What this module is about?

This module is about similar triangles, definition and similarity theorems. As you go over the exercises you will develop your skills in determining if two triangles are similar and finding the length of a side or measure of an angle of a triangle.

What are you expected to learn?

This module is designed for you to:
1. apply the definition of similar triangles in:
   a. determining if two triangles are similar
   b. finding the length of a side or measure of an angle of a triangle
2. verify the Similarity Theorems:
   a. AAA Similarity
   b. SAS Similarity
   c. SSS Similarity
3. apply the properties of similar triangles and the proportionality theorems to calculate lengths of certain line segments.
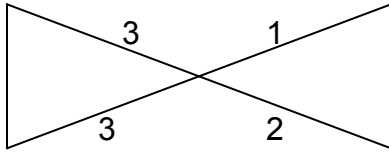
How much do you know?

A. For each pair of triangles, indicate whether the two triangles are similar or not. If they are similar, state the similarity theorem or definition that supports your answer.
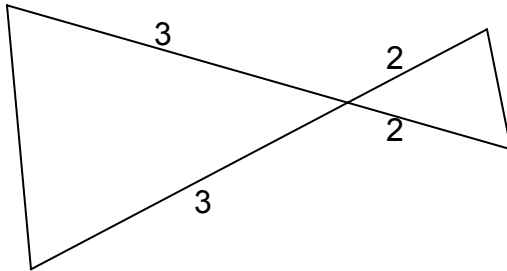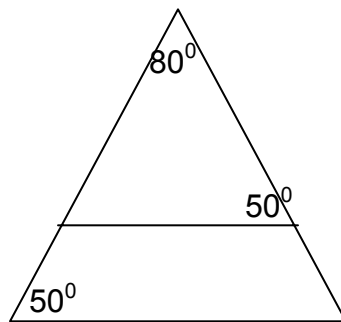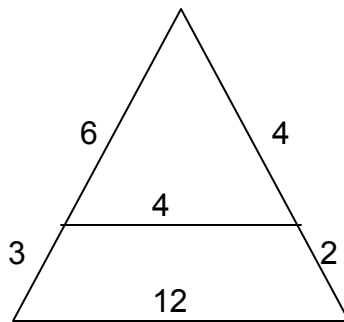
1.
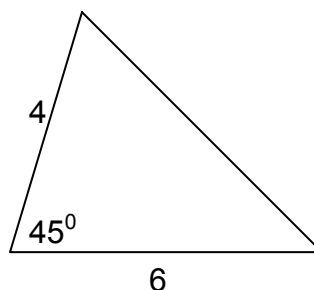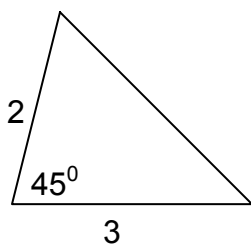
2.

3.

4.

5.

B.

6 – 10. The length of the sides of a triangle are 14, 8 and 6. Find the length of the two sides of a similar triangle if the length of the shortest side is 12.

Lesson 1
Definition of Similar Triangles

Similar Triangles

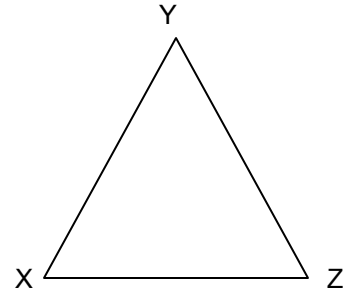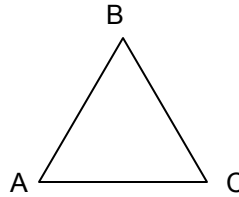Two triangles are similar if corresponding angles are congruent and corresponding sides are proportional.

Examples:
1. If in $\triangle ABC$ and $\triangle XYZ$

$\angle A \cong \angle X$
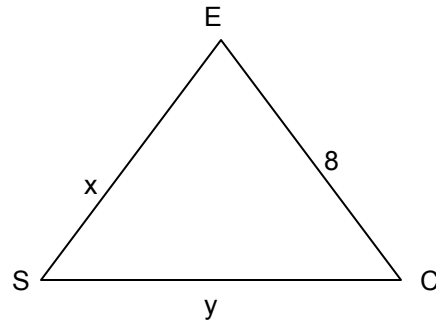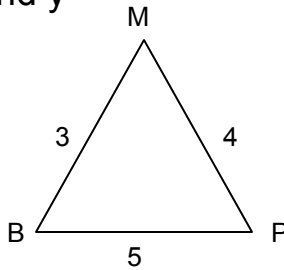$\angle B \cong \angle Y$
$\angle C \cong \angle Z$

$$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$$

then $\triangle ABC \sim \triangle XYZ$ (Triangle ABC is similar to triangle XYZ)

2. $\triangle BMP \sim \triangle SEC$
Find the value of x and y

Since the corresponding sides are proportional

$$\frac{BM}{SE} = \frac{MP}{EC} = \frac{BP}{SC}$$

$$\frac{3}{x} = \frac{4}{8} = \frac{5}{y}$$

$$\frac{3}{x} = \frac{1}{2} = \frac{5}{y}$$

$$\frac{3}{x} = \frac{1}{2} \; ; \; \frac{1}{2} = \frac{5}{y}$$

x = 6   y = 10

Exercises:

A. Given: $\triangle MOL \sim \triangle REY$

Fill the blanks.
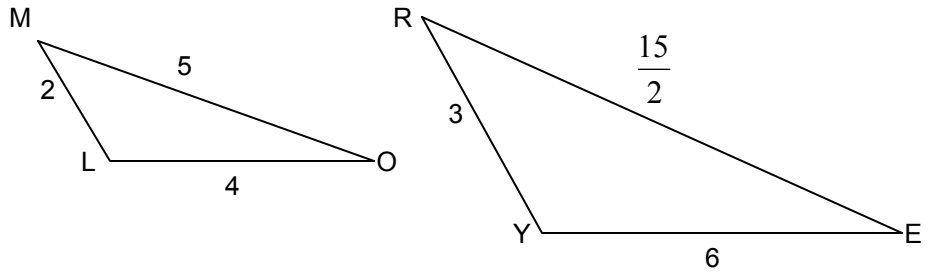
1) $\angle M \cong$ _____

2) $\angle O \cong$ _____

3) _____ $\cong \angle Y$

4) $\dfrac{ML}{RY} = \dfrac{MO}{?}$

5) $\dfrac{ML}{RY} = \dfrac{LO}{?}$

6) $\dfrac{2}{3} = \dfrac{?}{15}$

7) $\dfrac{4}{6} = \dfrac{2}{?}$

M

5

2

L

4

O

R

$\dfrac{15}{2}$

3

Y

6

E

B. State whether the proportion is correct for the indicated similar triangles

1. $\triangle RST \sim \triangle XYZ$

$\dfrac{RS}{XY} = \dfrac{ST}{YZ}$

2. $\triangle ABC \sim \triangle DEF$

$\dfrac{AB}{DE} = \dfrac{BC}{EF}$

3. $\triangle RST \sim \triangle LMK$

$\dfrac{RT}{LM} = \dfrac{ST}{MK}$

4. $\triangle DEF \sim \triangle HIS$

$\dfrac{DE}{HI} = \dfrac{EF}{IJ}$

5. $\triangle KLM \sim \triangle PQR$

$\dfrac{KM}{PR} = \dfrac{LM}{QR}$

6. $\triangle XYZ \sim \triangle UVW$

$\dfrac{XY}{UV} = \dfrac{XZ}{UW}$

Complete the proportions

7. $\triangle ABC \sim \triangle DEF$

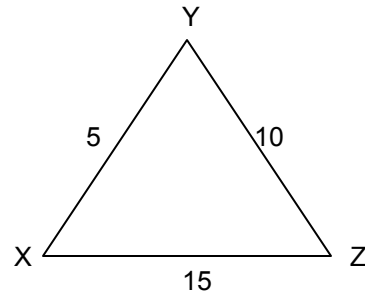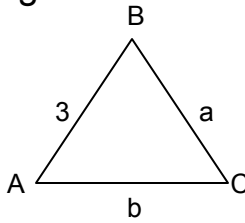$$\frac{AB}{?} = \frac{BC}{?} = \frac{AC}{?}$$

8. $\triangle KLM \sim \triangle RST$

$$\frac{?}{KL} = \frac{?}{LM} = \frac{?}{KM}$$

9. $\triangle RST \sim \triangle XYZ$

$$\frac{XY}{?} = \frac{XZ}{?} = \frac{YZ}{?}$$

10. $\triangle MNO \sim \triangle VWX$

$$\frac{VX}{?} = \frac{VW}{?} = \frac{WX}{?}$$

C. Find the missing length

1. $\triangle ABC \sim \triangle XYZ$
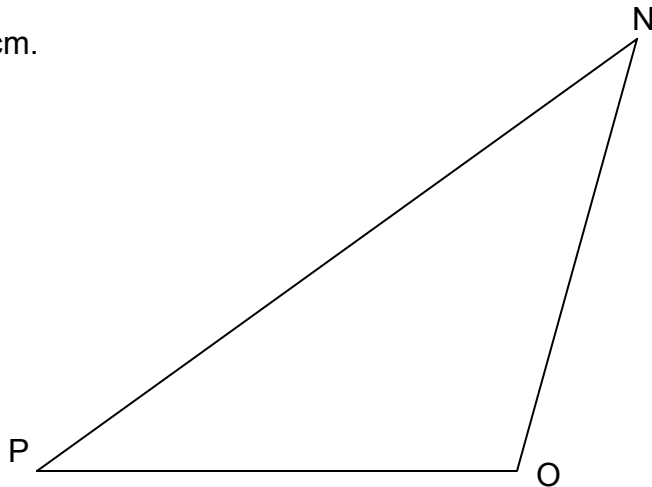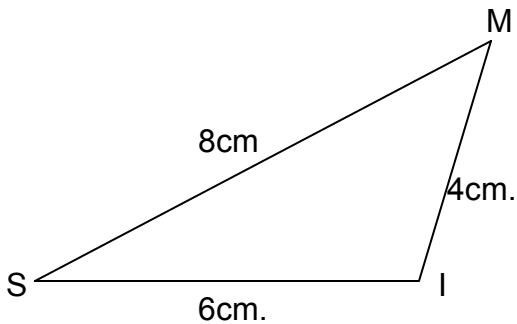   find b and a

2. $\triangle SIM \sim \triangle PON$

   SI = 6cm.   IM = 4cm.   SM= 8cm,

   Find the lengths of the sides of $\triangle PON$ if the ratio of the lengths of the corresponding sides is 1:3.
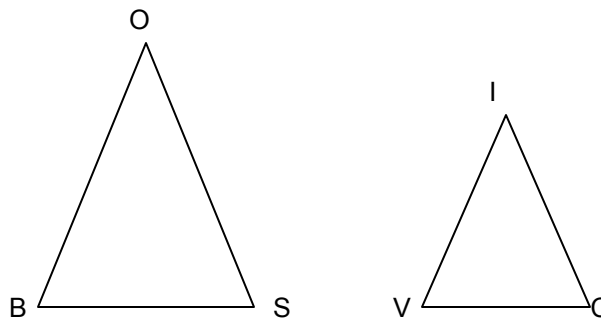
Lesson 2

AAA Similarity Theorem

If in two triangles the corresponding angles are congruent, then the two triangles are similar.

If $\triangle BOS \leftrightarrow \triangle VIC$

$\angle B \cong \angle V$, $\angle O \cong \angle I$, $\angle S \cong \angle C$
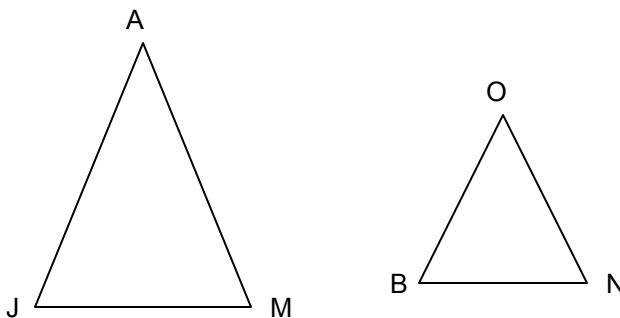
then $\triangle BOS \sim \triangle VIC$

AA Similarity

If two angles of one triangle are congruent to the corresponding two angles of another triangle, the triangles are similar.

Given: $\angle A \cong \angle O$
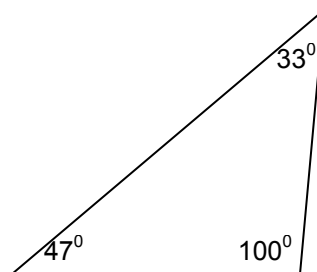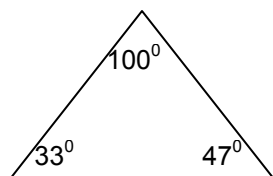
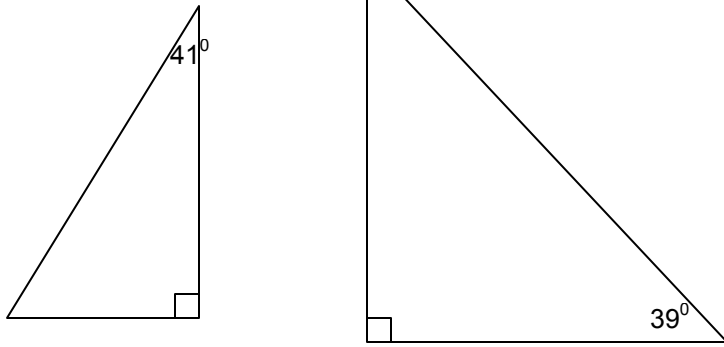$\angle J \cong \angle B$

Then: $\triangle JAM \sim \triangle BON$

Examples:

Are the triangles similar by AA Similarity?
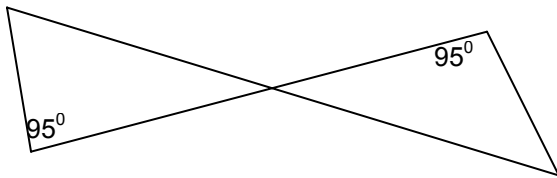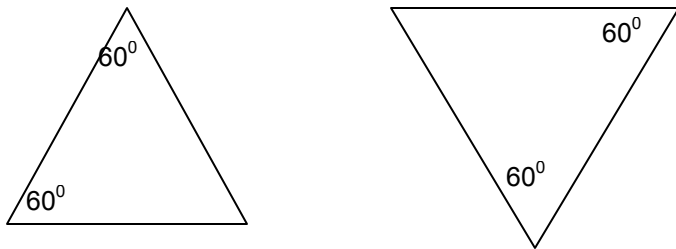
1.

2.

41⁰
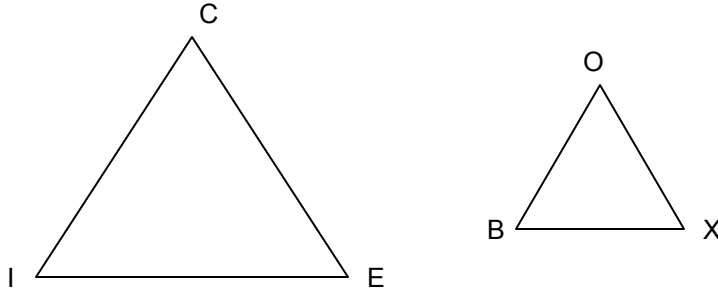
39⁰

3.

60⁰

30⁰
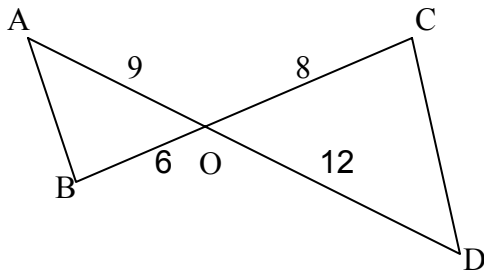
4.

95⁰

95⁰

5.

60⁰

60⁰

60⁰

60⁰
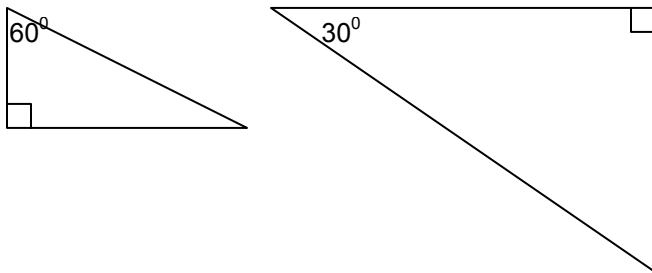
SAS Similarity Theorem

If in two triangles two pairs of corresponding sides are proportional and the included angles are congruent, then the triangles are similar.
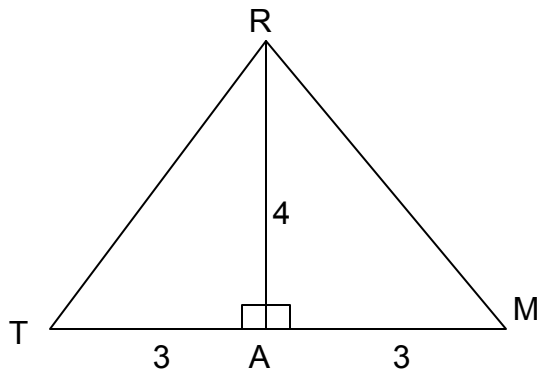
If $\triangle ICE \leftrightarrow \triangle BOX$

$$\frac{CI}{OB} = \frac{CE}{OX}$$

and $\angle C \cong \angle O$

then $\triangle ICE \sim \triangle BOX$

Examples:

$\triangle AOB \sim \triangle DOC$ by SAS

since $\dfrac{AO}{DO} = \dfrac{BO}{CO}$ and

$\angle AOB \cong \angle DOC$

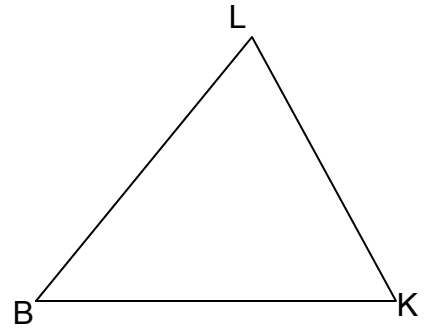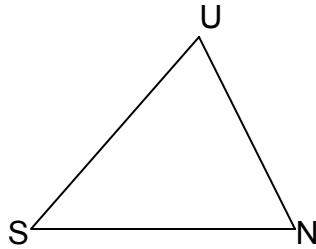Can you explain why is $\triangle RAT \sim \triangle RAM$ by SAS ?

SSS Similarity Theorem

If the two triangles three corresponding sides are proportional, then the triangles are similar.

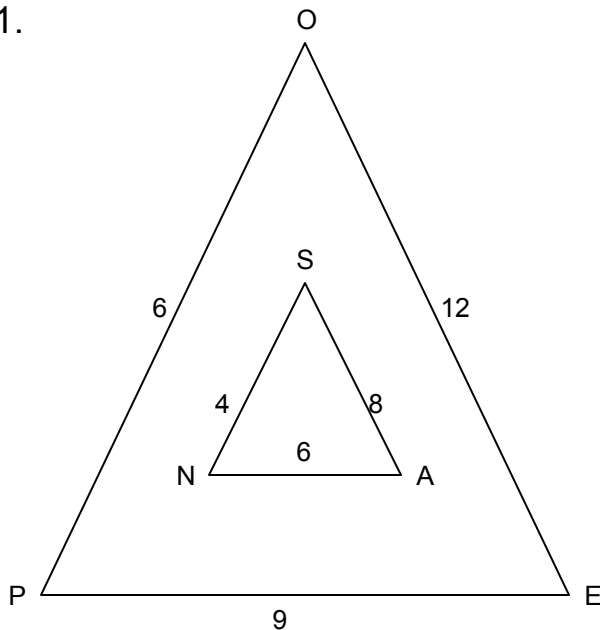If $\triangle SUN \leftrightarrow \triangle BLK$

$$\frac{SU}{BL} = \frac{UN}{LK} = \frac{SN}{BK}$$

then $\triangle SUN \sim \triangle BLK$

Example:

1.

$$\frac{SN}{OB} = \frac{NA}{PE} = \frac{SA}{DE}$$

$$\frac{4}{6} = \frac{6}{9} = \frac{8}{12}$$

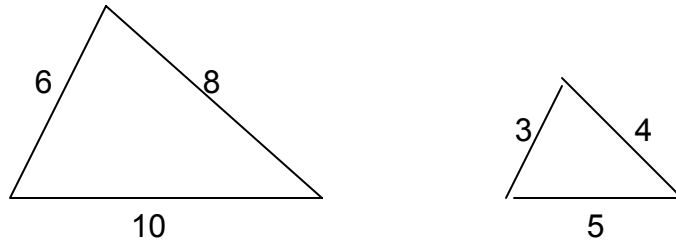since the corresponding sides are proportional then $\triangle POE \sim \triangle NSA$ by SSS Similarity.

2. Explain: Any two congruent triangles are similar.

Exercises:
   A. Tell whether the two triangles are similar. Cite the Similarity
      Postulate or Theorem to justify your answer. (Identical marks
      indicate $\cong$ parts.)

1.

2.

3.

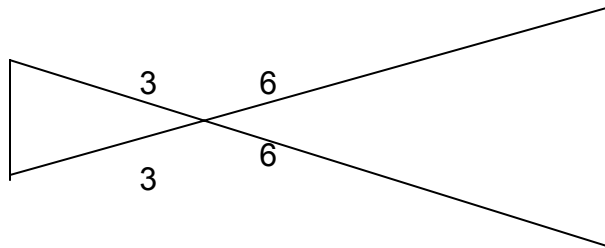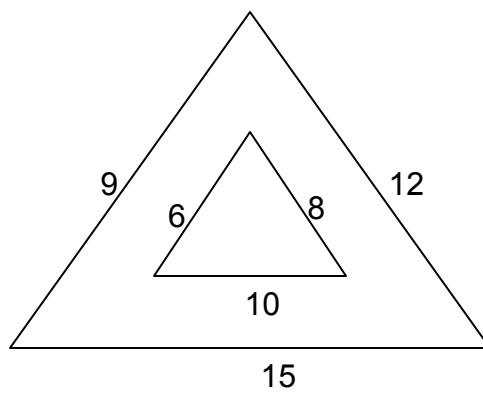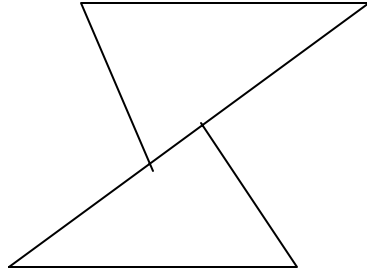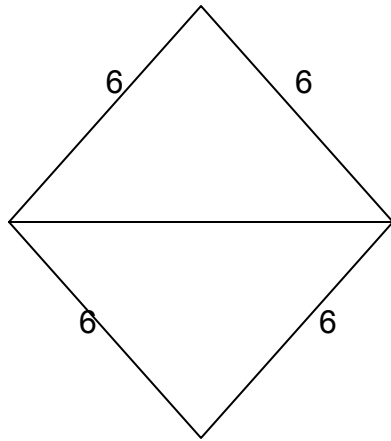
4.

5.

6.

6   6

6   6

7.

3

5

5

3

8.

2

8

4   8

9.
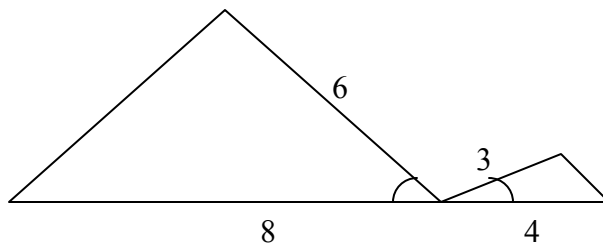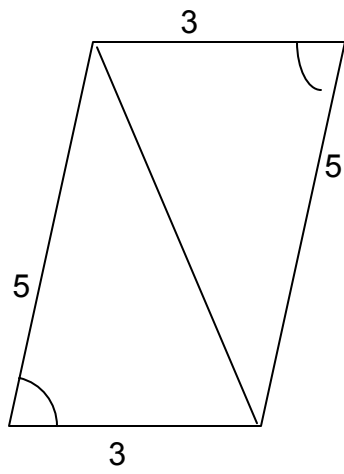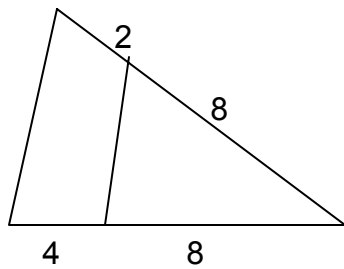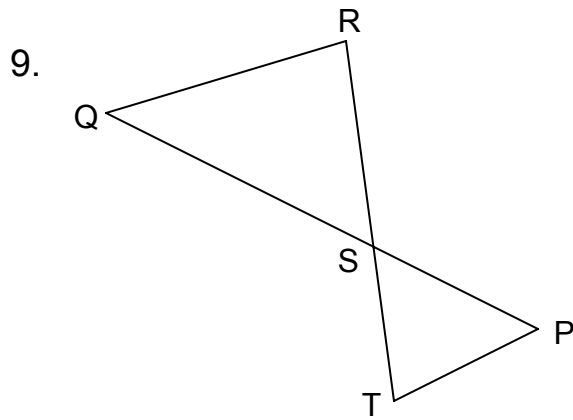
$$\frac{QS}{PS} = \frac{RS}{ST}$$

10.
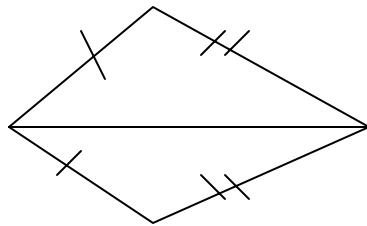
B. Determine whether each pair of the following triangles are similar by SAS, AAA, SSS or not at all.

1.
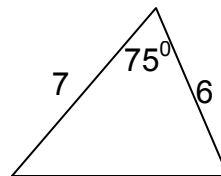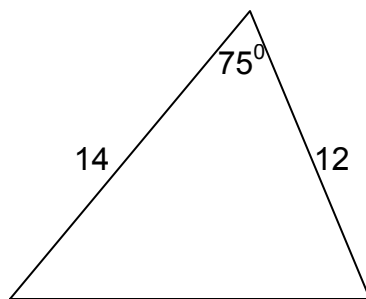
2.
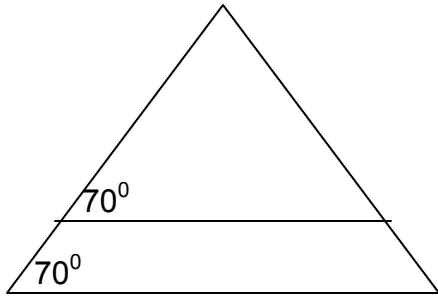
3.
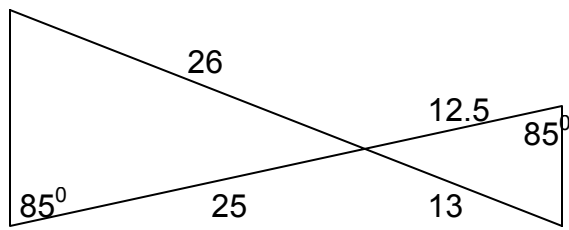
4.
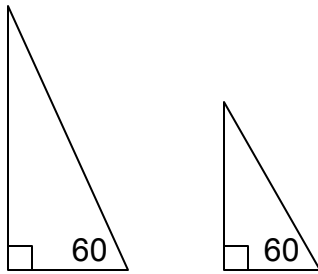
5.
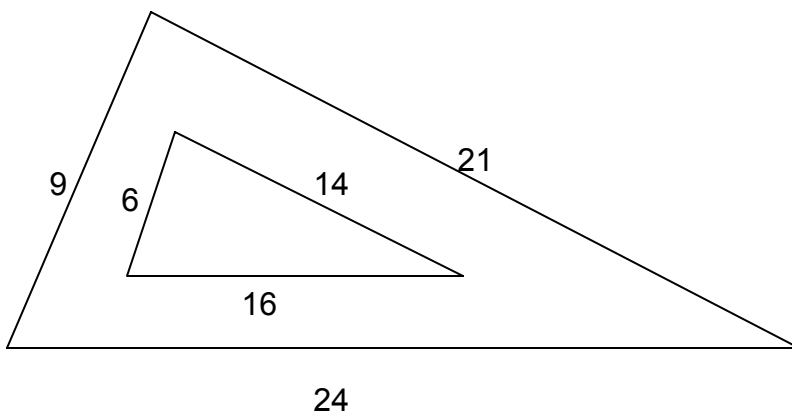
C. Answer the following:

1. Two isosceles triangles have an angle of 50^0. Does it follows that the triangles are similar?

2. Two angles of $\triangle BEL$ have measures of 20 and 50. Two angles of $\triangle JAY$ have measures of 30 and 100. IS $\triangle BEL \sim \triangle JAY$?

3. Is it possible for two triangles to be similar if two angles of one have measures 50 and 75, where as two angles of the other have measures 55 and 70?

4. Two angles of have measures 40 and 80, where as the two angles of the other have measures 60 and 80, are the two triangles similar?

5 – 6. The lengths of the sides of a triangle are 12 and 15. If the length of the shortest side of a similar triangle is 12, find the lengths of the other two sides.

7- 8. In the figure, if AE = 8
    AB=4, BC=10, ED=3
    Find BD and DC.
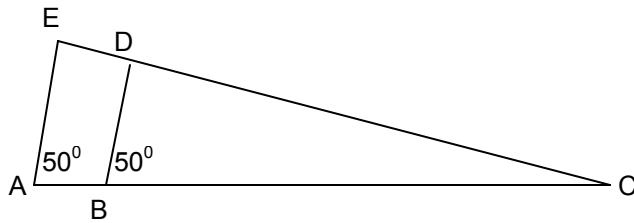
9 – 10. Explain: Any two equilateral triangle are similar.

Lesson 3

Application of the Properties of Similar Triangles to Calculate Lengths of certain line segments.

Examples:

$\triangle ABC \sim \triangle DEF$. Find the missing measure.

1. AB = 36, BC = 24, DE = 48 $m\angle B$ = 110, $m\angle E$ = 110; EF =_____.

$$\frac{AB}{DE} = \frac{BC}{EF}$$

$$\frac{36}{48} = \frac{24}{x}$$

36x = 1152
x = 32
EF = 32

2. AB = 38, BC =24, AC=30, DE=12, EF=16, DF= _____.

$$\frac{AB}{DE} = \frac{AC}{DF}$$

$$\frac{18}{12} = \frac{30}{x}$$

$$\frac{3}{2} = \frac{30}{x}$$

3x = 60
x = 20
DF = 20

3. AC = 15, DE=12, DF=20, $m\angle A = \ m\angle D = 35$, AB =_____

$$\frac{AB}{DE} = \frac{AC}{DF}$$

$$\frac{x}{12} = \frac{15}{20}$$

$$\frac{x}{12} = \frac{3}{4}$$

4x = 36
x = 9
AB = 9

4. The lengths of the sides of a triangle are 14, 8 and 6. Find the perimeter of a similar triangle if the length of its shortest side is 12.
Let x = the length of one side of a triangle
    x = the length of the longest side of the triangle

$$\frac{6}{12} = \frac{8}{x}$$

$$\frac{1}{2} = \frac{8}{x}$$

x = 16

$$\frac{6}{12} = \frac{14}{y}$$

$$\frac{1}{2} = \frac{14}{y}$$

y = 28

P = 12 + 16 + 28
P = 56

5. On a level ground, a 5ft. person and a flagpole cast shadows of 10 feet and 60 feet respectively. What is the height of the flagpole?
Let x = the height of the flagpole

$$\frac{5}{x} = \frac{10}{60}$$

$$\frac{5}{x} = \frac{1}{6}$$

x = 30 feet the height of the flagpole

Exercises

1. A yardstick casts a shadow of 24in. at the same time an electric post cast a shadow of 20ft. 8 in. What is the height of the electric post?
2. Two triangles are similar. The lengths of the sides of one triangle are 5, 12 and 13. Find the lengths of the missing sides of the other triangle if its longest side is 39.
3. The perimeter of a triangle is 32cm. and the ratio of the sides is 3:6:7. Find the length of each side of the triangle.

4. A tall building at Makati casts a shadow of 12m. at the same time a 7m. light pole cast a shadow of 3m. Find the height of the building.
5. If the shadow of the tree is 20m. long and the shadow of the person, who is 190cm. tall, is 250cm. long. How tall is the tree?

Let's Summarize

1. Two triangles are similar if their vertices can be paired so that corresponding angles are congruent and the lengths of corresponding sides are proportional.

2. The AAA Similarity. If the corresponding of two triangles are congruent, then the two triangles are similar.

3. The AA Similarity. If two pairs of corresponding angles of two triangles are congruent, then the two triangles are similar.

4. The SSS Similarity. If the lengths of corresponding sides of two triangles are proportional, then the two triangles are similar.

5. The SAS Similarity. If one pair of corresponding angles of two triangles are congruent and the lengths of the corresponding sides that include these angles are proportional, then the two triangles are similar.
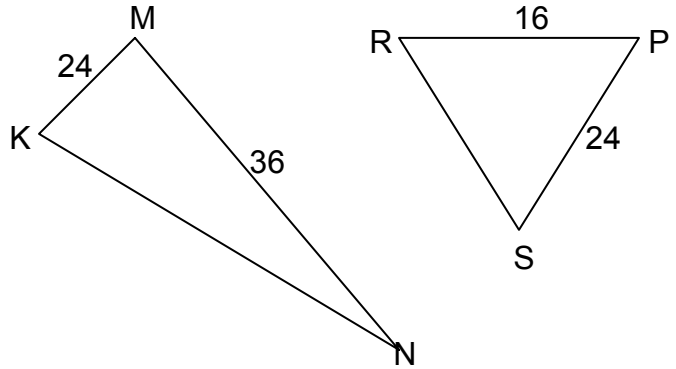
What have you learned?

A. Given the figures and the information below can you conclude that $\triangle KMN \sim \triangle RPS$?
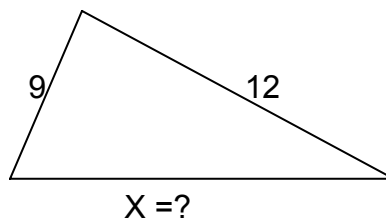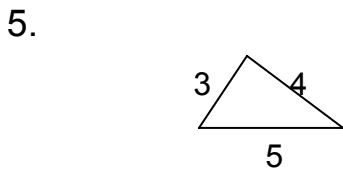
If so what Similarity Theorem?

1. $m\angle M = 80 \qquad m\angle P = 80$
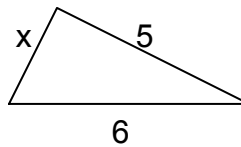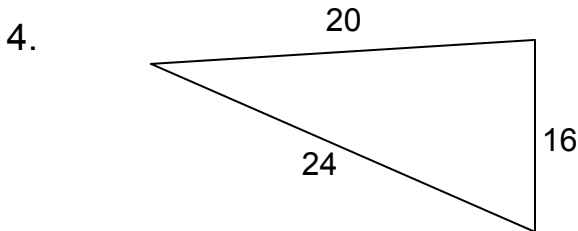
2. $m\angle M = 80$, $m\angle R + m\angle S = 100$

3. RS = 26
    KN = 40

B. Find the missing side

4.

5.

X =?

6 – 7. If the shadow of the tree is 14cm. long and the shadow of the person who is 1.8m. tall is 4m long, how tall is the tree?

8 – 10. A pole 3m. high has a shadow 5m long when the shadow of a nearby building is 110m. long. How tall is the building?

Key

How much do you know?

1. yes, SAS
2. No
3. yes, SAS
4. yes, AAA
5. yes, SSS
6. 16,28

Lesson 1 Exercises:

A.1. $\angle R$
  2. $\angle E$
  3. $\angle L$
  4. RE
  5. YE
  6. 5
  7. 3

B. 1. Correct
  2. correct
  3. Not correct
  4. correct
  5. correct
  6. correct
  7. DE, EF, DF
  8. RS, ST, RT
  9. RS, RT, ST
  10. MO, MN, NO

C. 1. a=6
     b=1

  2. NP=24
     NO=12
     PO = 18

Lesson 2

A.1. Similar, SSS
  2. Similar, SAS
  3. Similar, SAS
  4. Similar, SSS
  5. Similar, AAA
  6. Similar, SSS
  7. Similar, SAS
  8. not similar
  9. Similar, SAS
  10. Similar, SSS

B.1. Similar, SAS
  2. Similar, AA
  3. Similar, SAS, AA
  4. Similar, AA
  5. Similar, SSS

C.1. yes
  2. no
  3. no
  4. yes
  5. } 18, 22.5
  6. }
  7. } BD = 5.7'
  8. } DC = 7.5
  9. } Equilateral $\triangle$
  10. } is equiangular
       by AAA. Any
       2 equilateral $\triangle$
       are similar.

Lesson 3

Exercises

1. 372in.
2. 15,   36
3. 6cm.,   12cm.,   14cm.
4. 28m.
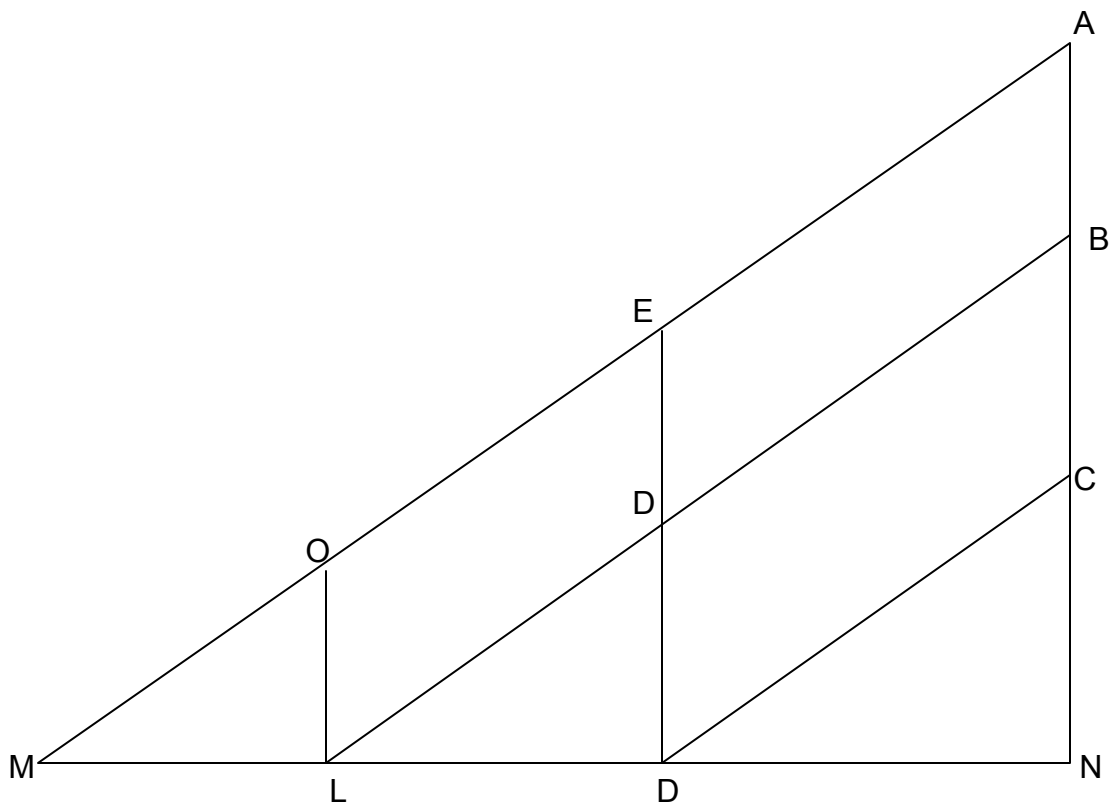5. 152cm.

What have your learned?

1. Similar, SAS
2. Similar, SAS
3. not similar
4.   4
5.   15
6-7.   6.3m
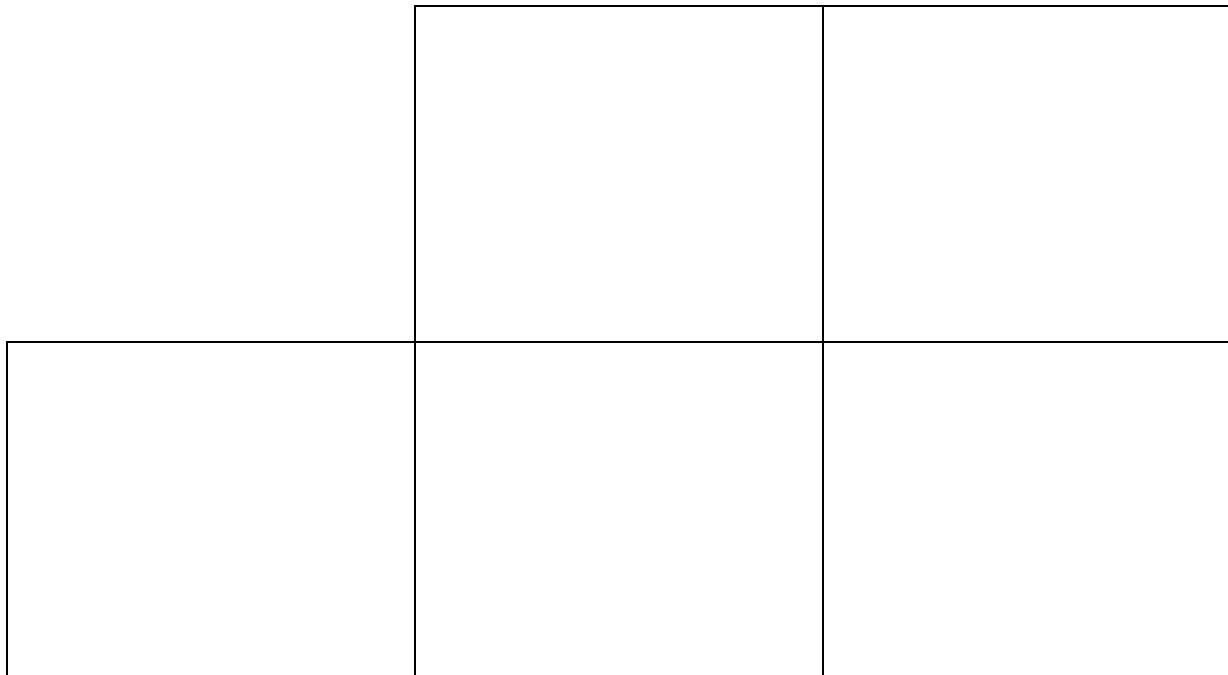8 - 10.   66m.

Fun with Math
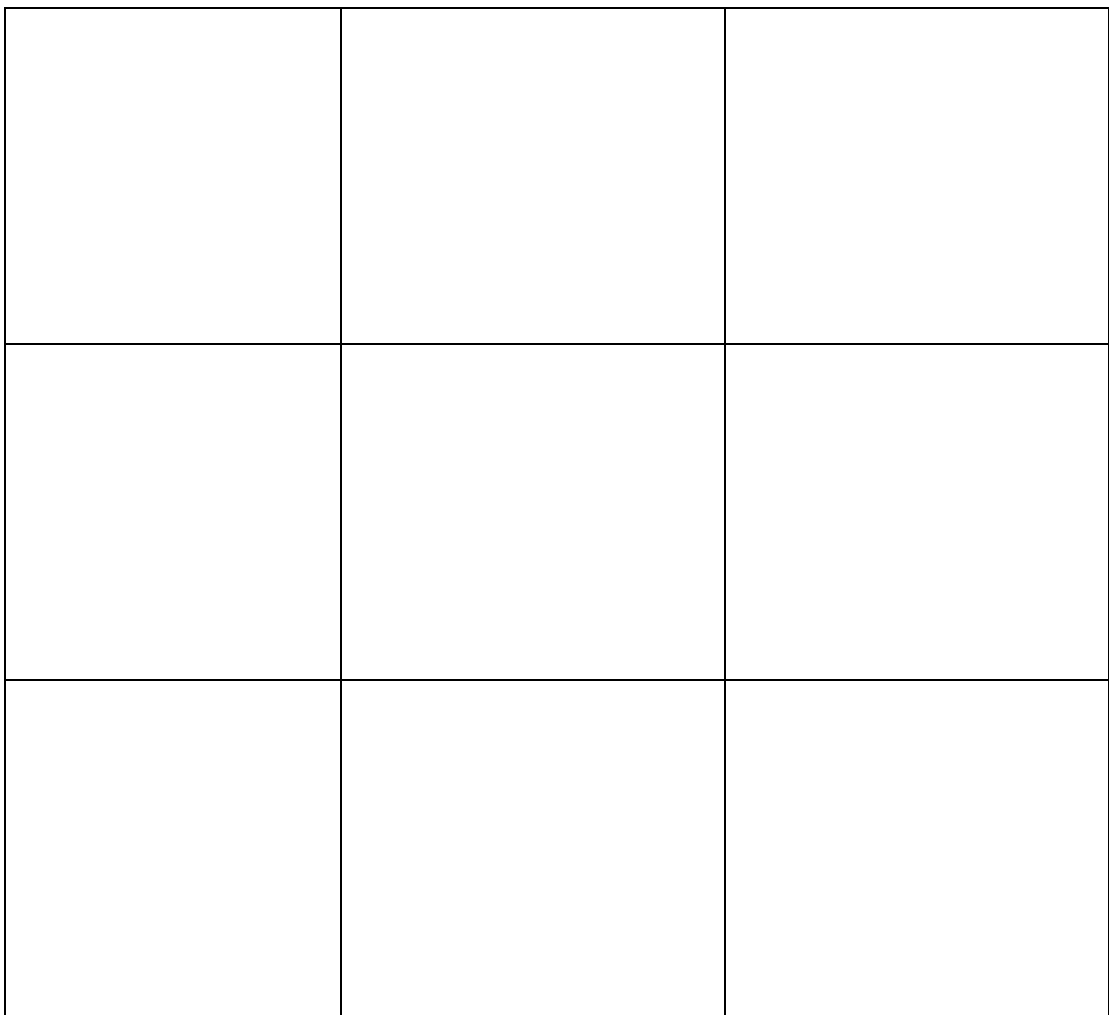
Activities

Related to

Similar Figure

List all the triangles in the figure that are similar to $\triangle MAN$.

Polymonies are made up of a number of squares connected by common sides. Thirteen sticks were used to make this one with four squares. Investigate the numbers of sticks needed to make others.

How many rectangles are there in this diagram?

Make a rectangular grid of squares. Cut it along the grid lines to make two identical pieces.