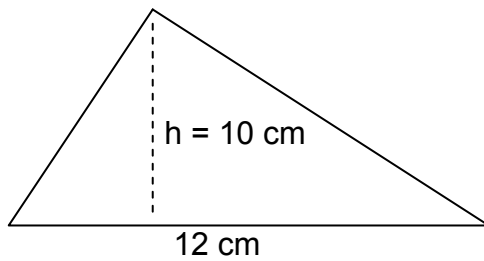
h = 10 cm

12 cm

Step 2. Substitute the data in the formula

$$A = \frac{1}{2} \, bh$$

$$= \frac{1}{2} \, (12) \, (10)$$

$$= 60 \text{ cm}$$

The area is 60 square centimeters

Example 5

Find the base of the triangle with an altitude of 12 cm and an area of 66 cm^2.

Solution

Step 1. Draw and label the figure

h = 12 cm

b = ?
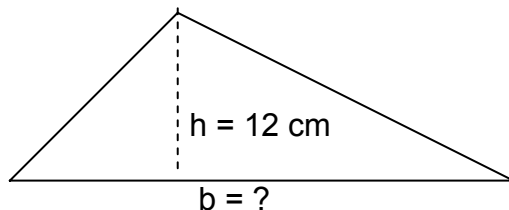
Step 2. Substitute the data in the formula.

$$A = \frac{1}{2} \, bh$$

$$66 = \frac{1}{2} \, (b) \, (12)$$

$$66 = 6 \, b$$

$$6 \, b = 66$$

$$b = 11$$

The base is 11 centimeters.

Try this out

Set A

Find the area of each parallelogram.

1.

4 m
6 m

3.

4 mm
13 mm

2.

6 cm
5.2 cm

4.

8 m
6 m

Find the area of each triangle

5.

2
4

8.

12 mm
15 mm

6.

5
3

9.

20 m
25 m

7.

5 cm
6 cm

10.

6 m
11 m
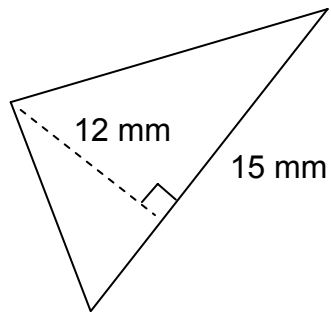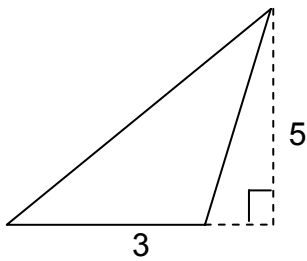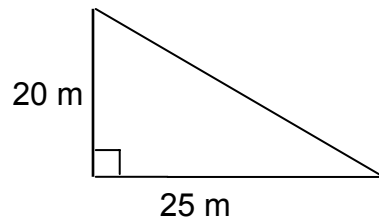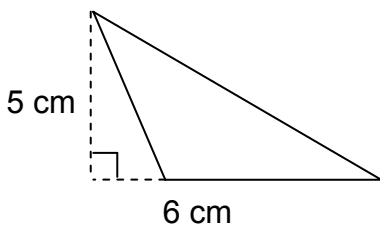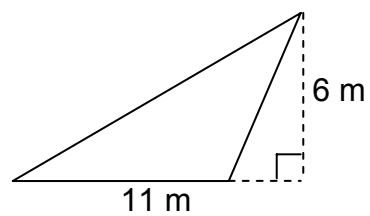
Set B.

Complete the table below

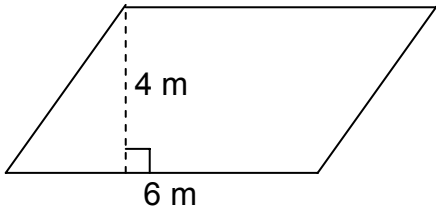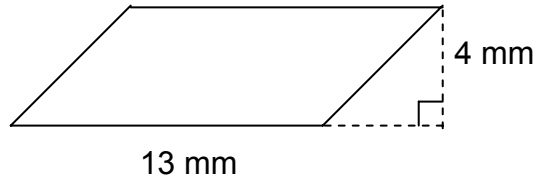|	Base	Altitude	Area of the Parallelogram
1	16	5	
2		10 cm	180 cm^2
3	14 km		168 km^2
4		8 m	92 m^2
5		4.5 cm	36 cm^2

.
Complete the table below

	base	Altitude	Area of the Triangle
6	14 cm		84 cm^2
7		16 m	80 m^2
8	16 cm		64 cm^2
9	7.6 cm	4.5 cm	__ cm^2
10		5.6 m	23.8 m^2

Set C

Find the area of the parallelogram in which

1. b = 30, h=6
2. b = $3\frac{1}{2}$, h = 6.5
3. b = 24. h = 12
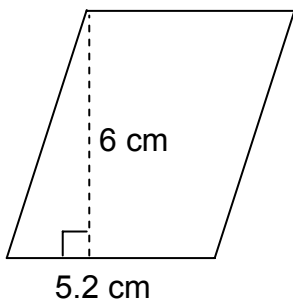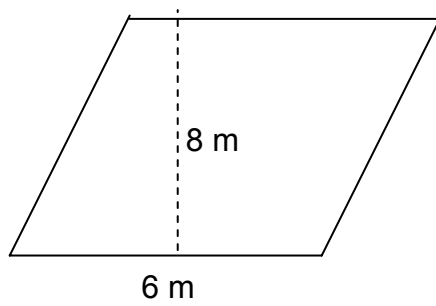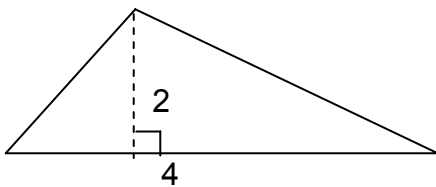
Find the area of each triangle described below.

4. b = 40, h = 24
5. b = 26, h = 14
6. b = 28, h = 16

Solve the following

7. Find the area of a parallelogram with base 17 m and height 14 m.
8. Find the area of a triangle with base of 16 m and height of 12 m.
9. A piece of cardboard is in the form of a parallelogram. Find its area if the base is 17 cm and the altitude is 12 cm.
10. A piece of paper is in the form of a triangle. What is its area if the base is 18 cm and its altitude is 8.6 cm?

Lesson 3

Area of a Trapezoid

Recall that a trapezoid is a quadrilateral with two bases which are parallel. The area of a trapezoid is one half the product of the length of an altitude and the sum of the lengths of the two bases.
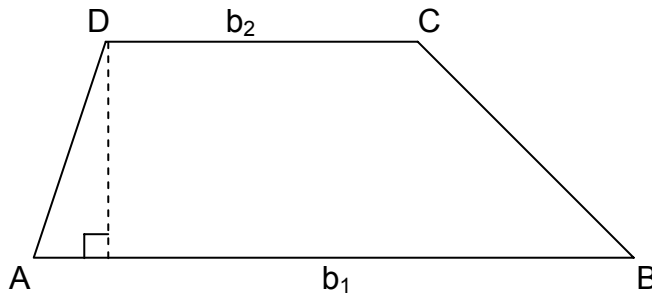
$$A = \frac{1}{2} h \left(b_1 + b_2 \right)$$

Where h = altitude
b_1 = lower base
b_2 = upper base

Example 1

Find the area of trapezoid ABCD.
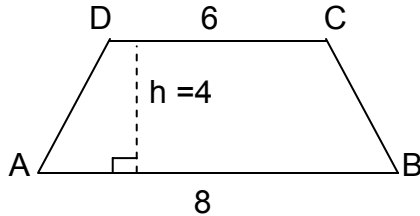
$$A = \frac{1}{2} h \left(b_1 + b_2 \right)$$

$$= \frac{1}{2} \left(4 \right) \left(8 + 6 \right)$$

$$= \frac{1}{2} \left(4 \right) \left(14 \right)$$

$$= 28$$

The area is 28 square units.

Example 2

In the following trapezoid, HG = 6 cm and EF = 12 cm. If its area is 36 cm^2, find its altitude h.
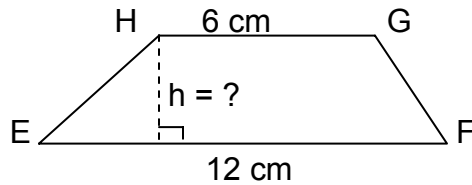
Solution:

$$A = \frac{1}{2} h \, (b_1 + b_2)$$

$$36 = \frac{1}{2} h \, (6 + 12)$$

$$36 = \frac{1}{2} h \, (18)$$

$$36(2) = 18 \, h$$
$$18h = 72$$
$$h = 4 \text{ cm}$$

The altitude h is 4 cm.

Example 3

Find the longer base of a trapezoid with shorter base 5, height 4 and area 24..
Step 1 Draw and label the figure. Represent the longer leg by x.
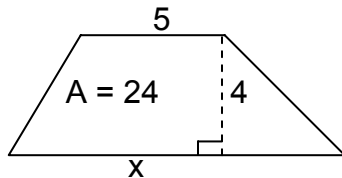
Step 2. Substitute the data in the formula.

$$A = \frac{1}{2} h(b_1 + b_2)$$

$$24 = \frac{1}{2} (4)(x + 5)$$

$$24 = 2(x + 5)$$
$$2(x + 5) = 24$$
$$2x + 10 = 24$$
$$2x = 24 - 10$$
$$2x = 14$$
$$x = 7$$

The longer base is 7 .

Try this out

Set A

ABCD is a trapezoid.
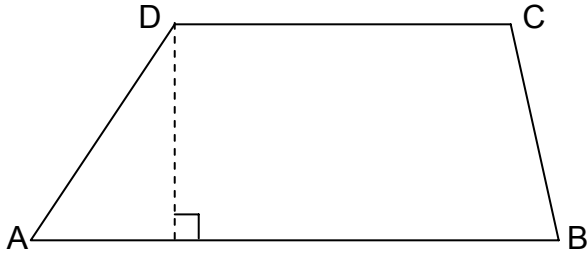
1. If AB = 16, DC = 8, DE = 6, find the area
2. If AB = 17, DC = 9, DE = 8, find the area
3. If AB = 20, DC = 18, DE = 10, find the area
4. If AB = 30, DC = 20, DE = 10, find the area
5. If AB = 40, DC = 30, DE = 20, find the area

EFGH is a trapezoid.
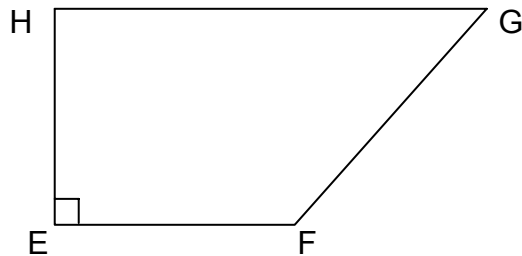
6. If EF = 8, HG = 10, HE = 6, find the area.
7. If EF = 6, HG = 8, HE = 4, find the area
8. If EF = 12, HG =16, HE = 6, find the area
9. If EF = 10, HG =12, HE = 9, find the area
10. If EF = 12, HG =14, HE = 7, find the area

Set B

Supply the missing information for each trapezoid

	H	b_1	b_2	Area
1	4	8	11	
2	8	10	13	
3	12	10	14	
4		12	16	140
5		10	18	112

Find the area of each trapezoid

6.
8

5

11

7.
7 cm

5 cm

13 cm
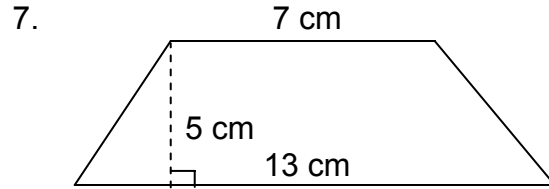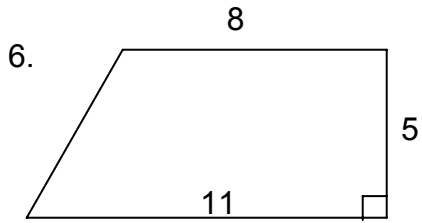
8. Given a trapezoid with bases 16 and 20 and height 9, Find the area.
9. The bases of a trapezoid are 8 and 20 and the area is 84. Find the height.

8

h = ?

A = 84

20
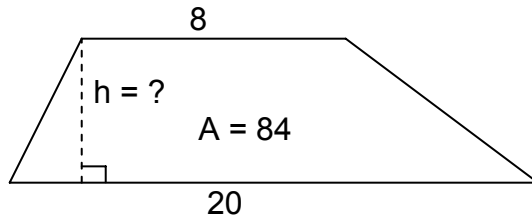
10. The height of a trapezoid is 12 and the area is 162. If one of the bases is 16, find the other.

Set C

1. The height of a trapezoid is 4. The bases are 5 and 7. Find the area.
2. The height of a trapezoid is 6. The bases are 7 and 9. What is the area?
3. The bases of a trapezoid are 4 and 8 and the area is 36. Find the height.
4. The bases of a trapezoid are 6 and 8 and the area is 56. Find the altitude.
5. The altitude of a trapezoid is 8 and the area is 64. If one base is 5, find the other base.
6. The height of a trapezoid is 2 and the area is 16. If the upper base is 9, find the lower base.
7. A trapezoid has bases 9 and 10 and the area 38. Find its altitude.
8. A trapezoid has bases 8 and 12 and area 80. Find its height.
9. Find the longer base of a trapezoid with shorter base 5, height 6, and area 48.
10. Find the shorter base of a trapezoid with longer base 12 , altitude 6, and area 54.

Lesson 4

Area of a Circle

Recall that a circle is a set of points in a plane that have the same distance from a given point in the plane. The given point is called the center, and the distance from the center to any given point on the circle is called the radius.

The area of a circle is the measure of the space bounded by it. The ratio of the circumference to the diameter of any circle is equal to the same number, represented by the Greek letter π (pi) The approximate value of π is 3.14 or 22/7.

The formula for the area of a circle with radius r units is

$$A = \pi r^2$$
Where A = area
r = radius

Example 1

The radius of a circle is 2 cm. Find its area.

Solution:

$A = \pi r^2$
$\approx 3.14 (2)^2$
$\approx 12.56 \text{ cm}^2$

2 cm
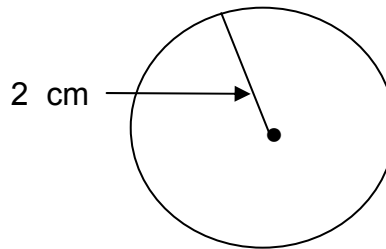
The area is 12.56 square cm.

Example 2

The diameter of a circle is 6 cm. Find its area.

Solution:

Step 1. Find the radius

Radius(r) = diameter (d) divided by 2
$r = 6 \div 2$
$r = 3 \text{ cm}$

$d = 6 \text{ cm}$

The radius is 3 cm.

Step 2. Find the area.
$A = \pi r^2$
$\approx 3.14 (3)^2$
$\approx 3.14 (9)$
$\approx 28.26 \text{ cm}^2$
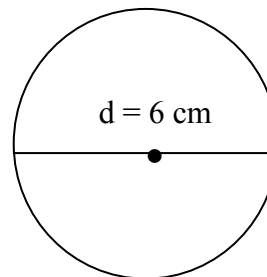
The area is 28.26 square centimeters

Example 3 Find the area of the circle in terms of π if its radius is 2 m.

Solution

$A = \pi r^2$
$\approx \pi(2)^2$
$\approx 4\pi$

2 m

Example 4 Find the area of the shaded region.
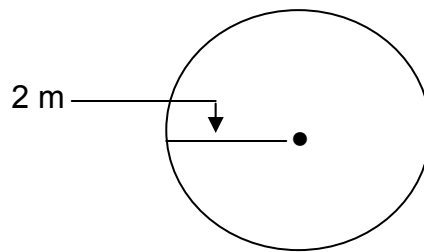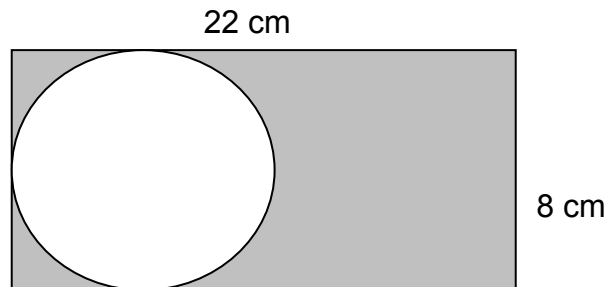
22 cm

8 cm

Solution:

Step 1. Find the area of the rectangle.
$A = lw$
$= (22)(8)$
$= 176 \text{ cm}^2$

Step 2. Find the area of the circle

The diameter of the circle is the width of the rectangle.
$r = \dfrac{d}{2}$
$r = \dfrac{8}{2}$
$r = 4 \text{ cm}^2$

Substitute 4 in the area formula for circles
$A = \pi r^2$
$\approx 3.14\,(4)^2$
$\approx 3.14\,(16)$
≈ 50.24

Step 3. Subtract the area of the circle from the area of the rectangle.
Area of the shaded region = area of the rectangle – area of the circle
$= 176 - 50.24$
$= 125.76 \text{ cm}^2$

The area of the shaded region is 125.76 square centimeters.

Try this out

Set A.

Find the area of each circle with the given diameter or radius Use 3.14 for π.

1. radius = 5 cm
2. radius = 1.5 mm
3. diameter = 4 cm
4. diameter = 12 dm
5. radius = 4.6 m
6. radius = 2.2 cm
7. diameter = 4.8 dm
8. diameter = 6.4 cm
9. radius = 4.8 m
10. radius = 3.4 dm

Set B

Find the area of each circle described below. Give the answers in terms of π.

1.

r = 8

3.

d = 10

2.

r = 9

4.

d = 11
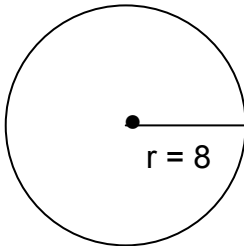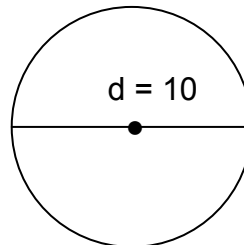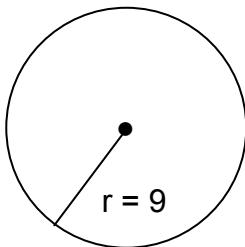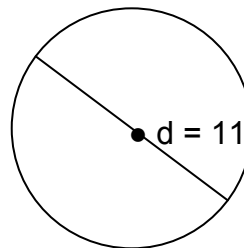
5. The radius is 7 cm
6. The radius is 12 cm
7. The diameter is 14 mm
8. The diameter is 26 mm
9. The radius is 3.4 km
10. The radius is 2.6 km

Set C.

Find the area of each circle in terms of π

1. r = 8 dm
2. r = 18 dm
3. d = 5 cm
4. d = 14 cm
5. r = 4.6 mm
6. r = 4.4 mm

7. The students performed their dance number in a circular platform 20 m in diameter. Find the area of the platform.

8. The radius of a circular garden is 40 m. Half of the garden will be planted with roses. How many square meters will be planted with roses.

Find the area of the shaded part of the figure. Give the answers in terms of π

9
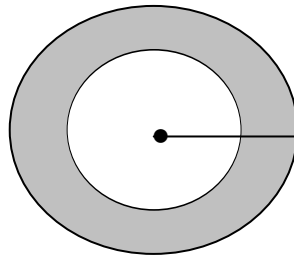
r of the bigger circle is 8 cm
r of the smaller circle is 6 cm.

10.

14 cm

12 cm

Let's Summarize

1. The area of a region is the number of square units contained in the region.
2. A square unit is a square with a side I unit in length.
3. The area (A) of a rectangle is the product of its length (*l*) and its width (w).
   A = lw
4. The area (A) of a square is the square of the length of a side (s). $A = s^2$
5. The area (A) of a parallelogram is equal to the product of the base (b) and the height (h).
   A = bh
6. The diagonal separates the parallelogram into two congruent triangles.
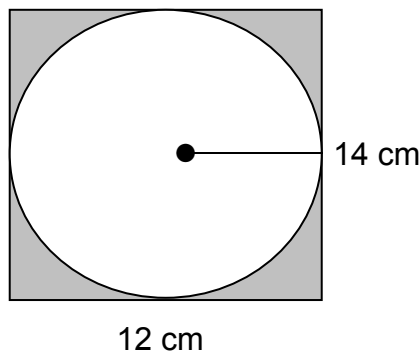7. The area (A) of a triangle equals half the product of the base (b) and the height (h).
   $A = \frac{1}{2}$ bh. Sometimes altitude is used instead of height.
8. The area (A) of a trapezoid is one half the product of the length of its altitude and the sum of the lengths of the two bases. $A = \frac{1}{2} h (b_1 + b_1)$.
9. A circle is a set of points in a plane that have the same distance from a given point in the plane.
10. In all circles the ratio of the circumference to the diameter is always equal to the same number, represented by the Greek letter π.
11. The formula for finding the area of a circle with a radius of r units is: $A = \pi r^2$

What have you learned

1. Find the area of a square whose side is 15 dm.

2. Find the area of the triangle below.

9 cm

10 cm
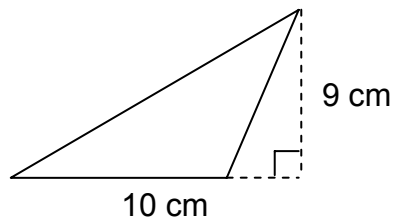
3. Find the area of a rectangle whose length and width are 14 dm and 8 dm respectively.

4. Find the area of the trapezoid ABCD below

B   8 cm   C

6 cm

A   12 cm   D
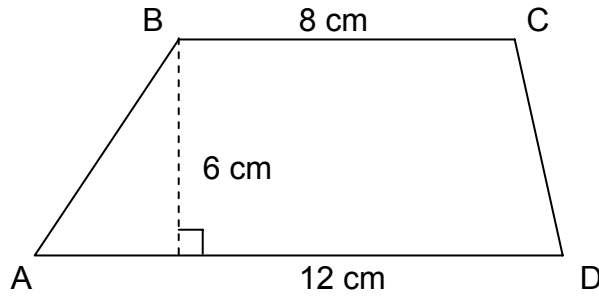
5. Find the area of a circle whose radius is 6 cm.
6. Find the base of a triangle if the altitude is 6 cm and the area is 36 cm^2.
7. Find the area of a parallelogram with base 14 cm and height 6 cm.
8. The area of the parallelogram ABCD below is 66 cm^2. Find h.

B   C

h = ?

A   11 cm   D

9. Find the area of the figure

7

5

3

2   3
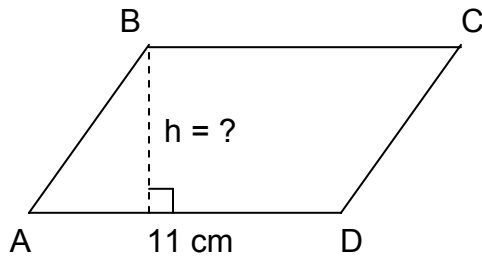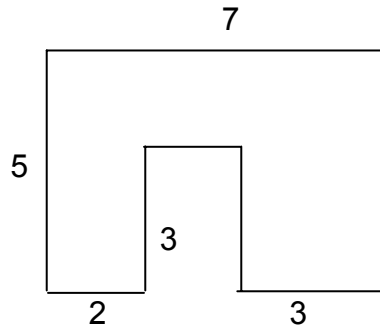
10. Gen's garden is 5 meters wide and 7 meters long. Find the area of the garden.

Answer Key

How much do you know

1. 81 cm^2
2. 24 cm^2
3. 60 cm^2
4. 32 cm^2
5. 153.86 mm^2

6. 8 cm
7. 96 cm^2
8. 8 cm
9. 44 cm^2
10. 24 m

Try this out

Lesson 1

Set A.

1. 10
2. 18
3. 14
4. 16
5. 25 cm^2

6. 128 cm^2
7. 304 cm^2
8. 30.25 m^2
9. 140
10. 49 mm^2

Set B.

1. 225 cm^2
2. 84 m^2
3. 72.25 m^2
4. 40 cm^2
5. 169 cm^2

6. 180 cm^2
7. 10 cm^2
8. 13 cm^2
9. 28 sq. units
10. 96 sq. units

Set C

1. 16 m^2
2. 20.25 cm^2
3. 88 cm^2
4. 72 cm^2
5. 6 cm

6. 9 cm
7. 14 cm
8. 15 cm
9. 11 cm
10. 12 cm

Lesson 2

Set A

1. 24 m^2
2. 31.2 cm^2
3. 52 mm^2
4. 48 m^2
5. 4 sq. units

6. 7.5 sq. units
7. 15 cm^2
8. 90 mm^2
9. 250 m^2
10. 33 m^2

Set B

1. 80 sq. units	6. 12 cm
2. 18 cm	7. 10 m
3. 12 km	8. 8 cm
4. 11.5 m	9. 17.1 cm
5. 8 cm	10. 8.5 m

Set C

1. 180 sq. units	6. 224 sq. units
2. 22.75 sq. units	7. 238 m^2
3. 288 sq. units	8. 96 m^2
4. 480 sq. units	9. 204 cm^2
5. 182 sq. units	10. 77.4 cm^2

Lesson 3

Set A

1. 72 sq. units	6. 54 sq. units
2. 104 sq. units	7. 28 sq. units
3. 190 sq. units	8. 84 sq. units
4. 250 sq. units	9. 99 sq. units
5. 700 sq. units	10. 91 sq. Units

Set B

1. 38 sq. units	6. 47.5 sq. units
2. 92 sq. units	7. 50 cm^2
3. 144 sq. units	8. 162 sq. units
4. 10	9. 6
5. 8	10. 11

Set C.

1. 24 sq. units	6. 7
2. 48 sq. units	7. 4
3. 6	8. 8
4. 8	9. 11
5. 11	10. 6

Lesson 4
 Set A

1. 78.5 cm^2	6. 15.1976 cm^2
2. 7.065 mm^2	7. 18.0864 dm^2
3. 12.56 cm^2	8. 32.1536 cm^2
4. 113.04 dm^2	9. 72.3456 m^2
5. 66.4424 m^2	10. 36.2984 dm^2

Set B

1. 64π
2. 81π
3. 25π
4. 30.25π
5. 49π cm^2

6. 144π cm^2
7. 49π cm^2
8. 169π mm^2
9. 11.56π km^2
10. 6.76π km^2

Set C

1. 64π dm^2
2. 324π dm^2
3. 6.25π cm^2
4. 49π cm^2
5. 21.16π mm^2

6. 19.36π mm^2
7. 100π m^2
8. 800π m^2
9. 64π cm$^2 - 36 \pi$ cm$^2 = 28 \pi$ cm^2
10. 196 cm$^2 - 49 \pi$ cm^2

What have you learned?

1. 225 dm^2
2. 45 cm^2
3. 112 dm^2
4. 60 cm^2
5. 36π cm^2

6. 12 cm
7. 84 cm^2
8. 6 cm
9. 29 sq. units
10. 35 m^2