Module 1
Geometry of Shape and Size

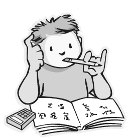
What this module is about

This module is about undefined terms and angles. As you go over the exercises you will learn to name the real-world objects around you that suggest points, lines and planes. You will develop skills in naming a point, a line and its subsets. You will also learn to name the parts of an angle and determine its measure in degrees.

What you are expected to learn

This module is designed for you to:

1. describe the ideas of
    - point
    - line
    - plane
2. name the subsets of a line
    - segment
    - ray
3. name the parts of an angle
4. determine the measure of an angle using a protractor
5. illustrate different kinds of angles
    - acute
    - right
    - obtuse



How much do you know

Identify the term described.

1. It has no length, width, or thickness.
2. It has length but no width and no thickness.
3. It is a flat surface that extends infinitely in all directions.
4. It is the union of two noncollinear rays with a common endpoint.
5. It is an instrument used to determine the approximate measure of an angle.

6. An angle with a measure greater than 0 but less than 90
7. It is a subset of a line with two endpoints.
8. An angle with a measure greater than 90 but less than 180.
9. The geometric figure suggested by the ceiling of your room.
10. It is the intersection of two distinct planes.

What you will do

Lesson 1

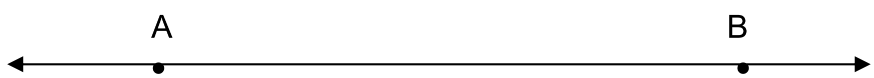
Undefined Terms

The three undefined terms are point, line and plane. These three undefined terms form the foundation of geometry. Although they will not be defined they will however be used in defining other important terms. For example, space is defined as a set of all points. A point is an exact location in space. It has no length, width or thickness. It is represented by a dot. Look at the tip of your pen. It suggests a point. A point is named by using a capital letter. The points below are named points P, Q and R respectively.

• P • Q • R

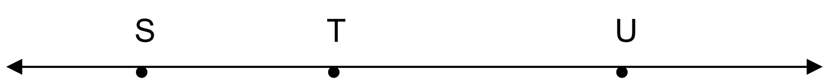
A line has infinite length, but no width and no thickness. It is an infinite set of points that extends infinitely in opposite directions. The pen or pencil you are holding right now is a real world object that suggests a line. A line is represented by ⟷ . The arrow suggests that the line continues without end in both directions.

You can name a line in two ways. One way of naming a line is by using two different capital letters. Observe the line below. It is named line AB written as $\overleftrightarrow{AB}$. The double-headed arrow placed over *AB* indicates that the line has no endpoints.



Example:

Give six names for the line below.



Answers:

$\overleftrightarrow{ST}$, $\overleftrightarrow{TS}$, $\overleftrightarrow{TU}$, $\overleftrightarrow{UT}$, $\overleftrightarrow{SU}$, $\overleftrightarrow{US}$

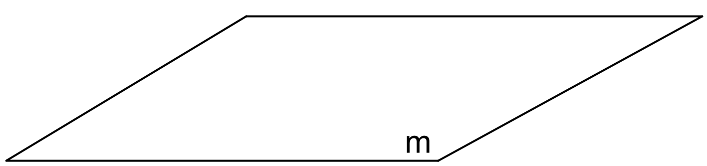
The second method of naming a line is by using a small letter. The line below is named line m.

Like a line, a plane is also a set of infinite points. However, a plane has infinite width and length but no thickness. It is a flat surface that extends infinitely in all directions. The top of your dining table, the wall of your room and even a page of this module are examples of real-world objects that suggest planes.

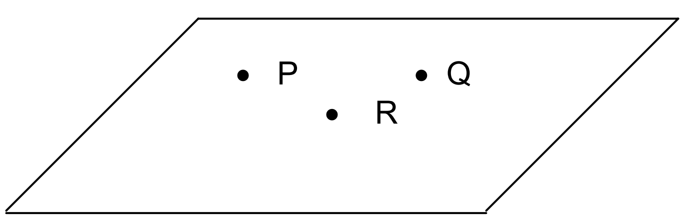
A slanted four sided figure similar to the one below is used to represent a plane.

You can name a plane in three ways. You may use a capital letter placed at one of its corners. The plane below is named plane P.

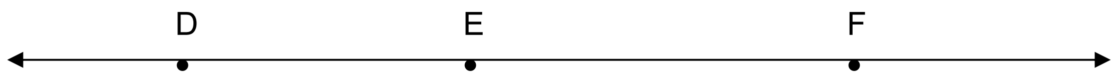
You may use a small letter placed at one of its corners. The plane below is named plane m.



You may named it by using three points not on a straight line. The plane below is named plane PRQ.

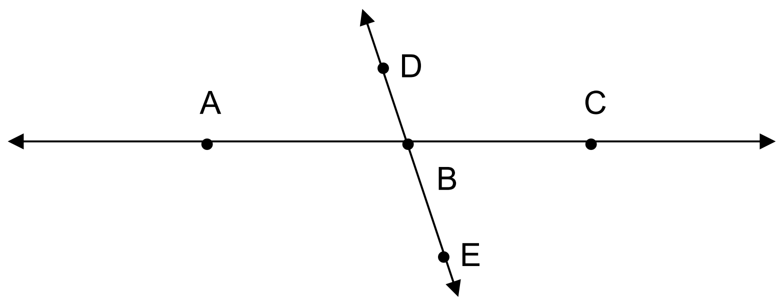


The three points below are collinear. Points are collinear if they are on the same line.



Example:

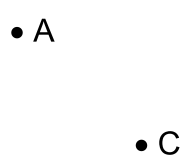
List all sets of three collinear points in the figure.



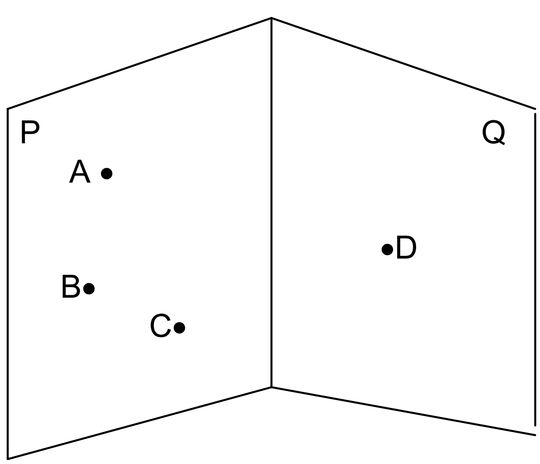
Answers:
:

A, B, C and D, B, E

Consider the three points below. It is not possible to draw one straight line through the three points A, B and C. These three points are non collinear points.



In the figure below, points A, B, and C are in the same plane. Points such as points A, B, and C, which are in the same plane are called coplanar points. In the same figure, points A, B, C and D are not coplanar because they do not lie in the same plane. Points A, B, C lie in plane P, whereas point D lies in plane Q.



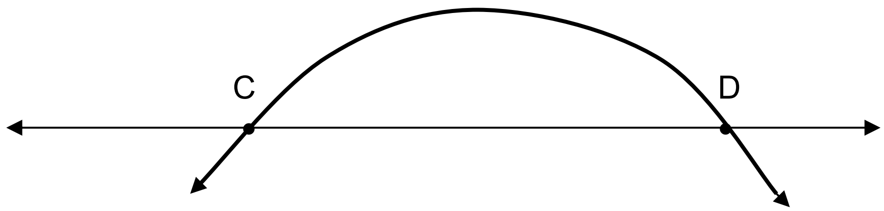
The following statements describe some basic relationships among points, lines and planes

1. Two points determine exactly one line.

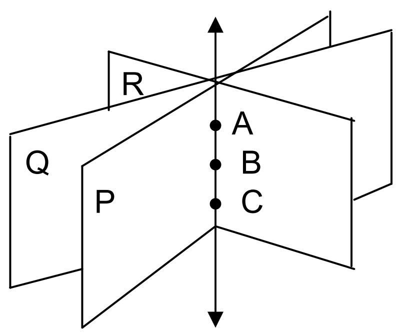
   a. Through two different points B and C below, you can draw one and only one line.

In geometry, line means straight line.

   b. It is not possible to draw more than one straight line through given two points. In the following illustration, there is only one straight line that passes through points C and D. The other line is a curve line.

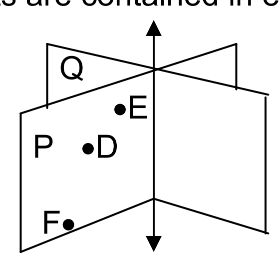


2. Three collinear points are contained in at least one plane.



In the figure, points A, B, and C are collinear. They lie in plane P, plane Q and plane R. In fact they can be contained in an infinite number of planes.

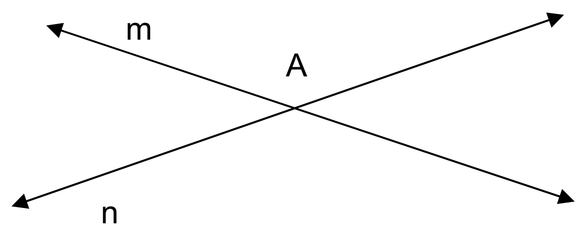
3. Three non collinear points are contained in exactly one plane.



In the figure, points D, E, and F are not collinear. They are contained in exactly one plane P.

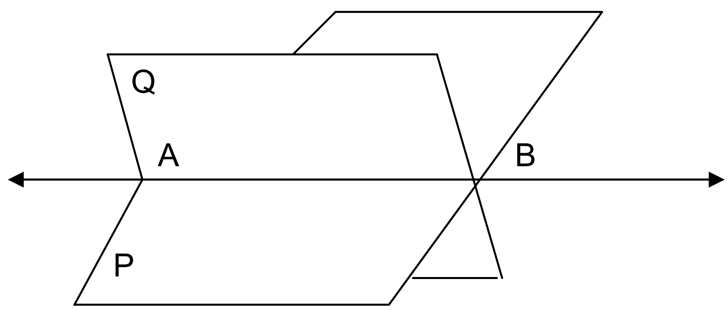
4. The intersection of two distinct lines is a point.

In the figure, line m and line n intersect and their intersection is point A.



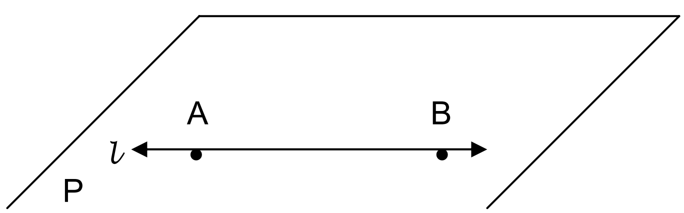
5. The intersection of two distinct planes is a line.

In the figure below, planes P and Q intersect and their intersection is line AB.



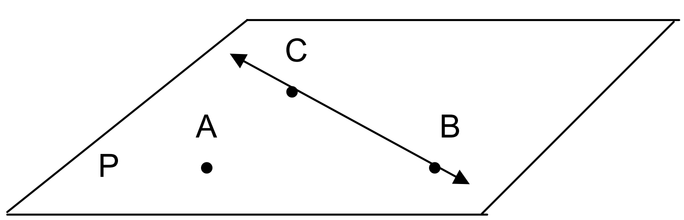
6. If two points are in a plane, then the line containing the points is in the same plane.

If the two points A and B are in plane P, then the line l which contains them lies also in plane P.



7. A line and a point not on the line are contained in exactly one plane.

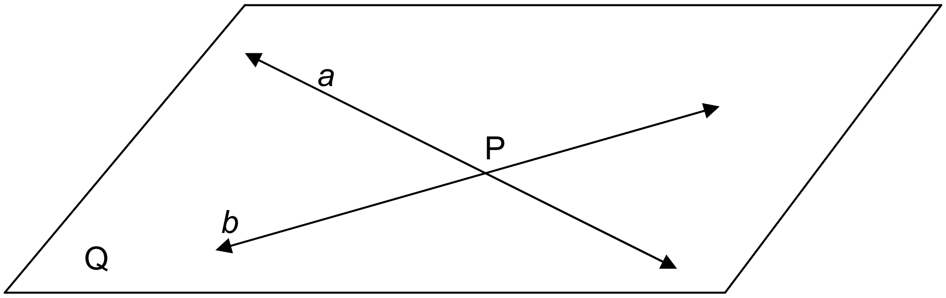
In the figure, point A does not lie on line BC. This point and line BC are contained in one plane P. This is the same as saying they determine exactly one plane P.



8. Two intersecting lines are contained in exactly one plane.

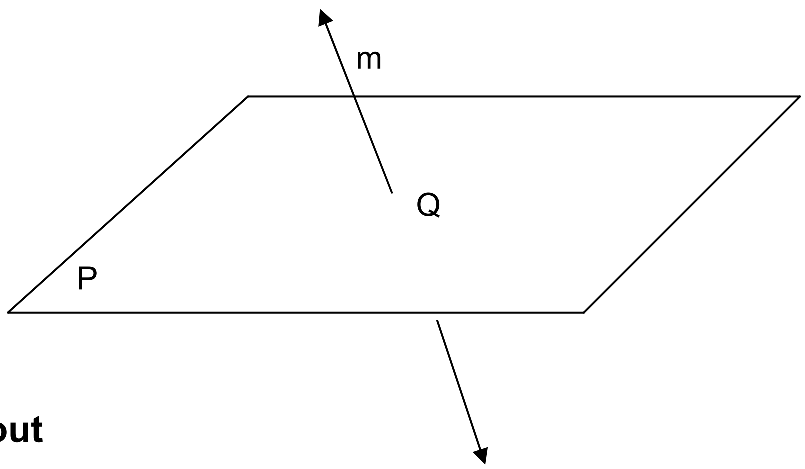
Example:

Lines a and b which intersect at point P are contained in exactly one plane Q. There is no other plane that can contain them.



9. If a line not contained in a plane intersects the plane, the intersection is a single point.

In the figure, plane P does not contain line m. The intersection of line m and plane P is a single point Q.



Try this out

Set A.

Determine the undefined term suggested by each of the following.

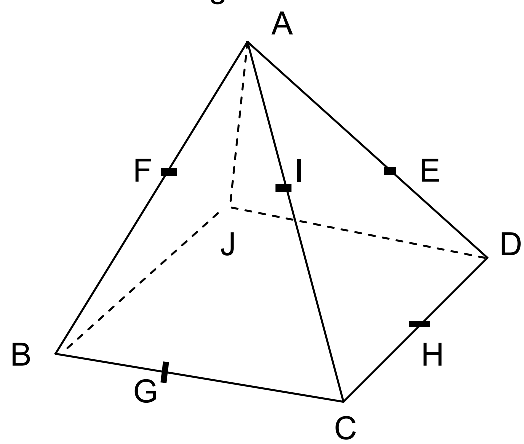
1. the tip of a pencil
2. the top of a coffee table
3. telephone wires
4. the wall of a room
5. the surface of the page of a book
6. the ruler's edge
7. the tip of a needle
8. a window pane

9. the floor of your bedroom
10. the string on a guitar

Set B.

Write True or False

Use the three-dimensional figure below for exercises 1-10.



1. Points A, F, B are collinear.
2. Points A, E, B are collinear
3. Points B, G and C are on the same line
4. Points G, C, D are not on the same line.
5. Points A, I, H are coplanar.
6. Points A, F, G are coplanar.
7. Points A, F, G , E are coplanar.
8. Points A, F, B, G are coplanar.
9. Points A. I, C are collinear and coplanar.
10. Points A, F, C are collinear and coplanar.

Set C.

Complete the following statements.

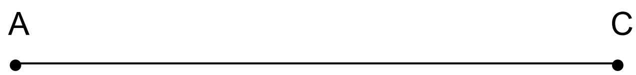
1. A ______ is an exact location in space.
2. A ______ has infinite length but no width and no thickness.
3. A ______ has infinite width and length but no thickness.
4. Two points determine exactly one ______
5. Three _________points are contained in at least one plane.
6. Three _________points are contained in exactly one plane.
7. The intersection of two distinct planes is a ________.
8. The intersection of two distinct lines is a ________.
9. Two intersecting lines determine a _________.
10. If a line not contained in a plane intersects the plane, the intersection is a single _____.

Lesson 2

The Subsets of a Line

The subsets of a line are segment and ray. A segment has two endpoints. It is named by its endpoints.

The segment below may be named $\overline{AC}$ or $\overline{CA}$. A vinculum is placed above its name to distinguish it from the name of a line where the same letters are used.



Example:

Write the name of each segment.

a. 

b. 

Answers:

a. $\overline{EF}$ or $\overline{FE}$ b. $\overline{MN}$ or $\overline{NM}$

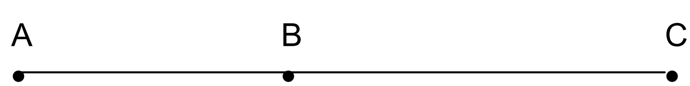
The length of a segment is the distance between its endpoints.

Example:

If the distance between points C and D below is 9 cm. then the length of segment $\overline{CD}$ is 9 cm. This is written as CD = 9 cm. Notice that there is no vinculum above CD.

A segment may be defined as the union of points A, C together with all the points between them.

Illustration:

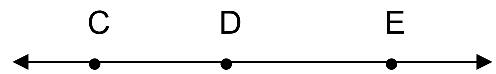


In the above segment, A and C are the endpoints of the segment. There are points between A and C. These points together with the endpoints A and C make a segment. In the above figure, point B is just one of the points between A and C.

A point such as point B above is between point A and C if and only if (1) A, B, and C are distinct points, (2) they are collinear and (3) AB + BC = AC. These three conditions must be satisfied before it can be said that B is between A and C. The word distinct in the first condition means that the three points are different from one another.

Examples:

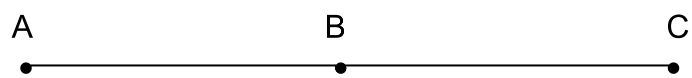
1. Draw points C, D, and E on a line. How many different segments are determined? Name them.



Answers:

$\overline{CD}$, $\overline{DE}$, $\overline{CE}$

2. If AB =5 cm, BC = 7 cm, and AC = 12 cm. Is B between A and C?



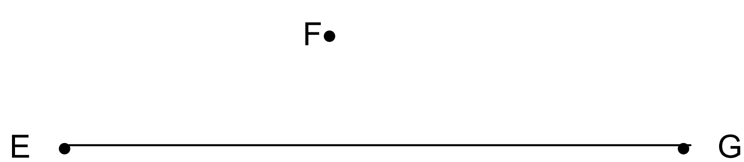
In the figure, A, B, and C are different points on the same line. The sum of the lengths of $\overline{AB}$ and $\overline{BC}$ is equal to the length of $\overline{AC}$.

AB + BC = AC
5 cm + 7 cm = 12 cm

Since the three conditions are satisfied, therefore B is between A and C.

Example:

Is F in the figure below between E and G?

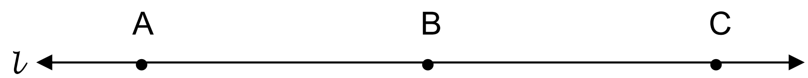


In the figure , points E, F and G are not collinear, hence F is not between point E and point G. Also, EF + FG $\neq$ EG.

A ray is a subset of a line that has one endpoint and extends forever in one direction.

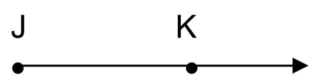
Example:

The part of the line from point B that goes on indefinitely to the right is a ray. The part of the line from point B that goes on indefinitely to the left is another ray.



The ray which starts from point B that goes on indefinitely to the right is named ray BC denoted by $\overrightarrow{BC}$. Its endpoint is B. Notice that when you name a ray, you use two capital letters, and its endpoint is written first. The other ray in the above figure is ray BA, denoted by $\overrightarrow{BA}$.

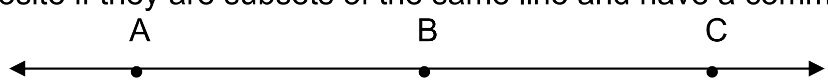
Example: Write a name for each figure.

a. 

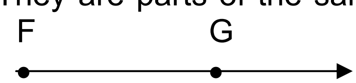
b. 

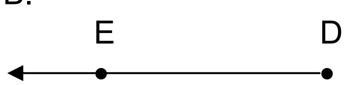
Answers:
a. $\overrightarrow{JK}$ b. $\overrightarrow{MN}$

Another term you should learn in this lesson is the term opposite rays. Two rays are opposite if they are subsets of the same line and have a common endpoint.

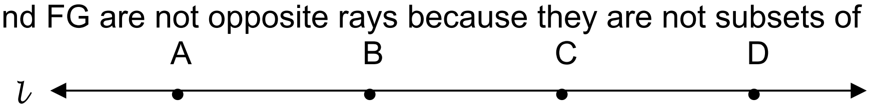


$\overrightarrow{BC}$ and $\overrightarrow{BA}$ are opposite rays. They are parts of the same line *l* and their common endpoint is B.





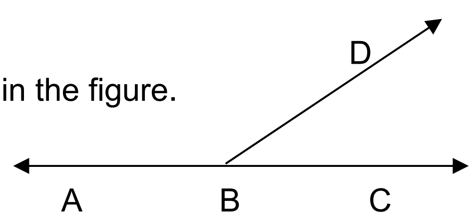
$\overrightarrow{DE}$ and $\overrightarrow{FG}$ are not opposite rays because they are not subsets of the same line.



$\overrightarrow{BA}$ and $\overrightarrow{CD}$ are not opposite rays because they do not have a common endpoint.

Example:

Name all the points, segments and rays in the figure.



Answers:

The points are A, B, C, and D. The segments are $\overline{AB}$, $\overline{BC}$, $\overline{AC}$, and $\overline{BD}$. The rays are $\overrightarrow{BA}$, $\overrightarrow{BC}$, and $\overrightarrow{BD}$

Try this out

Set A:

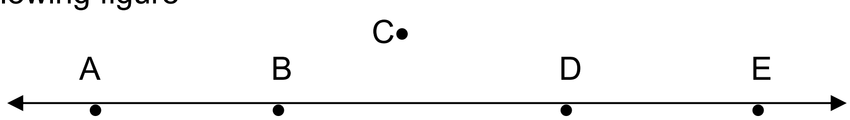
Use the figure below for exercises nos. 1-10

1. Name the ray with endpoint at B going in the direction of D.
2. Name the ray with endpoint at C going in the direction of A.
3. Name the segment joining point B with point D.
4. Give two opposite rays with common endpoint C.
5. What is the intersection of ray $\overrightarrow{BD}$ and ray $\overrightarrow{CA}$?
6. Name the ray opposite $\overrightarrow{BC}$.
6. Name the ray opposite $\overrightarrow{CA}$.
7. What point is between points B and D?
8. Give another name for $\overrightarrow{BC}$
9. Give another name for $\overrightarrow{CB}$

Set B

Write true or false

Use the following figure



1. AB + BD = AD
2. AB + BE = AE
3. AC + CD = AD
4. B is between A and D
5. C is between B and D
6. A, B, C, D are collinear
7. AB = AD – BD
8. $\overrightarrow{DE}$ and $\overrightarrow{BA}$ are opposite rays.
9. Ray BE can be named $\overrightarrow{BD}$.
10. Ray DA can be named $\overrightarrow{AD}$.

Set C

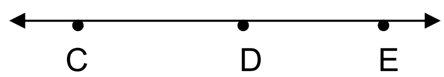
Fill in the blanks

1. A segment has _________endpoints

2. A __________is a subset of a line with one definite endpoint and extends infinitely in one direction.

3. _________are two collinear rays with a common endpoint.

Use the figure at the right for exercises nos. 4-8



4. CD + ______ = CE.
5. The ray opposite $\overrightarrow{DE}$ is ________
6. The ray with endpoint C going in the direction of D is _________
7. The ray with endpoint E going in the direction C is __________
8. The point between two other points is ________.

9. If two points P and Q are exactly the same point, then the distance between them is ______

10. The endpoint of each ray in the figure is _______

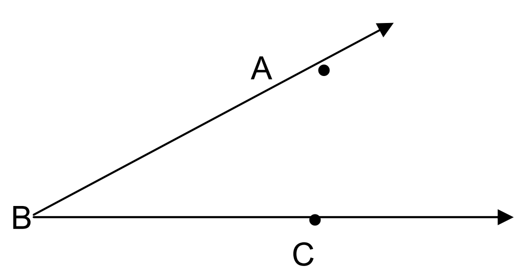
Lesson 3

Angles

An angle is a union of two noncollinear rays with a common endpoint. The common endpoint is called the vertex of the angle and the two rays are called sides.

Example:

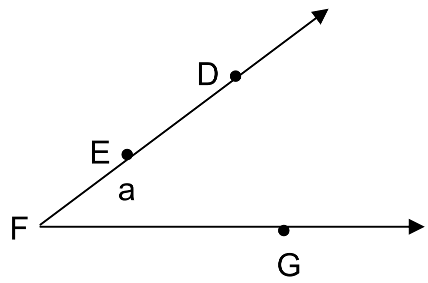
The figure below is an angle. Its vertex is point B and its two sides are BA and BC. The symbol used for an angle is ∠. The angle in the example can be named ∠ ABC. It can also be called ∠CBA. The letter representing the vertex is written between the other two letters.

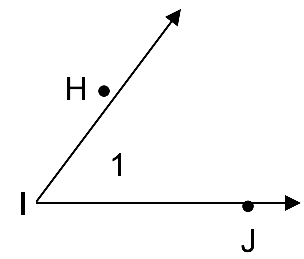


An angle may be written in other ways.

Example:

Angle DFG can also be named ∠EFG, ∠GFD, ∠GFE, ∠F and ∠a. Angle HIJ can be named ∠ 1.

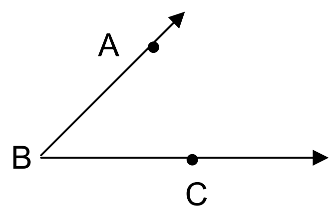




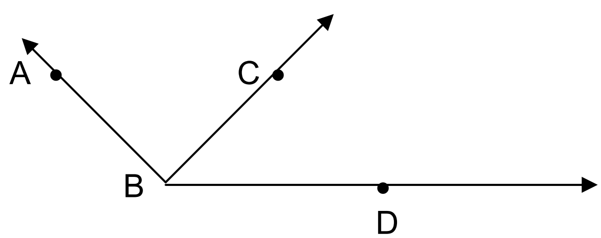
There are times when it is not advisable to use the vertex letter in naming an angle. Using it may result to confusion.

Example:

Angle ABC below may be named ∠ B

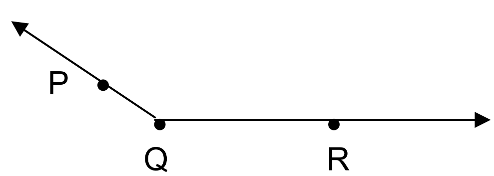


Angle ABC below should not be named ∠B. In the figure, there are three angles with vertex B. They are ∠ABC, ∠DBC and ∠ABD.



Example:

Give three different names for the angle shown below.



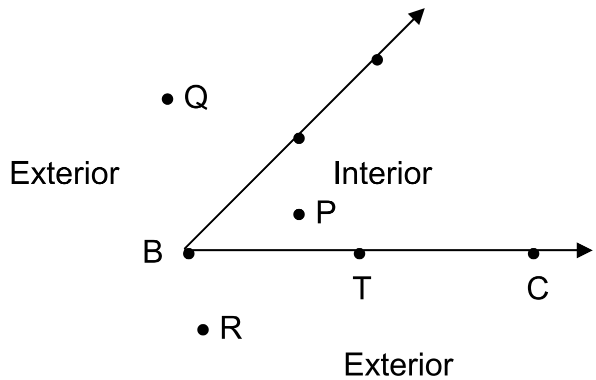
Answers:

∠ PQR, ∠ RQP, ∠Q

An angle separates a plane into three sets: the points on the angle, the interior of the angle, and the exterior of the angle.

Example:

In the figure, points T and S are on ∠ABC.. Point P is in the interior and points Q and R are in the exterior of the angle.

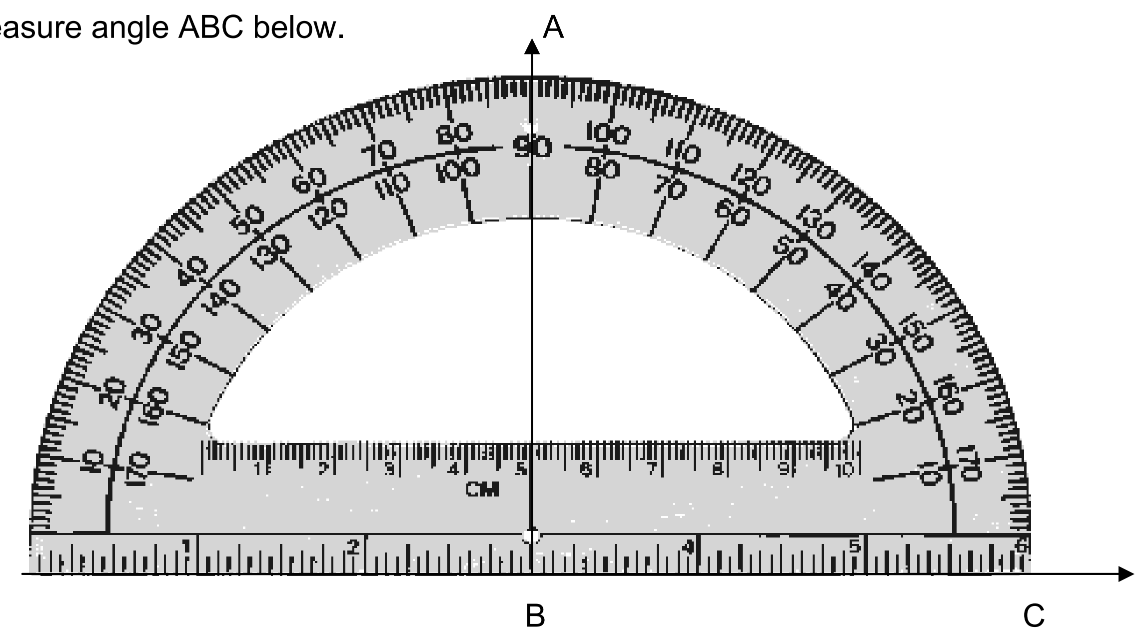


The Measure of an angle

You can determine the measure of an angle in degrees by means of a protractor. You can do this by placing the center mark of the protractor on the vertex of the angle you want to measure and then placing the 0 degree mark on one side of the angle. Then read the number where the other side crosses the scale. You can also use a protractor in constructing an angle of a given measure.

Example:

Measure angle ABC below.



The measure of ∠ABC as indicated in the protractor is 90 degrees. This can be written in two ways.

∠ ABC = 90^0 (Angle ABC equals 90 degrees.)
m∠ABC = 90. (The measure of ∠ABC is 90.)

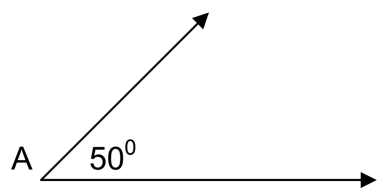
In this module the measure of an angle is always greater than 0 degree but less than 180 degrees. This restriction will be followed in this module because of the definition of an angle.

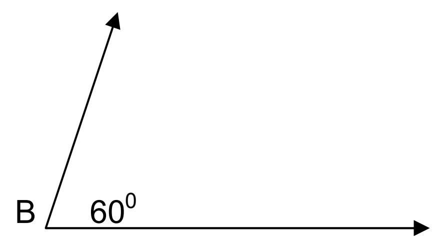
Addition of Angles

The measures of two or more angles can be added.

Example

The measure of $\angle$A is 50^0 and the measure of $\angle$ B is $60^{0.}$ Find the sum of their measures.

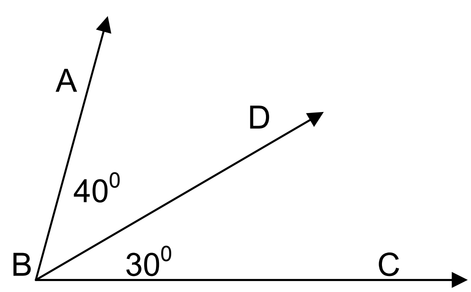




$$m \angle A + m \angle B = 50^0 + 60^{0.}$$
$$= 110^0$$

Example

$\angle$ABD and $\angle$CBD are two coplanar angles with a common side BD. If m $\angle$ ABD = 40 and m $\angle$ CBD = 30, find the measure of angle ABC.



$$m \angle ABD + m \angle CBD = 40^0 + 30^0$$
$$= 70^0$$

Example:

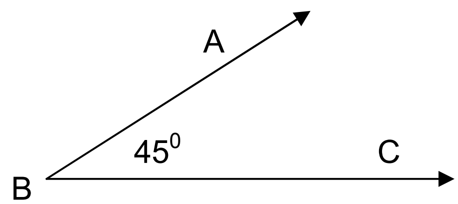
If m$\angle$ABC = 120, m$\angle$ABD = 2x + 10, and m$\angle$CBD =3 x [Use the preceding figure] Find m$\angle$ABD.

$$m\angle ABD + m\angle CBD = m\angle ABC$$
$$2x + 10 + 3x = 120$$
$$2x + 3x = 120 - 10$$
$$5x = 110$$
$$x = 22$$
$$2x + 10 = 2(22) + 10$$
$$= 44 + 10$$
$$= 54$$

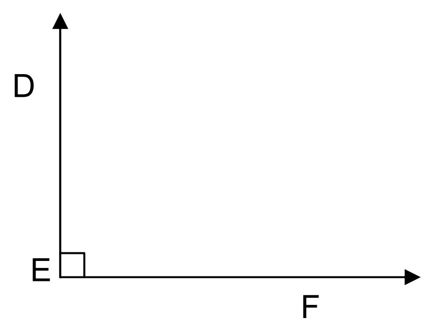
Kinds of Angles

There are three kinds of angles according to measure. They are the following.

1. Acute angle- is an angle with a measure grater than 0 but less than 90.
   $\angle ABC$ below is an acute angle.

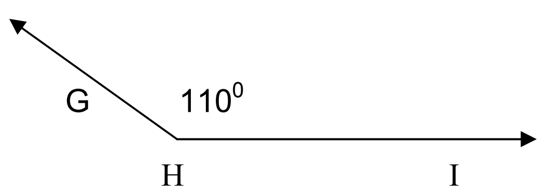


2. Right angle- is an angle with a measure of 90.
   $\angle ABC$ below is a right angle.



The symbol ┐ in the corner of a right of the figure indicates that the measure of the angle is 90.

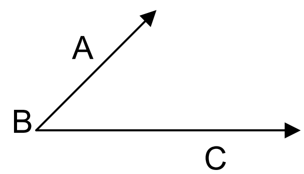
3. Obtuse angle – is an angle with a measure greater than 90 but less than 180,



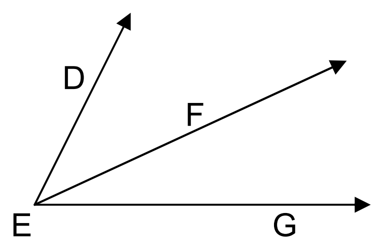
Try this out

Set A.

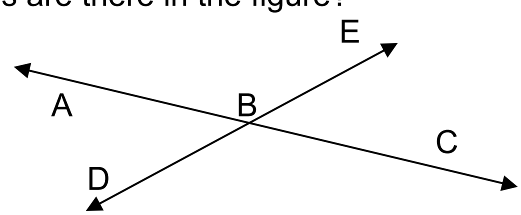
1. Name the angle below in three ways.



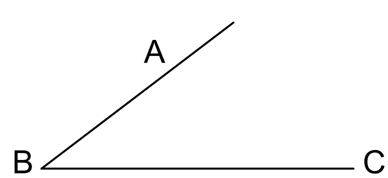
2. Which is the vertex letter in angle STG?
3. Name the three angles in the figure below.



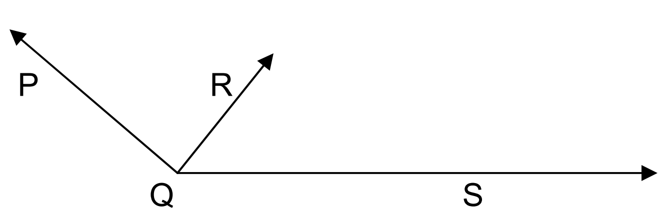
4. What are the sides of ∠ BET?
5. What is the common side of ∠ABD and ∠CBD?
6. Into how many sets does an angle separate a plane?
7. Is the vertex of an angle in its interior?
8. How many angles are there in the figure?



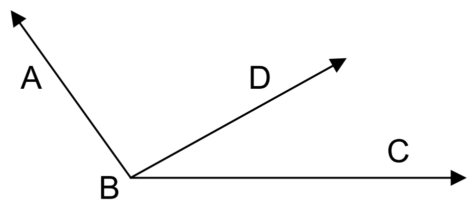
9. Is the figure below an angle? Why? Why not?



10. Explain why it is not correct to name the angle below ∠ Q?



Set B. Use the figure below for exercises 1-10. The three angles in the figure are coplanar.

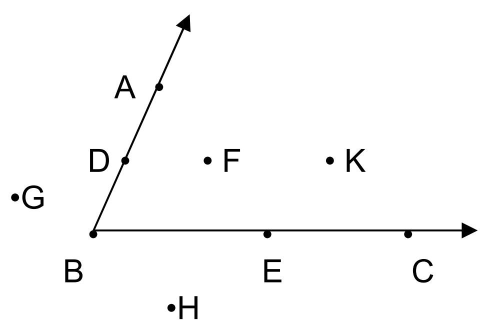


1. If m∠ABD = 80 and . m∠CBD = 40, find the m∠ ABC.
2. If m∠CBD = 30 and m∠ABD = 85, find the m∠ABC
3. If m∠ABD =45.5 and m∠CBD= 44, find the m∠ABC.
4. If the m∠CBD = 30.5 and m∠ ABD = 65, find the m∠ABC.
5. If m∠ABC =110 and . m∠CBD = 40, find the m∠ ABD.
6. If m∠ABC =115 and . m∠ABD = 40, find the m∠ CBD
7. If m∠ABC =84 and . m∠CBD = 2x, and m∠ABD = 4x, find m∠ABD
8. If m∠ABC =96 and . m∠CBD = x, and m∠ABD = 2x, find m∠CBD

Use a protractor for exercises 9-10

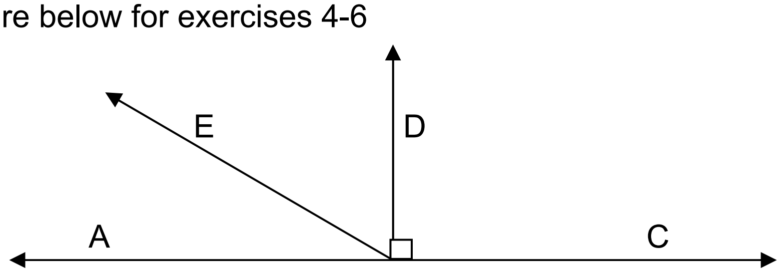
9. Construct an angle with a measure of 45 degrees.
10. Construct an angle with measure of 125 degrees.

Set C. Use the figure below for exercise 1 –3.



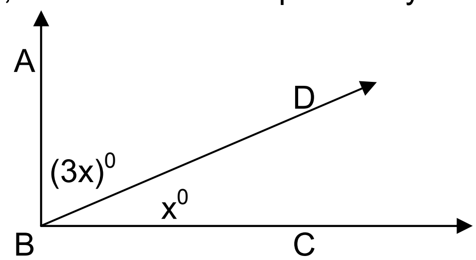
1. Name all the points in the interior of ∠ABC.
2. Name all the points in the exterior of ∠ABC.
3. Name all the points that are neither on the exterior nor interior of ∠ABC.

Use the figure below for exercises 4-6



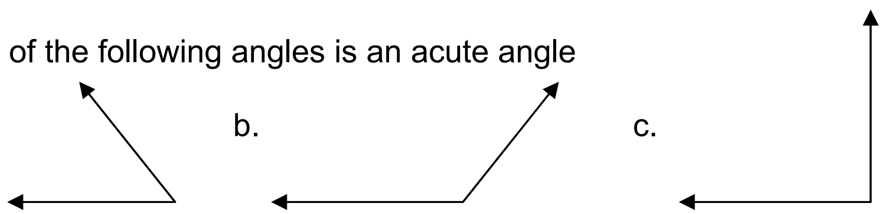
In the figure $\overrightarrow{BC}$ and $\overrightarrow{BA}$ are opposite rays.

4. Name all the angles determined in the figure.
5. Tell whether the angles in the figure are acute, right or obtuse.
6. Name the two angles with the same measure.
7. In the figure below, $\overrightarrow{BA}$, $\overrightarrow{BD}$ and $\overrightarrow{BC}$ are coplanar rays. If $\angle ABC$ is a right angle, find x



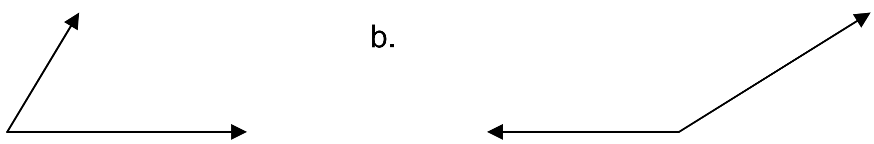
8. Which of the following angles is an acute angle

a. b. c.



9. Using your protractor, find the measure of each angle below.

a. b.

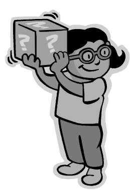


10. Draw angles with the following measures.
    a. 125^0 c. 90^0
    b. 35^0 d. 140^0

Let's summarize

1. The three undefined terms in geometry are point, line and plane.
2. A line is an exact location in space. It has no length, width or thickness.
3. A line has infinite length, but no width and no thickness.
4. A plane has infinite width and length but no thickness.
5. Two points determine exactly one line.
6. Two distinct lines intersect in only one point
7. Collinear points are points on the same line.
8. Coplanar points are points on the same plane.
9. Three collinear points are contained in at least one plane.
10. Three noncollinear point are contained in exactly one plane.

11. The intersection of two distinct planes is a line
12. If two points are in a plane, then the line containing the points is in the same line.
13. A line and a point not on the line, are contained in exactly one plane.
14. Two intersecting lines are contained in exactly one plane.
15. If a line not contained in a plane intersects the plane, the intersection is a single point.
16. A segment is a subset of a line that consists of two endpoints and all the points between them.
17. A ray is a subset of a line with a definite endpoint and extends infinitely in one direction.
18. An angle is the union of two noncollinear rays with a common endpoint.
19. An angle separates the plane into three sets: the points in the interior of the angle, the points in the exterior of the angle and the points on the angle itself.
20. A protractor is used to measure an angle in degrees.
21. An angle with a measure greater than 0 but les than 90 is an acute angle.
22. An angle with a measure of 90 is a right angle.
23. An angle with a measure greater than 90 but less than 180 is an obtuse angle.



What have you learned

Multiple Choice. Choose the letter of the correct answer.

1. It is flat surface that extends infinitely in all directions.
   A. Point C. Plane
   B. Line D. rectangle

2. It is a set of points that extends forever in opposite directions.
   A. Point C. Plane
   B. Line D. Space.

3. Which of the following is false?
   A. Exactly one plane contains two intersecting lines.
   B. Two points determine a line.
   C. The intersection of two distinct planes is a line
   D. Three collinear points are contained in exactly one plane

4. Which of the following real objects suggest a point?
   A. The edge of the beam of a building
   B. The corner of the Main street and the 1st Ave.
   C. The floor of a newly constructed building.
   D. The wall of your room.

5. It is a subset of a line with a definite endpoint and extends infinitely in one direction.
   A. Ray C. Opposite Rays

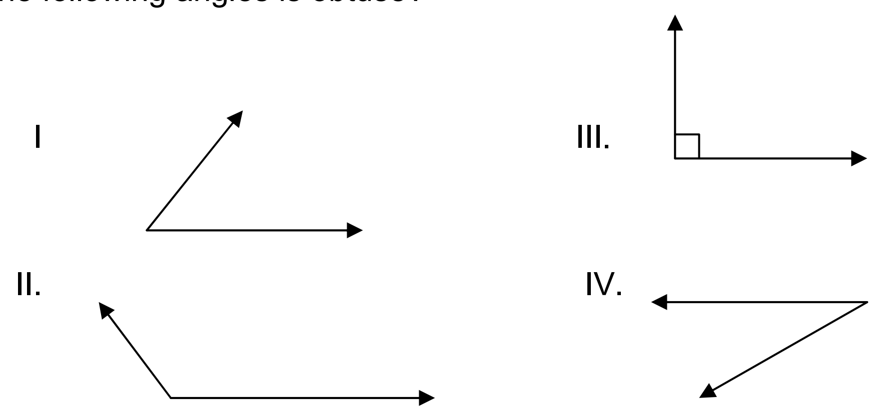
B. Segment
D. Plane

6. It is the union of two noncollinear rays with a common endpoint.
   A. Plane
   B. Angle
   C. Space
   D. Segment

7. It is an angle with a measure greater than 0 but less than 90.
   A. Acute angle
   B. Right Angle
   C. Obtuse angle
   D. non of these

8. It is angle with a measure of 90.
   A. Acute angle
   B. Right Angle
   C. Obtuse angle
   D. none of these

9. Which of the following angles is obtuse?



   A. I only
   B. I and II
   C. II only
   D. I and III

10. It is used to measure an angle in degrees.

   A. Compass
   B. Ruler
   C. protractor
   D. tape measure

Answer Key

How much do you know

1. point
2. line
3. plane
4. angle
5. protractor
6. acute angle
7. segment
8. obtuse angle
9. plane
10. line

Try this out

Lesson 1

Set A

1. point
2. plane
3. line
4. plane
5. plane
6. line
7. point
8. plane
9. plane
10. line

Set B

1. True
2. False
3. True
4. True
5. True
6. True
7. False
8. True
9. True
10. False

Set C

1. point
2. line
3. plane
4. line
5. non-collinear
6. collinear
7. line
8. point
9. plane
10. point

Lesson 2

Set A
.

1. BC or BD
2. CA or CB
3. BD
4. CD and CA or CD or CB
5. BC
6. BA
7. CD
8. C
9. CB
10. BC, AB

Set B

1. True
2. True
3. False
4. True
5. False
6. False
7. True
8. False
9. True
10. False

Set C

1. two
2. ray

3. opposite rays
4. DE
5. DC
6. CD or CE
7. EC or ED
8. D
9. 0
10. B

Lesson 3

Set A

1. ∠ABC, ∠CBA, ∠B
2. T
3. ∠DEF, ∠GEF, ∠DEG
4. EB, ET
5. BC
6. Three sets including itself
7. No.
8. 4
9. No
10. Q is the vertex of the three angles. ∠Q may mean ∠PQR, ∠RQS, and ∠PQS

Set B

1. 120
2. 115
3. 89.5
4. 95.5
5. 70
6. 75
7. 56
8. 32
9. Use your protractor
10. Use your protractor

Set C.

1. F, K
2. G, H
3. A, D, B, E, C
4. ∠ABE, ∠ABD, ∠DBE, ∠EBC, ∠DBC
5. ∠ABE and ∠DBE are acute angles
   ∠ABD and ∠DBC are right angles
   ∠EBC is obtuse

6. ∠ABD and ∠DBC. Both are right angles with measure of 90^0 each.
7. 22.5
8. a
9. Use your protractor
10. Use your protractor

What have you learned

1. C
2. B
3. D
4. B
5. A
6. B
7. A
8. B
9. C
10. C