Module 2
Plane Coordinate Geometry

What this module is about

This module will discuss how the distance between two points can be derived by applying the Pythagorean theorem. By using the derivation of distance, this module will also define and discuss the midpoint formula. Furthermore, this module will also define and verify figures and their characteristics on the coordinate plane using the coordinate proof. This will also enhance your knowledge of distances between two points and how to get the lengths of segments and sides of polygons.

What you are expected to learn

This module is written for you to

1. Derive the distance formula using the Pythagorean theorem.
2. Apply the distance formula in finding lengths of segments.
3. Verify congruence of segments by applying the distance formula.
4. Derive the midpoint formula.
5. Verify the midpoint of a segment using the distance formula.
6. Solve problems that are application of the distance and midpoint formula.

How much do you know

Answer the following questions as indicated.

1. If the coordinate of point X is - 2 , and the coordinate of point Y is 3, what is the length of $\overline{XY}$?

Find the distance between the following given pairs of points.

2. M(5, 5) and N(9, 8)
3. R(0, 6) and S(8, 0)
4. P(4, -3) and Q(4, 4)

Find the coordinate of the midpoint X of the segments whose endpoints are:

5. (3, 0), (7, 6)
6. (-2, -3), (-6, 9)
7. (8, 1), (-5, 5)

8. Find the perimeter of a triangle whose vertices are at the given points A(2, 3), B(5, 7), C(0, 1).
9. If A is the midpoint of $\overline{MN}$, determine the coordinates of A if the coordinates of the endpoints are M(-4, -3) and N(5, 7)
10. The coordinates of the vertices of a quadrilateral are (6, 0), (2, 3), (3, -4) and (-1, -1). What kind of quadrilateral is formed when you connect the vertices?

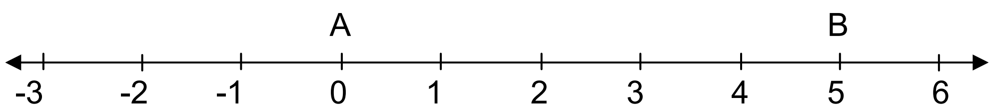
What you will do

Lesson 1

The Distance Formula

When you refer to the distance between any two points on the plane, either horizontally, vertically or any other positions, then what you mean is getting the length of the segment joining the two points.

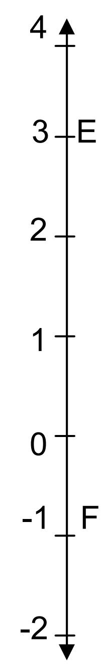
Illustrations: Horizontal Distance



The distance between A and B, which is denoted by AB is

$AB = |5 - 0| = |5| = 5.$

Vertical Distance



The distance between E and F, which is denoted by EF is

$EF = |3 - (-1)| = |4| = 4.$

Distance between two points or length of a segment is always a unique positive real number. That is why it is necessary to use the symbol | | for absolute value.

If x_1 and x_2 are the coordinates of A and B respectively on a horizontal number line, then the distance between A and B is denoted by

$$\boxed{AB = |x_2 - x_1|}$$

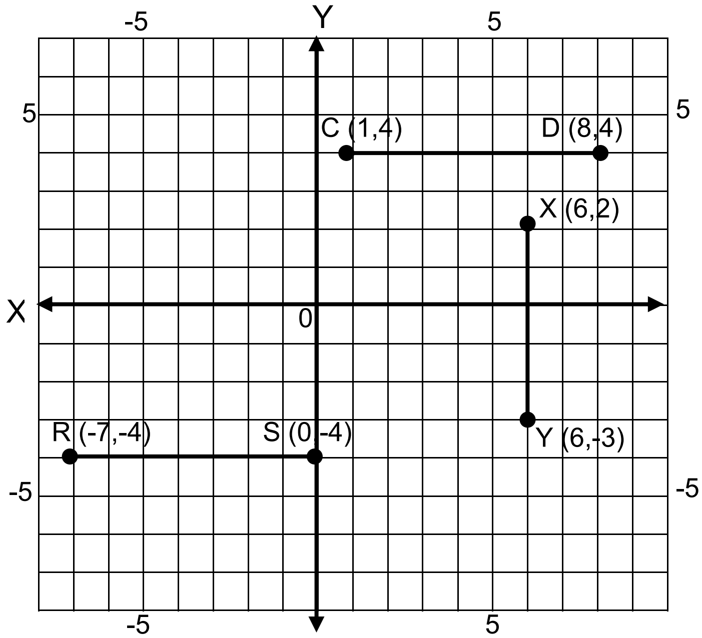
If y_1 and y_2 are the coordinates of E and F respectively on a vertical number line, then the distance between E and F is denoted by

$$\boxed{EF = |y_2 - y_1|}$$

Examples:

Find the length of the following segments:

a. CD
b. XY
c. RS



Solution:

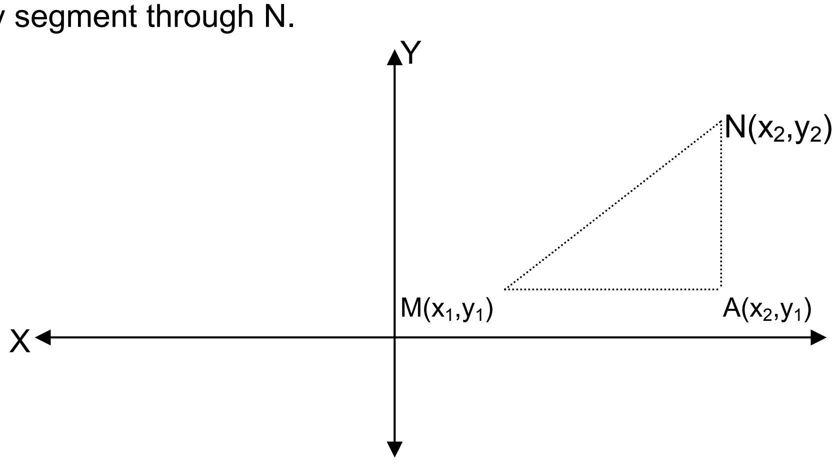
a. Points C and D have the same y-coordinate. Therefore, distance CD is denoted by
$CD = |8 - 1| = |7| = 7$

b. Points X and Y have the same x-coordinate. So distance XY can computed as
$XY = |2 - (-3)| = |2 + 3| = 5$

c. Points R and S have the same y-coordinate. So distance RS is
$RS = |-7 - 0| = |-7| = 7$

Suppose the given segment on a coordinate plane is neither horizontal nor vertical. How are you going to find the distance?

Let $M(x_1, y_1)$ and $N(x_2, y_2)$ be two points on a Cartesian coordinate plane. Let there be another point A where an imaginary horizontal segment through M intersects an imaginary segment through N.



Observe that right triangle MAN is formed on the coordinate plane. The distance between M and N is equal to the length of the hypotenuse MN of the right triangle. By the Pythagorean theorem,

$$(MN)^2 = (MA)^2 + (NA)^2$$

But in the earlier discussion, you were given the some formula how to get the distance on the horizontal number line and the vertical number line. You can just call them horizontal distance and vertical distance. Therefore if you substitute the previous formula to the above formula you will get

$$(MA)^2 = |x_2 - x_1| \quad \text{and}$$

$$(NA)^2 = |y_2 - y_1|$$

Putting together the formula above and the Pythagorean theorem, you will have

$$(MN)^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2.$$

Simplifying, you will get the distance formula. The distance between two points $M(x_1, y_1)$ and $N(x_2, y_2)$ is given by the formula,

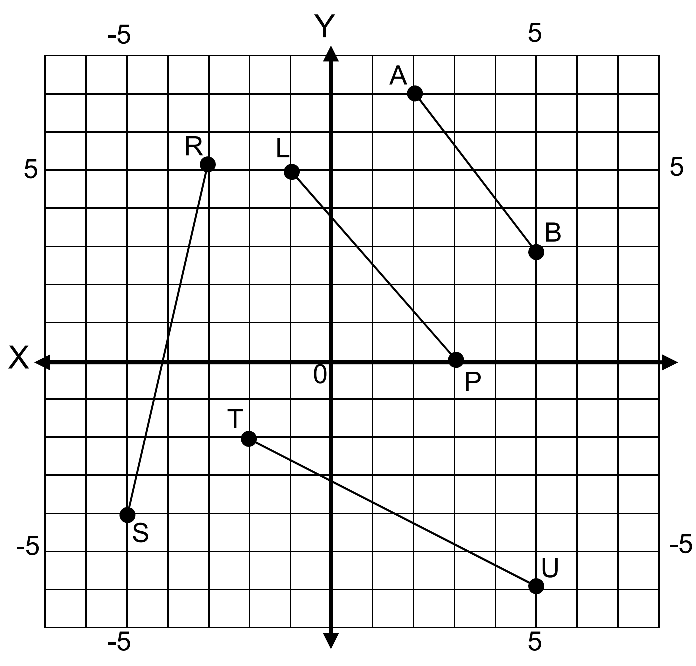
$$\boxed{MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

This we can do since the absolute value of a number is non negative, so is the square of a number is also non negative.

Examples:

1. Use the distance formula to find the length of the given segments in the coordinate plane.

a. AB
b. LP
c. RS
d. TU



Solutions: In each of the following segments, determine first the coordinates of the endpoints from the graphs.

a. AB ; A(2, 7), B(5, 3)

$x_1 = 2$ $x_2 = 5$

$y_1 = 7$ $y_2 = 3$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(5-2)^2 + (3-7)^2}$$
$$= \sqrt{(3)^2 + (-4)^2}$$
$$= \sqrt{9+16}$$
$$= \sqrt{25}$$
$$AB = 5$$

b. LP ; L(-1, 5) , P(3, 0)

$x_1 = -1$ $x_2 = 3$

$y_1 = 5$ $y_2 = 0$

$$LP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{[3-(-1)]^2 + (0-5)^2}$$
$$= \sqrt{(4)^2 + (-5)^2}$$
$$= \sqrt{16+25}$$
$$LP = \sqrt{41}$$

c. RS ; R(-3, 5) , S(-5, -4)

$x_1 = -3$ $x_2 = -5$

$y_1 = 5$ $y_2 = -4$

$$RS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{[-5-(-3)]^2 + (-4-5)^2}$$
$$= \sqrt{(-5+3)^2 + (-9)^2}$$
$$= \sqrt{(-2)^2 + (-9)^2}$$
$$= \sqrt{4+81}$$
$$RS = \sqrt{85}$$

d. TU ; T(-2, -2), U(5, -6)

$x_1 = -2$ $x_2 = 5$

$y_1 = -2$ $y_2 = -6$

TU = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

= $\sqrt{[5-(-2)]^2 + [-6-(-2)]^2}$

= $\sqrt{(5+2)^2 + (-6+2)^2}$

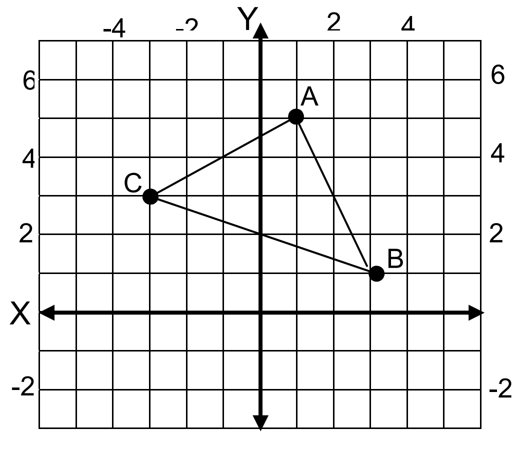
= $\sqrt{(7)^2 + (-4)^2}$

= $\sqrt{49+16}$

TU = $\sqrt{65}$

2. Draw a triangle with vertices A(1, 5), B(3, 1), C(-3, 3). Show that ΔABC is isosceles.

Solution: a. Plot the given points (vertices) on a Cartesian coordinate plan.



b. To show that ΔABC is isosceles, find the length of the sides. For the triangle to be isosceles, at least two of the sides must have the same length.

1. Find AC; A(1, 5), C(-3, 3)
$x_1 = 1$ $x_2 = -3$
$y_1 = 5$ $y_2 = 3$

AC = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

= $\sqrt{(-3-1)^2 + (3-5)^2}$

= $\sqrt{(-4)^2 + (-2)^2}$

= $\sqrt{16+4}$

= $\sqrt{20}$

AC = $2\sqrt{5}$

2. Find BC; B(3, 1) , C(-3, 3)
$x_1 = 3$ $x_2 = -3$
$y_1 = 1$ $y_2 = 3$

$$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-3-3)^2 + (3-1)}$$
$$= \sqrt{(-6)^2 + (2)^2}$$
$$= \sqrt{36+4}$$
$$= \sqrt{40}$$
$$BC = 2\sqrt{10}$$

3. Find AB; A(1, 5) B(3, 1
   $x_1 = 1$ $x_2 = 3$
   $y_1 = 5$ $y_2 = 1$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(3-1)^2 + (1-5)^2}$$
$$= \sqrt{(2)^2 + (-4)^2}$$
$$= \sqrt{4+16}$$
$$= \sqrt{20}$$
$$AB = 2\sqrt{5}$$

Since the length of AC equals the length of AB, then $\overline{AC} \cong \overline{AB}$. Therefore ΔABC is an isosceles triangle.

3. Find the perimeter of a quadrilateral whose vertices are P(-2,2), Q(5, 2), R(4, -3) and S(-3, -3). What kind of quadrilateral is PQRS?

Solution:

a. Find PQ; the y-coordinate of P and Q is the same

$$PQ = \sqrt{[5-(-2)]^2}$$
$$= \sqrt{(5+2)^2}$$
$$= \sqrt{(7)^2}$$
$$PQ = 7$$

b. Find QR

$$QR = \sqrt{(4-5)^2 + (-3-2)^2}$$
$$= \sqrt{(-1)^2 + (-5)^2}$$
$$= \sqrt{1+25}$$
$$QR = \sqrt{26}$$

c. Find RS; the y-coordinate of R and S is the same

RS = $\sqrt{(-3-4)^2}$

= $\sqrt{(-7)^2}$

= $\sqrt{49}$

RS = 7

d. Find PS

PS = $\sqrt{[-3-(-2)]^2+(-3-2)^2}$

= $\sqrt{(-1)^2+(-5)^2}$

= $\sqrt{1+25}$

= $\sqrt{26}$

The perimeter of quadrilateral PQRS = PQ + QR + RS + PS

= 7 + $\sqrt{26}$ + 7 + $\sqrt{26}$

= 14 + 2$\sqrt{26}$

Based on the computed lengths of the sides, PQ = RS and QR = PS which means that the opposite sides of the quadrilateral are congruent. Therefore, PQRS is a parallelogram.

4. Show by the distance formula that the following points R(3, 5), S(0, -1) and T(1,1) are collinear.

Solution: To illustrate that the given points R, S and T are collinear, you have to show that the sum of the lengths of the two short segments is equal to the length of the longer segment.

a. Find RS

RS = $\sqrt{(0-3)^2+(-1-5)^2}$

= $\sqrt{(-3)^2+(-6)^2}$

= $\sqrt{9+36}$

= $\sqrt{45}$

RS = 3$\sqrt{5}$

b. Find ST

ST = $\sqrt{(1-0)^2+[1-(-1)]^2}$

= $\sqrt{(1)^2+(2)^2}$

= $\sqrt{1+4}$

ST = $\sqrt{5}$

c. Find RT

$$RT = \sqrt{(1-3)^2 + (1-5)^2}$$
$$= \sqrt{(-2)^2 + (-4)^2}$$
$$= \sqrt{4+16}$$
$$= \sqrt{20}$$
$$RT = 2\sqrt{5}$$

Now add ST and RT. ST + RT = $\sqrt{5}$ + 2$\sqrt{5}$ = 3$\sqrt{5}$ which means that ST + RT = RS.

This conclusion satisfies the definition of betweenness and so R, S and T are collinear. To verify further, plot the three points on the Cartesian coordinate plane.

5. The endpoints of the base of an isosceles triangle are A(1, 2) and B(4, -1). Find the y-coordinate of the third vertex if its x-coordinate is 6.

Solution:

Let C(6, y) be the coordinates of the third vertex.

Since ∆ABC is isosceles, and AB is the base, then AC = BC.

$$AC = \sqrt{(6-1)^2 + (y-2)^2}$$
$$BC = \sqrt{(6-4)^2 + [y-(-1)]^2}$$

But AC = BC, therefore to solve for y, equate the values of AC and BC.

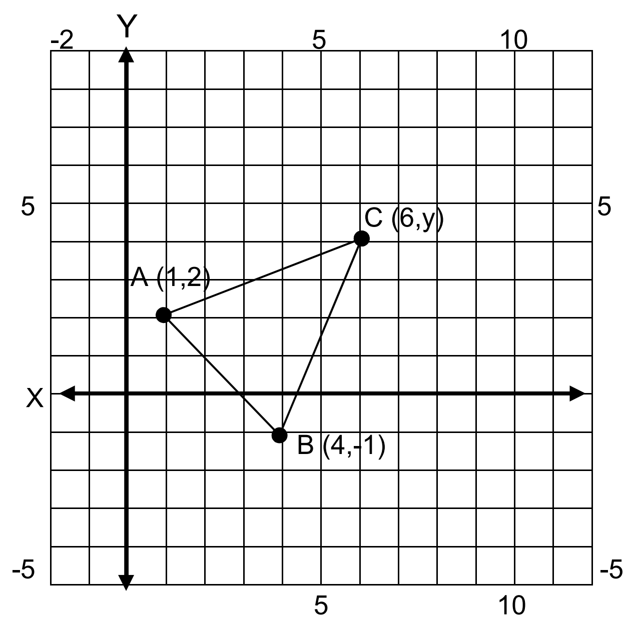
$$\sqrt{(6-1)^2 + (y-2)^2} = \sqrt{(6-4)^2 + [y-(-1)]^2}$$
$$\sqrt{5^2 + (y-2)^2} = \sqrt{2^2 + (y+1)^2}$$

Squaring both sides

$$25 + y^2 - 4y + 4 = 4 + y^2 + 2y + 1$$
$$-6y = -24$$
$$y = 4$$

So the third vertex is C(6, 4).

The triangle is shown in the figure at the right.



6. A point A(x, 1) is $\sqrt{29}$ units from B(8, 3). Find x.

Solution:

AB = $\sqrt{(8-x)^2+(3-1)^2}$ = $\sqrt{29}$

$\sqrt{64-16x+x^2+2^2}$ = $\sqrt{29}$

$64 - 16x + x^2 + 4 = 29$ Squaring both sides of the equation

$x^2 - 6x + 39 = 0$

$(x - 13)(x - 3) = 0$

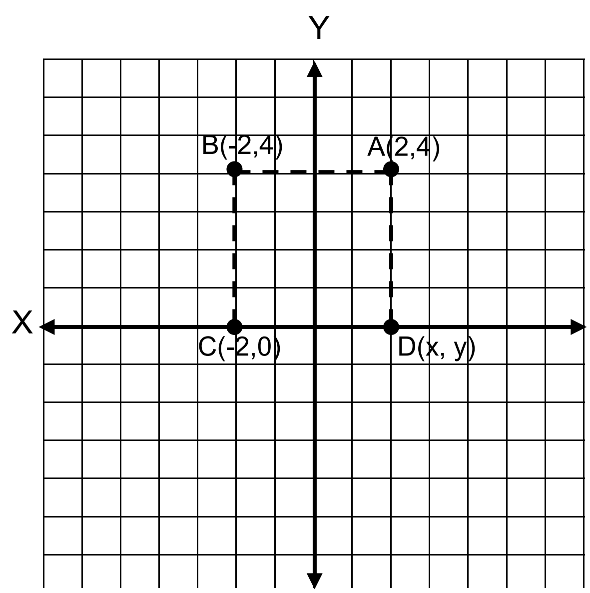
$x - 13 = 0$ $x - 3 = 0$

$x = 13$ $x = 3$

There are two values of x. Therefore the two points are (13, 1) and (3, 1)

7. Three of the vertices of a square are points A(2, 4), B(-2, 4), C(-2, 0). Find the fourth vertex D(x, y).

Solution: Plot the points on the coordinate plane.



Try this out

A. Find the distance between the following pairs of points.

1. (0, 4), (0, 6)
2. (2, -1), (7, -1)

3. (4, -3), (-7, -3)
4. (1, 5), (3, 8)
5. (-4, -7), (0, 5)
6. (2, 8), (-5, -1)
7. (-5, 4), (-3, -3)
8. (6, 2), (5, -2)
9. (-1, 6), (5, -1)
10. (-4, -5), (6, 0)

B. Find the perimeter of the polygons with vertices at the given points.

11. (1, 2), (4, 6), (7, 2)
12. (-1, 7), (-1, 1), (-9, 1)
13. (2, -2), (-1, -5), (-3, -1)
14. (2, -6), (2. -9), (-3, -6), (-3, 9)
15. (4, -1), (7, -2), (5, -6), (2, -5)
16. (-2, 4), (0, 6), (2, 4), (0, 0)
17. (-5, -4), (-3, -6), (-5, -9), (-8, 8), (-8, -5)

C. Given the distance(d) between two points D and F and the coordinates of one of the endpoints. Find the coordinates of the other endpoint if either x or y coordinate is given.

18. d = 13, D(-4, 1), F(x, -4)
19. d = 7, D(5, 0), F(1, y)
20. d = 5, D(-4, y), F(0, -3)
21. d = $\sqrt{5}$, D(x, 4), F(3, 5)

D. Solve the following problems:

22. Draw a triangle with vertices (6, 3), (2, 7), (10, 7). What kind of triangle is it?
23. Use the distance formula to show that (3, 0), (0, 4), (6, -4) are collinear.
24. The distance from (5, 7) to (x, 2) is $\sqrt{34}$. Find all possible values for x.
25. Find the fourth vertex S of a rectangle whose three vertices are p(-3, 2), Q(-3, 7) and R(2, 7).
26. The point (5, y) is $\sqrt{17}$ units from (6, 2). Find y.
27. A line segment 5 units long has one of its ends at (3, 1). The y-coordinate of the other end is 5. Find its x-coordinate.

Lesson 2

The Midpoint Formula

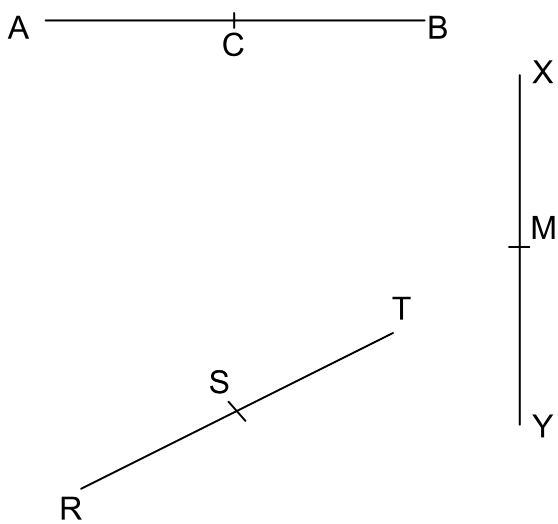
The midpoint of a segment is a point that divides a segment into two (2) congruent segments.

Illustrations:

1. C is the midpoint of $\overline{AB}$. Then $\overline{AC} \cong \overline{BC}$. So AC = CB.

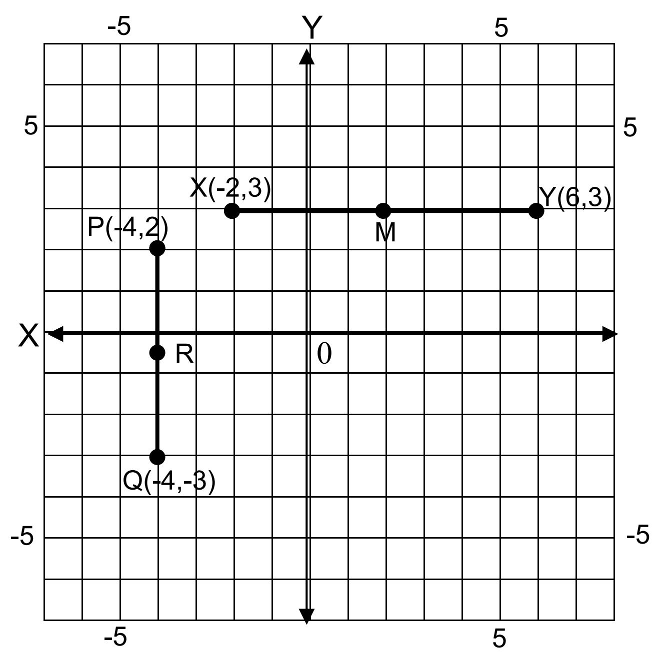
2. If M is the midpoint of $\overline{XY}$ then $\overline{XM} \cong \overline{MY}$, thus XM = MY.

3. S is the midpoint of $\overline{XY}$
   Therefore, $\overline{RS} \cong \overline{ST}$, and RS = ST.



From the illustrations given, you can say that the midpoint of a segment should lie between the endpoints of the segments and the three points must be collinear.

How do you get the midpoint of the segment on the coordinate plane? Consider the coordinate plane below and the segments on it.



If M and R are the midpoints of $\overline{XY}$ and $\overline{PQ}$ respectively, how do your determine the coordinates of M and R?

To get the midpoint of $\overline{XY}$, you have to consider that the segment is horizontal, thus the y-coordinate is the same. Since point M lies between the two endpoints and in the middle, the x-coordinate of M is the average of the x-coordinates of the two endpoints of the segment and the y-coordinate is 3.

So the coordinates of M is

$$M(x_m, y_m) = \left[\frac{-2+6}{2}, 3\right]$$

$$= \left(\frac{4}{2}, 3\right)$$

$$= (2, 3)$$

To get the coordinates of R, consider $\overline{PQ}$. Since the segment is vertical, the two endpoints have the same x-coordinate which is -4. To get the y-coordinate, get the average of the y-coordinates of the two endpoints. That is

$$R(x_m, y_m) = \left[-4, \frac{2+(-3)}{2}\right]$$

$$= \left(-4, -\frac{1}{2}\right)$$

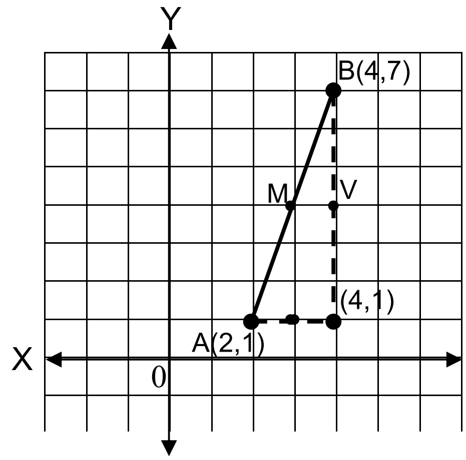
Therefore, to get the coordinates of the midpoint of horizontal segment the formula below is used. Since the two endpoints have the same y – coordinate the midpoint (M) is

$$M\left(\frac{x_1+x_2}{2}, y\right)$$

For vertical segments, since the two endpoints have the same x-coordinate, then the formula is

$$M\left(x, \frac{y_1+y_2}{2}\right)$$

How do you get the coordinates of the segment on the coordinate plane which is neither horizontal nor vertical? $\overline{AB}$ illustrated below is neither horizontal nor vertical.



The endpoints of $\overline{AB}$ are A(2, 1) and B(4, 7). Let M be the midpoint of $\overline{AB}$. To determine the coordinates of M, draw horizontal segment passing through A and a vertical segment passing through B. The two segments intersect at a point whose coordinates are (4, 1). Get the coordinates of the midpoint H of the horizontal segment.

$$H\left(\frac{2+4}{2},1\right) \text{ or } H(3, 1)$$

Then get the coordinates of the midpoint V of the vertical segment .

$$V\left(4,\frac{1+7}{2}\right) \text{ or } V(4, 4).$$

The points H and V suggest that the midpoint of M are (3, 4). To check if M is really the midpoint of $\overline{AB}$, we have to show that AM = MB.

$$\begin{aligned} AM &= \sqrt{(3-2)^2+(4-1)^2} \\ &= \sqrt{1^2+3^2} \\ &= \sqrt{1+9} \\ &= \sqrt{10} \end{aligned}$$

$$\begin{aligned} MB &= \sqrt{(4-3)^2+(7-4)^2} \\ &= \sqrt{1^2+3^2} \\ &= \sqrt{1+9} \\ &= \sqrt{10} \end{aligned}$$

Since AM = MB = $\sqrt{10}$, then M is really the midpoint of $\overline{AB}$.

For segments on the coordinate plane which are neither horizontal nor vertical, the formula for finding its midpoint M is given below.

The Midpoint Formula:

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are any two points in a coordinate plane, then the midpoint M of $\overline{AB}$ has coordinates

$$\boxed{M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)}$$

Examples:

1. Find the coordinates of the midpoint M of the segments whose endpoints are

   a. (3, 5), (7, 1)
   b. (0, 1), (-4, 3)
   c. (-3, -6), (2, -11)
   d. (4, -1), (-7, 3)

Solutions: Using the midpoint formula,

a. (3, 5), (7, 1)

$x_1 = 3,\quad x_2 = 7$
$y_1 = 5,\quad y_2 = 1$

$$M\left(\frac{7+3}{2},\frac{1+5}{2}\right) = M\left(\frac{10}{2},\frac{6}{2}\right)$$
$$= M(5,3)$$

b. (0, 1), (-4, 3)

$x_1 = 0\ ,\quad x_2 = -4$
$y_1 = 1\ ,\quad y_2 = 3$

$$M\left(\frac{-4+0}{2},\frac{3+1}{2}\right)$$
M(-2, 2)

c. (-3, 6), (2, 11)

$x_1 = -3,\quad x_2 = 2$
$y_1 = 6\ ,\quad y_2 = 11$

$$M\left(\frac{-3+2}{2},\frac{-6+(-11)}{2}\right)$$
$$M\left(\frac{-1}{2},\frac{-17}{2}\right)$$

d. (4, 1), (-7, 3)
$x_1 = 4$, $x_2 = -7$
$y_1 = 1$, $y_2 = 3$

$$M\left(\frac{4+(-7)}{2}, \frac{3+1}{2}\right)$$
$$M\left(\frac{-3}{2}, \frac{4}{2}\right) = \left(\frac{-3}{2}, 2\right)$$

2. M(-1, -3) is the midpoint of $\overline{ST}$, If the coordinates of S are (-3, 2), find the coordinates of T.

Solution:

Step 1. Let T have coordinates (x, y). By the midpoint formula, the midpoint of $\overline{ST}$ is

$$\left(\frac{x+(-3)}{2}, \frac{y+2}{2}\right) = \left(\frac{x-3}{2}, \frac{y+2}{2}\right)$$

Step 2. We are given that the coordinates of M is (-1, -3) and M is the midpoint of $\overline{ST}$. Therefore,

$$\frac{x-3}{2} = -1$$
$$x - 3 = -1(2)$$
$$x - 3 = -2$$
$$x = -2 + 3$$
$$x = 1$$

and $$\frac{y+2}{2} = -3$$
$$y + 2 = 2(-3)$$
$$y + 2 = -6$$
$$y = -6 - 2$$
$$y = -8$$

So the coordinates of T are (1, -6)

3. One endpoint P of segment PS and its midpoint R are given. Use the midpoint formula to find the coordinates of the second segment S.

a. P(3, -4), R(0, 0)
b. P(2, 5), R(5, -1)
c. P(-6, -3), R(0, 1)

Solutions:

a. Let S have the coordinates (x, y). Using the midpoint formula, the midpoint of $\overline{PS}$ is given as $\left(\frac{x+3}{2}, \frac{y-4}{2}\right)$

The coordinates of midpoint R are (0, 0)

$$\frac{x+3}{2} = 0$$

x + 3 = 0

x = -3

$$\frac{y-4}{2} = 0$$

y -4 = 0

y = 4

The coordinates of R are (-3, 4)

b. Coordinates of S are (x, y). Applying the midpoint formula, the coordinates of R is $\left(\frac{x+2}{2}, \frac{y+5}{2}\right)$

The coordinates of midpoint R are ((5, -1)

$$\frac{x+2}{2} = 5$$

x + 2 = 10

x = 8

$$\frac{y+5}{2} = -1$$

y + 5 = -2

y = -7

The coordinates of R are (8, -7)

c. Coordinates of S are (x, y). Using the midpoint formula, the coordinates of midpoint R is given as $\left(\frac{x+(-6)}{2}, \frac{y+(-3)}{2}\right)$

The coordinates of midpoint R are (0, 1). Solving for x and y,

$$\frac{x-6}{2} = 0$$

x – 6 = 0

x = 6

$$\frac{y-3}{2}=1$$

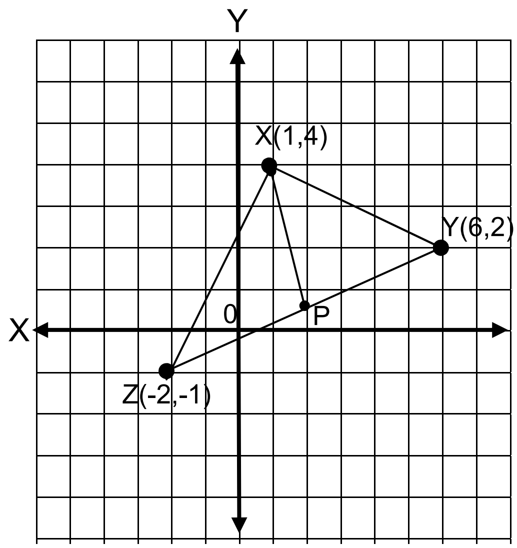
y – 3 = 2

y = 2 + 3

y = 5

Therefore the coordinates of S are (6, 5)

4. The vertices of ΔXYZ are X(1, 4), Y(6, 2) and Z(-2, -1). Find the length of the median to $\overline{ZY}$.



Solution:

The median of a triangle is a segment joining the vertex and the midpoint of the opposite side . Let the midpoint of XY be point P.

Step 1. Get the coordinates of P

$$P\left(\frac{6-2}{2},\frac{2-1}{2}\right)$$

$$P\left(\frac{4}{2},\frac{1}{2}\right)$$

$$P\left(2,\frac{1}{2}\right)$$

Step 2. Find the length of $\overline{XP}$. X(1, 4), $P\left(2,\frac{1}{2}\right)$

$$XP = \sqrt{(2-1)^2+\left(\tfrac{1}{2}-4\right)^2}$$

$$= \sqrt{1^2+\left(\tfrac{-7}{2}\right)^2}$$

$$= \sqrt{1+\frac{49}{4}}$$

$$= \sqrt{\frac{53}{4}}$$

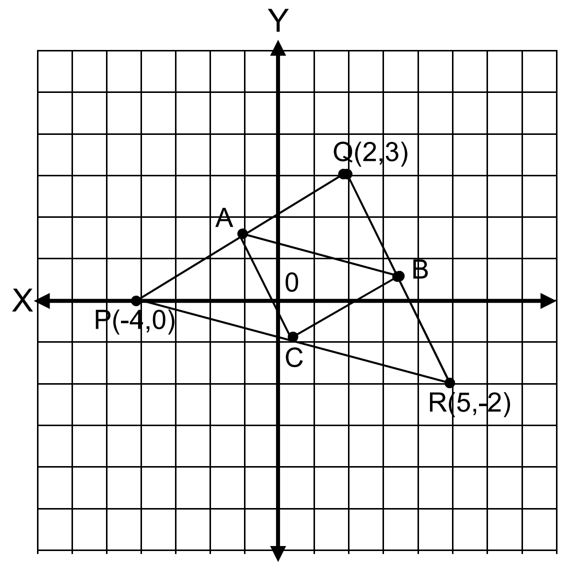
$$XP = \frac{\sqrt{53}}{2}$$

Hence, the length of median XP is $\frac{\sqrt{53}}{2}$.

5. Find the perimeter of a the triangle formed by joining the midpoints of the sides of a triangle whose vertices are P(-4, 0), Q(2, 3) and R(5, -2).

Solution:

Step 1. Let A, B and C be the midpoints of $\overline{PQ}$, $\overline{QR}$ and $\overline{PR}$. Get the coordinates of each midpoint.



For the coordinates of A

P(-4, 0), Q(2, 3)

$$A\left(\frac{-4+2}{2},\frac{3+0}{2}\right)$$

$$= A\left(\frac{-2}{2},\frac{3}{2}\right)$$

$$= A\left(-2,\frac{3}{2}\right)$$

For the coordinates of B

Q(2, 3), R(5, -2)

$$B\left(\frac{2+5}{2},\frac{3-2}{2}\right)$$

$$= B\left(\frac{7}{2},\frac{1}{2}\right)$$

For the coordinates of C

P(-4, 0), R(5, -2)

$$C\left(\frac{-4+5}{2},\frac{0-2}{2}\right)$$

$$= C\left(\frac{1}{2},-1\right)$$

Step 2. Find the lengths of $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$.

$$AB = \sqrt{\left(-1-\frac{7}{2}\right)^2 + \left(\frac{3}{2}-\frac{1}{2}\right)^2}$$

$$= \sqrt{\left(-\frac{9}{2}\right)^2 + \left(\frac{2}{2}\right)^2}$$

$$= \sqrt{\frac{81}{4} + 1^2}$$

$$= \sqrt{\frac{85}{4}}$$

$$AB = \frac{\sqrt{85}}{2}$$

$$BC = \sqrt{\left(\frac{1}{2}-\frac{7}{2}\right)^2 + \left(-1-\frac{1}{2}\right)^2}$$

$$= \sqrt{\left(-\frac{6}{2}\right)^2 + \left(-\frac{3}{2}\right)^2}$$

$$= \sqrt{\frac{36}{4} + \frac{9}{4}}$$

$$= \sqrt{\frac{45}{4}}$$

$$BC = \frac{3\sqrt{5}}{2}$$

$$AC = \sqrt{\left(-1-\frac{1}{2}\right)^2 + \left[\frac{3}{2}-(-1)\right]^2}$$

$$= \sqrt{\left(-\frac{3}{2}\right)^2 + \left(\frac{3}{2}+1\right)^2}$$

$$= \sqrt{\frac{9}{4} + \left(\frac{5}{2}\right)^2}$$

$$= \sqrt{\frac{9}{4} + \frac{25}{4}}$$

$$= \sqrt{\frac{34}{4}}$$

$$AC = \frac{\sqrt{34}}{2}$$

Step 3. Get the sum of the lengths AB + BC + AC

$$\text{Perimeter of } \Delta ABC = \frac{\sqrt{85}}{2} + \frac{3\sqrt{5}}{2} + \frac{\sqrt{34}}{2}$$

$$= \frac{\sqrt{85} + 3\sqrt{5} + \sqrt{34}}{2}$$

Try this out

A. What are the coordinates of the midpoint of the segment joining each pair of points.

1. (0, 0), (6, 0)
2. (0, 0), (-7, 0
3. (0, 0), (0, -8)
4. (1, 3), (5, 7)
5. (5, -1), (-1, -7)
6. (-8, -2), (0, 0)
7. (-3, -4), (3, -3)
8. (6, -1), (-4, 7)
9. (-1, -1), (-8, -9)
10. (a, b), (c, d)

B. If M is the midpoint of $\overline{AB}$, determine the coordinates of B.

11. A(3, 7), M(3, 0)
12. A(5, 2), M(-1, -1)
13. A(-4, -1), M(5, 2)
14. A(3, -4), M(5, 2)
15. A(-5, 6), M(7, 2)
16. A(0, -8), M(4, -4)
17. A(-1, 4), M(1, 1)
18. A(7, 0), M(0, 9)
19. A(-3, -5), M(3, -7)
20. A(a, b), M(c, d)

C. Solve the following problems:

21. Find the coordinates of the midpoint of each side of a triangle with vertices at (3, 5), (6,-4) and (-1, 1).
22. Find the coordinates of the midpoint of each side of a quadrilateral with vertices at (-2, -4), (7, -8), (4, -3) and (_5, 3).
23. Find the length of the median to side RP of ΔRPQ whose vertices are R(-3, 2), P(3, -3) and Q(-1, 6).

24. Find the length of each median of a triangle with vertices at (-1, 6), (-3, -2) and (7, -4).

25. A rectangle has vertices R(-3, 4), S(-3, -4), T(2, -4) and U(2, 4). Show that its diagonals have the same midpoint.

26. Use the distance formula to show that X(1, -1) is the midpoint of the segment with endpoints A(4, 1) and B(-2, -3).

27. Given R(5, 2), S(a, -2) and T(-3, b). Find a and b so that S is the midpoint of $\overline{RT}$.

28. Show that the points (-1, -2), (2, 1) and (-3, 6) are the vertices of a right triangle. Use the distance formula.

29. Given A(7, 1), R(2, x) and B(-x, 5), find x so that R is the midpoint of $\overline{AB}$.

30. Find the perimeter of the triangle in no. 28.

Let's summarize

1. The distance between two points on the plane is the length of the segment joining the two points. For horizontal distance between points A and B, the formula to be used is

   AB = $|x_2 - x_1|$, where the y-coordinate is the same.

   For vertical distance between points A and B, the distance is denoted by

   AB = $|y_2 - y_1|$, where the x-coordinate is the same.

2. The distance between two points A(x_1, y_1) and B(x_2, y_2) is given by the formula

   AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

3. The midpoint of a segment is a point that divides the segment into two congruent segments.

4. The midpoint of a horizontal segment can be determined by the formula

   M = $\frac{x_1 + x_2}{2}$

5. The midpoint of a vertical segment is determined by the formula

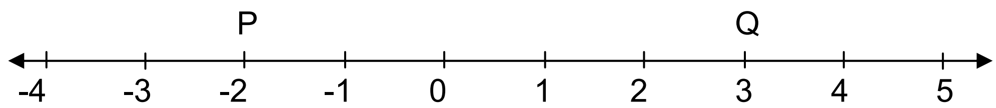
   M = $\frac{y_1 + y_2}{2}$

6. The midpoint M of a segment whose endpoints are A(x_1, y_1) and B(x_2, y_2) is given by the formula M$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

What have you learned

Answer the following questions as indicated.

1. What is the length of $\overline{PQ}$?

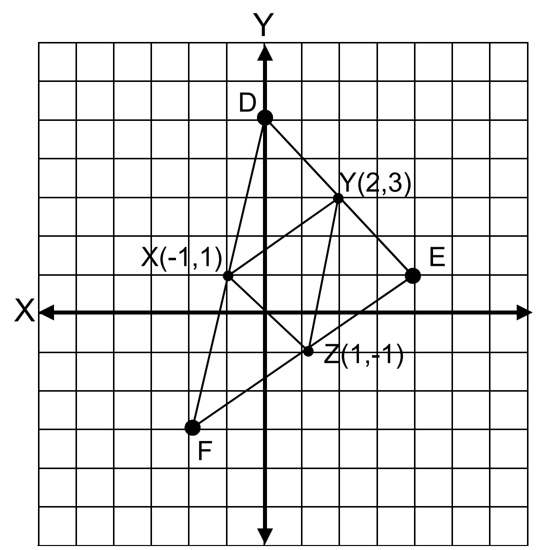


Find the length of each side of a triangle whose vertices are J(0, 3), K(-4, 0) and L(1, -1).

2. $\overline{JK}$
3. $\overline{KL}$
4. $\overline{JL}$

What are the coordinates of the midpoint of a segment whose endpoints are:

5. A(7, 3), and X(1, -11)
6. R(-6, 1) , and S(1, -10)
7. Find the perimeter of ΔXYZ in the figure if X, Y and Z are the midpoints of $\overline{FD}$, $\overline{DE}$ and $\overline{EF}$ respectively.

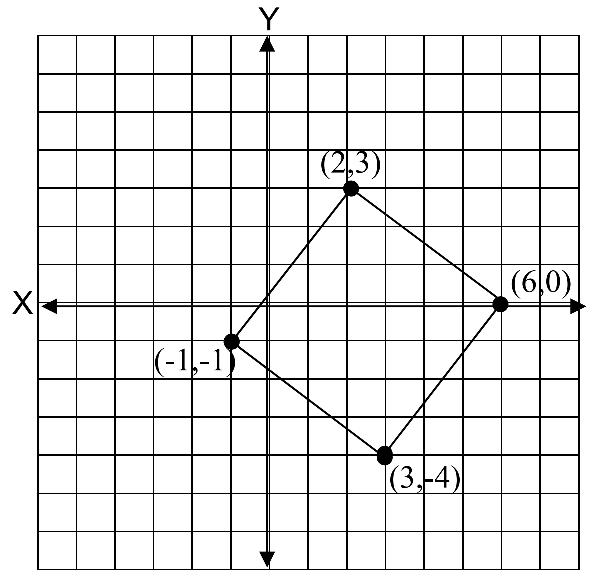


8. M is the midpoint of $\overline{PR}$. If the coordinates of M and P are given, find the coordinates of R. P(-3, 7) and M(1, 1)
9. If the length of $\overline{RS}$ is $\sqrt{29}$, and the coordinates of R are (-5, 1), find the x-coordinate of S if its y-coordinate is 3.
10. Find the coordinate of the intersection of the diagonal of a rectangle whose vertices are M(2, 7), N(6, 3), O(-1, -4) and P(-5, 0)

Answer Key

How much do you know

1. 5
2. 5
3. 10
4. 7
5. (5, 3)
6. (-4, 3)
7. $\left(\frac{3}{2}, 3\right)$
8. 5 + 2$\sqrt{2}$ + $\sqrt{61}$
9. $\left(\frac{1}{2}, 2\right)$
10. From the figure, you can easily conclude that the quadrilateral is a parallelogram. To determine what kind of parallelogram it is, compute for the slope of any two adjacent sides and compare. Their slopes are the negative reciprocal of each other.



Try this out

Lesson 1

A. 1. 2
2. 5
3. 11
4. $\sqrt{13}$
5. 4$\sqrt{10}$
6. $\sqrt{130}$
7. $\sqrt{53}$
8. $\sqrt{17}$
9. $\sqrt{85}$
10. 5$\sqrt{5}$

B. 11. 16
12. 24
13. $3\sqrt{2} + 2\sqrt{5} + \sqrt{26}$
14. $18 + 5\sqrt{10} + \sqrt{34}$

15. $4\sqrt{5}+2\sqrt{10}$

16. $4\sqrt{2}+4\sqrt{5}$

17. $2\sqrt{2}+\sqrt{13}+\sqrt{298}+\sqrt{10}+13$

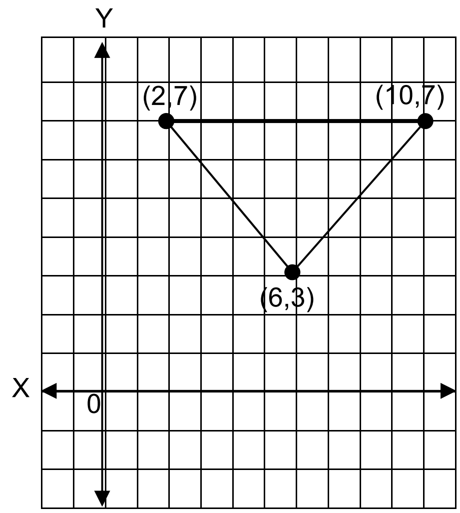
C. 18. x = -16 ; x = 8

19. y = $\pm\sqrt{33}$

20. y = -6 ; y = 0

21. x = 5 ; x = 1

D. 22. the triangle is isosceles



23. Let A(3, 0) , B(0, 4) and C(6, -4) be the given points. Find the lengths, AB, BC and AC.

Solution:

AB = $\sqrt{(0-3)^2+(4-0)^2}$

= $\sqrt{3^2+4^2}$

= $\sqrt{9+16}$

= $\sqrt{25}$

AB = 5

BC = $\sqrt{(6-0)^2+(-4-4)^2}$

= $\sqrt{6^2+(-8)^2}$

= $\sqrt{36+64}$

= $\sqrt{100}$

BC = 10

AC = $\sqrt{(6-3)^2+(-4-0)^2}$

= $\sqrt{3^2+(-4)^2}$

= $\sqrt{9+16}$

= $\sqrt{25}$

AC = 5

AB + AC = 5 + 5 = 10 = BC. By definition of betweenness, A is between B and C. Therefore, A, B and C are collinear.

24. x = 8; x = 2
25. S(2, 2)
26. y = 6; y = -2
27. x = 6; x = 0

Lesson 2

A. 1. (3, 0)
2. $\left(-\frac{7}{2},0\right)$
3. (0, -4)
4. (3, 5)
5. (2, -4)
6. (-4, -1)
7. $\left(0,-\frac{7}{2}\right)$
8. (1, 3)
9. $\left(-\frac{9}{2},-5\right)$
10. $\left(\frac{a+c}{2},\frac{b+d}{2}\right)$

11. (3, -7)
12. (-7, -4)
13. (14, 5)
14. (7, 8)
15. (19, -2)
16. (8, 0)
17. (3, -2)
18. (-7, 18)
19. (9, -9)
20. (2c – a, 2d – b)

21. $\left(\frac{9}{2},\frac{1}{2}\right)$, $\left(\frac{5}{2},-\frac{3}{2}\right),(1,3)$

22. $\left(\frac{5}{2},-6\right),\left(\frac{11}{2},-\frac{11}{2}\right),\left(-\frac{1}{2},0\right),\left(-\frac{7}{2},-\frac{1}{2}\right)$

23. Length of the median is $\frac{\sqrt{173}}{2}$

24. Length of the medians are $3\sqrt{5}$, $3\sqrt{10}$ and $\sqrt{117}$

25. Midpoint of RT = $\left(-\frac{1}{2},0\right)$

Midpoint of SU = $\left(-\frac{1}{2},0\right)$

26. AX = $\sqrt{13}$, BX = $\sqrt{13}$
Since AX = BX, therefore, X is the midpoint of AB.

27. a = 1, b = -6

28. $s_1 = \sqrt{[2-(-1)]^2+[1-(-2)]^2}$
$= \sqrt{3^2+3^2}$
$= \sqrt{9+9}$
$= \sqrt{18}$
$s_1 = 3\sqrt{2}$

$s_2 = \sqrt{(-3-2)^2+(6-1)^2}$
$= \sqrt{(-5)^2+(5)^2}$
$= \sqrt{25+25}$
$= \sqrt{50}$
$s_2 = 5\sqrt{2}$

$s_3 = \sqrt{[-1-(-3)]^2+(-2-6)^2}$
$= \sqrt{2^2+(-8)^2}$
$= \sqrt{4+64}$
$= \sqrt{68}$

$s_1^2 + s_2^2 = (3\sqrt{2})^2 + (5\sqrt{2})^2$
$= 18 + 50 = 68 = s_3^2$

The square of s_3 is equal to the sum of the squares of s_1 and s_2.

29. x = 3

30. $P = s_1 + s_2 + s_3$
$= 3\sqrt{2}+5\sqrt{2}+2\sqrt{17}$
$= 8\sqrt{2}+2\sqrt{17}$

What have you learned.

1. $|PQ| = 5$
2. 5
3. $\sqrt{26}$
4. $\sqrt{17}$
5. (4, -4)
6. $\left(-\frac{5}{2}, -\frac{9}{2}\right)$
7. $2\sqrt{2} + \sqrt{13} + \sqrt{17}$
8. R(5, -5)
9. x = 0; x = -10
10. $\left(\frac{1}{2}, \frac{3}{2}\right)$