Module 1
Plane Coordinate Geometry

What this module is about

This module will explain to you the relationship among lines on the plane. This will also tell you about intersecting line, non-intersecting lines, characteristics of parallel lines and perpendicular lines. Furthermore, this will tell you how to determine the intersection of two lines if there is any.

What you are expected to learn

This module will help you

1. Determine the point of intersection of two lines
2. Compute for the coordinates of the intersection of two lines.
3. Determine without graphing if the given lines are parallel, perpendicular or neither.
4. Define algebraically parallel and perpendicular lines.

How much do you know

Given each pair of lines, determine if they are a) intersecting but not perpendicular, b) perpendicular and c) parallel

1. $y = 3x - 7$
   $y = 3x + 1$

2. $2x + 3y = 5$
   $3x - 2y = 8$

3. $x + 2y = 9$
   $4x + 3y = 1$

4. $3x - 2y = 3$
   $3x + 5y = 14$

5. $2x - 4y = 1$
   $4x - 8y = 7$

6. The slopes of parallel lines are ___________.
7. What is the slope of the line parallel to $2x + 4y - 3 = 0$?
8. What is the slope of the line perpendicular to $x - 6y + 5 = 0$?
9. Write equation of the line parallel to $5x + 8y = 7$ and passing through $(2, 4)$.
10. At what point do the lines $3x - y = 7$ and $5x + y = 9$ intersect?

What you will do

Lesson 1

Intersection of Two Lines

Lines on the plane can either intersect or not. If two lines intersect, then there is a common point between them. That common point is where the two lines intersect. In a plane, this point has two coordinates, the x and y coordinates. The coordinates of the intersection can be solved algebraically. In second year algebra, you are taught how to solve systems of linear equations. That knowledge will help you a lot in understanding this lesson. Since every equation of the line assumes the general form $ax + by + c = 0$, to get the intersection of two lines, you solve for the value of x and y common to both equations.

To solve for the value of x and y algebraically, there are methods that can be used. The following steps can help you in solving systems of linear equations.

1. Eliminate one variable by using
   a. addition or subtraction
   b. finding the value of one variable in terms of the other variable
2. Solve for the value of the other variable
3. Substitute the computed value of one variable on either of the two equations to solve for the value of the remaining variable.

Examples:

1. Find the point of intersection of the lines whose equations are $3x - y = 10$ and $5x + y = 14$.

Solution:

To find the intersection of the two lines, solve the system by eliminating one variable through addition. Add the similar terms of the first equation to that of the second equation.

Equation 1	$3x - y = 10$
Equation 2	$\underline{5x + y = 14}$
Add	$8x + 0 = 24$
	$8x = 24$
	$x = 3$

Then replace x with 3 in either Equation 1 0r Equation 2

Equation 1 $3x – y = 10$
$3(3) – y = 10$
$9 - y = 10$
$- y = -1$
$y = 1$

Therefore the point of intersection of the two lines is (3, -1)

2. Find the point of intersection of the two lines whose equations are $x + 2y = 4$ and $3x + 5y = -21$.

Solution:

Another method of eliminating one variable is through substitution. You get the value of one variable in terms of the other variable.

Equation 1 $x + 2y = 4$
Equation 2 $3x – 5y = -21$

Using Equation 1, solve for x in terms of y

$x + 2y = 4$
$x = 4 – 2y$

Using Equation 2, replace x with $4 – 2y$

Equation 2 $3x – 5y = -21$
$3(4 – 2y) - 5y = -21$
$12 – 6y – 5y = - 21$
$- 11y = - 21 – 12$
$- 11y = - 33$
$y = 3$

Then replace y with 3 on either Equation 1 or Equation 2 to solve for the value of x.

Equation 1 $x + 2y = 4$
$x+ 2(3) = 4$
$x + 6 = 4$
$x = -2$

So the intersection of the two lines is the point whose coordinates are (-2, 3).

3. At what point do the lines $3x + 2y = 12$ and $4x – 3y = -1$ intersect?

Solution.

To eliminate a variable in this example, first find equivalent equations with equal but opposite coefficient in x or in y by multiplying one or both equations by a number factor.

Equation 1 $3x + 2y = 12$
Equation 2 $4x - 3y = -1$

$3(3x + 2y) = 12$ ⇨ $9x + 6y = 36$

$2(4x - 3y) = -1$ ⇨ $\underline{8x - 6y = -2}$
Add Equation 1 and 2 $17x + 0 = 34$
$7x = 34$
$x = 2$

Then replace x with 2 in either Equation 1 or 2

Equation 1 $3x + 2y = 12$
$3(2) + 2y = 12$
$6 + 2y = 12$
$2y = 6$
$y = 3$

Therefore, the point of intersection is (2, 3).

Example 4. Find the point of intersection of the lines $y = 2x - 2$ and $y - 2x = 1$.

Solution: Using any of the previous methods, solve the system of equations.

Equation 1 $y = 2x - 2$

Equation 2 $y - 2x = 1$

Using Equation 1, solve for y in terms of x

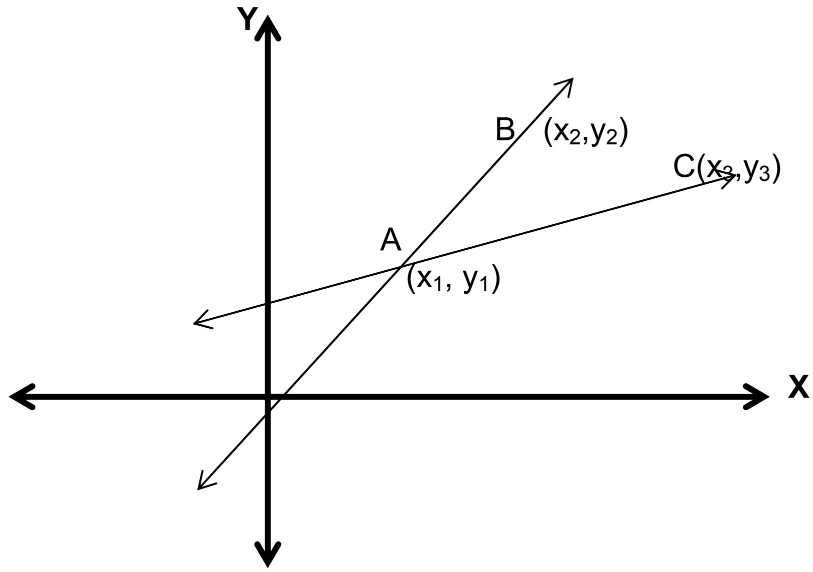
$$y = 2x - 2$$

Replace y by 2x - 2 in the second equation.
$(2x - 2) - 2x = 1$
$2x - 2 - 2x = 1$
$-2 = 1$, this is a false statement.

Solving the system led to a false statement. Therefore the system is inconsistent and has no solution . So the lines do not intersect. There is a way of determining whether the two lines intersect or not. The following theorem can be used to determine if two lines intersect or not.

Theorem: If two non-vertical lines intersect, then their slopes are not equal.

This theorem can be verified as follows.



Let A, B and C be three distinct points on a plane. By line determination postulate, line AB and line AC can be constructed with A as a common point or point of intersection.

Get the slopes of the two lines.

For line AB, the slope

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

For line AC, the slope

$$m_2 = \frac{y_3 - y_1}{x_3 - x_1}$$

Since the numerators and denominators in the two fractions are different, then the two slopes are not equal. Thus

$$m_1 \neq m_2$$

The converse of the theorem, " If two non-vertical lines have different slopes, then the lines are intersecting," is also true.

5. Without graphing, show that 5x – y = 7 and 3x + 2y = 4 are intersecting lines.

Solution: Transform each equation to slope-intercept form.

Equation 1 5x – y = 7
- y = -5x + 7
y = 5x - 7

slope (m_1) = 5

Equation 2 $3x + 2y = 4$

$2y = -3x + 4$

$y = -\frac{3}{2}x + 2$

slope (m_2) = $-\frac{3}{2}$

Since $m_1 \neq m_2$, then the two lines are intersecting.

Try this out

A. Without graphing, show that the following pairs of lines are intersecting.

1. $y = 3x – 4$
   $y = x + 7$

2. $y = \frac{1}{2}x - 4$
   $y = \frac{3}{2}x + 6$

3. $x + 4y = 3$
   $2x + y = 7$

4. $2x – y = 8$
   $2x + y = 7$

5. $3x – 5y = 4$
   $x + y = 4$

B. Find the intersection of the following pairs of lines.

1. $3x – 4y = 1$
   $3x + y = -4$

2. $x + 5y = 13$
   $2x – y = -7$

3. $x – 6y = 11$
   $3x + 3y = 12$

4. $2x – 5y = -7$
   $3x + 2y = 18$

5. $3x – 7y = -15$
   $2x + 6y = -10$

Lesson 2

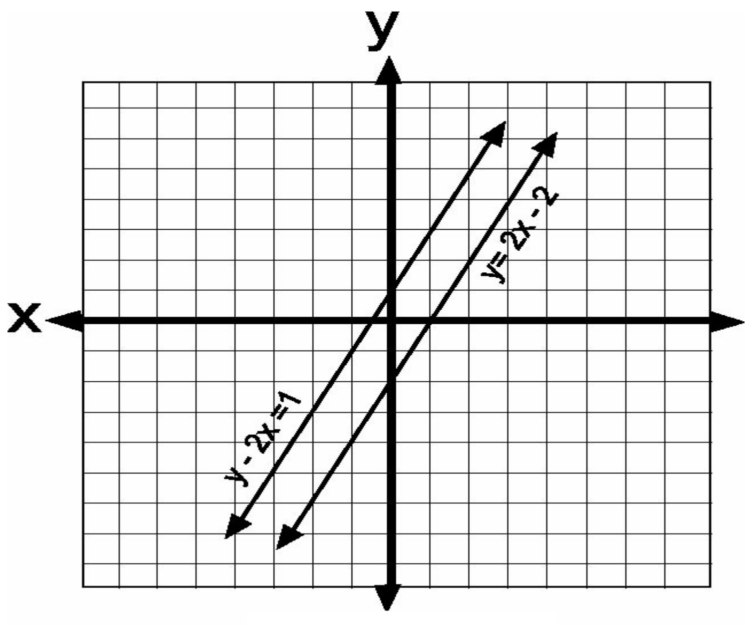
Parallel and Perpendicular Lines

In the previous lesson, there was an example in which the solution of the system of equations led to a false statement. This showed that the two lines do not intersect. In a plane, if two lines do not intersect, then they are parallel.

Let us consider the given in example 4.

Equation 1 $y = 2x - 2$
Equation 2 $y - 2x = 1$

If we graph the two lines on the same set of axes, they will look like this.



The graphs showed that the two lines are parallel. Without graphing, you can also determine if the lines are parallel or if they intersect. The following theorem can be used to prove that two lines are parallel.

Theorem: If two non vertical lines are parallel, then the slopes are equal. The converse of this theorem is also true.

Converse theorem: If two lines which are not coincident have the same slope, then the lines are parallel.

Consider the given lines above. To prove that they are parallel, their slopes must be equal, transform the equations into y-form or slope-intercept form.

Equation 1 $y = 2x - 2$

The equation is already in y - form.

slope(m_1) = -2

Equation 2 $y – 2x = 1$

Again, transform to slop-intercept form

$y = -2x + 1$

slope (m_2) = -2

You can see that $m_1 = m_2$. Therefore, the two lines are parallel.

The slope of horizontal line is zero (0), thus all horizontal lines are parallel to each other. Slope of vertical line is undefined and so all vertical lines are also parallel to each other.

In a plane, some lines are parallel, and others are intersecting. Intersecting lines may be perpendicular or not. The next theorem will help you determine if the pair of lines is perpendicular or not.

Theorem: If two non vertical lines are perpendicular, then their slopes are negative reciprocals.

Examples:

1. Show that the lines whose equations are $3x + y = 4$ and $x – 3y = 7$ are perpendicular.

Solution:

Equation 1 $3x + y = 4$

Transform to slope-intercept form

$y = -3x + 4$

slope (m_1) = -3

Equation 2 $x – 3y = 7$

Transform to slope-intercept form

$- 3y = -x + 7$

$$y = \frac{-1}{-3}x + \frac{7}{-3}$$

$$y = \frac{1}{3}x - \frac{7}{3}$$

slope (m_2) = $\frac{1}{3}$

Comparing their slopes, it is obvious that they are the negative reciprocal of each other. Hence, we can conclude that the two lines are perpendicular.

2. Determine which pairs of lines are parallel, perpendicular or just intersecting.

a. Line 1	5x – 2y = 7
b. Line 2	2x + 5y = 4
c. Line 3	4x + 10y = 6
d. Line 4	3x + y = 5
e. Line 5	x – 3y = 6
f. Line 6	2x + 6y =1

Solution:

Transform each equation to slope-intercept form

a. 5x – 2y = 7

$$-2y = -5x + 7$$

$$y = \frac{-5}{-2}x + \frac{7}{-2}$$

$$y = \frac{5}{2}x - \frac{7}{2}$$

$m_1 = \frac{5}{2}$

b. 2x + 5y = 4

$$5y = -2x + 4$$

$$y = \frac{-2}{5}x + \frac{4}{5}$$

$m_2 = -\frac{2}{5}$

c. 4x + 10 y = 6

$$10y = -4x + 6$$

$y = \frac{-4}{10}x + \frac{6}{10}$

$y = -\frac{2}{5}x + \frac{3}{5}$

$m_3 = -\frac{2}{5}$

d. 3x + y = 5

$y = -3x + 5$

$m_4 = -3$

e. x – 3y = 6

$-3y = -x + 6$

$y = \frac{-x}{-3} + \frac{6}{-3}$

$y = \frac{1}{3}x - 2$

$m_5 = \frac{1}{3}$

f. 2x + 6y = 1

$6y = -2x + 1$

$y = \frac{-2}{6}x + \frac{1}{6}$

$y = -\frac{1}{3}x + \frac{1}{6}$

$m_6 = -\frac{1}{3}$

By comparing the computed slopes, you can determine the relationship of a line with the other lines.

Since the slopes of line 1 and line 2 are negative reciprocals, then line 1 is perpendicular to line 2. Line 1 and line 3 are also perpendicular to each other since their slopes are negative reciprocals. The slopes of line 2 and line 3 are equal so line 2 is parallel to line 3. The slopes of line 4 and line 5 are also negative reciprocals so these lines are also perpendicular. On the other hand, other pairs of lines like line 4 and line 6, line 5 and line 6 are just intersecting line since their slopes are not equal. The summary of the relationship between each pair of lines is given below.

Pair of Lines	Relationship	Pair of Lines	Relationship
Line 1 and line 2	Perpendicular	Line 2 and line 6	Just intersecting
Line 1 and line 3	Perpendicular	Line 3 and line 4	Just intersecting
Line 1 and line 4	Just intersecting	Line 3 and line 5	Just intersecting
Line 1 and line 5	Just intersecting	Line 3 and line 6	Just intersecting
Line 1 and line 6	Just intersecting	Line 4 and line 5	Perpendicular
Line 2 and line 3	Parallel	Line 4 and line 6	Just intersecting
Line 2 and line 4	Just intersecting	Line 5 and line 6	Just intersecting
Line 2 and line 5	Just intersecting		

3. Prove that the quadrilateral whose vertices are given is a parallelogram.
A(-3, 3), B(-2, -2), C(8, 3), D(7, 8)

Solution:

Parallelogram is defined as quadrilateral whose opposite sides are parallel. To prove that ABCD is a parallelogram, show that opposite sides have equal slopes.

Slope of AB:

$$m_{AB} = \frac{-2-3}{-2-(-3)}$$

$$= \frac{-5}{1}$$

$$m_{AB} = -5$$

Slope of CD:

$$m_{CD} = \frac{8-3}{7-8}$$

$$= \frac{5}{-1}$$

$$m_{CD} = -5$$

Since the slopes of side AB and side CD are equal, then $AB \parallel CD$.

Slope of BC:

$$m_{BC} = \frac{3-(-2)}{8-(-2)}$$

$$= \frac{3+2}{8+2}$$

$$= \frac{5}{10}$$

$$m_{BC} = \frac{1}{2}$$

Slope of AD:

$$m_{AD} = \frac{8-3}{7-(-3)}$$

$$= \frac{5}{10}$$

$$m_{AD} = \frac{1}{2}$$

Since the slope of side BC is equal to the slope of side AD then $BC \parallel AD$.

Therefore we can conclude that ABCD is a parallelogram since the opposite sides are parallel.

4. Prove that the following points are vertices of a right triangle. A(-3, 5), B(6, 1) and C(-7, -4).

Solution:

Since one angle of a right triangle is right, then two sides must be perpendicular and you have to find the slopes of the three sides to prove this.

Slope of AB:

$$m_{AB} = \frac{1-5}{6-(-3)}$$

$$m_{AB} = \frac{-4}{9}$$

Slope of BC:

$$m_{BC} = \frac{-4-1}{-7-6}$$

$$= \frac{-5}{-13}$$

$$m_{BC} = \frac{5}{13}$$

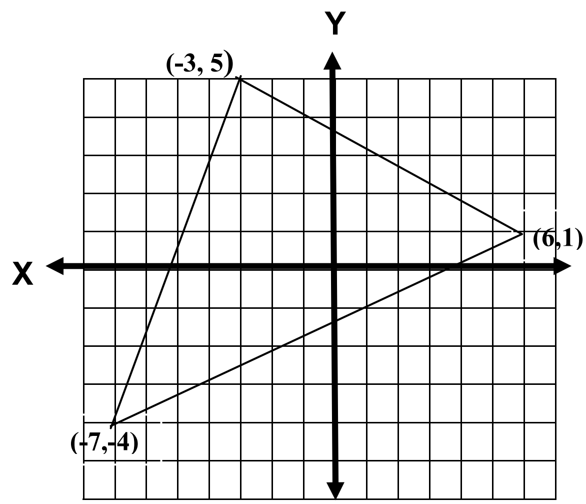
Slope of AC:

$$m_{AC} = \frac{-4-5}{-7-(-3)}$$

$$= \frac{-9}{-4}$$

$$m_{AC} = \frac{9}{4}$$

From the computed slopes of the three sides AB, BC and AC, it is clear that AB is perpendicular to AC since their slopes are negative reciprocals. Hence we can conclude that $\angle BAC$ is a right angle. It only proved that ΔABC is a right triangle. If we sketch the graph on the Cartesian plane, it will look like this.



5. Write an equation in standard form of the line passing through

   a. point (1, -2) and parallel to y = 3x + 2
   b. point (3, 4) and perpendicular to y = 2x – 7.

Solution:

a. Since parallel lines have equal slopes, then the slope of the required line is 3. Using the point slope form, determine the equation of the line.

$$y - y_1 = m(x - x_1)$$
$$y - (-2) = 3(x - 1)$$
$$y + 2 = 3x - 3$$
$$-3x + y + 2 + 3 = 0$$
$$-3x + y + 5 = 0 \quad \text{or}$$
$$3x - y - 5 = 0, \text{ this is the required equation of the line}$$

b. Perpendicular lines have slopes which are negative reciprocal of one another.

Since the slope of the given line is 2, then the slope of the required line is $-\frac{1}{2}$. Using the point-slope form of the line, determine the equation of the required line.

$$y - y_1 = m(x - x_1)$$
$$y - 4 = -\frac{1}{2}(x - 3)$$

$$2(y - 4) = -1(x - 3)$$
$$2y - 8 = -x + 3$$
$$x + 2y - 8 - 3 = 0$$
$x + 2y - 11 = 0$, the required equation of the line.

6. A right triangle has its right angle at (-4, 1) and the equation of one of its legs is $2x - 3y + 11 = 0$. Find the equation of the other leg.

Solution:

The two legs of a right triangle are perpendicular at the vertex of the right angle. Their slopes are the negative reciprocal of each other. Since the equation of one of its legs is $2x - 3y + 11 = 0$, then we have to compute for the slope first.

Get the slope of the given leg.

$$2x - 3y + 11 = 0$$

Transform the equation to slope intercept form.

$$-3y = -2x - 11$$
$$y = \frac{-2}{-3}x + \frac{-11}{-3}$$
$$y = \frac{2}{3}x + \frac{11}{3}$$

Since the slope of the given line is $\frac{2}{3}$, then the slope of the required line is $-\frac{3}{2}$.

Using the point - slope form, determine the equation of the line:

$$y - y_1 = m(x - x_1)$$
$$y - 1 = \frac{-3}{2}\left[x - (-4)\right]$$
$$2(y - 1) = -3(x + 4)$$
$$2y - 2 = -3x - 12$$
$$3x + 2y - 2 + 12 = 0$$

The equation of the other leg is $3x + 2y + 10 = 0$

Try this out

A. Determine if the pair of lines are parallel, perpendicular or neither.

1. $y = 2x + 5$

   $y = -\frac{1}{2}x - 7$

2. $y = 4x - 1$
   $y = 4x + 3$

3. $y = \frac{5}{2}x + \frac{3}{2}$
   $y = -\frac{2}{5}x + 3$

4. $y = 3x - 7$
   $y = x + 8$

5. $2y = x + 4$
   $y = 2x - 5$

6. $x + y = 5$
   $x - y = 3$

7. $2x + y = 9$
   $x - 2y = -4$

8. $2x + y = 8$
   $4x + 2y = -3$

9. $3x + 2y - 1 = 0$
   $2x - 3y - 7 = 0$

10. $2x + 7y + 6 = 0$
    $2x + 7y - 11 = 0$

B. Prove that the following points are vertices of a parallelogram or not. Justify.

1. A(3, 7), B(4, 3), C(-2, 1), D(-3, 5)
2. L(4, 2), O(8, 3), V(6, -5), E(2, -6)
3. P(-3, -2), L(1, -1), A(1, -8), N(-3, -6)

C. Verify if the following are vertices of a right triangle. Explain your answer.

1. E(1, 6), A(6, 2), r(1, 2)
2. A(1, 4), C(-3, -1), E(7, -5)
3. P(3, -1), E(6, -3), N(3, -7)

D. Find the equation of the line in standard form given the following conditions.

1. Passing through (-1, -1) and parallel to $y = 4x + 1$.
2. Passing through (0, 4) and parallel to $y = -3x - 2$.
3. Passing through (3, 0) and parallel to $x - 3y = 7$.

4. Passing through (1, -5) and parallel to 2x + 5y – 4 = 0.
5. Passing through (2, 1) and perpendicular to x – y = 3.
6. Passing through (3, -4) and perpendicular to y = 3x + 1.
7. Passing through (-2, -4) and perpendicular to 5x + 2y = 4.

E. Solve the following problems.

1. The line containing points (-6, k) and (4, 3) is perpendicular to the line containing points (9, 1) and (13, k+1). Find k.
2. A right triangle has its angle at (5, 7) and the equation of one of its legs is 2x – y – 3 = 0. Find the equation of the other leg.
3. Write the equations of three lines parallel to 2x + y = 4.
4. Write the equations of three lines perpendicular to 3x + y – 4 = 0
5. Show that the lines 6x + 7y – 6 = 0 and 7x – 6y + 9 = 0 are the legs of a right triangle.

Let's summarize

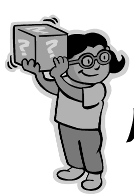
1. If two lines intersect, there is a common point between them which is the intersection of the two lines.

2. The coordinates of the point of intersection of two lines can be determined algebraically by using specific method according to the given problem.

3. If two non vertical lines intersect, their slopes are not equal.

4. If two non vertical lines are parallel, then their slopes are equal.

5. If two non vertical lines are perpendicular, then their slopes are negative reciprocals.



What have you learned

Nos. 1. – 5. Determine if the following pairs of lines are parallel, perpendicular or neither.

1. y = 2x + 3
   y = 2x – 8

2. y = x – 5
   y = -x + 5

3. y = 3x + 7

$y = 2x - 1$

4. $2x + y = 4$
   $x - 2y = 5$

5. $3x + 7y - 6 = 0$
   $3x + 7y + 1 = 0$

6. What is the slope of the line parallel to $5x + 4y - 3 = 0$?

7. Find the slope of the line perpendicular to $4x - 2y - 7 = 0$.

8. Find the equation of the line passing through (1, 3) and parallel to $y = 4x + 7$.

9. Find the equation of the line perpendicular to $3x + y - 7 = 0$ and passing through (4, 3).

10. At what point do lines $3x - 4y = 8$ and $3x + y = -2$ intersect?

Answer Key

How much do you know

1. parallel
2. perpendicular
3. intersecting but not perpendicular
4. intersecting but not perpendicular
5. parallel
6. equal
7. $-\frac{1}{2}$
8. -6
9. 5x + y – 14 = 0
10. (2, -1)

Lesson 1

1. Equation 1 $y = 3x - 4$
$m_1 = 3$

Equation 2 $y = x + 7$
$m_2 = 1$

Therefore $m_1 \neq m_2$, so the lines are intersecting.

2. Equation 1 $y = \frac{1}{2}x - 4$
$m_1 = \frac{1}{2}$

Equation 2 $y = \frac{3}{2}x + 6$
$m_2 = \frac{3}{2}$

Since $m_1 \neq m_2$, so the lines are intersecting.

3. Equation 1 $x + 4y = 3$
$4y = -x + 3$
$y = -\frac{1}{4}x + \frac{3}{4}$
$m_1 = -\frac{1}{4}$

Equation 2 $2x + y = 7$
$y = -2x + 7$
$m_2 = -2$

Since $m_1 \neq m_2$, so the lines are intersecting.

4. Equation 1 $2x - y = 8$

$-y = -2x + 8$

$y = 2x - 8$

$m_1 = 2$

Equation 2 $x + 3y = 1$

$3y = -x + 1$

$y = -\frac{1}{3}x + \frac{1}{3}$

$m_2 = -\frac{1}{3}$

Since $m_1 \neq m_2$, so the lines are intersecting.

5. Equation 1 $3x - 5y = 4$

$-5y = -3x + 4$

$y = \frac{-3}{-5}x + \frac{4}{-5}$

$y = \frac{3}{5}x - \frac{4}{5}$

$m_1 = \frac{3}{5}$

Equation 2 $x + y = 4$

$y = -x + 4$

$m_2 = -1$

Since $m_1 \neq m_2$, so the lines are intersecting.

B.
1. (-1, -1)
2. (-2, 3)
3. (5, -1)
4. (4, 3)
5. (-5, 0)

Lesson 2

A.

1. Equation 1 $y = 2x + 5$

$m_1 = 2$

Equation 2 $y = -\frac{1}{2}x - 7$

$m_2 = -\frac{1}{2}$

The two slopes are the negative reciprocals of each other, hence the two lines are perpendicular.

2. Equation 1 $y = 4x - 1$
$m_1 = 4$

Equation 2 $y = 4x + 3$
$m_2 = 4$

The two slopes are equal, therefore the two lines are parallel.

3. Equation 1 $y = \frac{5}{2}x + \frac{3}{2}$
$m_1 = \frac{5}{2}$

Equation 2 $y = -\frac{2}{5}x + 3$
$m_2 = -\frac{2}{5}$

Since the two slopes are negative reciprocals, then the two lines are perpendicular.

4. Equation 1 $y = 3x - 7$
$m_1 = 3$

Equation 2 $y = x + 8$
$m_2 = 1$

Since $m_1 \neq m_2$, and are also not the negative reciprocals, then the lines are neither parallel nor perpendicular.

5. Equation 1 $2y = x + 4$
$y = \frac{x}{2} + \frac{4}{2}$
$y = \frac{1}{2}x + 2$
$m_1 = \frac{1}{2}$

Equation 2 $y = 2x - 5$
$m_2 = 2$

Since $m_1 \neq m_2$, and are not negative reciprocals, then the lines are neither parallel nor perpendicular.

6. Equation 1 $x + y = 5$

$y = -x + 5$

$m_1 = -1$

Equation 2 $x - y = 3$

$-y = -x + 3$

$y = x - 3$

$m_2 = 1$

Since the negative reciprocal of 1 is -1, then the two lines are perpendicular.

7. Equation 1 $2x + y = 9$

$y = -2x + 9$

$m_1 = -2$

Equation 2 $x - 2y = -4$

$-2y = -x - 4$

$y = \frac{-x}{-2} + \frac{-4}{-2}$

$y = \frac{1}{2}x + 2$

$m_2 = \frac{1}{2}$

Since m_1 and m_2 , are negative reciprocals, then the lines are perpendicular.

8. Equation 1 $2x + y = 8$

$y = -2x + 8$

$m_1 = -2$

Equation 2 $4x + 2y = -3$

$2y = -4x - 3$

$y = \frac{-4}{2}x - \frac{3}{2}$

$m_2 = -2$

Since $m_1 = m_2$, then the two lines are parallel.

9. Equation 1 $3x + 2y - 1 = 0$

$2y = -3x + 1$

$y = \frac{-3}{2}x + \frac{1}{2}$

$m_1 = -\frac{3}{2}$

Equation 2 $2x - 3y + 7 = 0$

$$-3y = -2x - 7$$
$$y = \frac{-2}{-3}x + \frac{-7}{-3}$$
$$y = \frac{2}{3}x + \frac{7}{3}$$
$$m_2 = \frac{2}{3}$$

Since m_1 and m_2 , are negative reciprocals, then the lines are perpendicular.

10. Equation 1 $\quad 2x + 7y + 6 = 0$

$$7y = -2x - 6$$
$$y = \frac{-2}{7}x - \frac{6}{7}$$
$$m_1 = -\frac{2}{7}$$

Equation 2 $\quad 2x + 7y - 11 = 0$

$$7y = -2x + 11$$
$$y = \frac{-2}{7}x + \frac{11}{7}$$
$$y = -\frac{2}{7}x + \frac{11}{7}$$
$$m_2 = -\frac{2}{7}$$

Since $m_1 = m_2$, then the two lines are parallel.

B. To prove that the given quadrilateral is a parallelogram, show that the slopes of opposite sides are equal, otherwise, the quadrilateral is not a parallelogram.

Step 1. Get the slopes of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{AD}$

$$m_{AB} = \frac{3-7}{4-3} = \frac{-4}{1}$$
$$= -4$$

$$m_{BC} = \frac{1-3}{-2-4}$$
$$= \frac{-2}{-6}$$
$$= \frac{1}{3}$$

$$m_{CD} = \frac{5-1}{-3-(-2)}$$

$$= \frac{4}{-3+2}$$

$$= \frac{4}{-1} = -4$$

$$m_{AD} = \frac{5-7}{-3-3}$$

$$= \frac{-2}{-6}$$

$$= \frac{1}{3}$$

In this quadrilateral, the opposite sides are AB and CD. Likewise, BC and AD are also opposite sides. Note that the computed slopes of the opposite sides are equal, thus the opposite sides are parallel. Therefore by definition, ABCD is a parallelogram.

2. Compute for the slopes of the opposite sides.

$$m_{LO} = \frac{3-2}{8-4}$$

$$= \frac{1}{4}$$

$$m_{OV} = \frac{-5-3}{6-8}$$

$$= \frac{-8}{-2}$$

$$= 4$$

$$m_{VE} = \frac{-6-(-5)}{2-6}$$

$$= \frac{-6+5}{2-6}$$

$$= \frac{1}{4}$$

$$m_{LE} = \frac{-6-2}{2-4}$$

$$= \frac{-8}{-2}$$

$$= 4$$

Since the two pairs of opposite sides of LOVE are parallel, then LOVE is a parallelogram.

3. Compute for the slopes of the sides of quadrilateral PLAN.

$$m_{PL} = \frac{1-(-2)}{1-(-3)}$$
$$= \frac{-1+2}{1+3}$$
$$= \frac{1}{4}$$

$$m_{LA} = \frac{-8-(-1)}{1-1}$$
$$= \frac{-7}{0} \text{ , undefined}$$

$$m_{AN} = \frac{-6-(-8)}{-3-1}$$
$$= \frac{-6+8}{-4}$$
$$= \frac{2}{-4}$$
$$= -\frac{1}{2}$$

$$m_{PN} = \frac{-6-(-2)}{-3-(-3)}$$
$$= \frac{-6+2}{-3+3}$$
$$= \frac{-4}{0} \text{ , undefined}$$

From the computed values, LA and PN have equal slopes, hence they are parallel. On the other hand, the slopes of PL and AN are not equal, so they are not parallel. Therefore, PLAN is not a parallelogram.

C. To verify if the vertices were that of a right triangle, show that the slopes of a pair of adjacent sides are negative reciprocals.

1. Compute for the slopes of the sides EA, AR and ER.

$$m_{EA} = \frac{2-6}{6-1}$$
$$= \frac{-4}{5}$$

$$m_{AR} = \frac{2-2}{-5}$$
$$= \frac{0}{-5}$$
$$= 0$$

$$m_{ER} = \frac{2-6}{1-1}$$
$$= \frac{-4}{0}, \text{ undefined}$$

From the computed values, it is clear that the slopes of AR and ER are negative reciprocals, hence the two sides are perpendicular. Therefore, ∆EAR is a right triangle.

2. Compute for the slope of each side of the triangle.

$$m_{AC} = \frac{-1-4}{-3-(-1)}$$
$$= \frac{-5}{-3+1}$$
$$= \frac{-5}{-2}$$
$$= \frac{5}{2}$$

$$m_{CE} = \frac{-1-(-5)}{-3-7}$$
$$= \frac{-1+5}{-10}$$
$$= -\frac{4}{10}$$
$$= -\frac{2}{5}$$

$$m_{AE} = \frac{4-(-5)}{-1-7}$$

$$= \frac{4+5}{-8}$$

$$= -\frac{9}{8}$$

Since the slopes of two sides, AC and CE are negative reciprocals, then AC and CE are perpendicular. Hence ∆ACE is a right triangle because two of the sides form a right angle in between them.

3. Find the slopes of PE, EP and PN.

$$m_{PE} = \frac{-3-(-1)}{6-3}$$

$$= \frac{-3+1}{3}$$

$$= -\frac{2}{3}$$

$$m_{EN} = \frac{-3-(-7)}{6-3}$$

$$= \frac{-3+7}{3}$$

$$= \frac{4}{3}$$

$$m_{PN} = \frac{-1-(-7)}{3-3}$$

$$= \frac{-1+7}{0}$$

$$= \frac{6}{0}, \text{ undefined}$$

Based on the computation, no two slopes are negative reciprocals, hence no sides are perpendicular. Therefore, ∆PEN is not a right triangle.

D.

1. 4x – y + 7 = 0
2. 3x + y – 4 = 0
3. x – 3y – 3 = 0
4. 2x + 5y + 8 = 0
5. x + y – 3 = 0
6. x + 3y + 15 = 0
7. 2x – 5y – 16 = 0

E.

1. k = 8

2. Equation 1 $2x - y - 3 = 0$

$$-y = -2x + 3$$
$$y = 2x - 3$$
$$m_1 = 2$$
$$m_2 = -\frac{1}{2}$$

Using the point slope form

$$y - y_1 = m(x - x_1)$$
$$y - 7 = -\frac{1}{2}(x - 5)$$
$$2(y - 7) = -1(x - 5)$$
$$2y - 14 = -x + 5$$
$$x + 2y - 19 = 0$$, the required equation

3. Some of the possible equations of the line parallel to $2x + y = 4$ can be of this form.
$2x + y = c$, where c is any real number.

4. All equation of the line whose slope is $\frac{1}{3}$.

5. Equation 1 $6x + 7y - 6 = 0$

Rewrite to slope-intercept form

$$7y = -6x + 6$$
$$y = \frac{-6}{7}x + \frac{6}{7}$$
$$m_1 = -\frac{6}{7}$$

Equation 2 $7x - 6y + 9 = 0$

$$-6y = -7x - 9$$
$$6y = 7x + 9$$
$$y = \frac{7}{6}x + \frac{9}{6}$$
$$y = \frac{7}{6}x + \frac{3}{2}$$
$$m_2 = \frac{7}{6}$$

Their slopes are negative reciprocals.

What have you learned

1. Parallel
2. Perpendicular
3. Neither parallel nor perpendicular
4. Perpendicular
5. Parallel
6. $-\frac{5}{4}$
7. $-\frac{1}{2}$
8. $4x - y - 1 = 0$
9. $x - 3y + 5 = 0$
10. (0, -2)