Module 2 Geometríc Relations

What this module is about

This module will explain to you different characteristics of lines on a plane. In this module, you will discover that in a plane, lines may be parallel, perpendicular or just intersecting. This module will introduce you on the relationship among sides and angles of a triangle. You will also discover some theorems associated with perpendicularity and inequalities among sides and angles of a triangle. In addition, this module will define characteristics of lines in a plane.



This module is designed for you to

- 1. define and illustrate perpendicular and parallel lines
- 2. define and give examples of the perpendicular bisector of a segment.
- 3. define and illustrate exterior angles of a triangle and its relationships with other angles of a triangle.
- 4. define inequalities among angles and sides of a triangle.
- 5. illustrate lines that serve as transversal.



Answer the following questions.

1. If $l_1 \perp l_2$, then $\angle 1$ is _____ angle.

2. Given: $\overline{AB} \perp \overline{BC}$. If $\angle ABE = 2x + 15$, and $\angle EBC = x$, what is m $\angle ABE$?



- 3. An angle which is adjacent and supplementary to one of the angles of a triangle is a _____ angle.
- 4. In the figure, $m \angle EFD + m \angle EFG =$ _____
- 5. What property of inequality supports the statement, If m > 5 and 5 > n, then m > n.
- 6. If WY = 5 and YX = 8, what is the range of values of WX?



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- 7. If B is the midpoint of \overline{AX} , then \overline{AB} _____ \overline{BX} .
- 8. What is the length of the hypotenuse of a right triangle whose length of the legs are 3 cm and 4 cm respectively.?

9. Coplanar lines that do not intersect are called _____

In the figure, line *τ* intersects lines
 w and *w* at two points. *τ* is called ______

Vhat you will do

Lesson 1

Perpendicular Lines and Perpendicular Bisector of Segment

In the figure, two lines *a* and *b* intersect at point X, forming four angles. Two of these angles are $\angle 1$ and $\angle 2$. If all these angle are right angles, then <u>*a* is perpendicular to b</u>. The symbol to be used for perpendicular is " \perp ". The figure above can be represented in symbols as $a \perp b$.



We will define perpendicular lines as follows:

Two lines are perpendicular if and only if they intersect to form right angles. Since $a \perp b$ then $\angle 1$ and $\angle 2$ are right angles. The two other angles in the figure are also right angles. In the definition stated, there is the phrase if and only if (iff) which means that the definition is two way. 1) If the lines are perpendicular, then the angles formed are right angles and 2) if

the angles are right angles, then the lines or sides of the angle are perpendicular. This goes to show that one way of proving perpendicularity is to prove that the angles formed are right angles.

For you to recognize perpendicular lines, a small square is indicated at the foot of the perpendiculars which is the intersection.

Consider the given figure. $\overline{OD} \perp \overline{XM}$ at O. Hence there is a symbol of small square at the intersection. Notice that \overline{XM} is divided into two segments \overline{XO} and \overline{OM} which has similar markings. Those X _____ N Markings indicate that $\overline{XO} \cong \overline{OM}$ making O _____ O

The midpoint of the segment. Therefore *OD* is the <u>perpendicular bisector</u> of \overline{XM} .

The perpendicular bisector of a segment is a line, ray, segment or plane that is perpendicular to the segment at its midpoint. There are four cases by which a segment has its perpendicular bisector.



Case 4 Plane P is the perpendicular bisector of \overline{XY}



"Any two right angles are congruent". All right angles measure 90°.







Prove: $\overline{AE} \perp \overline{AT}$ Proof:

Statement

1.	\overline{AE} bisects $\angle DAS$
	\overline{AT} bisects \angle SAM
2.	$\angle 1 \cong \angle 2, \ \angle 3 \cong \angle 4$
3.	$m \angle 1 = m \angle 2, m \angle 3 = m \angle 4$
4.	$m \angle DAS + m \angle SAM = 180$
5.	$m \angle DAS = m \angle 1 + m \angle 2$
	$m \angle SAM = m \angle 3 + m \angle 4$
6.	$m \angle 1 + m \angle 2 + m \angle 3 + m \angle 4 = 180$
7.	$m \angle 2 + m \angle 2 + m \angle 3 + m \angle 3 = 180$
8.	$2 m \angle 2 + 2 m \angle 3 = 180$
9.	$m \angle 2 + m \angle 3 = 90$
10	$m \angle EAT = m \angle 2 + m \angle 3$
11	. m∠EAT = 90
12	. $\angle EAT$ is a right angle
13	$\overline{AE} \perp \overline{AT}$

Reason

- 1. Given
- 2. Definition of _____
- 3. Definition of congruent angles
- 4. Definition of _____
- 5. Angle _____ Postulate
- 6. _____ Property of Equality
- 7. _____ Property of Equality
- 8. Combining like terms
- 9. Multiplication Property of Equality
- 10. Angle Addition Postulate
- 11. Substitution
- 12. Definition of a _____.
- 13. Definition of

Lesson 2

Exterior Angle of Triangle and Triangle Inequality

Given $\triangle ABC$. If you extend side \overline{AC} through point D, then there is a new angle formed, $\angle BCD$. $\angle BCD$ is both adjacent and supplementary to one of the angles of $\triangle ABC$. $\angle BCD$ is called an <u>exterior angle</u> of $\triangle ABC$. Two angles, $\angle A$ and $\angle B$ are



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interior angles of $\triangle ABC$ which are not adjacent to $\angle BCD$. Therefore, $\angle A$ and $\angle B$ are called <u>remote interior angles</u> of the exterior angle. $\angle BCD$ is adjacent to $\angle BCA$. So $\angle BCA$ is called the adjacent interior angle.

An exterior angle of a triangle is an angle which is adjacent and supplementary to one of the angles of a triangle. The remote interior angles are angles of a triangle which are not adjacent to the given exterior angle of the triangle. Adjacent interior angle is an interior angle which forms a linear pair with the given exterior angle.

Illustration: Given $\triangle RST$.

 \angle STP is an exterior angle.

 \angle STP and \angle STR are adjacent and supplementary.

 \angle S and \angle R are the two remote interior angles. \angle STR is the adjacent interior angle to \angle STP.

Exterior Angle Equality Theorem

The measure of an exterior angle of a triangle is equal to the sum of its two remote interior angles. In $\triangle CEF$, $\angle FED$ is an exterior angle

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Conclusions:

Since $\angle C$ and $\angle F$ are the two remote interior angles.

Therefore, $m \angle FED = m \angle C + m \angle F$

Example:

If in the given triangle \triangle MRE, $m \angle M = 31$, $m \angle R = 34$ then, $m \angle RED = m \angle M + m \angle R$ $m \angle RED = 31 + 34$ $m \angle RED = 65$ and $m \angle REM = 180 - 65$ $m \angle REM = 115$



Since in the latter part of the lesson, you will be dealing with inequalities, it is necessary that you recall some properties of inequality which are of great help to you in proving statements in Geometry.

Let a, b, x and y be real numbers.

Trichotomy Property. Exactly one of the following is true: a < b; a = b; or a > bAddition Property of Inequality If a > b, then a + x > b + xSubtraction Property of Inequality If a > b, then a - x > b - x and x - b > x - aMultiplication Property of Inequality If a > b and y > 0, then ay > byIf a > b and y < 0, then ay < byTransitive Property of Inequality

If a > b and b > c, then a > c.

This properties of inequality can be applied to geometric figures since the measures of angles and segments are real numbers.

Illustrations: For segments: $\overline{AB} > \overline{CD}$ if and only if AB > CD $\overline{AB} < \overline{CD}$ if and only if AB < CD For angles: $\angle A > \angle B$ if and only if $m \angle A > m \angle B$ $\angle A < \angle B$ if and only if $m \angle A < m \angle B$

In Geometry, whenever possible, every statement is supported by reasons. In this lesson, you will find theorems are given but are not proven formally. Instead, illustrations are provided for easier understanding. You may opt to prove them though but you are allowed to use these theorems once you have gone through the illustrations and examples.

<u>Theorem:</u> The whole is greater than any of its parts. The meaning of this theorem is very clear to see.

Examples:

1. 10 = 7 + 3 Therefore: 10 > 7 or 10 > 3 2. Given PR with point Q in between P and R. By definition of betweenness, PR = PQ + QR Therefore: PR > PQ PR > QR



 Given ∠X and C is in the interior of ∠X By the Angle Addition Postulate, m∠X = m∠AXC + m∠CXB

> Therefore by using the theorem, $m \angle X > m \angle AXC$ $m \angle x > m \angle CXB$

<u>Theorem: Exterior Angle Inequality Theorem.</u> The measure of an exterior angle of a triangle is greater than the measure of either of the two remote interior angles.



Examples:

- 1. If $m \angle BAG = 113$, then $m \angle C < 113$ and $m \angle B < 113$ But $m \angle C + m \angle B = 113$.
- 2. If $m \angle C = 28$, then $m \angle BAG > 28$.

Theorem: Triangle Inequality Theorem. In any triangle, the sum of the lengths of any two of its sides is greater than the length of its third side.

Illustration:

In $\triangle ABC$, the following side inequalities hold

- a. AB + BC > AC
- b. AC + BC > AB
- c. AC + AB > BC



Examples:

Determine if the following segments whose given measures are sides of a triangle.

- 1. 5 cm, 3 cm, 4 cm
- 2. 10 cm, 11 cm, 6 cm
- 3. 3 cm, 2 cm, 1 cm

Solution:

- 5+3>4 3+4>5 5+4>3
 Conclusion: Since the sum of any two sides is greater than the third side, then 5cm, 3cm and 4cm are measures of the sides of a triangle.
 10+11>6 11+6>10 10+6>11
 Conclusion: Since the sum of any two sides is greater than the third side, then 10 cm, 11 cm and 6cm are measures of the sides of a triangle.
 3+2>1 3+1>2 2+1=3
 Conclusion: Since one of the sum is not greater than the third side then, 3cm, 2cm and 1 cm are not measures of sides of a triangle.
- 4. If in the given triangle, the lengths of the two sides are given, what is the range of the length of the third side, \overline{PQ} ?
 - a. 5+8 > PQ 13 > PQ
 b. 5+PQ > 8 PQ > 8 - 5 PQ > 3
 c. 8+PQ > 5 PQ > 5 - 8 PQ > - 3 (This cannot be since the side of triangle is always positive)
 - d. Combining statements a and b into a single statement will give you 13 > PQ and PQ > 3 which is written as 13 > PQ > 3.

Reversing the order, the statement can be written as 3 < PQ < 13.

So the range of the third side is 3 < PQ < 13. If you have noticed, that 3 is the difference between the lengths of the two given sides, while 13 is the sum of the lengths of the two given sides.

The Pythagorean Theorem states that "In any right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the two legs."

Illustration:

Given: $\triangle DEF$ s a right triangle. FE is the length of the hypotenuse DE and FD are the lengths of the legs Conclusion: DE² + FD² = FE²

Examples:

1. Given a right triangle, find x Solution: $c^2 = a^2 + b^2$ $x^2 = 3^2 + 4^2$ $x^2 = 9 + 16$ $x^2 = 25$ $x = \sqrt{25}$ x = 52. If c = 13, a = 12, find b. Solution: $c^2 = a^2 + b^2$ $13^2 = 12^2 + b^2$ $169 = 144 + b^2$ $b^2 = 169 - 144$ $b^2 = 25$ $b = \sqrt{25}$ b = 53. If c = 10, b = 6, find a. $c^2 = a^2 + b^2$







3. If c = 10, b = 6, find a.

$$c^2 = a^2 + b^2$$

 $10^2 = a^2 + 6^2$
 $100 = a^2 + 36$
 $a^2 = 100 - 36$
 $a^2 = 64$
 $a = \sqrt{64}$
 $a = 8$

4. The side of a square is 3 cm. Find the length of its diagonal. Solution:

x^2 x^2	$= 3^{2} + 3^{2}$ = 9 + 9	x
\mathbf{x}^2	= 18 3	
х	$=\sqrt{18}$	
х	$= 3\sqrt{2}$	3

In an isosceles right triangle like example no. 4, the legs are of equal lengths. Thus in order to find the length of the hypotenuse, you simply apply the Pythagorean formula.

In an isosceles right triangle, the length of the hypotenuse is equal to the length of the leg multiplied by $\sqrt{2}$.

Try this out

A. Justify each statement by stating the property, theorem, postulate, or definition that supports the statement.

- 1. If a < b, then 3a < 3b.
- 2. If $m \angle A = 65$ and $m \angle D > m \angle A$, then $m \angle D > 65$.

3. A is the midpoint of \overline{XY} . Therefore, AX = AY.

4. If x and y are the lengths of the legs of a right triangle and z is the length of the hypotenuse, then $x^2 + y^2 = z^2$.





B. Supply the missing statement

6. If $m \angle x = m \angle 1 + m \angle 2$, then $m \angle 1 _ m \angle x$

7. If \overrightarrow{AM} bisects $\angle DAY$, then $\angle DAM _ \angle MAY$



10. \angle LEN and \angle NEA are and



C. Determine if the following are lengths of the sides of a triangle.

- 11. 5, 5, 6
- 12. 3, 4, 5
- 13. 3, 3, 2
- 14. 1, 1, 2
- 15. 6, 8, 10
- 16. 7.5, 6.5, 14

D. Given the lengths of the two sides of a triangle, determine the range of the length of the third side.

17. 13, 10 18. 7, 11 19. 5, 5 20. 3.25, 6.1 21. 4.73, 8.92 22. Given the lengths of the four sides of quadrilateral KANE, determine the range of possible lengths of diagonals \overline{AE} and \overline{KN} .



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F. Given rt. ΔXYZ , compute for the missing side using the table below.

	XZ	ΥZ	XY
23.	7	5	
24	12		13
25.		6	12

G. Find the length of the diagonal of a rectangle given the measure of a side or sides. 26. 3, 5

27. 7

28. $\sqrt{10}$, $\sqrt{15}$

29. $3\sqrt{2}$

30. What is the length of the hypotenuse of an isosceles right triangle if one of the leg measures 13 cm?

Lesson 3

Introduction to Parallel Lines and Transversals

Lines may be classified as:

- a. coplanar and intersecting like lines a and b
- b. coplanar but not intersecting like lines m and n
- c. Non-coplanar and non-intersecting like lines k and l



<u>Illustration a represents intersecting lines</u>. The second illustration represents parallel lines and the third illustration showed skew lines.

Parallel lines are coplanar lines that do not intersect. <u>Skew lines</u> are non-coplanar lines.

Given coplanar lines *a* and *b*. A third line t intersects lines a and b at two distinct points X and Y. We call line t a <u>transversal</u>. A transversal is a line that intersect s two or more lines at two or more distinct points.



Try this out

Determine if the following statements define intersecting, parallel or skew lines.

- 1. The two frames of jalousie windows
- 2. Electric wires near the post
- 3. The iron base on the railroad tracks
- 4. The two flyovers at Nagtahan

Write true if the statement is always true, sometimes if the statement is sometimes true and false if it is never true.

- 5. Intersecting lines are coplanar.
- 6. Skew lines are non-coplanar.
- 7. Two parallel lines determine .a plane
- 8. Two non-intersecting lines are parallel.
- 9. Transversal intersect only two lines at a time.
- 10. Parallel lines are non coplanar.



Two lines are perpendicular if and only if they intersect to form right angles.

The perpendicular bisector of a segment is a line, ray, segment or plane that is perpendicular to the segment at its midpoint.

Any two right angles are congruent.

An exterior angle of a triangle is an angle, which is adjacent and supplementary to one of the angles of a triangle. The remote interior angles are angles of the triangle which are not adjacent to the given exterior angle. The measure of an exterior angle is equal to the sum of the measures of the two remote interior angles.

Properties of Inequality

- a. Trichotomy property of Inequality
- b. Addition property of Inequality
- c. Subtraction property of Inequality
- d. Multiplication property of Inequality
- e. Transitive property of Inequality

The whole is greater than any of its parts.

Exterior angle Inequality Theorem. The measure of an exterior angle of a triangle is greater than the measure of either of the two remote interior angles.

Triangle Inequality Theorem. In any triangle, the sum of the lengths of any two of its sides is greater than the length of its third side.

Pythagorean theorem. In any right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the two legs.

Parallel lines are coplanar lines that do not intersect. Skew lines are non-coplanar lines. Transversal is a line that intersects two or more lines at two or more distinct points.

What have you learned

Answer the following as indicated:

- 1. $\angle A$ is a right angle, $\angle X$ is a right angle. $\angle A ___ \angle X$
- 2. Using the figure at the right, $\angle RYP _ \angle R$.
- 3. $\overline{AB} \perp \overline{BC}$. \overline{BD} bisects $\angle B$. What is m $\angle 1$?
- 4. What property of inequality supports the Following statement.
 If x < 7, then 5x < 35.
- 5. $\triangle BET$ is a right triangle. m $\angle B < _$
- 6. In \triangle BET, if BE = 5, ET = 3, what is BT?



- 7. Using the measures given in Δ MAN, give the range of values or length of \overline{MN} .
- 8. Prison bars are examples of _____ lines
- 9. If $m \angle D = 23$, and $m \angle y = 19$, then $m \angle DAP =$ ____.



10. Find the perimeter of a square whose diagonal is $7\sqrt{2}$ cm long.



How much do you know

- 1. right
- 2. 65
- 3. an exterior
- 4. 180
- 5. transitive property of Inequality
- 6. 3 < WX < 13
- 7. ≅
- 8. 5 cm
- 9. parallel lines
- 10. transversal

Try this out

Lesson 1

- 1. right
- 2. 90
- 3. right
- 4. 90
- 5. Any two right angles are congruent
- 6. Angle Addition Postulate
- 7. Angle Addition Postulate
- 8. Transitive Property of Equality
- 9. Reflexive Property of Equality
- 10. Subtraction Property of Equality
- 11. Definition of Congruent Angles
- 12. *OP*
- 13. 13
- 14.46
- 15. 46
- 16. 92
- 17. 23
- 18.46
- 19. 26
- 20. 18
- 21.72
- 22.44
- 23.60
- 24. 30
- 25. $\angle 1 \cong \angle 3$

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26. m \angle ROM = m \angle NOW
2x + 22 = 3x - 12
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- x = - 34
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 $m \angle ROM = 2(34) + 22$ $m \angle ROM = 68 + 22$ $m \angle ROM = 90$ $m \angle NOW = 3(34) - 12$ $m \angle NOW = 102 - 12$ $m \angle NOW = 90$ Both m \angle ROM and m \angle NOW are 90 each So $\angle ROM$ and $\angle NOW$ are right angles Therefore $\overline{MW} \perp \overline{RN}$ 27. 7y - 71 = 90 $\overline{7y} = 90 + 71$

$$7y = 90 + 7$$

$$7y = 161$$

- y = 23
- 28. 2. angle bisector
 - 4. supplementary angles or linear pair postulate
 - 5. Addition
 - 6. Addition
 - 7. Transitive
 - 11. Right angle
 - 12. Perpendicular lines

Lesson 2

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- 1. Multiplication Property of Inequality
- 2. Transitive Property of Inequality
- 3. Definition of Midpoint
- 4. Pythagorean Theorem
- 5. Exterior Angle Inequality Theorem
- 6. <
- 7. ≅
- 8. >
- 9. 82
- 10. Adjacent and supplementary
- 11. Yes
- 12. Yes
- 13. Yes
- 14. No
- 15. Yes
- 16. No
- 17. 3 < third side < 23
- 18. 4 < third side < 18
- 19. 0 < third side < 10
- 20. 2.85 < third side < 9.35
- 21. 4.19 < third side < 13.65
- 22. a. 5 < AE < 17
 - b. 8 < KN < 20

- 23. $\sqrt{74}$ 24. 5 25. $6\sqrt{3}$ 26. $\sqrt{34}$ 27. $7\sqrt{2}$ 28. 5 29. 6
- 29.0
- 30. $13\sqrt{2}$ cm

Lesson 3

- 1. Parallel
- 2. Intersecting
- 3. Parallel
- 4. Skew
- 5. True
- 6. True
- 7. True
- 8. Sometimes
- 9. Sometimes
- 10. False

What have you learned

- 1. ≅
- 2. >
- 3. 45
- 4. Multiplication Property of Inequality
- 5.90
- 6. BT = $\sqrt{34}$
- 7. 5 < MN < 13
- 8. parallel
- 9. 42
- 10.7(4) = 28 cm.