Module 3
Geometric Relations

What this module is about

This module is about the angles formed by parallel lines (//) cut by a transversal. You will learn to determine the relation between pairs of angles formed by parallel lines cut by a transversal and solve problems involving segments and angles.

What you are expected to learn

This module is designed for you to:

1. identify the angles formed by parallel lines cut by a transversal.

2. determine the relationship between pairs of angles formed by parallel lines cut by a transversal:

    - alternate interior angles
    - alternate exterior angles
    - corresponding angles
    - angles on the same side of the transversal

3. solve problems using the definition and properties involving relationships between segments and between angles.
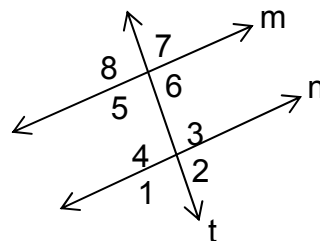
How much do you know

The figure below shows lines m // n with t as transversal.

Name:

1. 4 pairs of corresponding angles

2. 2 pairs of alternate interior angles

3. 2 pairs of alternate exterior angles

4. 2 pairs of interior angles on the same side of the transversal

5. 2 pairs of exterior angles on the same side of the transversal
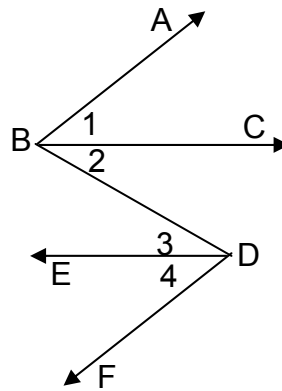
Using the same figure:

6. Name all numbered angles congruent to ∠7.

7. Name all numbered angles congruent to ∠4.

8. Name all numbered angles supplementary to ∠8, to ∠7.

9. Name all numbered angles supplementary to ∠3, to ∠4.

10. Name the pairs of equal angles and supplementary angles in the figure.

Given: $\overline{AB}$ // $\overline{CD}$
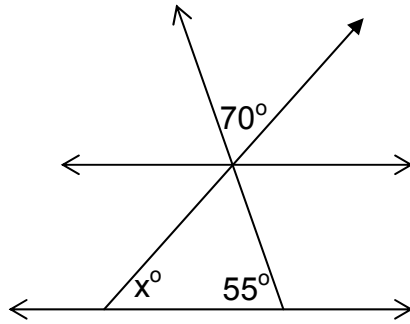 $\overline{AD}$ // $\overline{BC}$
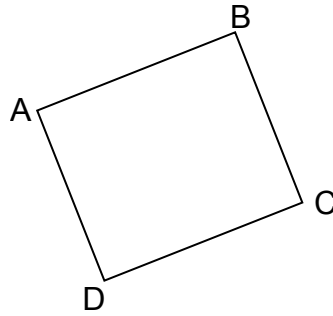
In the figure below,

$\overrightarrow{AB} \perp \overline{BD}$,

$\overrightarrow{DF} \perp \overline{BD}$,
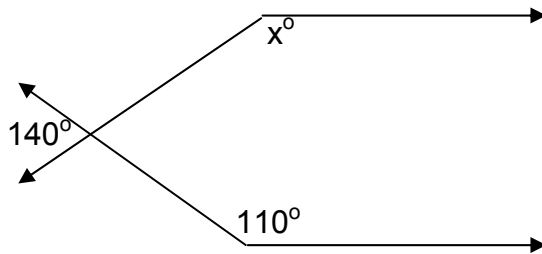
$\overrightarrow{BC}$ // $\overrightarrow{DE}$

11. Is m ∠2 = m ∠3? Why?

12. ∠2 is a complement of _____ and ∠3 is a complement of ____.

13. Is m ∠1 + m ∠2 = m ∠3 + m ∠4? Why?

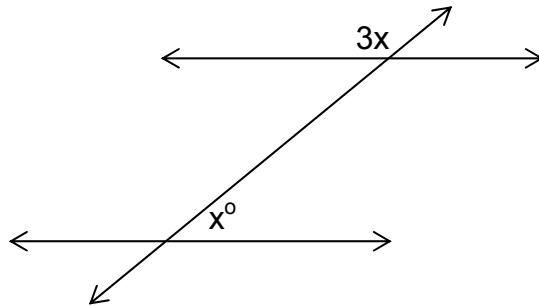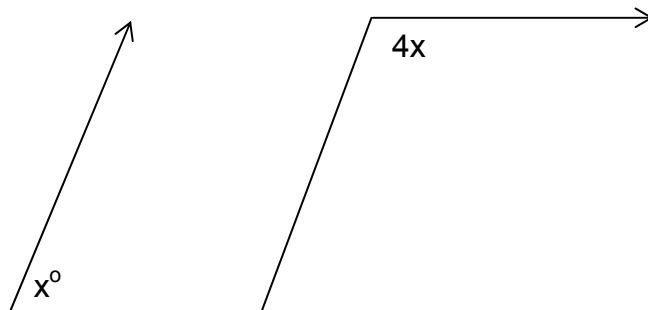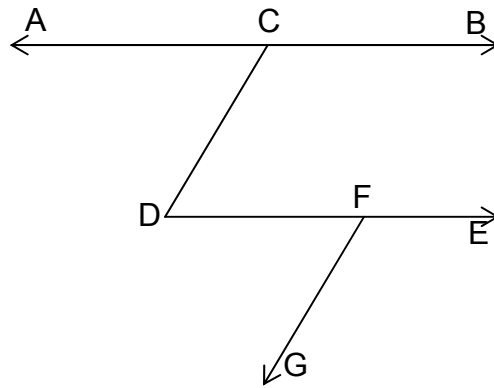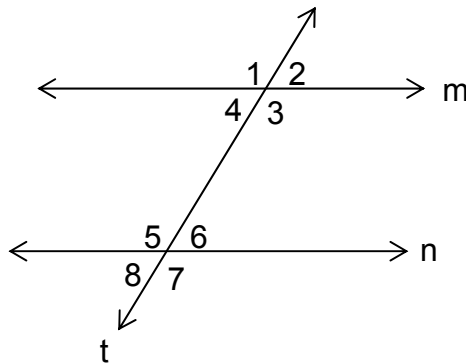Find the value of x in each of the following figure:

14.

15.

16.

17.

In the figure, write down the pairs of parallel lines and the pairs of congruent angles.
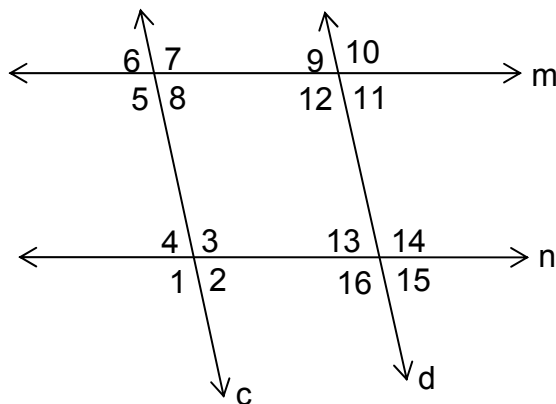
18.

19. If m∠3 = 135, find the measure of each angle in the figure.

20. If m∠6 = 85, find the measure of each numbered angle in the figure, a // b and c // d.
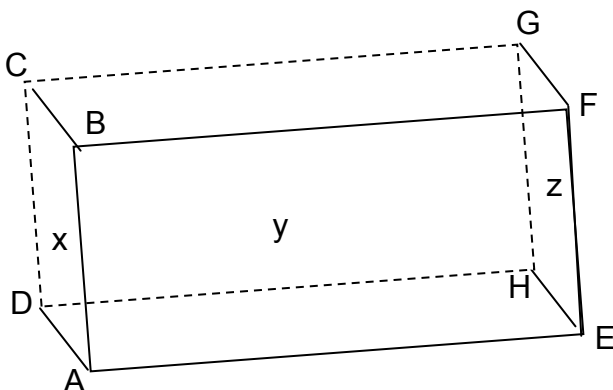
What you will do

Lesson 1

Angles Formed by Parallel Lines Cut by a Transversal

In the rectangular solid below, $\overline{AB}$ and $\overline{CD}$ are coplanar in plane x. $\overline{AB}$ and $\overline{EF}$ are coplanar in plane y. $\overline{EF}$ and $\overline{HF}$ are coplanar in plane z.

Line E intersect $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ at two different points. Line E is a transversal of lines AB and CD.

Definitions:

Coplanar lines are lines that lie in one plane.

Parallel lines are two lines that are coplanar and do not intersect.

Transversal is a line that intersects two or more lines at different points.
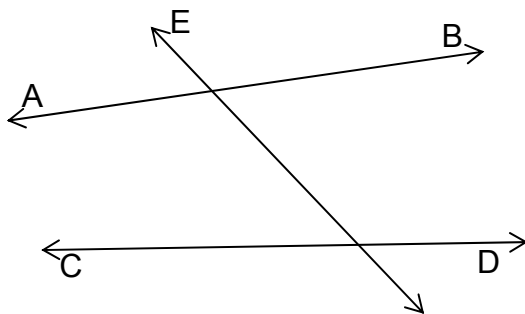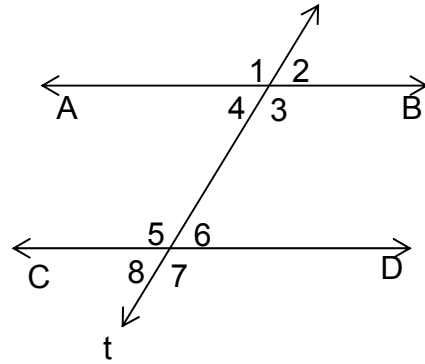

Line E intersect $\overline{AB}$ and $\overline{CD}$ at 2 different points. Line E is a transversal of lines AB and CD.

Examples:

1. In the figure, lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$
   are parallel lines cut by transversal
   line t. The angles formed are:

   Angles 1, 2, 7 and 8 are exterior angles

   Angles 3, 4, 5, and 6 are interior angles

   The pairs of corresponding angles are:

   > $\angle 1$ and $\angle 5$
   > $\angle 2$ and $\angle 6$
   > $\angle 4$ and $\angle 8$
   > $\angle 3$ and $\angle 7$

   The pairs of alternate interior angles are:

   > $\angle 3$ and $\angle 5$
   > $\angle 4$ and $\angle 6$

   The pairs of alternate exterior angles are:

   > $\angle 1$ and $\angle 7$
   > $\angle 2$ and $\angle 8$

   The pairs of exterior angles on the same side of a transversal (SST) are:

   > $\angle 1$ and $\angle 8$
   > $\angle 2$ and $\angle 7$

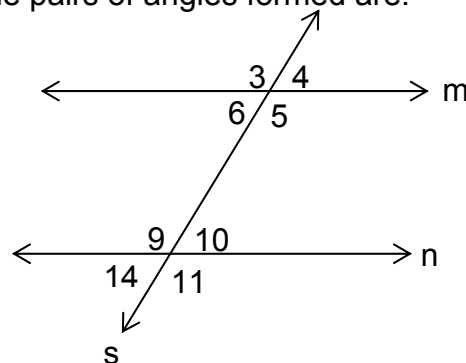   The pairs of interior angles on the same side of a transversal (SST) are:

   > $\angle 4$ and $\angle 5$
   > $\angle 3$ and $\angle 6$

2. Given: m // n, *s* is the transversal. The pairs of angles formed are:

   a. Corresponding angles:

   > $\angle 3$ and $\angle 9$
   > $\angle 6$ and $\angle 12$
   > $\angle 4$ and $\angle 10$
   > $\angle 5$ and $\angle 11$

b. Alternate exterior angles

∠3 and ∠11
∠4 and ∠12

c. Alternate interior angles

∠6 and ∠10
∠5 and ∠9

d. Interior angles on the same side of the transversal (SST).

∠6 and ∠9
∠5 and ∠10

e. Exterior angles on the same side of the transversal (SST).

∠3 and ∠12
∠4 and ∠11

3. m // n, t is the transversal.

In the figure, name and identify
    he pairs of angles formed:

```
              ↑
              |
   ←————————1 | 2————————→  m
            8 | 7
              |
              |
   ←————————9 |10————————→  n
           12 |11
              |
              ↓
              t
```

a. Corresponding angles:

∠1 and ∠9
∠8 and ∠12
∠2 and ∠10
∠7 and ∠11

b. Alternate interior angles
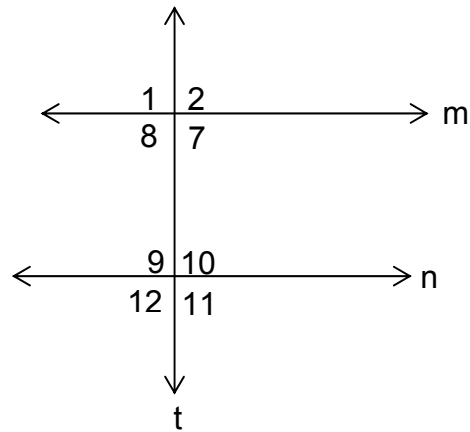
∠8 and ∠10
∠7 and ∠9

c. Alternate exterior angles

$\angle$1 and $\angle$11
$\angle$2 and $\angle$12

d. Exterior angles on the same side of the transversal (SST).
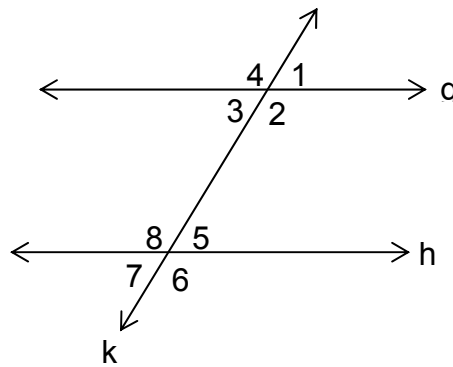
$\angle$1 and $\angle$12
$\angle$2 and $\angle$11

e. Interior angles on the same side of the transversal (SST).

$\angle$8 and $\angle$9
$\angle$7 and $\angle$10

Try this out

1. In the figure, lines g // h and is cut by line k.

Name and identify the pairs of angles formed

2. In the figure, q // r and s // t.
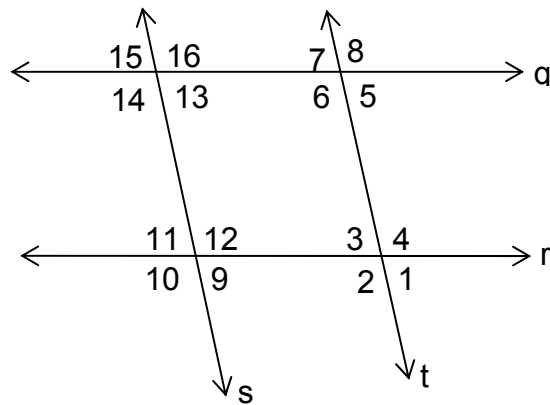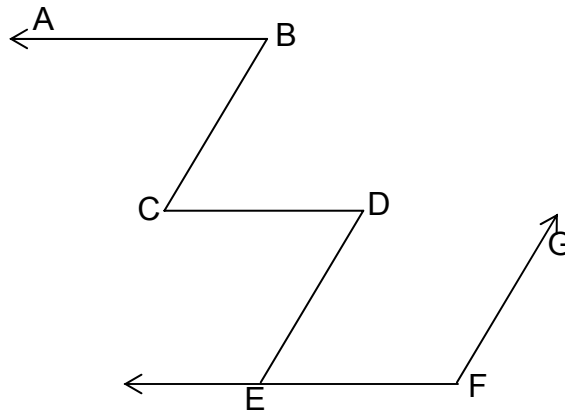

Name and identify the pairs of angles formed.

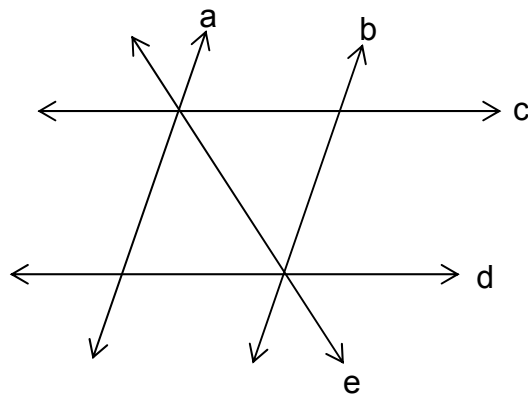3. In the figure, u // w. Lines x and y are transversals.
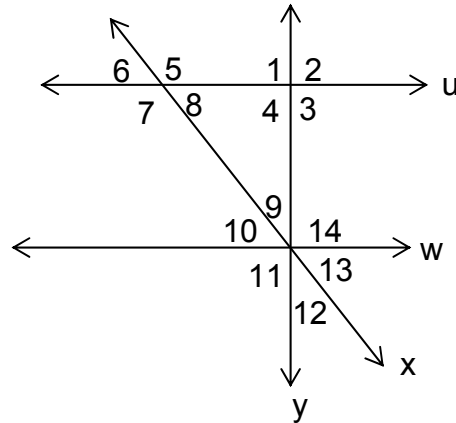
Name and identify the pairs of angles formed

4. In the figure, identify and name the pairs of parallel lines and its transversal.

5. In the figure, identify and name the pairs of parallel lines and its transversal / transversals.

Lesson 2

## Relationship Between Pairs of Angles Formed by Parallel Lines Cut by a Transversal

If two lines are cut by a transversal, then:

a. corresponding angles are congruent

b. alternate interior angles are congruent

c. alternate exterior angles are congruent

d. interior angles on the same side of a transversal are supplementary

e. exterior angles on the same side of a transversal are supplementary

Examples:

1. Given: p // q, r is a transversal

   Figure:
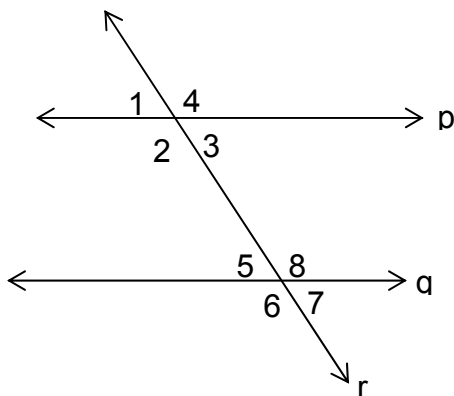
   What is the measure of each numbered angles if m∠1 = 120? Give the reason for your answer.

   Answers:

   If ∠1 = 120°, ∠5 = 120°   Corresponding angles are ≅ .
   If ∠5 = 120°, ∠3 = 120°   Alternate interior angles are ≅
   If ∠3 = 120°, ∠7 = 120°   Corresponding angles are ≅
   If ∠7 = 120°, ∠4 = 60°   Exterior angles on the same side
                                    of a transversal are supplementary
   If ∠4 = 60°, ∠8 = 60°   Corresponding angles are ≅
   If ∠8 = 80°, ∠2 = 60°   Alternate interior angles are ≅
   If ∠2 = 60°, ∠6 = 60°   Corresponding angles are ≅

2. Given: In the figure, if $m\angle 1 = 105$, determine the measures of the other numbered angles. Justify your answers.

Figure:

Answers:

If $\angle 1 = 105^{\circ}$, $\angle 5 = 105^{\circ}$	Corresponding angles are $\cong$.
If $\angle 5 = 105^{\circ}$, $\angle 3 = 105^{\circ}$	Alternate interior angles are $\cong$
If $\angle 3 = 105^{\circ}$, $\angle 7 = 105^{\circ}$	Corresponding angles are $\cong$
If $\angle 7 = 105^{\circ}$, $\angle 2 = 75^{\circ}$	Exterior angles on the same side of a transversal are supplementary
If $\angle 2 = 75^{\circ}$, $\angle 8 = 75^{\circ}$	Alternate exterior angles are $\cong$
If $\angle 8 = 75^{\circ}$, $\angle 4 = 75^{\circ}$	Corresponding angles are $\cong$
If $\angle 4 = 75^{\circ}$, $\angle 6 = 75^{\circ}$	Alternate interior angles are $\cong$

3. Given: m // n, s and t are the transversals

If $m\angle 5 = 110$ and $m\angle 12 = 90$, determine the measures of the other numbered $\angle$s. Justify your answer.

Answers:

If $m\angle 5 = 110$, $m\angle 12 + m\angle 13 = 110$
Since $m\angle 12 = 90$, $m\angle 13 = 20$.   Corresponding angles are $\cong$

$m\angle 4 = 70$ $\angle 4$ and $\angle 5$ are supplementary

If $m\angle 4 = 70$, $m\angle 11 = 70$ Corresponding angles are $\cong$

$m\angle 3 = 110$ $\angle 3$ and $\angle 5$ are vertical angles

11

If m∠3 = 110, m∠9 + m∠10 = 110 Corresponding angles are ≅

If m∠9 = 90, m∠7 = 90   Alternate interior angles are ≅

If m∠12 = 90, m∠2 = 90   Exterior angles on the same side
               of a transversal are supplementary
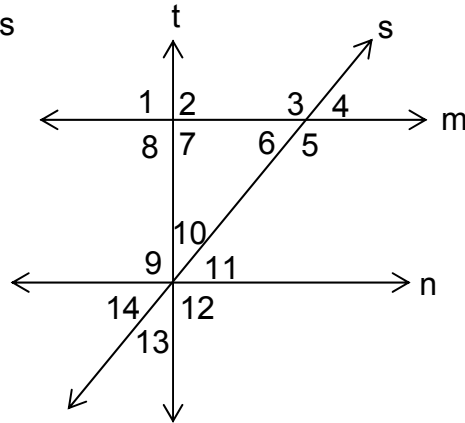
Since m∠9 = 90, m∠1 = 90   Corresponding angles are ≅

If m∠13 + m∠14 = m∠8, m∠8 = 90 Corresponding angles are ≅

Therefore, the measures of the numbered angles are:

∠1 = 90°	∠8 = 90°
∠2 = 90°	∠9 = 90°
∠3 = 110°	∠10 = 20°
∠4= 70°	∠11 = 70°
∠5 = 110°	∠12 = 90°
∠6 = 70°	∠13 = 20°
∠7 = 90°	∠14 = 70°

Try this out

1. If m∠6 = 85, find the measure of the other numbered angles. Justify your answers.

   Given: a // b, c is the transversal

   Figure:
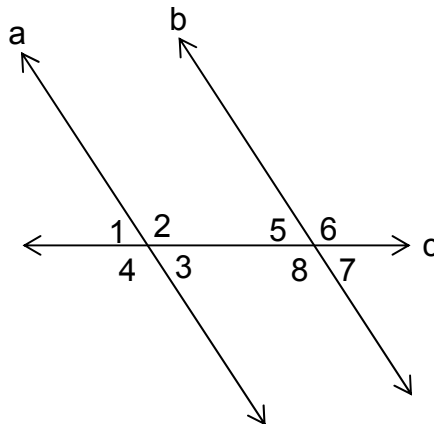
2. If $m\angle 10 = 118$ and $m\angle 4 = 85$, find the measures of the other numbered angles. Justify your answers.

Given: f // g, r // q

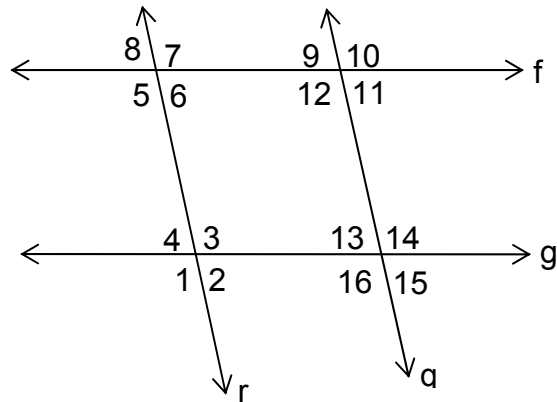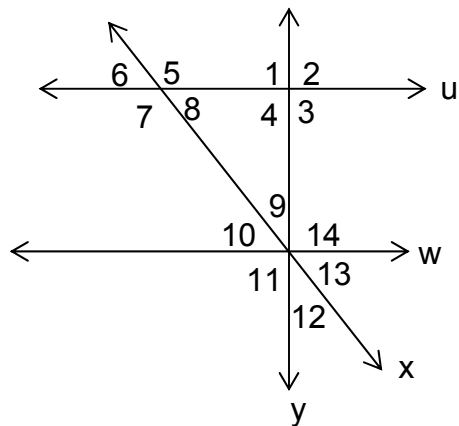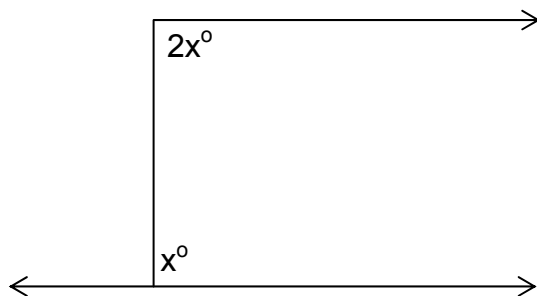3. In the figure, u // w. Lines x and y are transversals. If $m\angle 8 = 62$, $m\angle 14 = 90$, find the measures of the other numbered $\angle$s. Justify your answers.

Figure:

Form an equation in x and solve the equation.

4.

5.
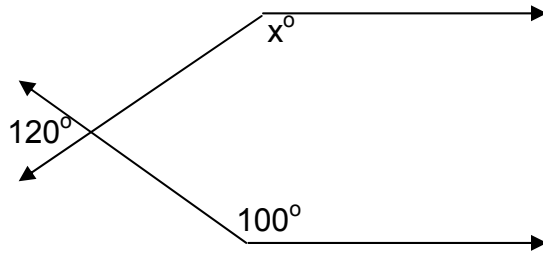
 Let' summarize

The angles formed by parallel lines cut by a transversal are:

1. corresponding angles
2. alternate interior angles
3. alternate exterior angles
4. exterior angles on the same side of a transversal
5. interior angles on the same side of a transversal

The relationship of the angles formed by parallel lines cut by a transversal are:
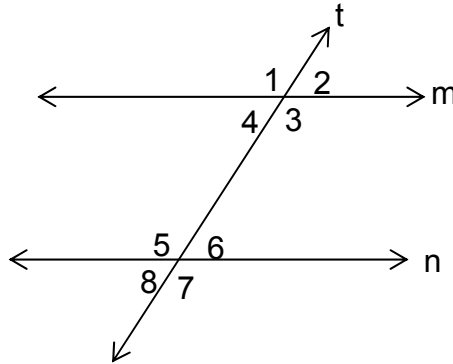
1. pairs of corresponding s are $\cong$
2. pairs of alternate interior angles are $\cong$
3. pairs of alternate exterior angles are $\cong$
4. pairs of interior angles on the same side of a transversal are supplementary
5. pairs of exterior angles on the same side of a transversal are supplementary

The figure below shows lines m // n with t as transversal.

Figure:

Name:

1. 4 pairs of corresponding angles.
2. 2 pairs of alternate interior angles.
3. 2 pairs of alternate exterior angles.
4. 2 pairs of interior angles on the same side of a transversal.
5. 2 pairs of exterior angles on the same side of a transversal.

Using the same figure:

6. Name all numbered angles congruent to ∠7.
7. Name all numbered angles congruent to ∠4.
8. Name all numbered angles supplementary to ∠8, ∠7.
9. Name all numbered angles supplementary to ∠3, ∠4.
10. Name the pairs of equal angles and supplementary angles in the figure.

Given: $\overline{AB}$ // $\overline{CD}$ Figure:
$\overline{AD}$ // $\overline{BC}$
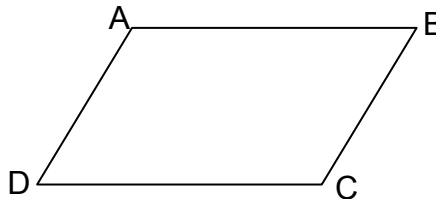
11. – 13. In the figure, $\overrightarrow{AB} \perp \overline{BD}$, $\overrightarrow{DF} \perp \overline{BD}$, $\overrightarrow{BC} \mathbin{//} \overrightarrow{DE}$

Figure:
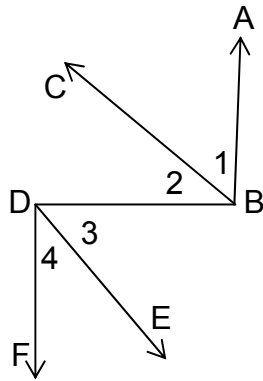
11. Is $m\angle 2 = m\angle 3$? Why?

12. $\angle 2$ is a complement of _____ and $\angle 3$ is a complement of _____.

13. Is $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$? Why?

Find the value of x in each of the following figures.

14.
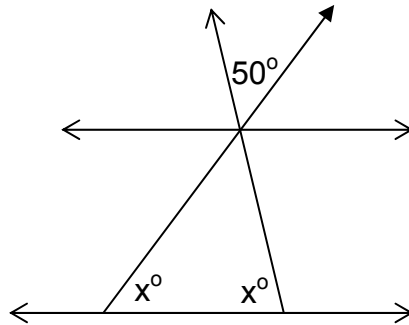
15.
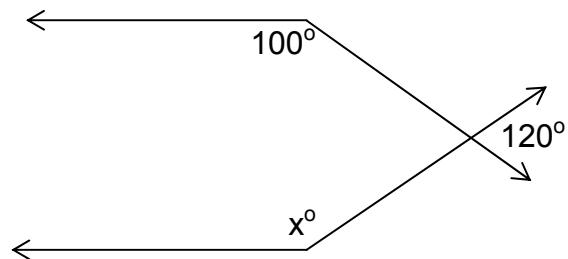
16.
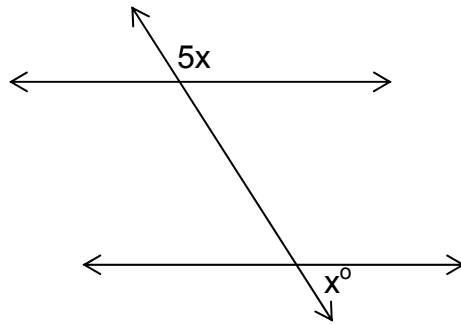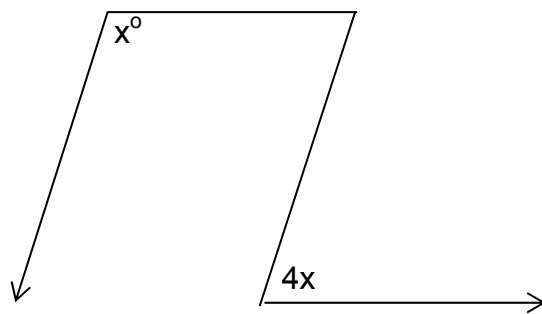
5x

x°

17.

x°

4x

In the figure, Write down the pairs of parallel lines and the pairs of congruent angles.

18.

B   A

F   D   C

E
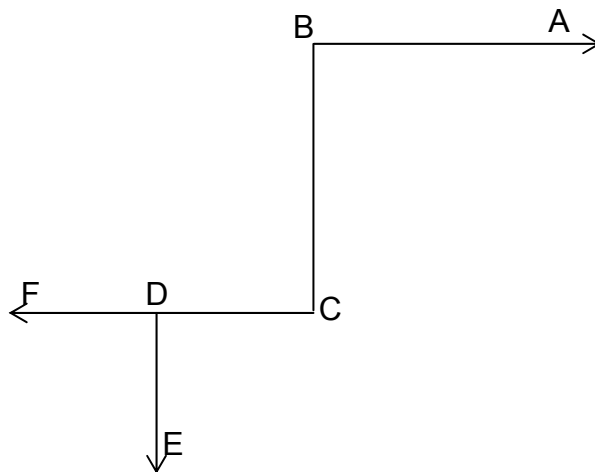
19. If m∠1 = 135, find the measure of each angle in the figure below:
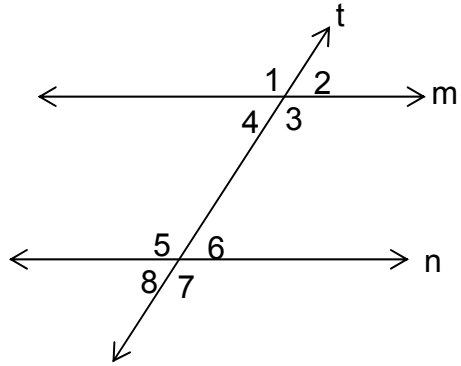
20. If m∠6 = 75, find the measure of each numbered angle in the figure, a //
b and c // d.
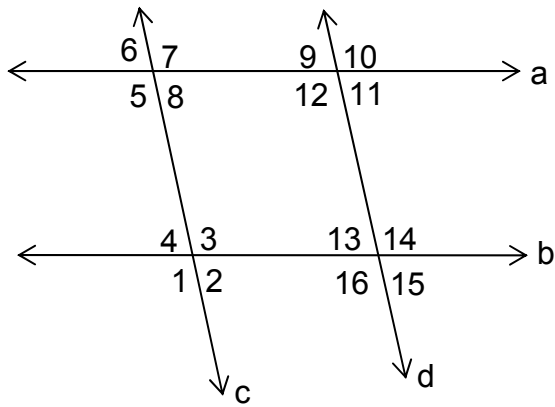
Answer key

How much do you know

1. ∠8 and ∠4,   ∠7 and ∠3

   ∠5 and ∠1,   ∠6 and ∠2

2. ∠5 and ∠3,   ∠4 and ∠6

3. ∠8 and ∠2,   ∠7 and ∠1

4. ∠5 and ∠4,   ∠6 and ∠3

5. ∠8 and ∠1,   ∠7 and ∠2

6. ∠3, ∠1

7. ∠8, ∠6

8. To ∠8: ∠1, ∠5, ∠7;   To ∠7: ∠2, ∠8, ∠6

9. To ∠3: ∠6, ∠4, ∠2;   To ∠4: ∠5, ∠3, ∠1

10. ∠A ≅ ∠C, ∠B ≅ ∠D, ∠A supplement ∠B, ∠B supplement ∠C

    ∠C supplement ∠D, ∠D supplement ∠A

11. They are alternate interior ∠s

12. ∠1, ∠4

13. Yes, because they are right angles.

14. x = 55°

15. x = 110°

16. x = 45°

17. x = 36°

18. $\overleftrightarrow{AB}$ // $\overrightarrow{DE}$; $\overline{CD}$ // $\overrightarrow{FG}$; ∠ACD ≅ ∠EDC; ∠EFG ≅ ∠CDF

19. ∠2 = 45°; ∠4 = 45°; ∠1 = 135°; ∠7 = 135°; ∠8 = 45°; ∠6 = 45°; ∠5

    = 135°

20. ∠4 = 85°; ∠5 = 95°; ∠1 = 95°; ∠7 = 95°; ∠3 = 95°; ∠8 = 85°; ∠2 = 85°

    ∠10 = 95°; ∠9 = 85°; ∠12 = 95°; ∠11 = 85°; ∠13 = 85°; ∠14 = 95°;

    ∠16 = 95°; ∠15 = 85°

Try this out

Lesson 1

1. $\angle 4$ and $\angle 8$; $\angle 3$ and $\angle 7$; $\angle 1$ and $\angle 5$; $\angle 2$ and $\angle 6$ are corresponding angles;

   $\angle 4$ and $\angle 7$; $\angle 1$ and $\angle 6$ are exterior angles on SST;

   $\angle 3$ and $\angle 5$; $\angle 2$ and $\angle 8$ are alternate interior angles;

   $\angle 4$ and $\angle 7$; $\angle 1$ and $\angle 6$ are exterior angles on SST;

   $\angle 3$ and $\angle 5$; $\angle 2$ and $\angle 8$ are alternate interior angles;

   $\angle 3$ and $\angle 8$; $\angle 2$ and $\angle 5$ are interior angles on SST;

   $\angle 4$ and $\angle 6$; $\angle 1$ and $\angle 8$ are exterior angles on SST;

2. $\angle 1$ and $\angle 5$; $\angle 4$ and $\angle 8$; $\angle 2$ and $\angle 6$; $\angle 3$ and $\angle 7$ are corresponding angles;

   $\angle 4$ and $\angle 6$; $\angle 3$ and $\angle 5$ are alternate interior angles;

   $\angle 7$ and $\angle 2$; $\angle 8$ and $\angle 1$ are exterior angles on SST;

   $\angle 6$ and $\angle 3$; $\angle 5$ and $\angle 4$ are interior angles on SST;

   $\angle 5$ and $\angle 13$; $\angle 6$ and $\angle 14$; $\angle 7$ and $\angle 15$; $\angle 8$ and $\angle 16$ are corresponding angles;

   $\angle 8$ and $\angle 15$; $\angle 5$ and $\angle 14$ are exterior angles on SST;

   $\angle 7$ and $\angle 16$; $\angle 6$ and $\angle 13$ are interior angles on SST;

   $\angle 16$ and $\angle 6$; $\angle 13$ and $\angle 7$ are alternate interior angles;

   $\angle 15$ and $\angle 5$; $\angle 14$ and $\angle 8$ are alternate exterior angles;

   $\angle 15$ and $\angle 11$; $\angle 14$ and $\angle 10$; $\angle 16$ and $\angle 12$; $\angle 13$ and $\angle 9$ are corresponding angles;

   $\angle 14$ and $\angle 11$; $\angle 13$ and $\angle 12$ are interior angles on SST;

   $\angle 15$ and $\angle 10$; $\angle 16$ and $\angle 9$ are exterior angles on SST;

   $\angle 4$ and $\angle 12$; $\angle 13$ and $\angle 11$ are alternate interior angles;

   $\angle 15$ and $\angle 9$; $\angle 16$ and $\angle 10$ are alternate exterior angles;

   $\angle 3$ and $\angle 11$; $\angle 4$ and $\angle 12$; $\angle 2$ and $\angle 10$; $\angle 1$ and $\angle 9$ are corresponding angles;

   $\angle 12$ and $\angle 2$; $\angle 3$ and $\angle 9$ are alternate interior angles;

   $\angle 11$ and $\angle 1$; $\angle 4$ and $\angle 10$ are alternate exterior angles;

$\angle$ 1and $\angle$ 10; $\angle$ 4 and $\angle$ 11 are exterior angles on SST;

$\angle$ 3 and $\angle$ 12; $\angle$ 2 and $\angle$ 9 are interior angles on SST;

3. $\angle$ 1 and $\angle$ 9 + $\angle$ 10; $\angle$ 4 and $\angle$ 11 are corresponding angles;

$\angle$ 2 and $\angle$ 14; $\angle$ 3 and $\angle$ 12 + $\angle$ 13 are corresponding angles;

$\angle$ 4 and $\angle$ 14; $\angle$ 3 and $\angle$ 9 + $\angle$ 10 are alternate interior angles;

$\angle$ 1 and $\angle$ 12 +$\angle$ 13; $\angle$ 2 and $\angle$ 11 are alternate exterior angles;

$\angle$ 1 and $\angle$ 11; $\angle$ 2 and $\angle$ 12 + $\angle$ 13 are exterior angles on SST;

$\angle$ 4 and $\angle$ 9 +$\angle$ 10; $\angle$ 3 and $\angle$ 14 are interior angles on SST;

$\angle$ 6 and $\angle$ 10; $\angle$ 7 and $\angle$ 11; $\angle$ 5 and $\angle$ 9 + $\angle$ 14; $\angle$ 8 and $\angle$ 13 are corresponding angles;

$\angle$ 7 and $\angle$ 9 + $\angle$ 14; $\angle$ 8 and $\angle$ 10 are alternate interior angles;

$\angle$ 6 and $\angle$ 13; $\angle$ 5 and $\angle$ 11 + $\angle$ 12 are alternate exterior angles;

$\angle$ 6and $\angle$ 11 + $\angle$ 12; $\angle$ 5 and $\angle$ 13 are exterior angles on SST;

$\angle$ 7 and $\angle$ 10; $\angle$ 8 and $\angle$ 9 + $\angle$ 14 are interior angles on SST;

4. $\overleftrightarrow{AB}$ // $\overline{CD}$ with $\overline{BC}$ as transversal

$\overline{CD}$ // $\overrightarrow{FE}$ with $\overline{DE}$ as transversal

$\overline{DE}$ // $\overrightarrow{FG}$ with $\overline{FE}$ as transversal

5. a // b, e, c and d are the transversals

6. c // d, a, b and c are the transversals

Lesson 2

1. $\angle$ 2 = 85°; $\angle$ 5 = 95°; $\angle$ 1 = 95°; $\angle$ 7 = 95°; $\angle$ 3 = 95°; $\angle$ 8 = 85°;

$\angle$ 4 = 85°

2. $\angle$ 12 = 118°; $\angle$ 9 = 62°; $\angle$ 11 = 62°; $\angle$ 16 = 118°; $\angle$ 13 = 62°;

$\angle$ 14 = 118°; $\angle$ 15 = 62°

$\angle$ 8 = 85°; $\angle$ 6 = 85°; $\angle$ 2 = 85°; $\angle$ 1 = 95°; $\angle$ 5 = 95°; $\angle$ 3 = 95°;

$\angle$ 1 = 95°

3. $\angle$ 8 = 62°; $\angle$ 6 = 62°; $\angle$ 10 = 62°; $\angle$ 13 = 118°; $\angle$ 9 = 28°;

$\angle$ 12 = 28°; $\angle$ 11 = 90°; $\angle$ 13 = 62°; $\angle$ 7 = 118°; $\angle$ 5 = 118°; $\angle$ 1 = 90°;

$\angle$ 2 = 90° ; $\angle$ 3 = 90°; $\angle$ 4 = 90°

4. $x = 60^o$

5. $x = 140^o$

What have you learned

1. $\angle 1$ and $\angle 5$, $\angle 4$ and $\angle 8$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$

2. $\angle 4$ and $\angle 6$, $\angle 3$ and $\angle 5$

3. $\angle 1$ and $\angle 7$, $\angle 2$ and $\angle 8$

4. $\angle 4$ and $\angle 5$, $\angle 3$ and $\angle 6$

5. $\angle 1$ and $\angle 8$, $\angle 2$ and $\angle 7$

6. $\angle 3$, $\angle 5$, $\angle 1$

7. $\angle 8$, $\angle 6$, $\angle 2$

8. $\angle 8$: $\angle 5$, $\angle 7$, $\angle 1$;　　　$\angle 7$: $\angle 2$, $\angle 6$, $\angle 8$

9. $\angle 3$: $\angle 6$, $\angle 4$, $\angle 2$;　　　$\angle 4$: $\angle 5$, $\angle 3$, $\angle 1$

10. $\angle A \cong \angle C$;　$\angle B \cong \angle D$;　$\angle A$ supplementary $\angle B$
　　　　　　　　　　　　　　　　$\angle B$ supplementary $\angle C$
　　　　　　　　　　　　　　　　$\angle C$ supplementary $\angle D$
　　　　　　　　　　　　　　　　$\angle A$ supplementary $\angle D$

11. Yes, they are alternate interior angles

12. $\angle 1$, $\angle 4$

13. Yes, because their sum is equal to 90°

14. $x = 65^o$

15. $x = 140^o$

16. $x = 30^o$

17. $x = 36^o$

18. $\overrightarrow{BA} \mathbin{/\mkern-5mu/} \overrightarrow{CF}$, $\overline{BC} \mathbin{/\mkern-5mu/} \overrightarrow{DE}$, $\angle ABC \cong \angle BCF$;　$\angle BCD \cong \angle CDE$

19. $\angle 2 = 45^o$, $\angle 3 = 135^o$, $\angle 4 = 45^o$, $\angle 5 = 135^o$, $\angle 6 = 45^o$, $\angle 7 = 135^o$, $\angle 8 = 45^o$

20. $\angle 4 = 75^o$, $\angle 1 = 105^o$, $\angle 5 = 105^o$, $\angle 7 = 105^o$, $\angle 3 = 105^o$, $\angle 8 = 75^o$, $\angle 2 = 75^o$, $\angle 9 = 75^o$, $\angle 10 = 105^o$, $\angle 12 = 105^o$, $\angle 11 = 75^o$