

## Module 2

# Properties of Quadrilaterals



### *What this module is about*

This module is about the properties of the diagonals of special quadrilaterals. The special quadrilaterals are rectangles, square, and rhombus. The conditions sufficient to guarantee that a quadrilateral is a parallelogram are also discussed in this module.



### *What you are expected to learn*

This module is designed for you to

1. apply inductive/deductive skills to derive the properties of the diagonals of special quadrilaterals
  - rectangle
  - square
  - rhombus
2. verify sets of sufficient conditions which guarantee that a quadrilateral is a parallelogram
3. apply the conditions to prove that a quadrilateral is a parallelogram
4. solve routine and non routine problems



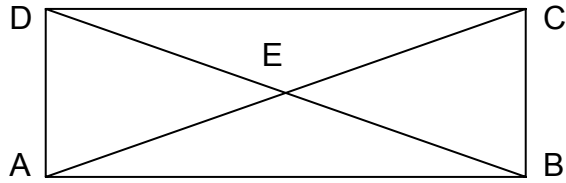
### *How much do you know*

True or False

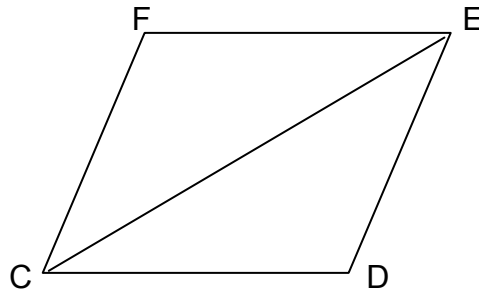
1. The diagonals of a square are congruent.
2. The diagonals of a rectangle are perpendicular.
3. The diagonals of a rhombus bisect each other.
4. A square is a rhombus.
5. If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

6. If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

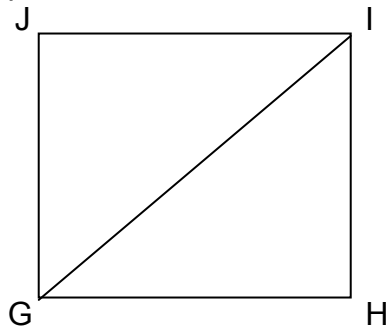
Quadrilateral ABCD is a rectangle. Its diagonals AC and BD intersect at E.



7. If  $AC = 2(x + 10)$  and  $BD = x + 60$ , what is AC?
8. If  $AE = 4x - 5$  and  $CE = 10 + x$ , what is AE?
9. Quadrilateral CDEF is a rhombus. If  $m\angle FCE = 3x - 5$  and  $m\angle DCE = 2x$ , find  $m\angle FCD$ .



Quadrilateral GHIJ is a square.



10. If  $m\angle HGI$  is  $3(x + 5)$ , what is  $x$ ?

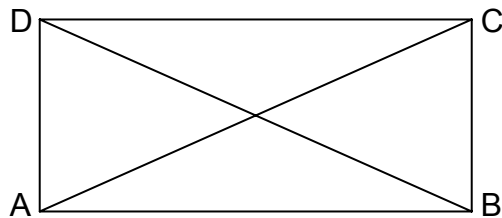


## *What you will do*

### Lesson 1

## The Properties of the Diagonals of Special Quadrilaterals

A diagonal of a quadrilateral is a segment which connects any two non-consecutive vertices. In the following quadrilateral,  $\overline{AC}$  and  $\overline{BD}$  are the diagonals.



The following are the properties of the diagonals of special quadrilaterals.

1. The diagonals of a rectangle are congruent.
2. The diagonals of a square are congruent.
3. The diagonals of a square are perpendicular
4. Each diagonal of a square bisects a pair of opposite angles.
5. The diagonals of a rhombus are perpendicular.
6. Each diagonal of a rhombus bisects a pair of opposite angles

You can apply inductive skills to derive these properties of the diagonals of special quadrilaterals. In the following activities you need a ruler, a pencil, a protractor and pieces of graphing paper.

1. Do the following activity:

- a. On a graphing paper, draw a rectangle.
- b. Name your rectangle  $\overline{ABCD}$ .
- c. Draw diagonals  $\overline{AC}$  and  $\overline{BD}$ .
- d. Find the lengths of  $\overline{AC}$  and  $\overline{BD}$ . Are their lengths equal? Are the diagonals congruent?

Conclusion: The diagonals of a rectangle are congruent

2. Do the following activity:

- a. On a graphing paper, draw a square.
- b. Name your square  $\overline{ABCD}$ .
- c. Draw diagonals  $\overline{AC}$  and  $\overline{BD}$ .

- d. Find the lengths of the diagonals. Are their lengths equal? Are the diagonals of the square congruent?

Conclusion: The diagonals of a square are congruent.

3. Do the following activity:

- a. Construct a square on a graphing paper
- b. Name your square  $EFGH$ .
- c. Draw its diagonals  $\overline{EG}$  and  $\overline{HF}$ .
- d. Label the intersection of the diagonals,  $M$ .
- e. Using a protractor, find the measures of  $\angle HME$ , and  $\angle HMG$ .
- f. What kind of angles are the two angles?
- g. Are the diagonals perpendicular?

Conclusion: The diagonals of a square are perpendicular

4. Do the following activity.

- a. Draw a square on a graphing paper.
- b. Name your square  $ABCD$ .
- c. Draw diagonal  $\overline{AC}$ .
- d. What do you notice? Into how many angles are the two opposite vertex angles divided?
- e. What do you conclude?

Conclusion: Each diagonal of a square bisects a pair of opposite angles.

5. Do the following activity.

- a. Draw a rhombus on a graphing paper.
- b. Name your rhombus  $ABCD$ .
- c. Draw the diagonals and name the point of intersection,  $E$ .
- d. Find the measures of  $\angle AED$  and  $\angle CED$ .
- e. What kind of angles are they?
- f. What can you say about the diagonals?

Conclusion: The diagonals of a rhombus are perpendicular.

6. Do the following activity.

- a. Draw a rhombus on a graphing paper.
- b. Name your rhombus  $ABCD$ .
- c. Draw diagonal  $\overline{AC}$ .
- d. What do you notice? Into how many angles are the two opposite vertex angles divided?
- e. What do you conclude?

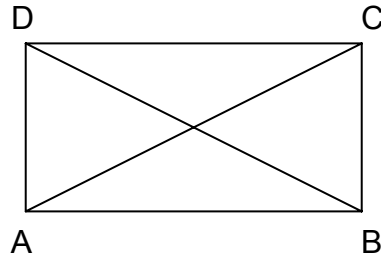
Conclusion: Each diagonal of a rhombus bisects a pair of opposite angles.

These properties of the diagonals of special quadrilaterals can also be proven deductively. Let us prove the first three properties deductively.

1. The diagonals of a rectangle are congruent.

Given: Rectangle  $ABCD$   
with diagonals  $\overline{AC}$  and  $\overline{BD}$ .

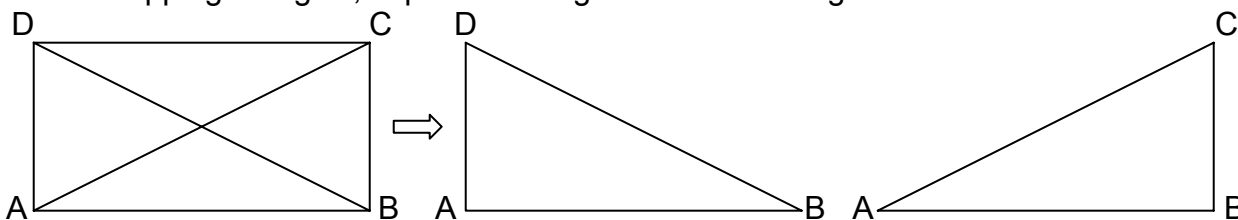
Prove:  $\overline{BD} \cong \overline{AC}$



Proof:

Statements	Reasons
1. Rectangle $ABCD$ with diagonals $\overline{AC}$ and $\overline{BD}$	1. Given
2. $\overline{AD} \cong \overline{BC}$ (S)	2. Opposite sides of a parallelogram are congruent (Remember, a rectangle is a parallelogram)
3. $\angle DAB$ and $\angle CBA$ are right angles	3. A rectangle is a parallelogram with four right angles
4. $\angle DAB \cong \angle CBA$ (A)	4. Any two right angles are congruent
5. $\overline{AB} \cong \overline{AB}$ (S)	5. Reflexive Property of Congruence
6. $\triangle DAB \cong \triangle CBA$	6. SAS Congruence
7. $\overline{BD} \cong \overline{AC}$	7. Corresponding Parts of Congruent Triangles are Congruent

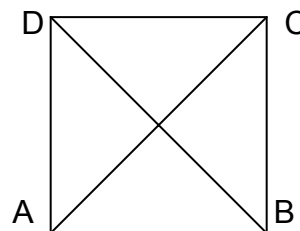
Triangles  $\triangle DAB$  and  $\triangle CBA$  overlap. If you find difficulty visualizing the two overlapping triangles, separate the figure into two triangles.



2. The diagonals of a square are congruent.

Given:  $\square ABCD$  is a square with diagonals  $\overline{AC}$  and  $\overline{BD}$

Prove:  $\overline{BD} \cong \overline{AC}$



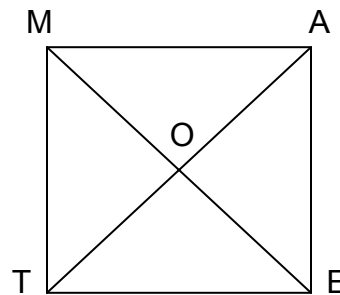
Proof

Statements	Reasons
1. $\square ABCD$ is a square with diagonals $\overline{AC}$ and $\overline{BD}$	1. Given
2. $\overline{AD} \cong \overline{BC}$ (S)	2. Opposite sides of a parallelogram are congruent (Remember, a square is a parallelogram)
3. $\angle DAB$ and $\angle CBA$ are right angles	3. A rectangle has four right angles (Remember that a square is a rectangle with four congruent sides and a rectangle has four right angles.)
4. $\angle DAB \cong \angle CBA$ (A)	4. Any two right angles are congruent
5. $\overline{AB} \cong \overline{AB}$ (S)	5. Reflexive Property of Congruence
6. $\triangle DAB \cong \triangle CBA$	6. SAS Congruence
7. $\overline{BD} \cong \overline{AC}$	7. Corresponding Parts of Congruent Triangles are Congruent

3. The diagonals of a square are perpendicular

Given:  $\square TEAM$  is a square with diagonals  $\overline{AT}$  and  $\overline{ME}$

Prove:  $\overline{ME} \perp \overline{AT}$



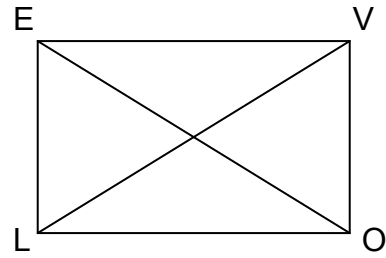
A proof can also be written in paragraph form.

Proof:

Side  $\overline{TM}$  and side  $\overline{EA}$  are congruent since they are sides of a square. A square is a rectangle with four congruent sides.  $\overline{MO} \cong \overline{EO}$  by Reflexive Property of Congruence. The diagonals of a parallelogram bisect each other. Since a square is a parallelogram therefore  $\overline{TO} \cong \overline{AO}$ .  $\triangle MOT \cong \triangle MOA$  by SSS congruence. Since  $\angle MOT$  and  $\angle MOA$  are supplementary and congruent, then each of them is a right angle. Therefore  $\overline{ME} \perp \overline{AT}$  by the definition of perpendicular.

### Example 1

The figure at the right is a rectangle.  
If the diagonal  $\overline{LV} = 2x$  and the diagonal  $\overline{OE} = 12$  cm, find  $x$ .



Solution:

Step 1. The diagonals of a rectangle are congruent.

$$\begin{aligned}\overline{LV} &\cong \overline{OE} \\ \overline{LV} &= \overline{OE} \quad (\text{Congruent segments have equal lengths})\end{aligned}$$

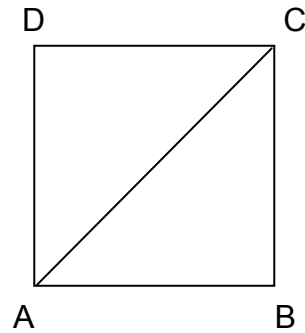
Step 2. Substitute  $2x$  for  $\overline{LV}$  and  $12$  for  $\overline{OE}$ . Then solve for  $x$ .

$$\begin{aligned}2x &= 12 \\ x &= 6\end{aligned}$$

Answer: The value of  $x$  is 6 cm.

### Example 2

Quadrilateral ABCD at the right is a square.  
Find  $m\angle CAB$



Solution:

Step 1, Quadrilateral ABCD is a square and a square is a rectangle.

Therefore:  $m\angle DAB = 90$ .

Step 2. But each diagonal of a square bisects a pair of opposite angles.

$$\text{Hence: } m\angle CAB = \frac{1}{2} m\angle DAB$$

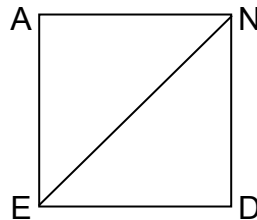
Step 3. Substitute 90 for  $m\angle DAB$ .

$$\begin{aligned}m\angle CAB &= \frac{1}{2} (90) \\ &= 45\end{aligned}$$

Answer:  $m\angle CAB = 45$

**Example 3**  $\square EDNA$  is a square.

If  $m\angle END$  is  $3(x + 5)$ , what is  $x$ ?



Solution:

- a.  $m\angle DCB = 90$  since  $\square ABCD$  is a square
- b. Each diagonal of a square bisects a pair of opposite angles.

Hence:  $m\angle ACB = 45$

$$3(x + 5) = 45$$

$$3x + 15 = 45$$

$$3x = 45 - 15$$

$$3x = 30$$

$$x = 10$$

#### Example 4

The figure at the right is a rhombus.  
If  $m\angle CAB = 30$ , what is the  $m\angle CAD$ ?

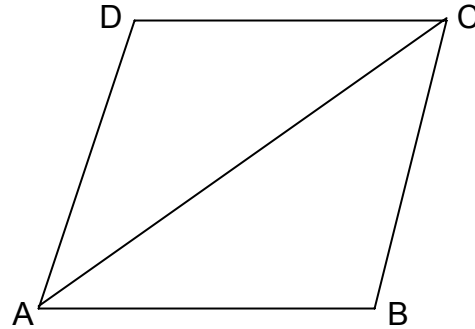
Step 1. Each diagonal of a rhombus bisects pair of opposite angles.

$$m\angle CAD = m\angle CAB$$

Step 2. Substitute 30 for  $m\angle CAB$  in the above equation.

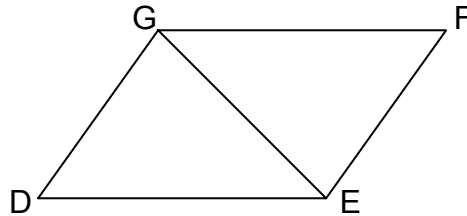
$$m\angle CAD = 30$$

Answer: The measure of  $\angle CAD$  is  $30^\circ$ .



#### Example 5

$\square DEFG$  is a rhombus. If  $m\angle FGE = 5x - 8$  and  $m\angle DGE = 3x + 22$ , find the measure of (a)  $m\angle FGE$  (b)  $m\angle DGE$  and (c)  $m\angle FGD$



Solution:

Step 1. Each diagonal of a rhombus bisects a pair of opposite angles.

$$m\angle FGE = m\angle DGE$$

$$5x - 8 = 3x + 22$$

$$5x - 3x = 22 + 8$$

$$2x = 30$$

$$x = 15$$



Step 2. Substitute 15 for  $x$

$$\begin{aligned} \text{a. } m\angle FGE &= 5x - 8 \\ &= 5(15) - 8 \\ &= 67 \end{aligned}$$

$$\begin{aligned} \text{b. } m\angle DGE &= 3x + 22 \\ &= 3(15) + 22 \\ &= 67 \end{aligned}$$

$$\begin{aligned} \text{c. } m\angle FGD &= m\angle FGE + m\angle DGE \\ &= 67 + 67 \\ &= 134 \end{aligned}$$

Answers: (a)  $m\angle FGE = 67$   
(b)  $m\angle DGE = 67$   
(c)  $m\angle FGD = 134$

### Example 6

- BETH is a rhombus. If  $m\angle TBE = 35$ , what is  $m\angle HEB$ ?

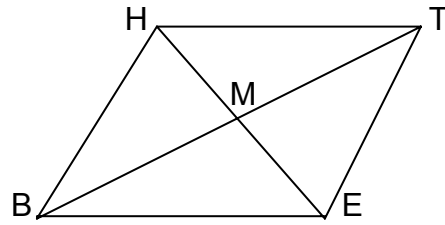
Solution:

Step 1. The diagonals of a rhombus are perpendicular. Hence,  $\angle BME$  is a right angle and its measure is  $90^\circ$ .  
 $m\angle BME = 90$

Step 2. The sum of the measures of the angles of a triangle is  $180^\circ$   
 $m\angle TBE + m\angle BME + m\angle HEB = 180$

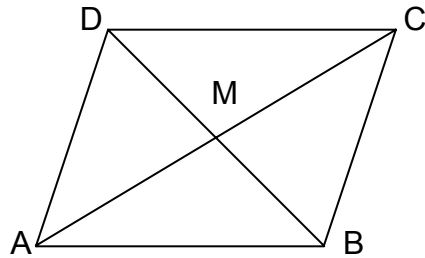
Step 3. Substitute 35 for  $m\angle TBE$  and 90 for  $m\angle BME$  in the above equation.  
 $35 + 90 + m\angle HEB = 180$   
 $125 + m\angle HEB = 180$   
 $m\angle HEB = 180 - 125$   
 $m\angle HEB = 55.$

Answer:  $m\angle HEB = 55$



### Example 7

- ABCD is a rhombus. If  $\overline{AM} = 16$  cm, what is  $\overline{CM}$ ?



Solution:

Step 1. The diagonals of a rhombus

Bisect each other.

$$\overline{CM} = \overline{AM}$$

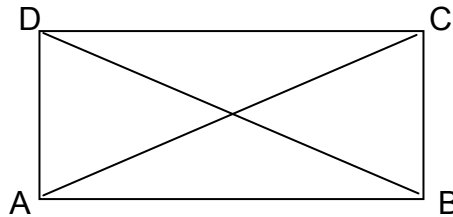
Step 2. Substitute  $\overline{16}$  cm for  $\overline{AM}$  in the above equation.

$$\overline{CM} = 16 \text{ cm}$$

Answer:  $\overline{CM} = 16 \text{ cm}$

Try this out

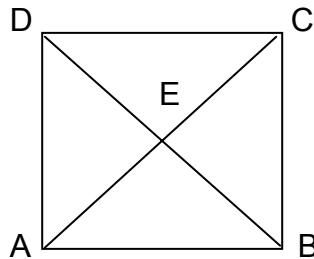
Set A. □ ABCD is a rectangle.



True or False

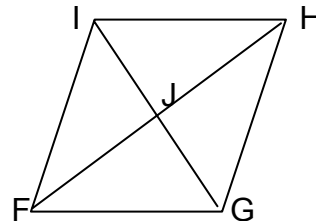
1. The lengths of  $\overline{AC}$  and  $\overline{BD}$  are equal.
2. Diagonals  $\overline{AC}$  and  $\overline{BD}$  are perpendicular.
3. The diagonal  $\overline{AC}$  bisects  $\angle DCB$ .
4. A rectangle is a parallelogram.

□ ABCD is a square



5.  $\angle EAB \cong \angle EBC$
6.  $\angle DEC$  is a right angle.

□ FGHI is a rhombus.

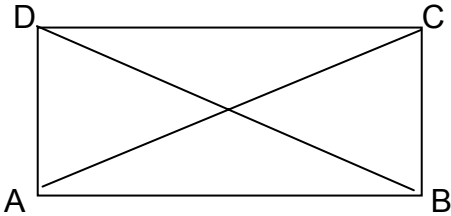


7.  $m\angle FIG = m\angle HIG$
8. The sum of  $m\angle JFG$  and  $m\angle JGF$  is 45.
9.  $\triangle FIG \cong \triangle HGI$

10. The diagonal  $\overline{FH}$  bisects the rhombus into two congruent triangles.

Set B

□ ABCD is a rectangle



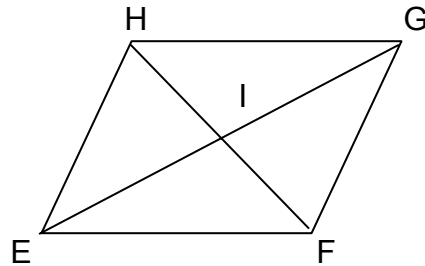
Find the indicated measure

1.  $\overline{AC} = 15$  dm. Find  $\overline{BD}$
2.  $\overline{BD} = 23$  cm. Find  $\overline{AC}$

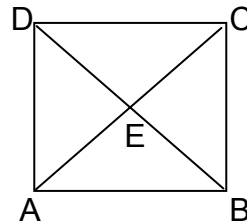
□ EFGH is a rhombus

Find the indicated measure.

3.  $m\angle HGE = 35$ . Find  $m\angle FGE$ .
4.  $m\angle HEI = 20$ . Find  $m\angle FEI$ .
5.  $m\angle IEF = 30$ . Find  $m\angle IFE$
6.  $m\angle IHE = 58$ . Find  $m\angle IEH$
7.  $m\angle IEF = 20$ . Find  $m\angle IEF + m\angle EIF$
8.  $m\angle IGH = 25$ . Find  $m\angle IGH + m\angle HIG$



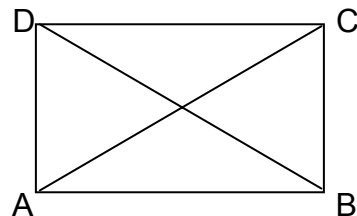
9. If □ ABCD is a square,  
then  $m\angle ACB =$  \_\_\_\_\_
10. If □ ABCD is square,  
then  $m\angle DEC =$  \_\_\_\_\_



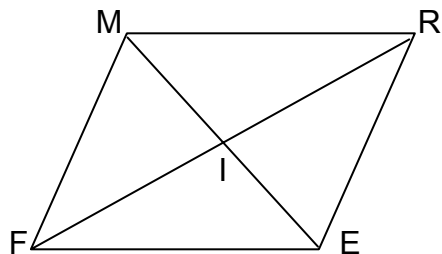
Set C

□ ABCD is a rectangle with diagonals  $\overline{AC}$  and  $\overline{BD}$ .

1.  $\overline{AC} = 2x + 15$ ,  $\overline{BD} = 3x + 10$ . Find  $\overline{AC}$ .
2.  $\overline{BD} = 6x + 5$ ,  $\overline{AC} = 5x + 14$ . Find  $\overline{BD}$ .

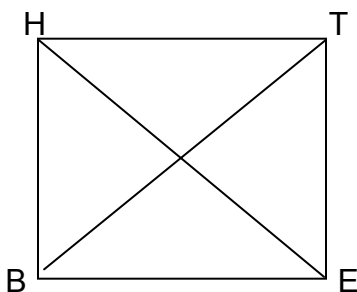


□ FERM is a rhombus.



3. If  $m\angle IFE = x + 20$ ,  $m\angle IEF = x + 26$ , find  $x$ .
4. If  $m\angle IMR = 4x + 20$ ,  $m\angle IRM = 2x + 10$ , find  $x$ .
5.  $m\angle IFE + m\angle IEF =$  \_\_\_\_\_
6.  $m\angle IMR + m\angle IRM =$  \_\_\_\_\_

□ BETH is a square.



7. If  $\overline{HM} = x + 15$ ,  $\overline{HE} = 40$ , what is  $x$ ?
8. If  $\overline{EM} = x + 9$ ,  $\overline{HE} = 30$ , what is  $x$ ?
9. If  $\overline{BM} = x + 12$ ,  $\overline{EM} = 2x - 20$ , what is  $x$ ?
10. If  $\overline{HM} = 44 - x$ ,  $\overline{TM} = 4 + 3x$ , what is  $x$ ?

## Lesson 2

### Conditions for a Parallelogram

The following are some conditions which guarantee that a given quadrilateral is a parallelogram.

1. If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
2. If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
3. If one pair of opposite sides of a quadrilateral are both congruent and parallel, then the quadrilateral is a parallelogram.
4. If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

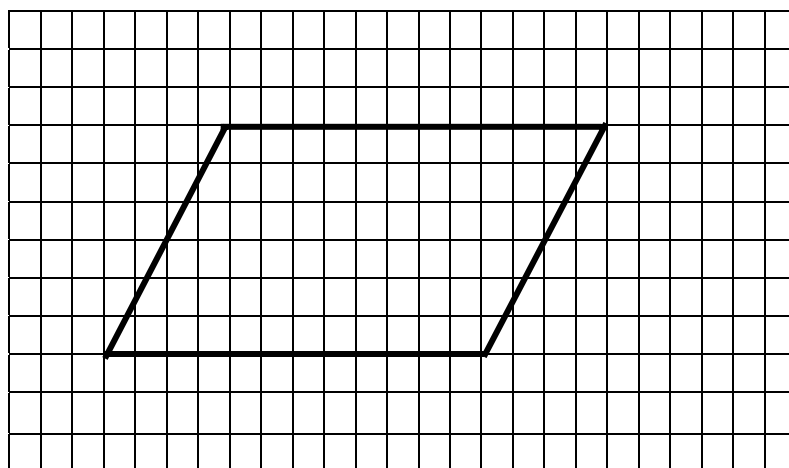
5. If the non-opposite angles of a quadrilateral are supplementary, then the quadrilateral is a parallelogram.

You can verify these sets of sufficient conditions which guarantee that a quadrilateral is a parallelogram. In the following activities you need a pencil, a ruler, a protractor and pieces of bond paper and graphing paper.

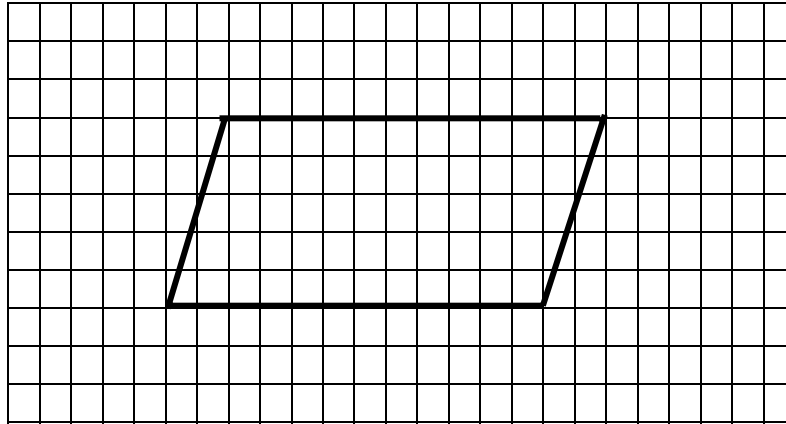
1. If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Do this activity:

- a. On a graphing paper, draw a quadrilateral such that both pairs of opposite sides are congruent. ( See the illustration.)



- b. Are the opposite sides equidistant? Find this out by using a ruler.
  - c. Are both pairs of sides parallel? (Remember, parallel lines are everywhere equidistant.)
  - d. Can you now conclude that the quadrilateral is a parallelogram? Why?
2. If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
    - a. On a graphing paper, with the aids of a ruler and a protractor, construct an quadrilateral such that both pairs of opposite angles are congruent. (See illustration)

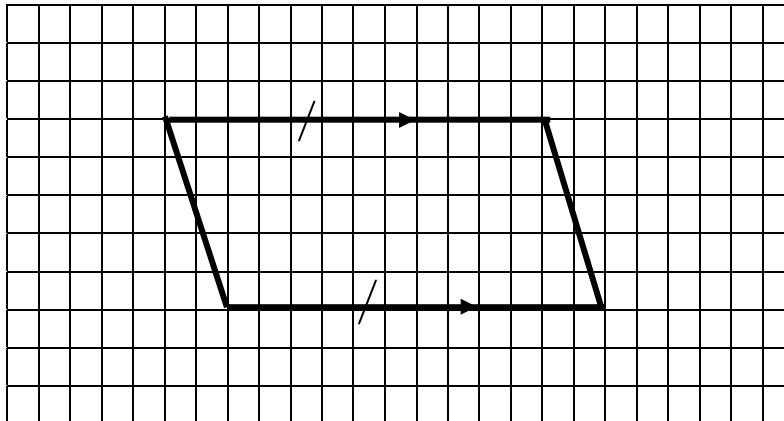


- b. Are the opposite sides congruent?
  - c. Can you now conclude that the quadrilateral is a parallelogram? Why?
3. If one pair of opposite sides of a quadrilateral are both congruent and parallel, then the quadrilateral is a parallelogram.

Do this activity

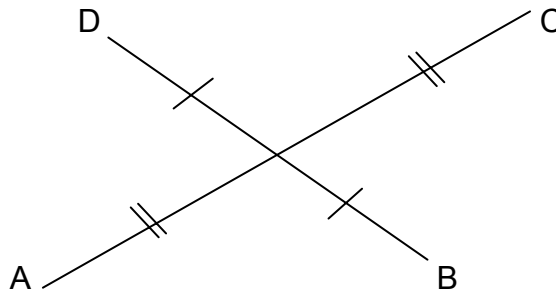
- a. On a graphing paper, draw a quadrilateral such that one pair of opposite sides are both congruent and parallel.

( See illustration below)

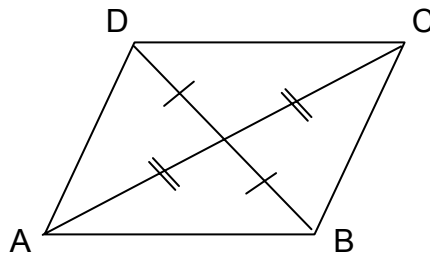


- b. Are the other two opposite sides congruent?
  - c. Can you now conclude that the quadrilateral is a parallelogram? Why?
4. If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

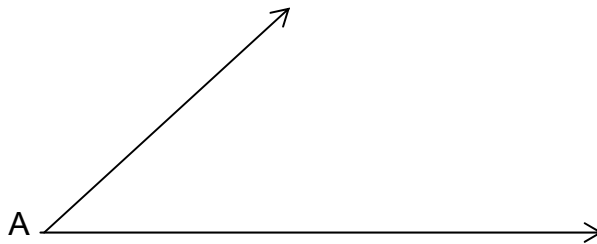
- a. On a bond paper, draw segments  $\overline{AC}$  and  $\overline{BD}$  bisecting each other. (See the illustration below.)



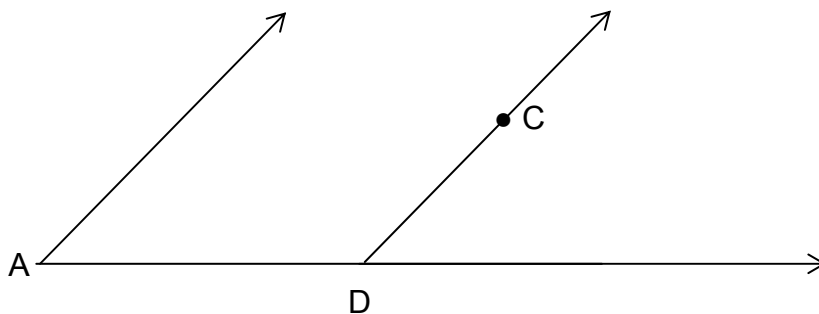
- b. Connect A to B, B to C, C to D and D to A .



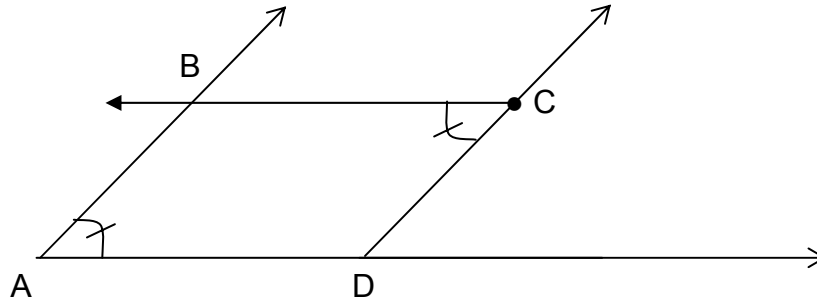
- c. Using a ruler, find the lengths of  $\overline{AB}$  and  $\overline{CD}$ . Are they equal?  
 d. Using a ruler, find the lengths of  $\overline{AD}$  and  $\overline{BC}$ . Are the lengths equal?  
 e. What kind of quadrilateral is  $\square ABCD$ ?
5. If the non-opposite angles of a quadrilateral are supplementary, then the quadrilateral is a parallelogram.
- a. On a bond paper, draw angle A. (See the illustration below.)



- b. Draw angle ADC such that its measure is supplementary to that of angle A.



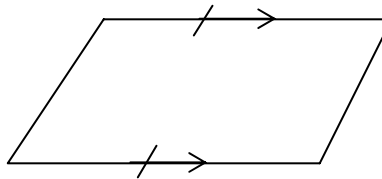
c. Draw angle DCB such that its measure is equal to that of angle A.



- d. Find the measure of angle CBA. Is it equal to the measure of angle ADC? Are  $\angle A$  and  $\angle B$  supplementary? How about  $\angle B$  and  $\angle C$ ? How about  $\angle D$  and  $\angle C$ ? Why?
- e. What kind of quadrilateral is  $\square ABCD$ ?

### Example 1

Determine whether the figure is a parallelogram. Identical “tick marks” indicate that the sides are congruent and identical “arrowheads” indicate the lines are parallel.



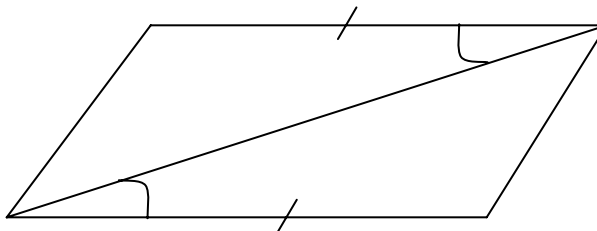
Solution:

If one pair of opposite sides of a quadrilateral are both congruent and parallel, then the quadrilateral is a parallelogram.

Hence the geometric figure is a parallelogram.

### Example 2

Determine whether the figure is a parallelogram.



Solution:

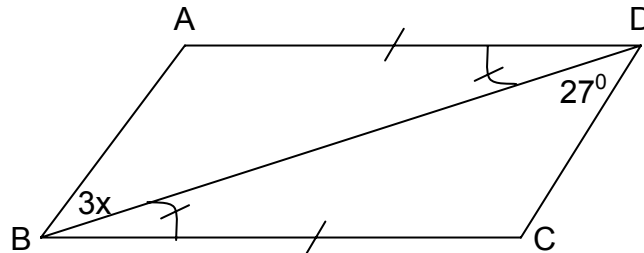


A pair of alternate interior angles are congruent, therefore a pair of opposite sides are parallel. These parallel sides are also congruent. As can be seen in the figure, they have the same length.

Hence the figure is a parallelogram.

Example 3.

Find the value of  $x$  for which  $\square ABCD$  is a parallelogram.



Solution:

If two lines are cut by a transversal and a pair of alternate interior angles are congruent, then the lines are parallel.

$$\begin{aligned} \overline{AD} & // \overline{BC} \text{ since } \angle ADB \cong \angle CBD \\ \overline{CD} & // \overline{AB} \text{ if } 3x = 27 \\ & x = 9 \end{aligned}$$

Hence the value of  $x$  should be 9.

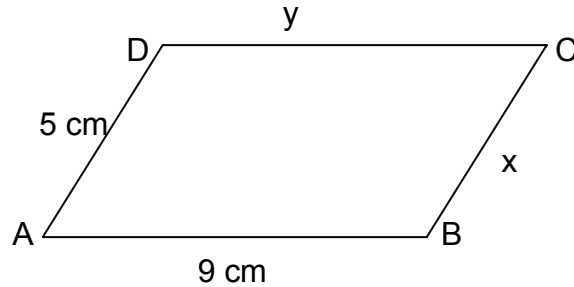
Try this out

Set A

True or False

1. If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
2. If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
3. If one pair of opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram.
4. If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
5. If the opposite angles of a quadrilateral are supplementary, then the quadrilateral is a parallelogram.

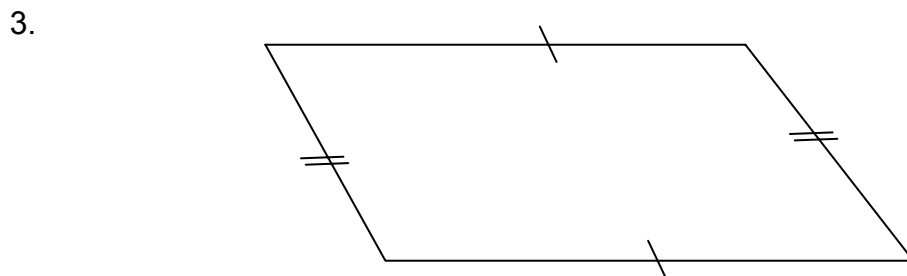
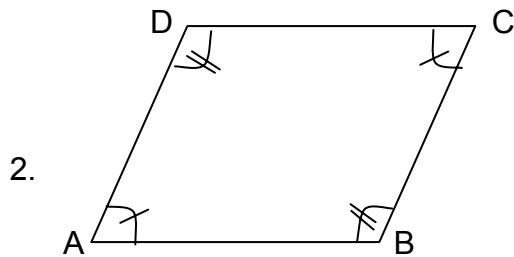
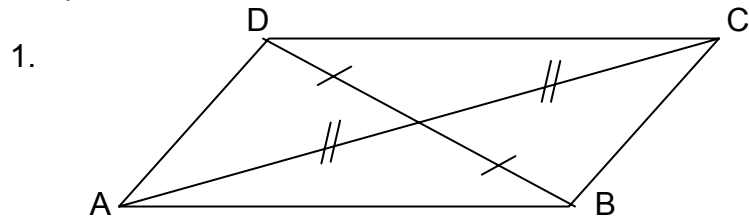
□ ABCD is a quadrilateral.  $\overline{AD} = 5$  cm and  $\overline{AB} = 9$  cm.



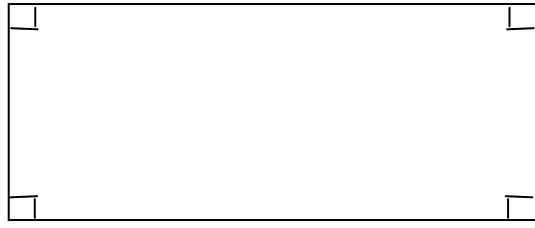
6. □ ABCD is a parallelogram if  $x = 5$  cm and  $y = 9$  cm.
7. □ ABCD is a parallelogram if  $m\angle C = 60$  and  $m\angle B = 120$ .
8. □ ABCD is a parallelogram if  $\overline{AB} \parallel \overline{DC}$ .
9. □ ABCD is a parallelogram if  $m\angle B \cong m\angle D$
10. □ ABCD is a parallelogram if  $\overline{AB} \cong \overline{DC} \cong \overline{AD} \cong \overline{BC}$ .

Set B.

Determine whether each quadrilateral is a parallelogram. Identical “tick marks” indicate that the sides or angles are congruent and identical “arrowheads” indicate the lines are parallel.



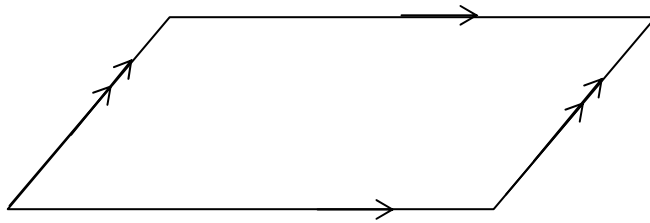
4.



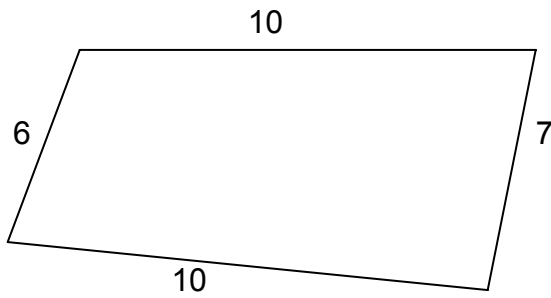
5.



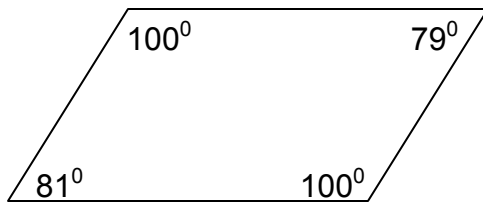
6.



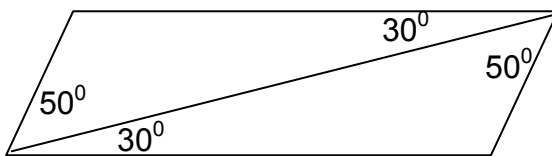
7.

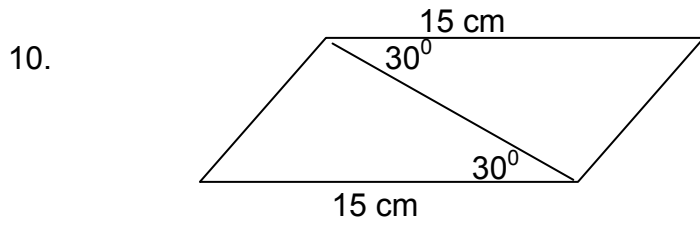


8.



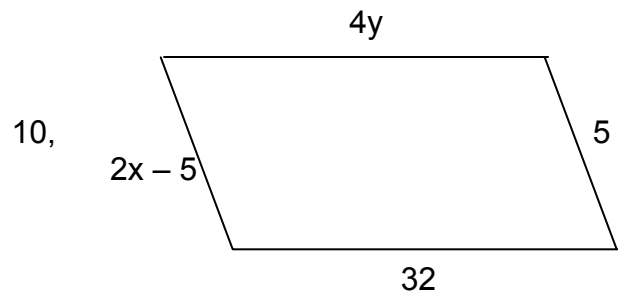
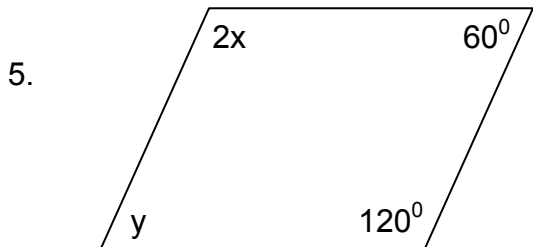
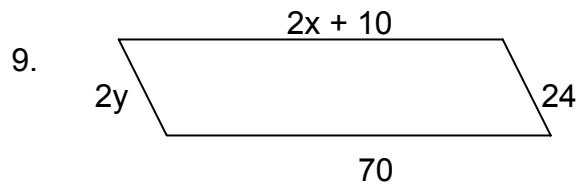
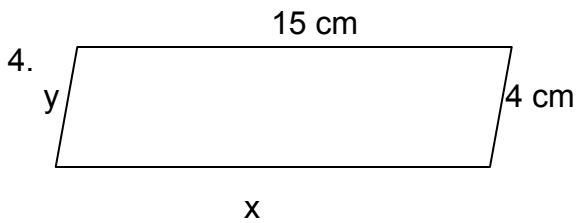
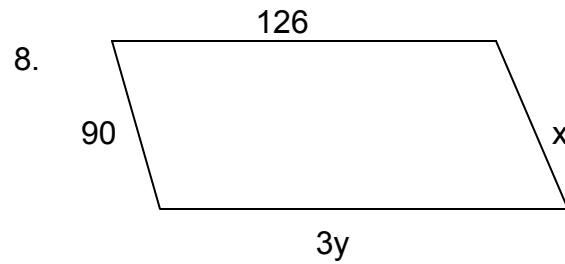
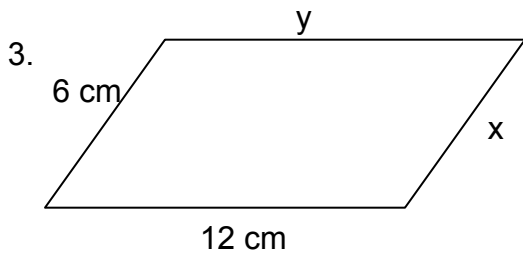
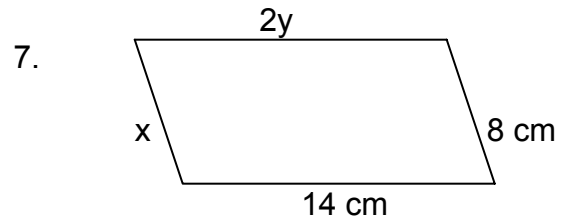
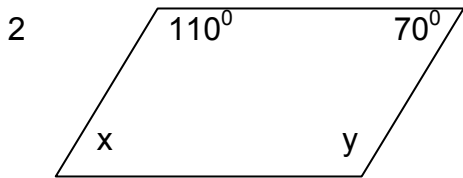
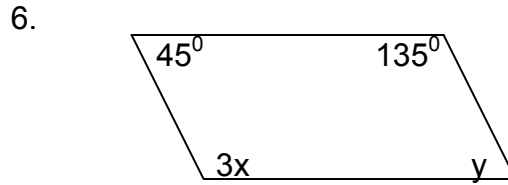
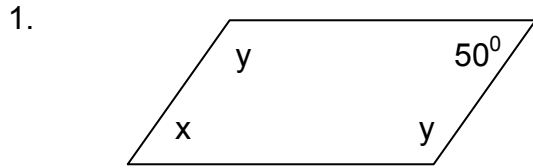
9.





Set C.

What values of  $x$  and  $y$  guarantee that each quadrilateral is a parallelogram.





## *Let's summarize*

1. A diagonal of a quadrilateral is a segment which connects any two non-consecutive vertices.
2. The diagonals of a rectangle are congruent.
3. The diagonals of a square are congruent.
4. The diagonals of a square are perpendicular
5. Each diagonal of a square bisects a pair of opposite angles.
6. The diagonals of a rhombus are perpendicular.
7. Each diagonal of a rhombus bisects a pair of opposite angles
8. A square is a special type of rhombus.
9. If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
10. If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
11. If one pair of opposite sides of a quadrilateral are both congruent and parallel, then the quadrilateral is a parallelogram.
12. If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
13. If the non-opposite angles of a quadrilateral are supplementary, then the quadrilateral is a parallelogram.
14. A quadrilateral is a parallelogram if both pairs of opposite side are parallel



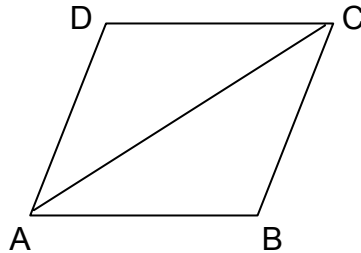
## *What have you learned*

Multiple Choice. Choose the letter of the correct answer.

1. A parallelogram is a rhombus if
  - A. The diagonals bisect each other
  - B. The diagonals are perpendicular.
  - C. Two consecutive angles are supplementary.
  - D. The opposite sides are parallel.
2. Which of the following is sufficient to guarantee that a quadrilateral is a parallelogram?
  - A. The diagonals are perpendicular
  - B. A pair of adjacent sides are congruent
  - C. Two consecutive angles are congruent
  - D. The diagonals bisect each other

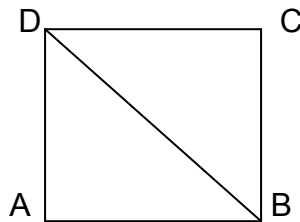
3. □ ABCD is a rectangle. if diagonal  $\overline{AC} = 2x + 6$  and diagonal  $\overline{BD} = 10$ , what is x?  
 A. 1  
 B. 2  
 C. 3  
 D. 4

4. □ ABCD is a rhombus.



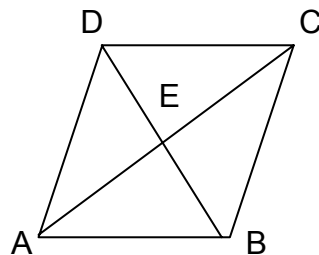
If  $m\angle DCA = 2(x+8)$  and  $m\angle BCA = 3x + 9$ , what is  $m\angle DCB$ ?

- A. 40  
 B. 50  
 C. 60  
 D. 70
5. □ ABCD is a square.



If  $m\angle ABD = 3(x + 10)$ , what is x?

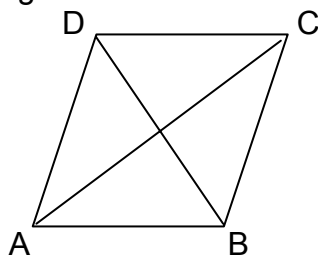
- A. 1  
 B. 3  
 C. 5  
 D. 7
6. □ ABCD is a rhombus. Diagonals  $\overline{AC}$  and  $\overline{BD}$  intersect each other at E.



If  $AE = 12$  and  $CE = 3x$ , what is x?

- A. 2  
 B. 4  
 C. 6  
 D. 8

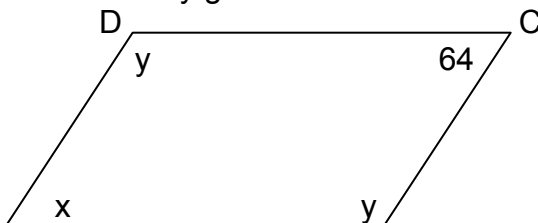
7.  $\square$  ABCD is a rhombus . Diagonals  $\overline{AC}$  and  $\overline{BD}$  intersect at E.



What is  $m\angle AED$ ?

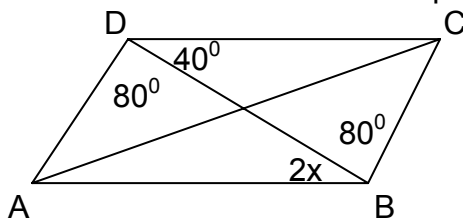
- A. 30  
B. 45  
C. 60  
D. 90

8. What values of  $x$  and  $y$  guarantee that  $\square$  ABCD is a parallelogram.



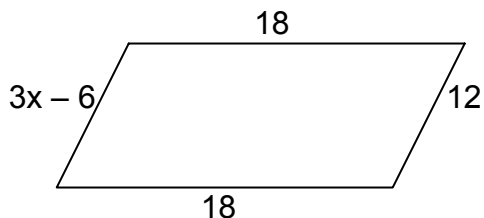
- A.  $x = 64$  ,  $y = 116$   
B.  $x = 32$ ,  $y = 116$   
C.  $x = 64$ ,  $y = 64$   
D.  $x = 32$ ,  $y = 64$

9. Find the value of  $x$  for which  $\square$  ABCD is a parallelogram.



- A. 10  
B. 20  
C. 30  
D. 40

10. Find the value of  $x$  for which  $\square$  ABCD is a parallelogram.



- A. 8  
B. 6  
C. 4  
D. 2



## Answer Key

How much do you know

1. True
2. False
3. True
4. True
5. True
6. True
7.  $\overline{AC} = 100$
8.  $\overline{AE} = 15$
9.  $m\angle FCD = 20$
10.  $x = 10$

Lesson 1

Set A

1. True
2. False
3. False
4. True
5. True
6. True
7. True
8. False
9. True
10. True

Set B

1. 15
2. 23
3. 35
4. 20
5. 60
6. 32
7. 110
8. 115
9. 45
10. 90

Set C

1.  $\overline{AC} = 25$
2.  $\overline{BD} = 59$
3.  $x = 22$
4.  $x = 10$
5. 90

6. 90

7.  $x = 5$

8.  $x = 6$

9.  $x = 32$

10.  $x = 10$

Lesson 2

Set A

1. True
2. True
3. False
4. True
5. False
6. True
7. True
8. True
9. True
10. False

Set B

1. Parallelogram
2. Parallelogram
3. Parallelogram
4. Parallelogram
5. Parallelogram
6. Parallelogram
7. Not a parallelogram
8. Not a parallelogram
9. Parallelogram
10. Parallelogram

Set C

1.  $x = 50^\circ$   
 $y = 130^\circ$
2.  $x = 70^\circ$   
 $y = 110^\circ$
3.  $x = 6$  cm  
 $y = 12$  cm
4.  $x = 15$  cm  
 $y = 4$  cm
5.  $x = 60^\circ$   
 $y = 60^\circ$
6.  $x = 45^\circ$   
 $y = 45^\circ$
7.  $x = 8$  cm  
 $y = 7$  cm
8.  $x = 90$  units  
 $y = 42$  units
9.  $x = 30$  units  
 $y = 12$  units
10.  $x = 5$  units  
 $y = 8$  units

What have you learned

1. B
2. D
3. B
4. C
5. C
6. B
7. D
8. A
9. B
10. B