Module 3
Similarity

What this module is about

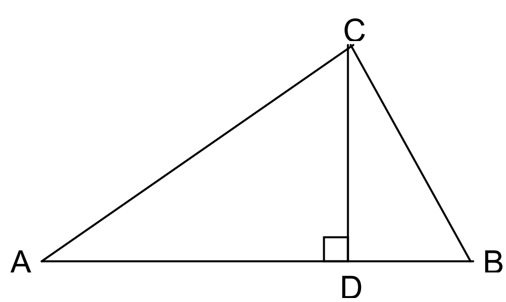
This module is about similarities on right triangles. As you go over the exercises you will develop skills in applying similarity on right triangles and solve for the missing lengths of sides using the famous Pythagorean theorem.

What you are expected to learn

1. Apply AA similarity on Right triangles
2. In a right triangle, the altitude to the hypotenuse separates the triangle into two triangles each similar to the given triangle and similar to each other.
3. On a right triangle, the altitude to the hypotenuse is the geometric mean of the segments in which it divides, each leg is the geometric mean of the hypotenuse and the segment of the hypotenuse adjacent to it.
4. Pythagorean Theorem and its application to special right triangles

How much do you know

Use the figure to answer each of the following:



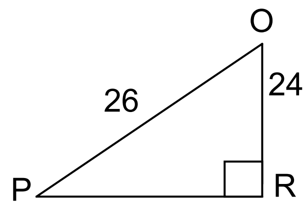
1. What is the hypotenuse of rt. $\triangle ABC$?

2. If $\angle C$ is the right angle of ΔABC and $\overline{CD} \perp \overline{AB}$ then $\Delta ABC \sim \Delta BDC \sim$ ___.

3. Complete the proportion: $\frac{AD}{CD} = \frac{?}{BD}$

4. In rt. ΔPRO, $\angle R$ is a right angle $\overline{OR} = 24$ and $\overline{PO} = 26$,

   find $\overline{PR}$:



5. In a 30°-60°-90° triangle the length of the hypotenuse is 14. Find the length of the longer leg.

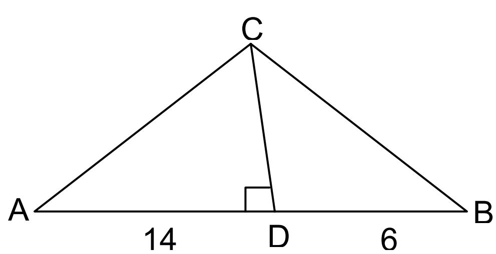
6. In a 30°-60°-90° triangle, the length of the hypotenuse is $11\frac{1}{2}$. Find the length of the shorter leg.

7. In a 45° - 45° - 90° triangle, the length of the hypotenuse is 16. Find the length of a leg.

8. Find the length of the altitude of an equilateral triangle if the length of a side is 6.

9. Find the length of the diagonal of a square if the length o a side is 10 cm.

10. ΔBAC is a right triangle $\angle C$ is right angle $\overline{CD} \perp \overline{AB}$. Find $\overline{CD}$ if $\overline{AD} = 14$, $\overline{DB} = 6$



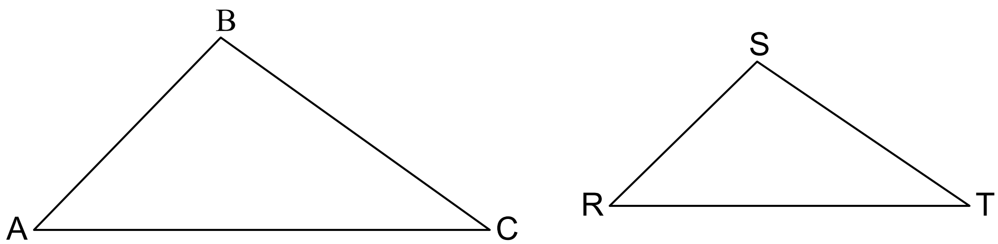
What will you do

Lesson 1

Similarity on Right Triangle

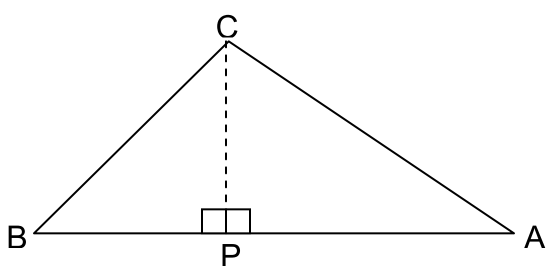
Let us recall the AA Similarity Theorem.

Given a correspondence between the vertices of two triangles. If two pairs of corresponding angles are congruent, then the triangles are similar.



From the theorem, if ABC $\leftrightarrow$ RST and $\angle A = \angle R$, $\angle B = \angle S$ then $\Delta ABC \sim \Delta RST$. We can apply this theorem to prove another theorem, this time in a right triangle.

Theorem: In a right triangle, the altitude to the hypotenuse separates the triangle into to two triangles each similar to the given triangle and similar to each other.



Given: Right ΔABC with altitude $\overline{CP}$

Prove: $\Delta ACP \sim \Delta CBP \sim \Delta ABC$

To prove this theorem, we apply the AA Similarity Theorem

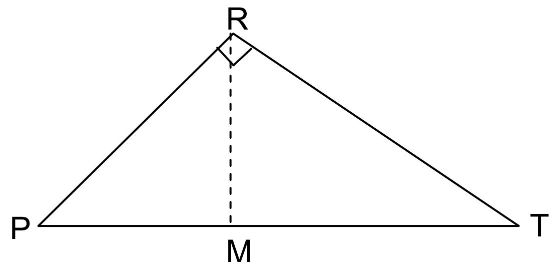
Examples:

If you are given ΔPRT a right triangle and RM an altitude to the hypotenuse then we can have three pairs of similar triangles.

ΔRMP ~ ΔTRP

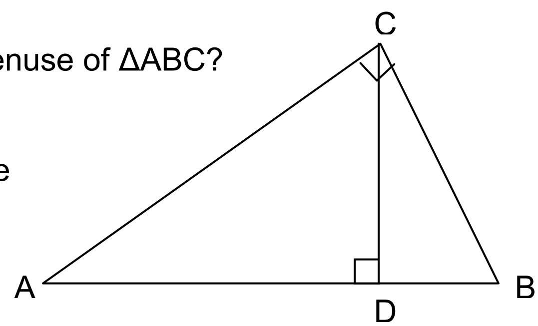
ΔTMR ~ ΔTRP

ΔRMP ~ ΔTMR



Try this out

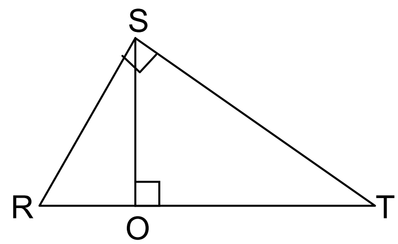
A. Use the figure to answer each of the following:



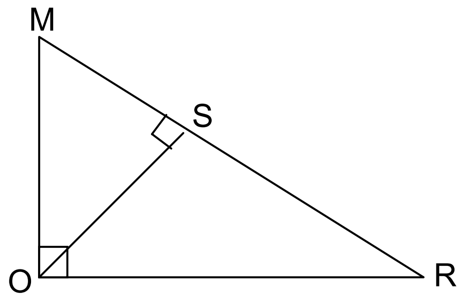
1. Name the right triangle of ΔABC
2. What is the altitude to the hypotenuse of ΔABC?
3. Name the hypotenuse of ΔABC
4. Two segments of the hypotenuse Are $\overline{AD}$ and ____.
5. The hypotenuse of ΔBCD if $\overline{CD} \perp \overline{AB}$ is ____.
6. Name the right angle of ΔACD
7. Name the hypotenuse of right ΔBCD
8. ΔADC ~ ______
9. ΔABC ~ ______
10. ΔABC ~ ______

B. Name the pairs of right triangles that are similar.

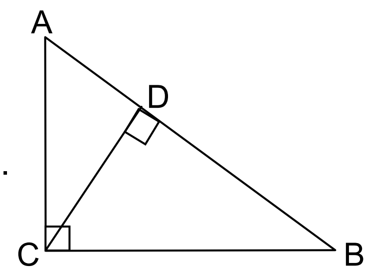
1.
2.
3.



4.
5.
6.



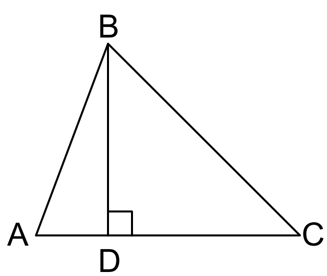
C. Use the figure at the right.



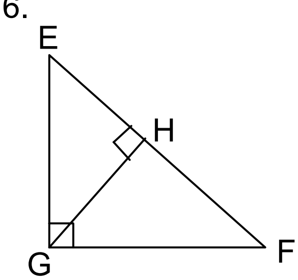
1. Name all the right triangles.
2. In ΔABC, name the altitude to the hypotenuse.
3. Name the hypotenuse in ΔADC.
4. Name the hypotenuse of ΔACB.

One of the segments shown is an altitude to the hypotenuse of a right triangle. Name the segment.

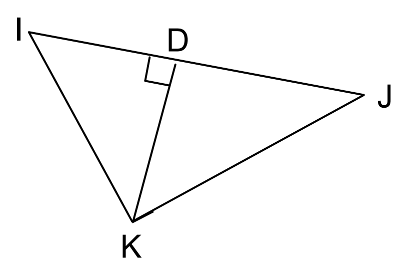
5.



6.

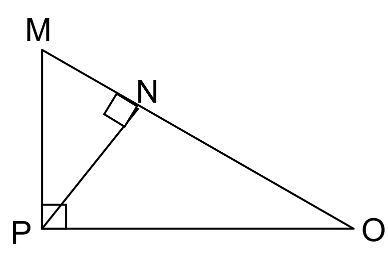


7.



Name the three pairs of similar triangles:

8.
9.
10.



Lesson 2

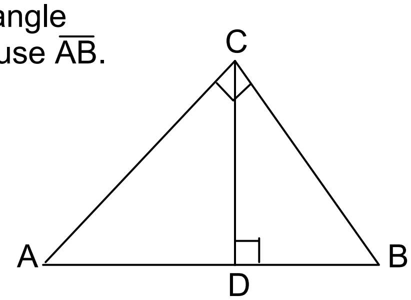
Geometric Mean in Similar Right Triangles

The previous theorem states that:

In a right triangle, the altitude to the hypotenuse divides the triangle into similar triangles, each similar to the given triangle.

If : ΔACB is right with $\angle C$, the right angle
$\overline{CD}$ is the altitude to the hypotenuse $\overline{AB}$.

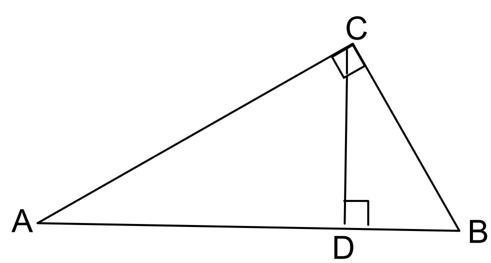
Then : $\Delta ADC \sim \Delta ACB$
$\Delta CDB \sim \Delta ACB$
$\Delta ADC \sim \Delta CDB$



Corollary: 1. In a right triangle, the altitude to the hypotenuse is the geometric mean of the segments into which divides the hypotenuse

In the figure:

$$\frac{AD}{CD} = \frac{CD}{DB}$$

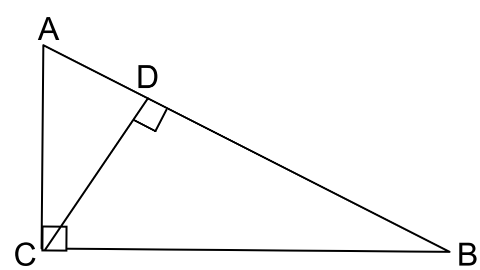


Corollary 2: In a right triangle, each leg is the geometric mean of the hypotenuse and the segment of the hypotenuse adjacent to it.

In the figure:

$$\frac{AB}{AC} = \frac{AC}{AD}$$

$$\frac{AB}{CB} = \frac{CB}{AD}$$



Examples:

1. How long is the altitude of a right triangle that separates the hypotenuse into lengths 4 and 20?

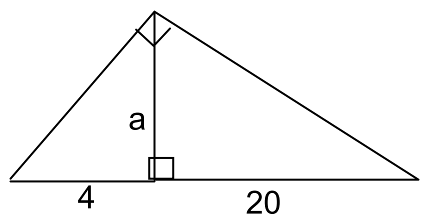
$\frac{4}{a} = \frac{a}{20}$

$a^2 = 80$

$a = \sqrt{80}$

$a = \sqrt{16 \cdot 5}$

$a = 4\sqrt{5}$



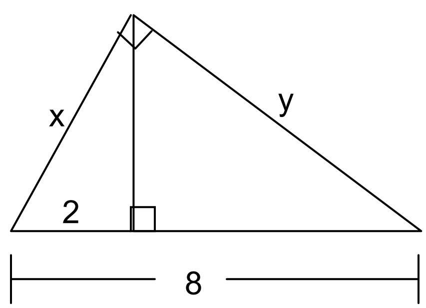
2. Use the figure at the right to solve for x and y.

$\frac{2}{x} = \frac{x}{8}$

$x^2 = 16$

$x = \sqrt{16}$

$x = 4$



Solve for y

$\frac{6}{y} = \frac{y}{8}$

$y^2 = 48$

$y = \sqrt{48}$

$y = \sqrt{16 \cdot 3}$

$y = 4\sqrt{3}$

Try this out

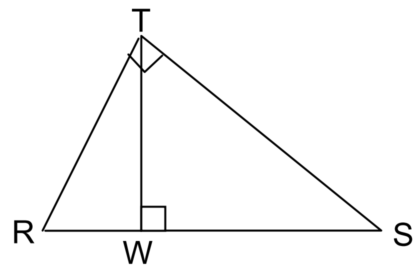
A. Supply the missing parts:

1. $\frac{RW}{RT} = \frac{RT}{?}$

2. $\frac{WS}{?} = \frac{TS}{RS}$

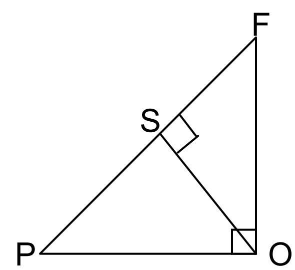
3. $\frac{?}{TS} = \frac{TS}{RS}$

4. $\frac{RS}{RT} = \frac{RT}{?}$



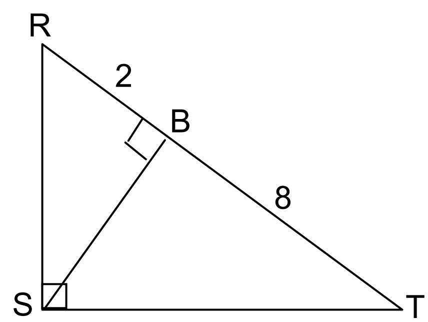
Give the indicated proportions.

5. The altitude is the geometric mean
6. The horizontal leg is the geometric mean
7. The vertical leg is the geometric mean



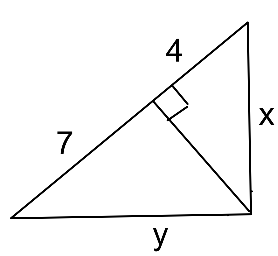
Find:

8. $\overline{BS}$
9. $\overline{RS}$
10. $\overline{ST}$



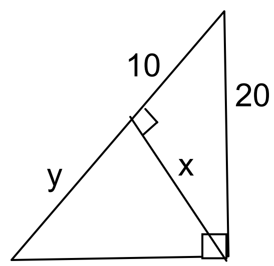
B. Solve for x and y:

1.



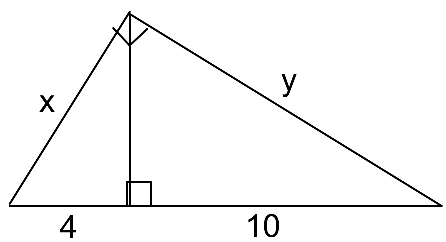
x =
y =

2.



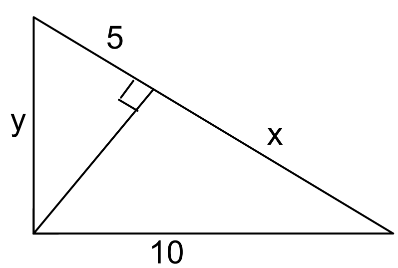
x =
y =

3.



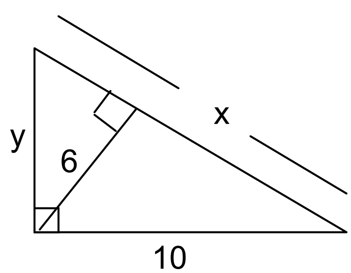
x =
y =

4.



x =
y =

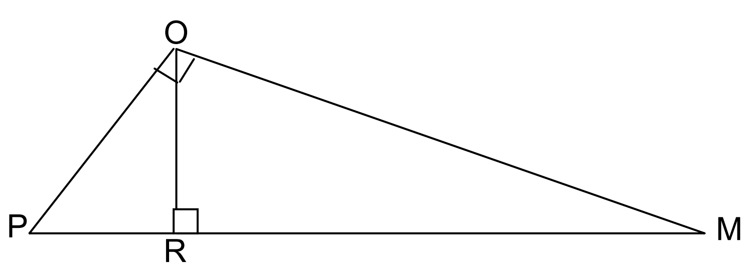
5.



x =

y =

C.



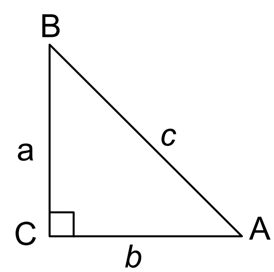
Given: Right ΔPOM $\overline{OR} \perp \overline{PM}$, Find the missing parts:

1. $\overline{PR}$ = 5, $\overline{RM}$ = 10, $\overline{OR}$ =
2. $\overline{OR}$ = 6, $\overline{RM}$ = 9, $\overline{PR}$ =
3. $\overline{PR}$ = 4, $\overline{PM}$ =12, $\overline{PO}$ =
4. $\overline{RM}$ = 8, $\overline{PM}$ =12, $\overline{OM}$ =
5. $\overline{PO}$ = 9, $\overline{PR}$ = 3, $\overline{PM}$ =
6. $\overline{PR}$ = 6, $\overline{RM}$ = 8, $\overline{PO}$ =
7. $\overline{PR}$ = 4, $\overline{RM}$ = 12, $\overline{OM}$ =
8. $\overline{PR}$ = 4, $\overline{PO}$ = 6, $\overline{RM}$ =
9. $\overline{PR}$ = 8, $\overline{OR}$ = 12, $\overline{RM}$ =
10. $\overline{PM}$ =15, $\overline{OM}$ = 12, $\overline{RM}$ =

Lesson 3

The Pythagorean Theorem

In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



In the figure:

ΔBCA is right with leg lengths,
a and *b* and hypotenuse length, *c*.

The Pythagorean Theorem in symbol:
$c^2 = a^2 + b^2$

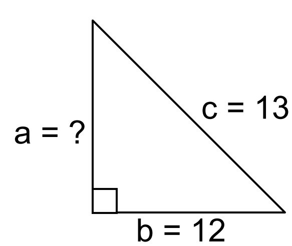
Pythagorean Theorem is named after Pythagoras, a Greek Mathematician of the sixth century $\overline{BC}$. This theorem can be used to find a missing side length in a right triangle.

Examples:

1. In the figure c = 13, b = 12

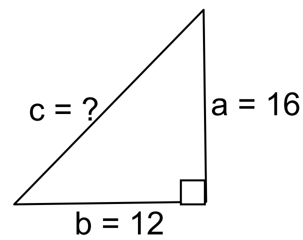
   Find a:

   $c^2 = a^2 + b^2$
   $13^2 = a^2 + 12^2$



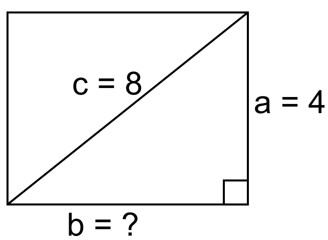
$a^2 = 13^2 - 12^2$
$a^2 = 169 - 144$
$a^2 = 25$
$a = \sqrt{25}$
$a = 5$

2. Find c, if a = 16 and b = 12



$c^2 = a^2 + b^2$
$c^2 = 16^2 + 12^2$
$c^2 = 256 + 144$
$c = \sqrt{400}$
$c = 20$

3.

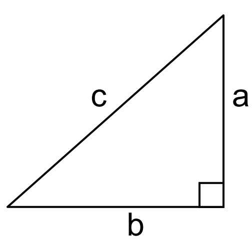


$c^2 = a^2 + b^2$
$8^2 = 4^2 + b^2$
$b^2 = 8^2 - 4^2$
$b^2 = 64 - 16$
$b^2 = 48$
$b = \sqrt{48}$
$b = \sqrt{16 \cdot 3}$
$b = 4\sqrt{3}$

Try this out:

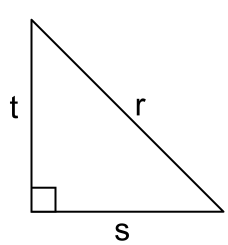
A. State whether the equation is correct or not

1.



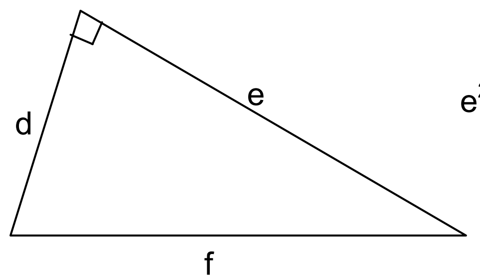
$a^2 + b^2 = c^2$

2.



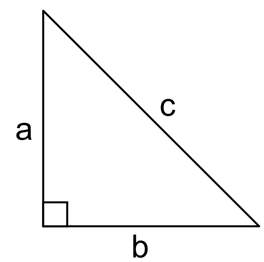
$r^2 = s^2 + t^2$

3.



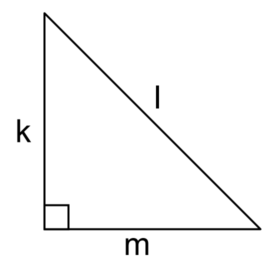
$e^2 = f^2 - d^2$

4.



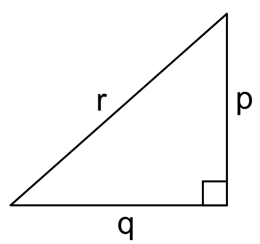
$a^2 = c^2 - b^2$

5.



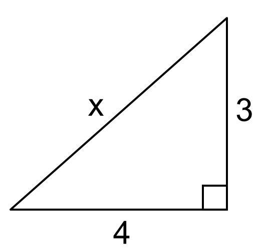
$k^2 = l^2 - m^2$

6.



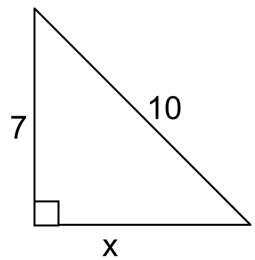
$p^2 = r^2 + q^2$

7.



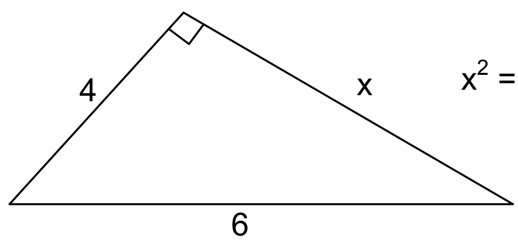
$x^2 = 3^2 + 4^2$

8.



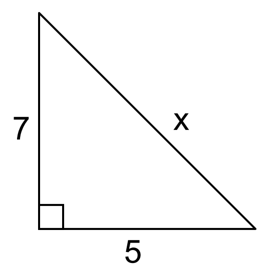
$x^2 = 10^2 - 7^2$

9.



$x^2 = 4^2 + 6^2$

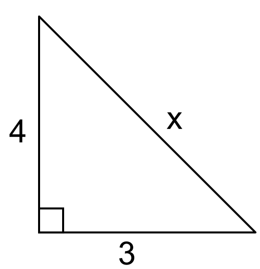
10.



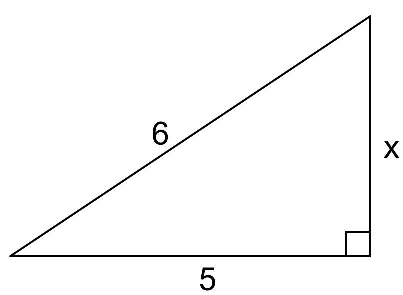
$x^2 = 7^2 - 5^2$

B. Write the equation you would use to find the value of x.

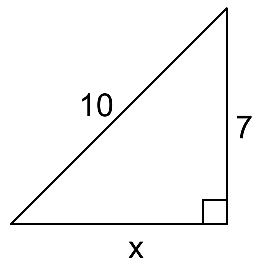
1.



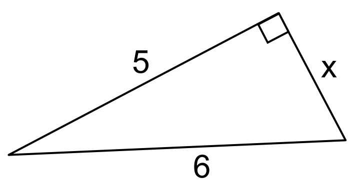
2.



3.



4.

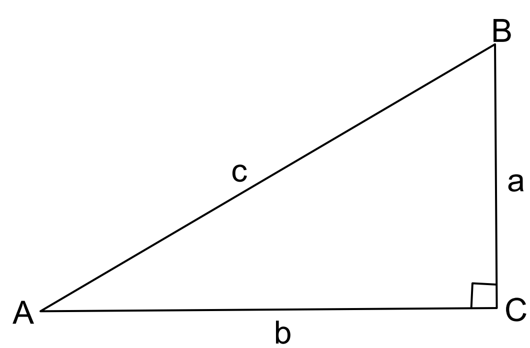


Classify each statement as true or false

5. $3^2 + 4^2 = 5^2$
6. $10^2 - 6^2 = 8^2$
7. $1^2 + 1^2 = 2^2$
8. $2^2 + 2^2 = 4^2$
9. $7^2 - 5^2 = 5^2$
10. $9^2 + 12^2 = 15^2$

C. Given the lengths of two sides of a right triangle. Find the length of the third side

1. a = 6, b = 8, c =
2. a = 5, b = 12, c =
3. a = 12, c = 15, b =
4. b = 4, c = 5, b =
5. a = 24, c = 26, b =
6. b = 16, c = 20



	a	b	c
7.	7	24	?
8.	4	6	?
9.	7	9	?
10	$6\sqrt{3}$	?	12

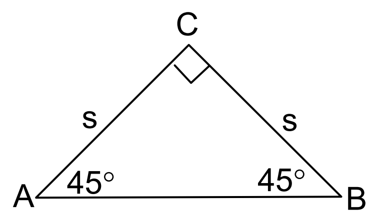
Lesson 4

Special Right Triangle

Isosceles Right Triangle or 45° – 45° – 90° Theorem:

In a 45° – 45° – 90° triangle, the length of the hypotenuse is equal to the length of a leg times $\sqrt{2}$.

In the figure:
If ΔABC, a 45° – 45° – 90° triangle when $\overline{AC} = \overline{BC} = s$ then $\overline{AB} = s\sqrt{2}$.

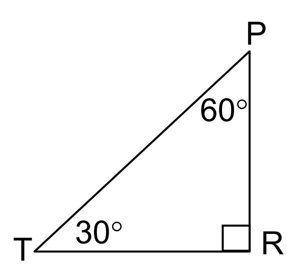


30° - 60° - 90° Theorem:

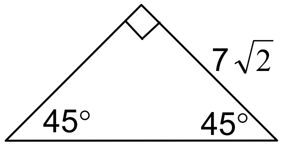
In a 30° - 60° - 90° triangle, the length of the hypotenuse is twice the length of the shorter leg, and the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg.

In the figure:

If ΔPRT where $\angle R$ is a right angle and $\angle T = 30°$,
Then:
a. $\overline{PT} = 2\overline{PR}$
b. $\overline{RT} = \overline{PR}\sqrt{3}$

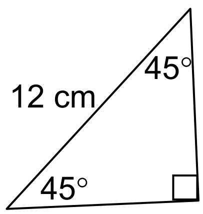


Examples:



1.Find the length of the hypotenuse of an isosceles right triangle with a leg $7\sqrt{2}$ cm long.

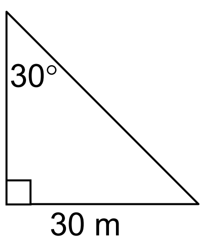
$$\text{Hypotenuse} = \text{leg} \cdot \sqrt{2}\,.$$
$$= 7\sqrt{2} \cdot \sqrt{2}$$
$$= 7 \cdot 2$$
$$= 14$$



2. Find the length of each leg of a 45° - 45° - 90° triangle with a hypotenuse 12 cm long.

$$\text{Leg} = \frac{hypotenuse}{\sqrt{2}}$$

$$= \frac{12}{\sqrt{2}} = \frac{12}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{12\sqrt{2}}{2} = 6\sqrt{2} \text{ cm}$$



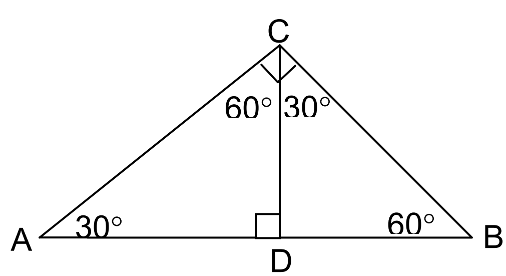
3. Find the length of the longer leg and the length of the hypotenuse.

$$\text{Longer leg} = \text{shorter leg} \cdot \sqrt{3}$$
$$= 30 \cdot \sqrt{3}$$
$$= 30\sqrt{3} \text{ m}$$

$$\text{hypotenuse} = \text{shorter leg} \cdot 2$$
$$= 30 \cdot 2$$
$$= 60 \text{ m}$$

Try this out

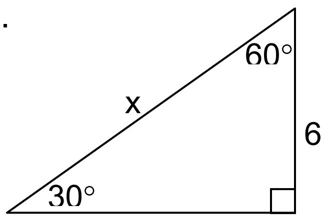
A. Use the figure to answer the following:

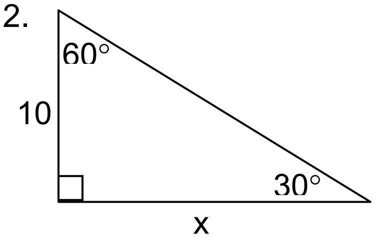


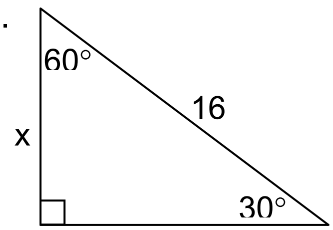
1. The hypotenuse of a rt. ΔABC is ____________.
2. The shorter leg of rt. ΔABC is ____________.

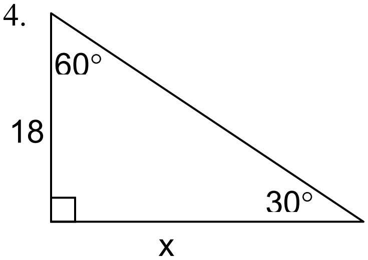
3. The shorter leg of rt. ΔADC is ____________.
4. The longest side of rt. ΔADC is __________.
5. The altitude to the hypotenuse of ΔACD is _________.
6. The longer leg of rt. ΔACB is __________.
7. The longer leg of rt. ΔADC is _________.
8. When $\overline{CD} = 2$ then ____ $= 4$.
9. When $\overline{CB} = 6$ then _____ $= 6\sqrt{3}$
10. When $\overline{CB} = 6$ then _____ $= 3$

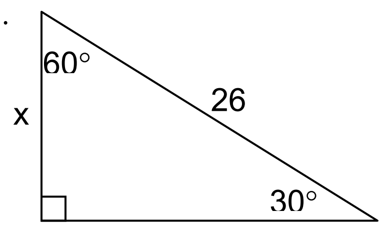
B. Find the value of x in each of the following:

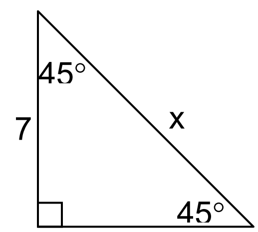
1.


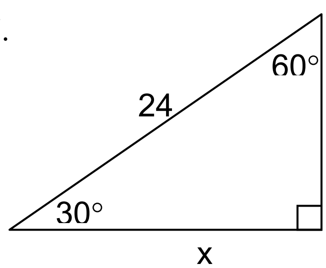
2.


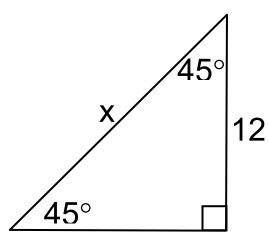
3.


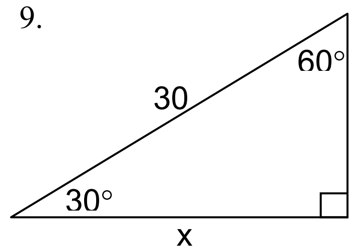
4.


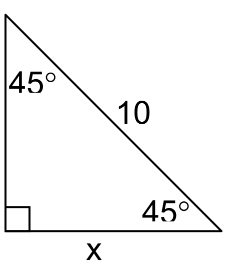
5.


6.


7.


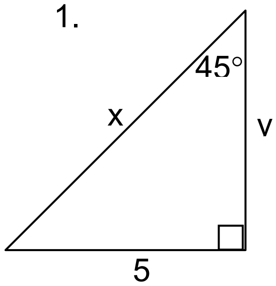
8.


9.


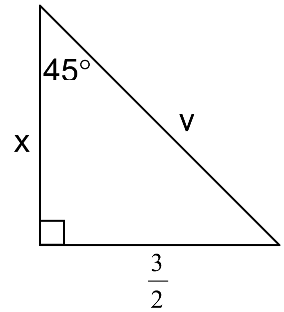
10.


C. Find the missing lengths, x and y.

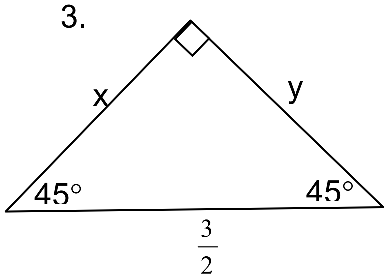
1.



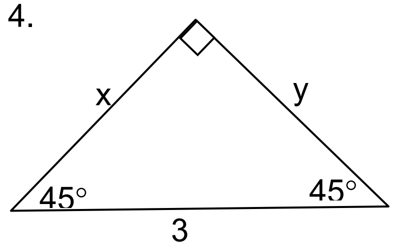
2.



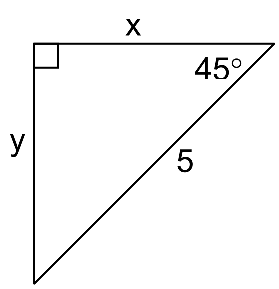
3.



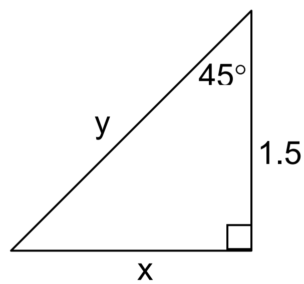
4.



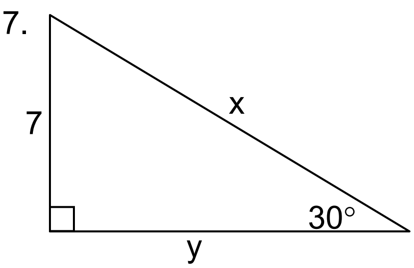
5.



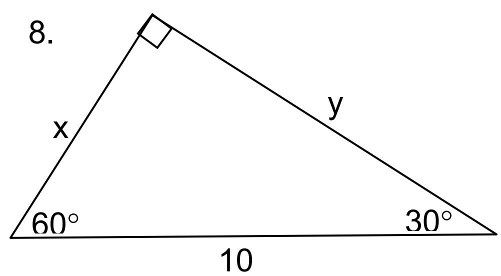
6.



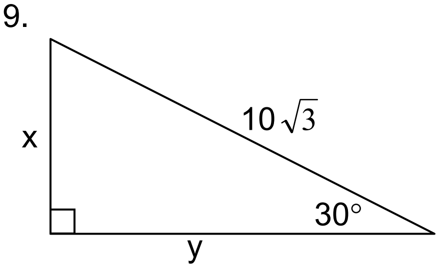
7.



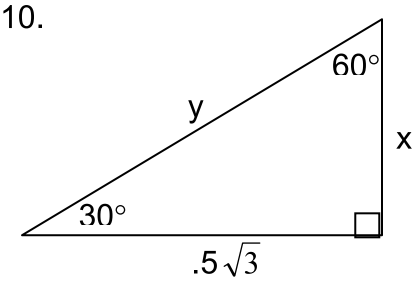
8.



9.



10.



Beyond the Pythagorean Theorem

In symbol $c^2 = a^2 + b^2$, where c is the hypotenuse and a and b are the legs of a right triangle.

Figure shows acute triangles

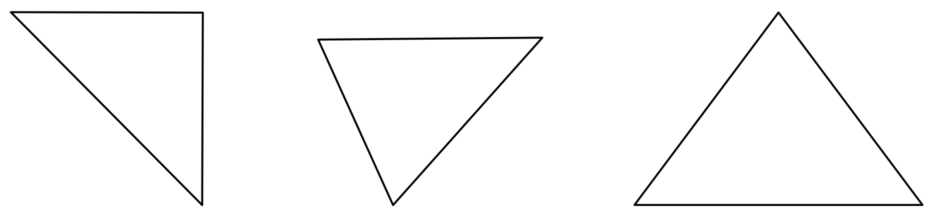


Figure shows obtuse triangles

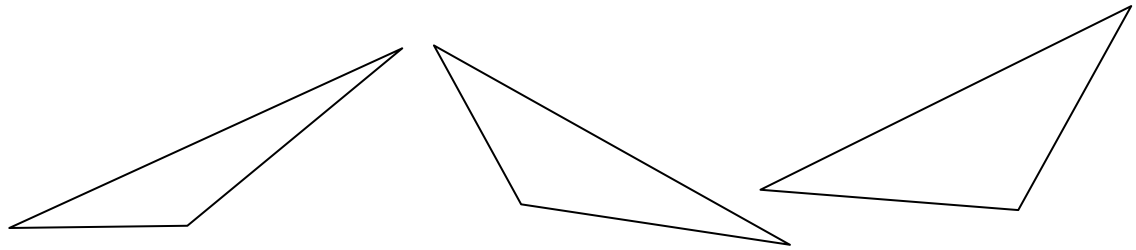
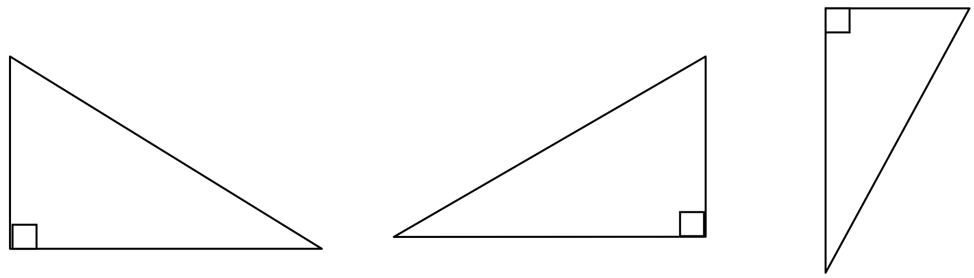


Figure shown right triangle



Activity:

This activity will help you extend your understanding of the relationship of the sides of a triangle.

Materials: Strips of paper cut in measured lengths of 2, 3, 4, 5, 6 and 8 units.

Procedure:

1. Form triangles with strips indicated by the number triplets below.
2. Draw the triangle formed for each number triple.
3. Fill out the table:

Number triplets	What kind of triangle	Compute c^2	Compute $a^2 + b^2$
1. 3 4 5	Right	$5^2 = 25$	$3^2 + 4^2 = 25$
2. 2 3 4			
3. 2 4 5			
4. 5 4 8			
5. 6 5 8			
6. 4 5 6			
7. 2 3 3			
8. 3 3 4			

After the computation, the completed table will look like this

Number triplets	Kind of triangle	c^2	$a^2 + b^2$	Comparison of c with $(a^2 + b^2)$
1. 3 4 5	Right	25	25	Equal to
2. 2 3 4	Obtuse	16	13	Greater than
3. 2 4 5	Obtuse	25	20	Greater than
4. 5 4 8	Obtuse	64	41	Greater than
5. 6 5 8	Obtuse	64	61	Greater than
6. 4 5 6	Acute	36	41	Smaller than
7. 2 3 3	Acute	9	13	Smaller than
8. 3 3 4	Acute	16	18	Smaller than

1. What kind of Δ did you get from triplet no. 1?
2. In triplet no. 1, what is the relation between c^2 and $(a^2 + b^2)$?
3. Which triplets showed obtuse triangle?

4. For each obtuse triangle compare the result from c^2 and $(a^2 + b^2)$.
5. For acute triangles how will you compare the result of c^2 and $(a^2 + b^2)$

Fill in the blanks with <, =, >:

6. In a right triangle, c^2 ____ $a^2 + b^2$
7. In an obtuse triangle, c^2 ______ $a^2 + b^2$
8. In an acute triangle, c^2 ____$a^2 + b^2$

Let's Summarize

Theorem: In a right triangle, the altitude to the hypotenuse separates the triangle into two triangles each similar to the given triangle and similar to each other.

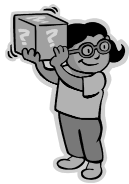
Corollary 1: In a right triangle, the altitude to the hypotenuse is the geometric mean of the segments into which it divides the hypotenuse.

Corollary 2: In a right triangle, each leg is the geometric mean of the hypotenuse and the segment of the hypotenuse adjacent to it.

Pythagorean Theorem: The square of the length of the hypotenuse is equal to the sum of the squares of the legs.

45°-45°-90° Theorem: In a 45°-45°-90° triangle, the length of the hypotenuse is equal to the length of a leg times $\sqrt{2}$.

30°- 60° - 90° Theorem: In a 30°- 60°- 90° triangle, the length of the hypotenuse is twice the length of the shorter leg, and the length of the leg is $\sqrt{3}$ times the length of the shorter leg.



What have you Learned

Fill in the blanks:

1. The _______ to the hypotenuse of a right triangle forms two triangles each similar to the given triangle & to each other.

2. The lengths of the ________ to the hypotenuse is the geometric mean of the lengths of the segments of the hypotenuse.

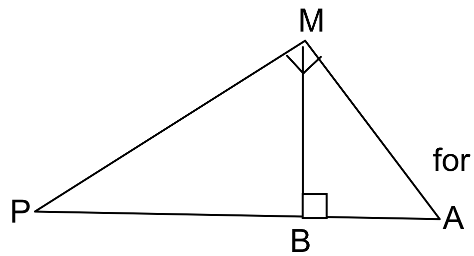
3. In the figure

   $\frac{AB}{MA} = \frac{MA}{?}$

4. If $\overline{BP} = 8$

   $\overline{AB} = 4$

   Find $\overline{PM}$ ___



for nos. 3 & 4

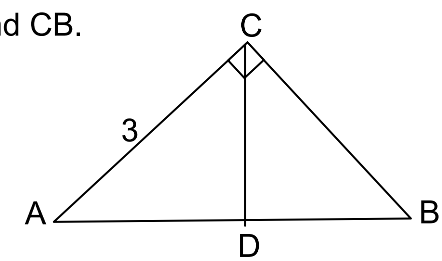
5. If in a right triangle the lengths of the legs are 8 and 15, the length of the hypotenuse is _______

6. Find the length of an altitude of an equilateral triangle if the length of a side is 10.

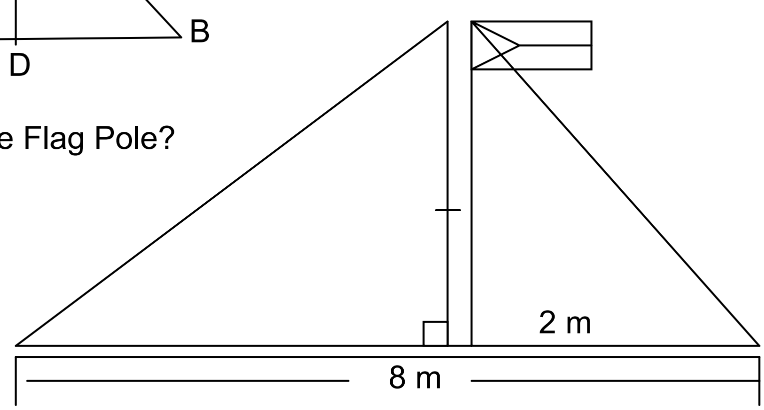
7. In a 30° – 60° – 90° triangle, the length of the hypotenuse is 8. Find the length of the shorter leg.

8. - 9. ΔACB is an isosceles right triangle. CD bisects $\angle C$, the right angle.

Find AB and CB.



10. What is the height of the Flag Pole?



Answer key

How much do you know

1. $\overline{AB}$
2. ΔACB
3. $\overline{CD}$
4. 10
5. $7\sqrt{3}$
6. $5\ \frac{3}{4}$
7. $8\sqrt{2}$
8. $3\sqrt{3}$
9. $10\sqrt{2}$
10. $2\sqrt{21}$

Lesson 1:

A.

1. $\angle C$ or $\angle ACB$
2. $\overline{CD}$
3. $\overline{AB}$
4. $\overline{BD}$ or $\overline{DB}$
5. $\overline{BC}$
6. $\angle ADC$
7. $\overline{BC}$
8. ΔBDC
9. ΔADC
10. ΔBDC

B.

1. $\Delta ROS \sim \Delta RST$
2. $\Delta TOS \sim \Delta RST$
3. $\Delta ROS \sim \Delta TOS$

4. $\Delta MST \sim \Delta MOR$
5. $\Delta RSO \sim \Delta MOR$
6. $\Delta MST \sim \Delta RSO$

C.

1. ΔADC, ΔBDC, ΔACB
2. $\overline{CD}$
3. $\overline{AC}$
4. $\overline{AB}$
5. $\overline{BD}$
6. $\overline{GH}$
7. $\overline{OK}$
8. $\Delta MNR \sim \Delta MPO$
9. $\Delta ONP \sim \Delta MPO$
10. $\Delta MNP \sim \Delta ONP$

Lesson 2

A.

1. WS
2. TS

B.

1. $x = 2\sqrt{11}$
   $y = 2\sqrt{7}$

C.

1. $5\sqrt{2}$
2. 4

3. WS
4. RW
5. $\frac{SF}{OS} = \frac{OS}{SP}$
6. $\frac{SP}{PO} = \frac{PO}{PF}$
7. $\frac{FS}{OF} = \frac{OF}{PF}$
8. 4
9. $2\sqrt{5}$
10. $4\sqrt{5}$

2. x = $10\sqrt{3}$
   y = 30
3. x = $2\sqrt{14}$
   y = $2\sqrt{35}$
4. x = 20
   y = $5\sqrt{5}$
5. x = $\frac{100}{6}$ or $16\frac{2}{3}$
   y = 10

3. $4\sqrt{3}$
4. $4\sqrt{6}$
5. 27
6. $4\sqrt{3}$
7. $8\sqrt{3}$
8. 9
9. 18
10. 9.6

Lesson 3

A.
1. correct
2. correct
3. not
4. correct
5. correct
6. not
7. correct
8. correct
9. not
10. not

B.
1. $x^2 = 3^2 + 4^2$
2. $x^2 = 6^2 - 5^2$
3. $x^2 = 10^2 - 7^2$
4. $x^2 = 6^2 - 5^2$
5. true
6. true
7. false
8. false
9. false
10. true

C.
1. 10
2. 13
3. 9
4. 3
5. 10
6. 12
7. 25
8. $2\sqrt{13}$
9. $\sqrt{130}$
10. 6

Lesson 4

A.
1. $\overline{AB}$
2. $\overline{BD}$
3. $\overline{CD}$
4. $\overline{AC}$

B.
1. 12
2. $10\sqrt{3}$
3. 8
4. $18\sqrt{3}$

C.

	x	y
1.	$5\sqrt{3}$	5
2.	$\frac{3}{2}$	$\frac{3\sqrt{2}}{2}$
3.	3	3
4.	$\frac{3\sqrt{2}}{2}$	$\frac{3\sqrt{2}}{2}$

5. $\overline{CD}$	5. 13	5. $\frac{5\sqrt{2}}{2}$	$\frac{5\sqrt{2}}{2}$
6. $\overline{AC}$	6. $7\sqrt{2}$	6. $\frac{1.5\sqrt{2}}{2}$	$\frac{1.5\sqrt{2}}{2}$
7. $\overline{AD}$	7. $12\sqrt{3}$	7. 14	$7\sqrt{3}$
8. $\overline{AC}$	8. $12\sqrt{2}$	8. 5	$5\sqrt{3}$
9. $\overline{AC}$	9. $15\sqrt{3}$	9. $5\sqrt{3}$	15
10. $\overline{DB}$	10. $\frac{10\sqrt{3}}{3}$	10. 5	10

What have you learned

1. altitude
2. altitude
3. $\overline{AP}$
4. $4\sqrt{2}$
5. 17
6. $5\sqrt{3}$
7. 4
8. $3\sqrt{2}$
9. 3
10. $2\sqrt{3}$