# MODULE 1 Geometríc Relatíons

# **What this module is about**

This module is about relations of segments and angles. As you go over the exercises, you will develop your skills involving points, segments and angle pairs and solve problems on the relationships between segments and between angles. Treat the lessons with fun and take time to go back if you feel you are at loss.



This module is designed for you to:

- 1. illustrate betweenness and collinearity.
- 2. illustrate the following:
  - congruent segments
  - midpoint of a segment
  - congruent angles
  - bisector of an angle
  - complementary angles
  - supplementary angles
  - adjacent angles
  - linear pair
  - vertical angles



Answer the following questions asked from the given figure:



Answer the following questions:

- 1. What is /IP/?
- 2. If /IM/ = /MH/ = /IH/, then what point is between the other two?
- 3. If M is the midpoint of  $\overline{AC}$ , what segments are congruent?
- 4. Name the coordinate of the midpoint of IJ
- 5.  $\angle$  EMF and  $\angle$  FMC are \_\_\_\_\_ angles.
- 6.  $\angle$  EMF form a linear pair with \_\_\_\_\_.
- 7. If  $\angle$  EMF is the complement of  $\angle$  FMC and on  $\angle$  FMC = 75, what is on  $\angle$  EMF?
- 8. If  $\angle AMD \cong \angle DMC$  and are supplementary, what kind of angle is each?
- 9. What is the measure of each angle if the measure of the angles in a supplementary pair is twice that of the other?
- 10. What is the measure of each angle if the two angles are both vertical and complementary?



## **Collinearity of Points**

You may ask this question, "how many points are there in a line?" "How many points are there in a plane?" Do you know the answer? Yes, it is infinite or many points. Now you take a look at BP in the figure.



Μ



The points of a line can be placed in correspondence with the real numbers in such a way that:

- 1. to every point of the line there corresponds exactly one real number;
- 2. to every real number there corresponds exactly one point of the line;
- 3. the distance between any two points is the absolute value of the corresponding numbers.

The number corresponding to a given point is called the <u>coordinate</u> of the point.

#### Examples:

1. The coordinate of M is 6, the coordinate of G is 0, the coordinate of B is -5.



2. If the coordinate of B is x and the coordinate of C is y then, /BC/ = /X-Y/ (read as distance BC equals the absolute value of X minus Y).



# Try this out

Which set of points are collinear:



Give the coordinate of each of the following points:

- 6. G
- 7. D
- 8. M
- 9. A
- 10. K

B. The coordinates of P and Q are listed. Find /PQ/.

1. P:	0	6. P:	7
Q:	7	Q:	3



Let A, B, C be three points. B is between A and C. If A, B and C are on one line and |AB| + |BC| = |AC|. This definition of betweenness means that:

1. If B is between A and C, then A, B and C then, A, B, C are collinear and /AB/ + /BC/ = /AC/.

2. If A, B and C are collinear and |AB| + |BC| = |AC| then, B is between A and C.

#### Examples:



# Try this out





4. A, B, and C are three points on a line with coordinates 8, 4, and 13 respectively.

5. R, S, and T have coordinates x, y, and z respectively x < y < z.

From each of the following equations determine which point is between the other two.

6. /AB/ + /BC/ = /AC/ 7. /PO/ + /QR/ = /PR/ 8. /LM/ + /LN/ = /MN/9. /BC/ - /AB/ = /AC/ 10. /QR/ - /PQ/ = /RP/ Ρ Q R STU V W Х Β. -4 -3 -2 2 3 -5 -1 0 1 4 1. Is it true that /QS/ + /ST/ = /QT/?2. Is it true that /PS/ + /ST/ = /PT/?3. Is it true that /PW/ - /PR/ = /RW/? 4. Is it true that O is the coordinate of the midpoint of  $\overline{PX}$ ? 5. Is it true that /XP/ - /WX/ = /PW/? BCDEFGHIJKLM Α 6. - 10. -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 6. Give the distance between points G and J. 7. Give the distance between points G and D. 8. Find the distance between  $\overline{G}$  and the midpoint  $\overline{JL}$ . 9. Find the coordinate of the midpoint of DK. 10. Find the midpoint of AM. C. E 1 0 U В Α -10 -5 10 0 5 15 Using the figure complete the statement in nos. 1-5. If E is between A and B,

- 1. /AB/ = /AE/ + \_\_\_\_\_ 2. /AE/ = /AB/ - \_\_\_\_\_ 3. /E/ = /AB/ - \_\_\_\_\_ 4. /AB/ = \_\_\_\_\_ 5. /IB/ = \_\_\_\_\_
- 6. What is the coordinate of the midpoint of AU?

- 7. What two segments are congruent if O is the midpoint of  $\overline{BE}$ ?
- 8. Find a segment congruent to  $\overline{AO}$  with B as one endpoint.
- 9. O is the midpoint of a segment with I as one of the endpoints. Find the segments.
- 10. <del>AU</del> ≅\_\_\_\_.

#### Congruent Segments and Midpoint of a Segment

Segments are congruent if and only if they have equal measures.

#### Examples:

$1 \overline{DS} \sim \overline{VV}$					R					0			
1. RS = AT				0					4				
If <u>you</u> look at $\overline{RS}$ and m RS = /0-4/ = 4	XY	,		X					Y	,			
$m \frac{\overline{XY}}{\overline{XY}} = /4 - \frac{8}{8} = 4$ $m \frac{\overline{RS}}{\overline{RS}} = m \frac{\overline{XY}}{\overline{XY}}$ $\therefore \overline{RS} \cong \overline{XY}$				4					8	3			
2. $\overline{AD} \cong \overline{EH}$													
	A	В	С	D	Е	F	G	Н	Ι	J			
•	-4	-3	-2	-1	0	1	2	3	.4	5		-	
3. $\overline{AI} \cong \overline{BJ}$ 4. if m MB = 25 cm. 5. RM = MB = 6 ther	ano 1, R	d_m M ≘	ST : ≝ M	<u>=</u> 25 B	i cm	the	n Ml	Ē ≅	ST	:			

Midpoint – is a midpoint of the segment which divides the segment into two congruent parts.

#### Examples:

1. P is the midpoint of 
$$\overrightarrow{BC}$$
  
if and only if  $\overrightarrow{BP} \cong \overrightarrow{PC}$ .



Any line that passes through the midpoint of a segment is called a <u>bisector</u> of the segment.

## Try this out

Α.

D	Е	F	G	Н	Ι	J	K	L	Μ	Ν	0	Ρ	Q	R	S		
-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7		
	1. 2. 3. 4. 5. 6. 7. 8.	Wha Wha Wha Wha Given etwe uch Wha Wha Is /M	at is t at is at can at two at can at can een p that / at is / IA/ =	he <u>di</u> m LC n you p poir n you D with points /MA/ MA/? /AD/?	$\frac{1}{2}$ star $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	nce b nclud are a are a and c /MD/	etwe t a <u>di</u> out L( ) / and	een l out star D ar	D an DG a nc <u>e 3</u> nd LI	d G? and Ē fron ? M 12	<u>.</u> 0? n L?	A			D	*	

9. Is  $\overline{MA} \cong \overline{AD}$ ?

10. Is A the midpoint of MD?

Given  $\overline{ME}$ ,  $\overline{SO}$ ,  $\overline{PA}$ , and  $\overline{RU}$  with their respective measures



1. /ME/ \_\_\_\_\_ /SO/ 2. /SO/ \_\_\_\_\_ /PA/ 3. /PA/\_\_\_\_\_/RU/ 4. /RU/\_\_\_\_\_/PA/ 5. /RU/ \_\_\_\_ /ME/ M N O P Q R S T U V W X Y Z A B C -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 6. How far is it from M to P? 7. How far is it from C to Z? 8. Find /RZ/ 9. Find /CU/ 10. Give two pairs of congruent segments. ABCDEFGHIJK C. -10 -8 -4 -2 0 2 4 6 1. /A<u>D</u>/ = \_\_\_\_\_\_ m\_DG 3. AD DG 4. If D is between A and G, /AD/ + /DG/ = \_\_\_\_\_. 5. D is the midpoint of 6. If G is the midpoint of  $\overline{EI}$  then  $\cong$ 7. If  $\overline{AD} \cong \overline{DG}$  then is the midpoint of  $\overline{AG}$ 8. Suppose point B is on  $\overline{AC}$  and B is not a midpoint of  $\overline{AC}$ , then |AB| + |BC| = Given: Point M is the midpoint of  $\overline{JK}$  and  $\overline{JM}$  = 5. Complete the following:

9. /MK/ = \_\_\_\_\_ and JM ≅ \_\_\_\_\_.

#### Lesson 4

Congruent Angles and Angle Bisector Congruent Angles are two angles with equal measure

Angle bisector is a ray from the vertex of an angle to a point in its interior which divides the angle into two congruent parts.





You can also state the definition of angle bisector this way:

If D is in the interior of  $\angle$  BAC and  $\angle$  BAD  $\cong \angle$  DAC, then AD bisects  $\angle$  BAC and AD is called the bisector of LBAC



#### Try this out

- A. Complete the following statements:
  - 1. Two angles are said to be congruent if the angles have \_\_\_\_\_ measures.
  - 2. When two angles have equal measures, the angles are\_\_\_\_\_
  - 3. If  $m \ge 3 = 57$  and  $\ge 4 = 57$  then  $\ge 3$  and  $\ge 4$  are \_\_\_\_\_\_ angles.
  - 4. A ray from the vertex to the interior of the angle which bisects an angle is called \_\_\_\_\_.



6. ∠MON ≅ \_\_\_\_\_ 7. ∠SOT ≅ \_\_\_\_\_



Β.

A

►

#### Complementary and Supplementary Angles

The sum of the measures of complementary angles and two angles is equal to 90.



The sum of the measures of supplementary angles are two angles is equal to 180.

Each of two supplementary angles is called a supplement of the other angle. You can see that  $\angle B$  is the supplement of  $\angle E$ .  $m \angle B + m \angle E = 180$  $\angle B$  and  $\angle E$  are supplementary angles.

Examples:

- 1. 15° + 75° = 90°
   ∠ AOB and ∠ BOC are complementary angles.
- 2.  $140^{\circ} + 40^{\circ} = 180^{\circ}$  $\angle$  SER and  $\angle$  TER are supplementary angles



 Given m∠LI = ∠70°, find its complement.
 Since the sum of the measures of complementary angles is equal to 90, subtract the given angle from 90 to get the complement.
 Solution: 90 - m∠I = complement 90 - 70 = 20 4.  $\angle X$  and  $\angle Y$  are supplementary. Find  $m \angle Y$  if  $m \angle X = 100$ . Solution:  $m \angle X + m \angle Y = 180$  $100 + m \angle Y = 180$  $m \angle Y = 180 - 100$  $m \angle Y = 80$ 

#### Try this out

A. In the given figure,  $\angle AOB$  and  $\angle DOC$  are right angles



What is the complement of each of the angles whose measures are given:

- 5. 12° 8. 67°
- 6. 39° 9. 79° 7. 41° 10. 84.5°
- 7. 41° 10. 84.5°
- B. What is the supplement of each of the angles whose measures are:
  - 1. 11°
  - 2. 44°
  - 3. 121°
  - 4. 152.5°
  - 5. 78.6°
  - 6.  $\angle A$  and  $\angle B$  are complementary. If  $\angle A = 78^{\circ}$  then,  $\angle B =$ \_\_\_\_\_.
  - 7.  $\angle C$  and  $\angle D$  are supplementary. If  $\angle C = 110.5^{\circ}$  then,  $\angle D = 10.5^{\circ}$
  - 8. An angle has a measure x. Find the measure of its complement.
  - 9. An angle has a measure 2x. Find the measure of its supplement.
  - 10. Two supplementary angles have measure of 2x 15 and x + 30.
- C. Find the measure of each angle.
  - 1. If  $\angle X$  and  $\angle Y$  are supplementary then,  $m \angle X + m \angle Y =$ \_\_\_\_\_.
  - 2. If the m  $\angle O$  + m  $\angle P$  = 90 then,  $\angle O$  and  $\angle P$  are \_\_\_\_\_

- 3. What is the complement of  $\angle K$  whose measure is *m*?
- 4. Find the supplement of an angle whose measure is *a*.
- 5. If two congruent angles are complementary then, each angle has a measure of \_\_\_\_\_.
- 6. If two angle are complementary then, each is a \_\_\_\_\_of the other.
- 7-8. The measure of an angle is 15 greater than twice the measure of its complement.

Find the measure of each angle:

9-10. The measure of an angle is 20 less than three times the measure of its supplement. Find the measure of each angle.

#### Lesson 6

#### Adjacent Angles, Linear Pair, Vertical Angles

Adjacent angles are two angles which have a common side and a common vertex but no interior points common. **Examples:** 



A linear pair are two adjacent angles whose non common sides are opposite rays.





If you try to measure each angle forming a linear pair, you will find out that the sum of their measures is 180. So angles forming a linear pair are supplementary.

Vertical Angles are two nonadjacent angles formed by two intersecting lines:

**Examples:** 



 $\angle 1$  and  $\angle 3$  are vertical angles  $\angle 2$  and  $\angle 4$  are vertical angles

 $\angle$  AOB are DOC vertical angles  $\angle$  BOC and  $\angle$  AOD are vertical angles

Use your protractor to measure each angle in the figure

Find:

m∠1=	m∠2 =
m∠3 =	m∠4 =
m∠AOB =	m∠BOC =
m∠DOC =	m∠AOD =

Have you found out that they have the same measure? Therefore, vertical angles are congruent.

# Try this out

Α.

1-2~ Name 2 angles adjacent to  $\ensuremath{{\sc \ }} 1$ 



- 3. Name an angle which form a linear pair with  $\angle 4$ .
- 4 5. Name two pairs of vertical angles
- 6. Are  $\angle 1$  and  $\angle 5$  adjacent?
- 7. Are  $\angle 4$  and  $\angle 5$  adjacent?
- 8. What pair of  $\angle S$  are  $\angle 3$  and  $\angle 4$ ?
- 9. Are  $\angle 1$  and  $\angle 3$  vertical angles?
- 10.  $\angle$  SOU form a linear pair with  $\angle$  \_\_\_\_\_.
- B. Answer with Yes or No.
  - 1. Are  $\angle 3$  and  $\angle 6$  vertical angles?
  - 2. Are  $\angle 2$  and  $\angle 3$  adjacent angles?
  - 3. Are  $\angle 1$  and  $\angle 8$  vertical angles?
  - 4. Are  $\angle 7$  and  $\angle 8$  linear pair?
  - 5. Are  $\angle 4$  and  $\angle 8$  linear pair?
  - 6. Are  $\angle 1$  and  $\angle 8$  adjacent?
  - 7. Are  $\angle 1$  and  $\angle 7$  linear adjacent?
  - 8. Are  $\angle 1$  and  $\angle 7$  linear pair?
  - 9. Are  $\angle 5$  and  $\angle 6$  adjacent?
- 10. Are  $\angle 1$  and  $\angle 4$  vertical angles?

#### C. True or False

- 1. Complementary angles are always adjacent.
- 2. Supplementary angles are sometimes adjacent.
- 3. The angles of a linear pair are always adjacent.
- 4. Vertical angles are sometimes adjacent.
- 5. If two angles are vertical then, they are either both acute or both obtuse.
- 6. Two adjacent right angles are supplementary.
- 7. Two vertical angles are always congruent.
- 8. If two angles form a linear pair, they are supplementary.
- 9. If one of the angles in a linear pair is 90, then the other angle has a measure greater than 90.
- 10. If two adjacent angles are congruent and complementary, the measure of each angle is 90.





Collinear points is a set of points which are contained in a line. The points of a line can be place in correspondence with the real numbers in such a way that:

- to every point of the line there corresponds exactly one real number;
- to every real number there corresponds exactly one point of the line;
- the distance between any two points is the absolute value of the difference of the corresponding numbers.

The number corresponding to a given point is called the coordinate of the point.

Let A, B, C be three points. If A, B, C are on one line and. B is between A and C then, /AB + /BC/ = /AC/

Congruent segments are segments with equal measure.

Midpoint is a point of a segment which divides the segment into 2 congruent parts.

Congruent angles are angles with equal measure.

Angle bisector is a ray from the vertex of an angle to a point In its interior which divides the angle into two congruent parts.

Complementary angles are two angles whose measures have the sum equal to 90.

Supplementary angles are two angles whose measures have the sum equal to 180.

Adjacent angles are two angles which have a common side and a common vertex but no interior points in common.

Linear Pair are two adjacent angles whose noncommon sides are opposite rays.

Vertical angles are two nonadjacent angles formed between two intersecting lines.



- 1. What is /BD/?
- 2. If  $\overline{RA} \cong \overline{SA}$  what do you call point A?
- 3. Name a supplement of  $\angle$  NAS.
- What angle pair is Illustrated by ∠ RAN and ∠ NAS?
- 5. If  $\angle BAM \cong \angle SAM$  what do you call  $\overrightarrow{AM}$ ?
- 6.  $\angle$  RAN is vertical to  $\angle$  \_\_\_\_.
- 7. The measure of  $\angle$  SAM = \_\_\_\_.
- 8.- 9. If the measure of an angle is twice the measure of its complement, what is the measure of each angle?
- The measure of ∠B is 9 more than twice the measure of ∠C. If ∠B and ∠C are supplementary angles, what is the measure of ∠B?





How much do you know

1.	9	6. ∠FMD
2.	M is between and H	7.15
3.	$\overline{AM} \cong \overline{MC}$	8. right
4.	I	9. 60°, 120°
5.	adjacent	10. 45°Try this out

Try this out Lesson 1

A.	<ol> <li>collinear</li> <li>not collinear</li> <li>collinear</li> <li>not collinear</li> <li>not collinear</li> <li>collinear</li> </ol>	6. 0 73 8. 6 96 10. 4
B.	1. 7 2.12 3.11 4.7 5.9	6. 4 7. 94 8. 10 9. 74 10. 39
$\sim$	1 2	6 7

C.	1. 3	6. 7
	2. 2	7. 4.5
	3. 4	8. 6
	4.5	9. 5
	5. 5.5	10. 7.64

#### Lesson 2

A.	<ol> <li>O is between P and M</li> <li>E is between C and U</li> <li>O is between B and A</li> <li>A is between B and C</li> <li>S is between R and T</li> </ol>	<ul> <li>6. B is between A and C</li> <li>7. Q is between P and R</li> <li>8. L is between M and N</li> <li>9. A is between B and C</li> <li>10. P is between R and Q</li> </ul>
B.	1. Yes 2. Yes 3. Yes	6.3 7.3 8.4

- 4. Yes 9. 0.5

5. Yes	10. G
C. 1. /EB/	6. O
2. /EB/	7. <u>E</u> O ≡ OB
3. /AE/	8. <u>IB</u>
4. 25	9. IU
5. 15	10. BE

A. 1. 36. 62. 37. 63.  $DG \cong LO$ 8. Yes4. O and I9. Yes5.  $LO \cong U$ 10. Yes

B. 1. <	6. 3
2. =	7. 3
3. <	8. 8
4. >	9. 8
5. >	10. $\overline{\text{MD}} \cong \overline{\text{CZ}}$ and $\overline{\text{RZ}} \cong \overline{\text{CU}}$ (answers may vary)
C. 1. 6	6. EG≅ GI

).	1. 6	6. EG≅ G
	2. =	7. D
	3. ≅	8. /AC/
	4. 12	9.5
	5. /AG/ (answers may vary)	10. 10, MK

#### Lesson 4

<ul> <li>A. 1. equal or the same</li> <li>2. congruent</li> <li>3. congruent</li> <li>4. angle bisector</li> <li>5. EM</li> </ul>	6. ∠SOR 7. ∠NOP 8. ∠QOR 9. <u>∠T</u> OQ 10. OQ
<ul> <li>B. 1. ∠PJK and ∠QJK</li> <li>2. ∠RJH and ∠HJS</li> <li>3. angle bisector</li> <li>4. angle bisector</li> <li>5. ≅</li> </ul>	6. $\angle PIT \cong \angle SIT$ 7. 45 8. 66 9. 120 10. 60

#### Lesson 5

A. 1.  $\angle$  3 and  $\angle$  4 6. 51°

2. $\angle$ 5 and $\angle$ 6 3. $\angle$ 1 and $\angle$ 2 4. $\angle$ 7 and $\angle$ 8 5. 78	7. 49 <sup>°</sup> 8. 23° 9. 11° 10. 5.5°
<ul> <li>B. 1. 169°</li> <li>2. 136°</li> <li>3. 59°</li> <li>4. 27.5°</li> <li>5. 101°</li> </ul>	6. 12° 7. 69.5° 8. 90 - x 9. 180 - 2x 10. 95, 85
C. 1. 180	6. complem

1.	180	6. complement
2.	complementary	7.65
3.	90 - m	8. 25
4.	180 - a	9. 130
5.	45°	10. 50

Α.	1.	∠2	6.	Yes
	2.	∠5	7.	Yes
	3.	$\angle 5$ and $\angle ROQ$	8.	adjacent
	4.	$\angle 1$ and $\angle 4$	9.	No
	5.	$\angle SOT$ and $\angle ROQ$	10.	∠ROU

В.	1.	No
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- 2. No
- 3. No
   4. Yes
- 4. Yes 5. Yes
- 0. 100

# C. 1. False

- 2. True
   3. True
   4. False
- 5. False
- J. 1 alse

What have you learned

12	6.	$\angle SAM$
Midpoint	7.	40°
∠RAN	8.	60°
Linear pair	9.	30°
Angle bisector	10	. 123º
	12 Midpoint ∠ RAN Linear pair Angle bisector	$\begin{array}{ccc} 12 & & 6.\\ Midpoint & & 7.\\ \angle RAN & 8.\\ Linear pair & 9.\\ Angle bisector & 10 \end{array}$

6. Yes 7. Yes

8. Yes

9. No

10. Yes

6. True

7. True

8. True

9. False

10. False