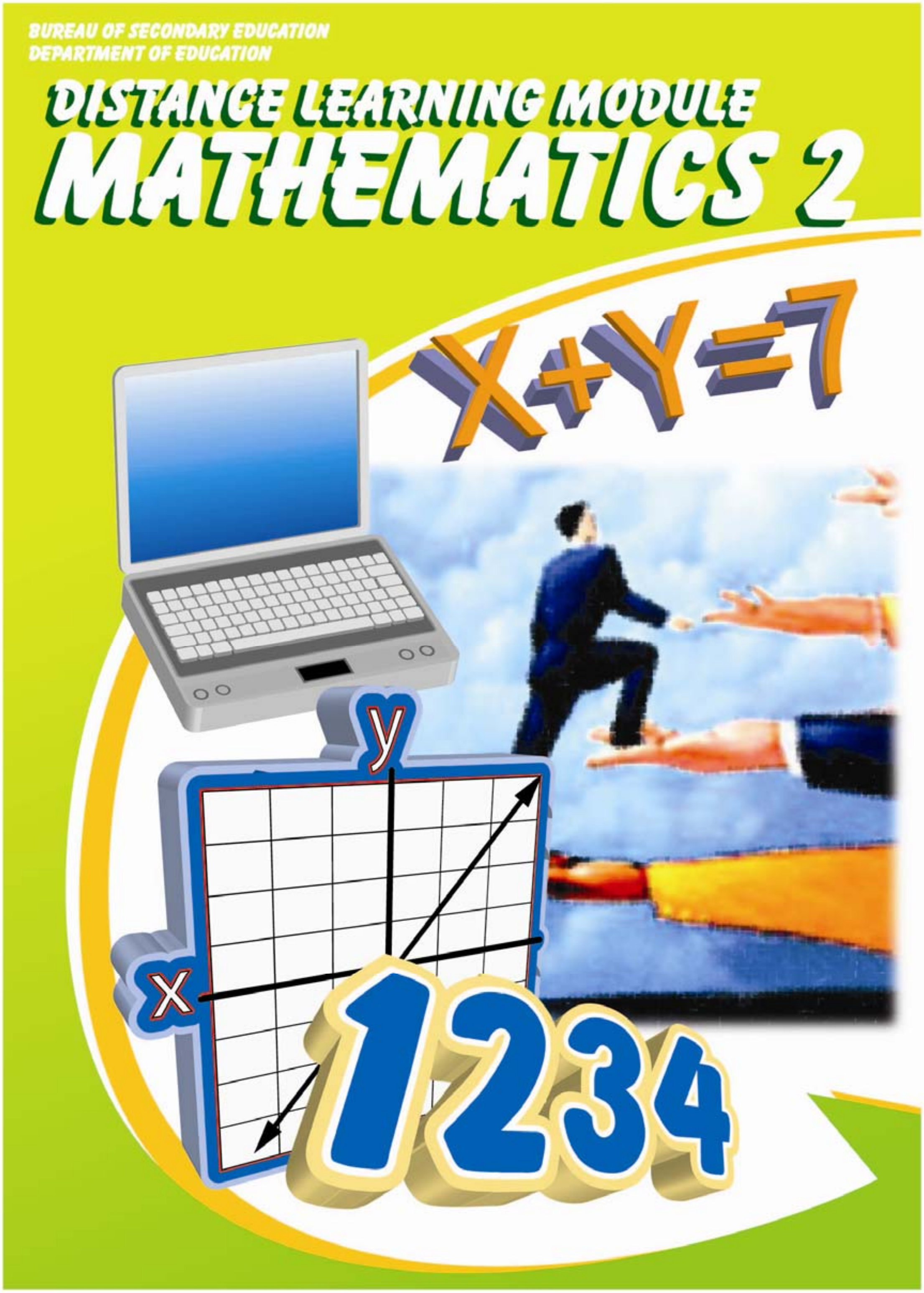

BUREAU OF SECONDARY EDUCATION
DEPARTMENT OF EDUCATION
DISTANCE LEARNING MODULE
MATHEMATICS 2
X+Y=7
y
x
1234

RADICAL EXPRESSIONS

Knowledge on radicals and its applications will surely help anyone understand and maybe conquer the things around him. One formula that may help a man in determining the efficiency of his motorcycle's engine is $e = \frac{c - \sqrt{c}}{c}$, where c is the compression ratio.

The work you have done with exponents and polynomials in Mathematics I will be very useful in understanding the concepts to be developed in this unit. You will notice that radical expressions behave like polynomials. In the last unit, you learned to use exponents to give you the square root, cube root or any root of a number or expression. In this unit, you will learn to use the $\sqrt{\ }$ symbol as another notation for the roots of numbers or expressions. You will also learn how to simplify, add, subtract, multiply or divide radicals and how to solve radical equations. And finally apply these skills in solving real-life problems.

U N I T V

Lesson 5.1 Definitions and Basic Notations

As discussed in Lesson 4.2, the "*n*th root of *a*" can be written as $a^{\frac{1}{n}}$. Another common notation, called radical notation, is

$$\sqrt[n]{a}$$

The symbol $\sqrt{\ }$ is called a radical sign;

n is called the index or the order of the radical,

a is called the radicand.

If no index is given, it is understood to be 2.

$a^{\frac{1}{2}} = \sqrt{a}, a^{\frac{1}{3}} = \sqrt[3]{a}, a^{\frac{1}{4}} = \sqrt[4]{a}$ and so on.

> If *a* is a real number and *n* is a positive integer, then
>
> $$a^{\frac{1}{n}} = \sqrt[n]{a}$$
>
> and $\sqrt[n]{a}$ read as "the *n*th root of *a*".

In Lesson 4.3, we noted that if $b^n = a$ then $a^{\frac{1}{n}} = b$. We can now say that

> If $b^n = a$ then $a^{\frac{1}{n}} = b$ or $\sqrt[n]{a} = b$.

Similarly, as discussed in Lesson 4.3, since $\left(a^{\frac{1}{n}}\right)^m = \left(a^m\right)^{\frac{1}{n}}$, it follows that

$\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$.

Example Write two equivalent notations for

a. $3^4 = 81$ b. $2^3 = 8$

Solution: Using the definition just described, we have

a. $3^4 = 81 \longrightarrow 81^{\frac{1}{4}} = 3$ or $\sqrt[4]{81} = 3$

b. $2^3 = 8$ $8^{\frac{1}{3}} = 2$ or $\sqrt[3]{8} = 2$

Changing Expressions with Rational Exponents to Radical Expressions and Vice Versa

Study how an expression in exponential notation is transformed into radical notation.

Example 1 Rewrite the following in radical notation.

a. $5^{\frac{1}{2}}$ b. $8^{\frac{2}{3}}$ c. $(7x)^{\frac{1}{4}}$ d. $(-3)^{\frac{1}{5}}$

Solution:

a.) $5^{\frac{1}{2}} = \sqrt{5}$

b.) $8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2$

c.) $(7x)^{\frac{1}{4}} = \sqrt[4]{7x}$

d.) $(-3)^{\frac{1}{5}} = \sqrt[5]{-3}$

Example 2 Rewrite the following in exponential notation

a. $\sqrt{16}$ b. $\sqrt[3]{5^2}$ c. $\left(\sqrt[4]{27}\right)^3$ d. $-\sqrt[6]{81}$

Solution:

a. $\sqrt{16} = 16^{\frac{1}{2}}$

b. $\sqrt[3]{5^2} = 5^{\frac{2}{3}}$

c. $\left(\sqrt[4]{27}\right)^3 = 27^{\frac{3}{4}}$

d. $-\sqrt[6]{81} = -81^{\frac{1}{6}}$

Let's Practice For Mastery! 1

A. Write each in radical form.

1. $7^{\frac{1}{2}}$
2. $v^{\frac{1}{6}}$
3. $x^{\frac{3}{4}}$
4. $w^{\frac{2}{5}}$
5. $(2x)^{\frac{1}{3}}$

B. Write each in exponential form.

6. $\sqrt[3]{11}$
7. $\sqrt[4]{5}$
8. $\sqrt[3]{w^2}$
9. $(\sqrt{x})^3$
10. $\sqrt{\frac{25}{16}}$

Let's Check Your Understanding! 1

A. Write each in radical form.

1. $(3x^3)^{\frac{1}{4}}$
2. $2x^{\frac{3}{2}}$
3. $(64a)^{\frac{3}{7}}$
4. $(2^9 p^3)^{\frac{1}{5}}$
5. $\left(\frac{16}{49}\right)^{\frac{2}{3}}$

B. Write each in exponential form.

6. $\sqrt{40a}$
7. $\sqrt[3]{2^5}$
8. $(\sqrt[5]{8})^2$
9. $\sqrt[4]{2x}$
10. $\sqrt[7]{2^3}$

Square Roots

This section discusses square roots in detail. We now use $\sqrt{a}$ to indicate $a^{\frac{1}{2}}$. Recall the definitions from Lesson 4.2 and 4.3, the equivalent notations $\sqrt{a}$ and $a^{\frac{1}{2}}$, and study the following.

1. Since $7^2 = 49$ and $(\sqrt{49})^2 = 49$, then $7 = \sqrt{49}$ or $\sqrt{49} = 7$. Thus, $7^2 = 49$ or $\sqrt{49} = 7$.

2. Since $3^2 = 9$ and $\left(\sqrt{9}\right)^2 = 9$, then $3 = \sqrt{9}$ or $\sqrt{9} = 3$. Thus, $3^2 = 9$ or $\sqrt{9} = 3$.

3. Since $5^2 = 25$ and $\left(\sqrt{25}\right)^2 = 25$, then $5 = \sqrt{25}$ or $\sqrt{25} = 5$. Thus, $5^2 = 25$ or $\sqrt{25} = 5$.

From the above examples, it should be seen that the reverse of squaring is called finding the square root. Note that what we have been squaring are positive numbers. What about negative numbers?

1. $7^2 = 49$, but $(-7)^2 = 49$
2. $3^2 = 9$, but $(-3)^2 = 9$
3. $5^2 = 25$, but $(-5)^2 = 25$

This leads us to the fact that every positive number x has 2 square roots, $\sqrt{x}$ and $-\sqrt{x}$.

> $\sqrt{x}$ is the positive square root and is called the principal square root while $-\sqrt{x}$ is the negative square root.

Now, if all positive numbers have square roots, are there negative numbers that have square roots? For example, what is the $\sqrt{-9}$? Is it correct to write $\sqrt{-9} = -3$? Since $(-3)^2 \neq -9$, there is no such number equal to $\sqrt{-9}$. Thus

> The square root of a negative number is not a real number.

Example 3 Find the square roots of the following:

a. $\sqrt{81}$ b. $-\sqrt{64}$ c. $\sqrt{-64}$

d. $\sqrt{0}$ e. $\pm\sqrt{64}$ f. $\sqrt{11^2}$

Solutions:

a. $\sqrt{81} = 9$ since $9^2 = 81$

b. $-\sqrt{64} = -8$ since $(-8)^2 = 64$

c. $\sqrt{-64}$ = not real since $(?)^2 = -64$

d. $\sqrt{0} = 0$

e. $\pm\sqrt{64} = \pm 8$ since $8^2 = 64$ or $(-8)^2 = 64$

f. $\sqrt{11^2} = 11$ since $\sqrt{11^2} = \sqrt{121}$

Let's Practice For Mastery! 2

Determine whether each statement is true or false. Color the box with the right answer. Then write the letters of the colored squares on the grid to decode the word. What is the word?

1. $-\sqrt{\frac{49}{36}} = -\frac{7}{6}$
2. $\sqrt{121} = \sqrt{11}$
3. $\sqrt{-4} = -2$
4. $\sqrt{90000} = 300$
5. $-\sqrt{0.366} = -0.06$
6. $\sqrt{196} = 14$
7. $\sqrt{0.0144} = 0.12$
8. $\pm\sqrt{.09} = \pm.3$

TRUE	FALSE
R	E
A	X
P	D
I	O
C	N
A	E
T	N
N	D

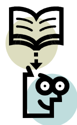
The missing word is _______________.

Let's Check Your Understanding! 2

Find each square root.

1. $\sqrt{196}$
2. $-\sqrt{256}$
3. $\pm\sqrt{324}$
4. $\sqrt{-100}$
5. $\sqrt{0.0225}$

The square roots of perfect square algebraic expressions can also be obtained. Study how they are extracted.



Example 4 Find the square roots of the following.

a. $\sqrt{c^4}$ b. $-\sqrt{100a^6}$ c. $\sqrt{h^8 s^{12}}$

Solutions:

a. $\sqrt{c^4} = c^2$ since $(c^2)^2 = c^4$

b. $-\sqrt{100a^6} = -(10a^3) = -10a^3$ since $-(10a^3)^2 = -100a^6$

c. $\sqrt{h^8 s^{12}} = h^4 s^6$ why?

Note that to find the square roots of variables, we simply divide the exponents of the variables by 2.

It is helpful to memorize the squares of the integers from 1 to 15 and also to recognize the square roots of these perfect squares.

Let's Practice For Mastery! 3

Find the square roots.

1. $-\sqrt{f^{10} t^4}$
2. $\sqrt{81t^8}$
3. $-\sqrt{64c^4}$
4. $\pm\sqrt{25d^4}$
5. $(\sqrt{25})^2$
6. $\sqrt{\dfrac{196p^{10}}{49}}$
7. $\pm\sqrt{.64b^{12}}$
8. $\sqrt{\dfrac{144}{169c^6}}$
9. $-\sqrt{h^8 s^{12}}$
10. $\sqrt{1.69a^2}$

Let's Check Your Understanding! 3

Find the square roots.

1. $\pm\sqrt{d^6}$
2. $\sqrt{r^{12}t^4}$
3. $-\sqrt{9a^2b^2}$
4. $\sqrt{\left(\frac{81}{100}\right)^2 b^2}$
5. $\sqrt{6.25t^{10}}$

Cube Roots

We also discuss cube roots aside from square roots. Knowledge of these two roots will make our work with radicals easier.

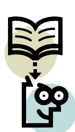
When working with square roots, we think of the perfect squares such as 1, 4, 9, 16, and 25. Similarly, when working with cube roots, we should think of the perfect cubes such as 1, 8, 27, 64, and 125.

Study the following.

1. If $2^3 = 8$, then $\sqrt[3]{8} = 2$.
2. If $(-2)^3 = -8$, then $\sqrt[3]{-8} = -2$
3. If $3^3 = 27$, then $\sqrt[3]{27} = 3$
4. If $(-3)^3 = -27$, then $\sqrt[3]{-27} = -3$

From the above examples, it should be noted that there is a cube root for every real number.

The cube root of a positive number is also a positive; and the cube root of a negative number is also a negative number.



Example 5 Find the cube roots of the following:

a. $\sqrt[3]{\frac{1}{8}}$ b. $-\sqrt[3]{27}$ c. $\sqrt[3]{-\frac{1}{8}}$ d. $\sqrt[3]{3^3}$

Solution:

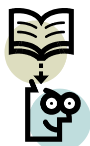
a. $\sqrt[3]{\frac{1}{8}} = \frac{1}{2}$ since $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$

b. $-\sqrt[3]{27} = -3$ since $(-3)^3 = -27$

c. $\sqrt[3]{-\frac{1}{8}} = -\frac{1}{2}$ since $\left(-\frac{1}{2}\right)^3 = -\frac{1}{8}$

d. $\sqrt[3]{3^3} = 3$ since $\sqrt[3]{3^3} = \sqrt[3]{27}$

Finding the cube roots of expressions is just like finding the square roots of expressions, but instead of dividing the exponents of the variables by 2, with what number do you divide the exponents?



Example 6 Find the cube roots of each.

a. $\sqrt[3]{x^{12}}$ b. $\sqrt[3]{-x^6y^{15}}$ c. $\frac{\sqrt[3]{x^9}}{27}$

Solutions:

a. $\sqrt[3]{x^{12}} = x^4$ since $\left(x^4\right)^3 = x^{12}$

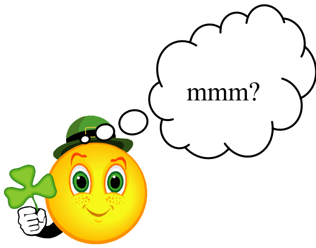
b. $\sqrt[3]{-x^6y^{15}} = -x^2y^5$ since $\left(-x^2y^5\right)^3 = -x^6y^{15}$

c. $\frac{\sqrt[3]{x^9}}{27} = \frac{x^3}{3}$ why?

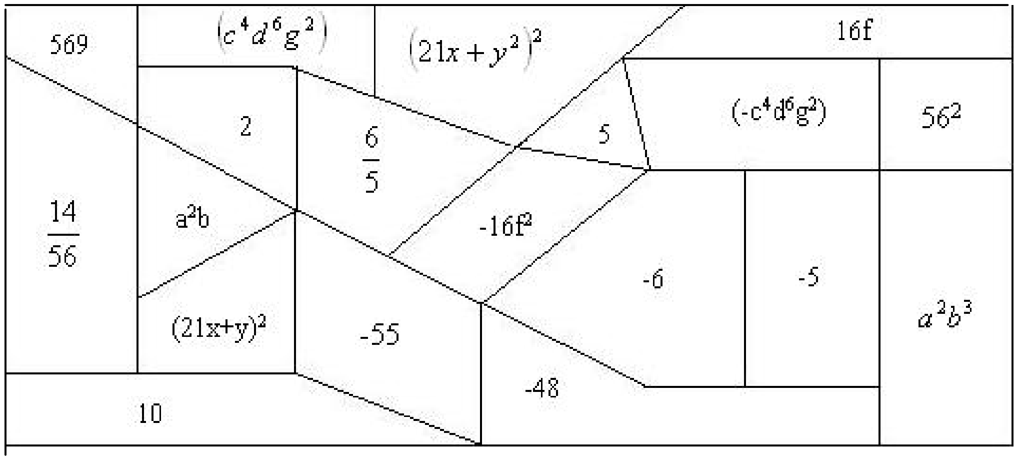
Let's Practice For Mastery! 4

Find the cube roots. Color the area where the answer is found. The colored region gives the answer to the riddle.

1. $\sqrt[3]{125}$
2. $\sqrt[3]{-125}$
3. $-\sqrt[3]{-8}$
4. $\sqrt[3]{\frac{216}{125}}$
5. $\sqrt[3]{a^6b^3}$
6. $-\sqrt[3]{c^{12}d^{18}g^6}$
7. $\sqrt[3]{\left(-16f^2\right)^3}$
8. $\sqrt[3]{(21x+y)^6}$



"I occur once in every minute, twice in every moment but not in a hundred thousand years. What am I?"



Let's Check Your Understanding! 4

Find the cube roots of the following.

1. $\sqrt[3]{-27}$ 2. $-\sqrt[3]{(-3p)^3}$ 3. $\sqrt[3]{r^9s^3t^6}$

2. 4. $\sqrt[3]{(23d^4)^3}$ 5. $\sqrt[3]{8x^9y^{36}}$

Even and Odd Roots

Radical expressions do not only involve square roots and cube roots. They also include other roots such as 4^{th} root, 5^{th} root, 6^{th} root and so on. We write $\sqrt[n]{a}$ for the *n*th root.

Study the following examples:

1. $\sqrt[5]{32} = 2$ since $2^5 = 32$
2. $\sqrt[7]{-128} = -2$ since $(-2)^7 = -128$
3. $\sqrt[4]{16} = 2$ since $2^4 = 16$
4. $-\sqrt[6]{64} = -2$ since $-(2)^6 = -64$
5. $\sqrt[8]{-256}$ not real since $(?)^8 = -256$

From the above, Examples 1 and 2, are radicals with odd roots since their indices, 5 and 7 respectively, are odd. Note that they are just like cube roots.

For odd roots, every number has just one root.

(1) If a number is positive its root is positive.

(2) If a number is negative, its root is negative.

Examples 3-5 are radicals with even roots. Why? What kind of indices do the radicals have? Note that looking for their roots works the same way as looking for square roots.

For even roots -

(1) every positive real number has two real nth roots, the positive root and the negative root.

(2) $\sqrt[n]{a}$ indicates the positive (principal) root and $-\sqrt[n]{a}$ indicates the negative root.

(3) Negative numbers do not have real roots.

Example 7 Find the indicated roots.

a. $-\sqrt[5]{-32}$ b. $\sqrt[7]{x^7}$ c. $\sqrt[4]{-16}$ d. $\sqrt[6]{64x^{12}y^{24}}$

Solutions:

a. $-\sqrt[5]{-32} = -(-2) = 2$

b. $\sqrt[7]{x^7} = x$

c. $\sqrt[4]{-16} = \text{not real}$

d. $\sqrt[6]{64x^{12}y^{24}} = 2x^2y^4$

Let's Practice For Mastery! 5

Find the indicated roots:

1. $\sqrt[4]{256}$
2. $\sqrt[4]{625}$
3. $\sqrt[4]{-81}$
4. $\sqrt[4]{\left(14b^2\right)^4}$
5. $\sqrt[5]{-32}$
6. $-\sqrt[7]{128}$
7. $\sqrt[6]{729}$
8. $-\sqrt[6]{a^6b^{12}c^{30}}$
9. $\sqrt[5]{\dfrac{x^{10}}{243}}$
10. $\sqrt[9]{-\left(8a^2\right)^9}$

Let's Check Your Understanding! 5

Find the indicated roots:

1. $\sqrt[4]{-16x^4}$
2. $\sqrt[5]{(3x)^5}$
3. $-\sqrt[4]{16a^{12}}$
4. $\sqrt[7]{-\dfrac{a^{14}b^{21}}{c^{35}}}$
5. $\left(-\sqrt[5]{3x^3}\right)^{20}$

Lesson 5.2 Simplifying Radicals

Just as it had always been with the different types of algebraic expressions, radical expressions need to be in simplified form too. But what does it mean for a radical expression to be in its simplest form?

A radical expression is in simplest form when all three statements below are true.

1. The expression under the radical sign has no perfect nth power factors other than one.
2. The expression under the radical sign does not contain a fraction.
3. The denominator does not contain a radical expression.

Example: Explain why each expression is not in simplest form.

Solution:

1. $\sqrt{20}$ — 20 under the radical sign contain a perfect square factor which is 4.

2. $\sqrt{\dfrac{2}{7}}$ — This clearly violates statement 2 of the definition as $\dfrac{2}{7}$ under the radical sign is a fraction.

3. $\dfrac{5}{\sqrt{3}}$ — This clearly does not satisfy statement 2 of the definition as the denominator contains a radical $\sqrt{3}$.

In order to simplify radicals that will satisfy the three statements, we will divide this lesson into two sections. The first section will discuss how to simplify radical expressions that contain perfect nth power factors while the second section will discuss how to simplify radical expressions that do not satisfy statements 2 and 3 of the definition.

Multiplication Property of Radicals

To simplify radical expressions that satisfy statement 1, we need to be familiar with property 1 of radicals.

Look at the following:

$$\sqrt{4 \cdot 25} = \sqrt{100} = 10 \text{ and } \sqrt{4} \cdot \sqrt{25} = 2 \cdot 5 = 10$$

Do you see that $\sqrt{4 \cdot 25} = \sqrt{4} \cdot \sqrt{25}$?

Let us consider another example,

$$\sqrt[3]{8 \cdot 27} = \sqrt[3]{216} = 6 \text{ and } \sqrt[3]{8} \cdot \sqrt[3]{27} = 2 \cdot 3 = 6$$

Notice again that $\sqrt[3]{8 \cdot 27} = \sqrt[3]{8} \cdot \sqrt[3]{27}$.

Clearly, the square root of a product is equal to the square roots of the factors and the cube root of a product is equal to the cube roots of the factors. In general:

Property 1 of Radicals

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

where a and b are positive numbers.

While this property is true for radicals of any order, we will first consider working with square roots.

Example 1 Simplify $\sqrt{12}$.

Solution: $\sqrt{12}$ is not in simplified form since it contains one perfect square factor which is 4. Thus,

$\sqrt{12} = \sqrt{4(3)}$ factoring 12 as 4(3)

$= \sqrt{4}\sqrt{3}$ using property 1 of radicals

$= 2\sqrt{3}$

Note that it would not have helped us to write $\sqrt{12}$ as $\sqrt{6 \cdot 2}$ since neither 6 nor 2 is a perfect square. It is easy to simplify radicals which contain perfect squares if we know the numbers that are perfect squares. The number 81 is a perfect square because $9^2 = 81$.

The first 20 perfect squares are listed for your easy reference.

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400

Can you identify the next 10 perfect squares? What are they?

With these perfect squares in mind, we are ready to simplify even more difficult radicals.

Example 2 Simplify the following.

a. $\sqrt{150}$ b. $\sqrt{72}$

Solution:

a. $\sqrt{150}$ contains 25 as its perfect square factor. Thus,

$\sqrt{150} = \sqrt{25 \cdot 6}$

$= \sqrt{25}\sqrt{6}$ using property 1 of radicals

$= 5\sqrt{6}$

b. $\sqrt{72}$ can be solved in two ways

Method 1

$\sqrt{72} = \sqrt{9 \cdot 8}$

$= \sqrt{9}\sqrt{8}$ why?

$= 3\sqrt{8}$ → not yet simplified so use property 1 of radicals

$= 3\sqrt{4 \cdot 2}$

$= 3 \cdot \sqrt{4}\sqrt{2}$ why?

$= 3 \cdot 2\sqrt{2}$

$= 6\sqrt{2}$

Method 2

$\sqrt{72} = \sqrt{36.2}$

$= \sqrt{36}\sqrt{2}$ why?

$= 6\sqrt{2}$

As we can see, Method 2 is shorter since it used the largest square factor while Method 1 may be longer but it could be easier since there are times when the largest square factor of a number is difficult to find.

Let's Practice For Mastery! 6

A. Simplify the following.

1. $\sqrt{24}$
2. $-\sqrt{18}$
3. $\sqrt{200}$
4. $\sqrt{75}$
5. $\sqrt{80}$
6. Verify that if a = 18 and b = 10, then $\sqrt{a} \cdot \sqrt{b} = 6\sqrt{5}$.

B. Join the dot next to each radical with its simplified form. The letter each line passes through is the code for the answer in the puzzle.

"If I had forty mangoes in one hand and fifty mangoes in the other hand, what would I have?"

$\sqrt{108}$ •		• $8\sqrt{2}$
$\sqrt{98}$ •	G	• $6\sqrt{5}$
$\sqrt{128}$ •	A	• $6\sqrt{6}$
$3\sqrt{48}$ •	D B N	• $8\sqrt{5}$
$\sqrt{180}$ •		• $10\sqrt{2}$
$\sqrt{320}$ •	H	• $6\sqrt{3}$
$\sqrt{216}$ •	S I	• $12\sqrt{3}$
$\sqrt{200}$ •		• $7\sqrt{2}$

Let's Check Your Understanding! 6

Simplify:

1. $\sqrt{32}$
2. $-\sqrt{27}$
3. $\sqrt{288}$
4. $\sqrt{48}$
5. $\sqrt{500}$

Radical expressions with variables can also be simplified. In Lesson 5.1, we were able to extract the roots of radicals with variables. For example, $\sqrt{x^6} = x^3$ since $(x^3)^2 = x^6$. This time we will simplify radicals like $\sqrt{x^7}, \sqrt{8a^{11}}$ and $\sqrt{10y^3}$.

Example 3 Simplify the following:

a. $\sqrt{x^7}$ b. $\sqrt{8x^{12}}$ c. $\sqrt{24x^4y^6}$

Solution:

a. The largest perfect square factor contained in x^7 is x^6.

So that $\sqrt{x^7} = \sqrt{x^6 x}$

$= \sqrt{x^6}\sqrt{x}$ using property 1 of radicals

$= x^3 \cdot \sqrt{x}$ since $(x^3)^2 = x^6$

b. x^{12} is already a perfect square so that

$$\sqrt{8x^{12}} = \sqrt{4 \cdot 2 \cdot x^{12}}$$

$$= \sqrt{4x^{12} \cdot 2}$$

$$= \sqrt{4x^{12}} \cdot \sqrt{2} \quad \text{using property 1 of radicals}$$

$$= \sqrt{4}\sqrt{x^{12}}\sqrt{2} = 2x^6\sqrt{2}$$

Notice that in step 3 of the solution, the first part consists of the perfect square factors while the second part consists of factors which are not perfect squares.

c. For this example, let us write $\sqrt{24x^9y^6}$ as the product of two radicals, one containing the perfect square factors and with the other containing the non- perfect square factor. Thus;

$$\sqrt{24x^9y^6} = \sqrt{(\ )} \cdot \sqrt{(\ )}$$

↑ perfect squares factors ↑ non- perfect squares factors

$= \sqrt{4} \cdot \sqrt{6}$ factoring 24 first

$= \sqrt{4x^8} \cdot \sqrt{6x}$ factoring x^9 next

$= \sqrt{4x^8y^6} \cdot \sqrt{6x}$ since y^6 is a perfect square it goes w/ the first radical

$= \sqrt{4} \cdot \sqrt{x^8} \cdot \sqrt{y^6} \cdot \sqrt{6x}$

$= 2x^4y^3\sqrt{6x}$

Let's Practice For Mastery! 7

Simplify the following. Then, write the letter in the spaces provided corresponding to the simplified form of each number to answer the puzzle.

"*What do you call a boomerang that does not come back?*"

1. $\sqrt{x^3}$ K
2. $\sqrt{25x^9}$ I
3. $\sqrt{18x^5}$ A
4. $\sqrt{12x^4y^7}$ C
5. $\sqrt{32x^9y^{13}}$ T
6. $\sqrt{24x^{15}}$ S

$3x^2\sqrt{2x}$		$2x^7\sqrt{6x}$	$4x^4y^6\sqrt{2xy}$	$5x^4\sqrt{x}$	$2x^2y^3\sqrt{3y}$	$x\sqrt{x}$

Let's Check Your Understanding! 7

Simplify:

1. $\sqrt{a^{11}}$
2. $\sqrt{36p^{17}}$
3. $\sqrt{48x^2y^9}$
4. $\sqrt{108x}$
5. $\sqrt{125x^5}$

We now consider simplifying radicals of order higher than 2. If in simplifying square roots, you were asked to find perfect square factors, what do you think should you find in simplifying cube roots, fourth roots and so on?

So, to simplify a cube root we need to find its perfect cube factors while in a fourth root, we need to find its perfect 4th root and so on.

Example 4 Simplify:

a. $\sqrt[3]{54}$ b. $\sqrt[3]{16x^3y^4}$ c. $\sqrt[4]{32x^{15}}$

Solution:

a. The perfect cube factor of $\sqrt[3]{54}$ is 27. Thus,

$\sqrt[3]{54} = \sqrt[3]{27 \cdot 2}$

$= \sqrt[3]{27}\sqrt[3]{2}$ why?

$= 3\sqrt[3]{2}$ why?

b. Since this example involves variables, let us use the method of rewriting the radicals as a product of two other radicals.

$\sqrt[3]{16x^3y^4} = \sqrt[3]{(\ )} \cdot \sqrt[3]{(\ )}$

↑ perfect cube factors ↑ non- perfect cube factors

$= \sqrt[3]{8} \cdot \sqrt[3]{2}$ factor 16 first (perfect cube factor of 16 is 8)

$= \sqrt[3]{8x^3}\sqrt[3]{2}$ factor x^3 next (x^3 is already a perfect cube)

$= \sqrt[3]{8x^3y^3}\sqrt[3]{2y}$ factor y^4 lastly (y^4 has y^3 as its perfect cube factor and $y^4 = y^3y$)

$= \sqrt[3]{8}\sqrt[3]{x^3}\sqrt[3]{y^3}\sqrt[3]{2y}$ why?

$= 2xy\sqrt[3]{2y}$ why?

c. For this example, we need to know the perfect fourth power of 32. Since $2^4 = 16$ and 16 is a factor of 32, then 16 is the perfect 4^{th} power of 32. So that

$\sqrt[4]{32x^{15}} = \sqrt[4]{16} \cdot \sqrt[4]{2}$ factoring 32 first

$= \sqrt[4]{16x^{12}} \cdot \sqrt[4]{2x^3}$ factoring x^{15} next (x^{15} has x^{12} as its perfect 4^{th} power factor and $x^{15} = x^{12}\, x^3$

$= \sqrt[4]{16} \cdot \sqrt[4]{x^{12}} \cdot \sqrt[4]{2x^3}$

$= 2x^3\sqrt[4]{2x^3}$ why?

Let's Practice For Mastery! 8

Simplify each of the following:

1. $\sqrt[3]{80x^8}$
2. $\sqrt[4]{96a^8}$
2. $\sqrt[5]{-a^6b^{11}c^{17}}$
4. $\sqrt[3]{-16x^6}$
5. $\sqrt[3]{54x^{10}}$
6. $\sqrt[4]{162c^4d^6}$
7. $\sqrt[3]{(x+y)^4}$
8. $\sqrt[5]{x^{13}y^8z^{22}}$
9. $\sqrt{x^2+6x+9}$ by first writing $x^2 + 6x + 9$ as $(x + 3)^2$
10. $\sqrt{x^2-10x+25}$

Let's Check Your Understanding! 8

Simplify the indicated roots.

1. $\sqrt[3]{800x^4}$
2. $\sqrt[3]{-32a^6}$
3. $\sqrt[4]{160a^8b}$
4. $\sqrt[3]{(a-b)^5}$
5. $\sqrt[5]{a^3b^{10}c^{18}}$

The Division Property of Radicals

We are now ready to simplify radicals which satisfy statements 2 and 3 of the definition for simplified radicals.

As in the last section, you need to know first the second property of radicals. Study the following:

$$\sqrt{\frac{144}{9}} = \sqrt{16} = 4 \text{ and } \frac{\sqrt{144}}{\sqrt{9}} = \frac{12}{3} = 4$$

What can you say about $\sqrt{\frac{144}{9}}$ and $\frac{\sqrt{144}}{\sqrt{9}}$?

Here is another example:

$$\sqrt[3]{\frac{1000}{8}} = \sqrt[3]{125} = 5 \text{ and } \frac{\sqrt[3]{1000}}{\sqrt[3]{8}} = \frac{10}{2} = 5$$

Since 5 is the answer for each case, then $\sqrt[3]{\frac{1000}{8}} = \frac{\sqrt[3]{1000}}{\sqrt[3]{8}}$.

The results above suggest the division property of radicals.

> Property 2 of Radicals
>
> $$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$
>
> where a and b are positive numbers and b $\neq 0$.

In words, the *n*th root of a fraction is equal to the quotient of the *n*th roots of the numerator and denominator.

Example 5 Write in simplest form.

a. $\sqrt{\frac{2}{25}}$ b. $\sqrt{\frac{8}{9}}$ c. $\sqrt{\frac{20x^2}{81y^2}}$

Solution:

a. The radicand contains a fraction $\sqrt{\frac{2}{25}}$ which does not satisfy condition 2.

Thus,

$$\sqrt{\frac{2}{25}} = \sqrt{\frac{2}{25}}$$

$$= \frac{\sqrt{2}}{5}$$

b. $\sqrt{\frac{8}{9}}$ does not satisfy condition 2. Thus,

$$\sqrt{\frac{8}{9}} = \frac{\sqrt{8}}{\sqrt{9}}$$

$$= \frac{\sqrt{8}}{3}$$

$\sqrt{8}$ is not yet simplified. It contains a perfect square factor. Hence,

$$= \frac{\sqrt{8}}{3} = \frac{\sqrt{4 \cdot 2}}{3}$$

$$= \frac{\sqrt{4}\sqrt{2}}{3}$$ why?

$$= \frac{2\sqrt{2}}{3}$$

c. $\sqrt{\frac{20x^2}{81y^2}} = \frac{\sqrt{20x^2}}{\sqrt{81y^2}}$ why?

$$= \frac{\sqrt{4x^2}\sqrt{5}}{\sqrt{81y^2}}$$

$$= \frac{2x\sqrt{5}}{9y}$$

Let's Practice For Mastery! 9

Simplify the following.

1. $\sqrt{\frac{5}{4}}$
2. $\sqrt{\frac{50}{49}}$
3. $\sqrt{\frac{8}{49}}$
4. $\sqrt{\frac{30}{81x^2}}$
5. $\sqrt{\frac{7x^5}{100}}$
6. $\sqrt{\frac{x^5y^2z^3}{25}}$
7. $\sqrt{\frac{19}{64x^2y^2}}$
8. $\sqrt{\frac{60x^2}{121}}$

Let's Check Your Understanding! 9

Simplify:

1. $\sqrt{\frac{8}{9}}$
2. $\sqrt{\frac{7}{144x^6}}$

3. $\sqrt{\dfrac{x^2y^4}{169}}$

4. $\sqrt{\dfrac{49x^2}{81y^2}}$

5. $\sqrt{\dfrac{75y}{81x^4}}$

What about radical expressions which do not satisfy statement 3 of the definition of a radical in simplest form? How are we going to simplify them?

Look at the example.

$\sqrt{\dfrac{3}{7}}$ does not satisfy statement 2. Using property 2 of radicals,

$$\sqrt{\frac{3}{7}} = \frac{\sqrt{3}}{\sqrt{7}}$$

There is a radical in the denominator; it does not satisfy condition 3. To get rid of the radical denominator, we apply a technique called "rationalizing the denominator".

Study the example below to see how this is done.

$$\sqrt{\frac{3}{7}} = \frac{\sqrt{3}}{\sqrt{7}} \underset{1}{=} \frac{\sqrt{3}}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} \underset{2}{=} \frac{\sqrt{21}}{\sqrt{49}} \underset{3}{=} \frac{\sqrt{21}}{7}$$

What happened in step 1?

Look at step 2. What did we multiply to $\dfrac{\sqrt{3}}{\sqrt{7}}$? Did the value of the fraction change when it was multiplied by $\dfrac{\sqrt{7}}{\sqrt{7}}$? Why is it necessary to multiply the fraction by $\dfrac{\sqrt{7}}{\sqrt{7}}$?

Look at step 3. What kind of radicand resulted in the denominator?

Notice that two things happened when we multiplied the numerator and denominator by the same quantity, $\left(\sqrt{7}\right)$; first, it did not change the value of the fraction since $\frac{\sqrt{7}}{\sqrt{7}}$ has a value of 1; and second, the denominator now has a radicand which is a perfect square (see $\sqrt{49}$ in step 3).

Let us have some more examples.

Example 6 Simplify the following:

a. $\frac{8}{\sqrt{3}}$ b. $\sqrt{\frac{7}{6}}$ c. $\frac{4}{\sqrt{12}}$ d. $\frac{10}{\sqrt{x}}$

Solution: All of Example 2 contains radicals in the denominators.

a. $\frac{8}{\sqrt{3}} = \frac{8}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{8\sqrt{3}}{\sqrt{9}} = \frac{8\sqrt{3}}{3}$

b. $\sqrt{\frac{7}{6}} = \frac{\sqrt{7}}{\sqrt{6}} = \frac{\sqrt{7}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{42}}{\sqrt{36}} = \frac{\sqrt{42}}{6}$

Notice that both numerator and denominator are multiplied by $\sqrt{3}$ and $\sqrt{6}$ respectively, because they will eventually make the denominator perfect squares. This makes simplification easy.

> In *rationalizing the denominator*, we need to find a radical expression which when multiplied to both numerator and denominator will make the denominator (in particular) a perfect square which then simplifies to a rational expression.

c. By inspection, notice that the denominator $\sqrt{12}$ is not simplified.

Hence, we need to simplify it first.

$$\frac{5}{\sqrt{12}} = \frac{5}{\sqrt{4(3)}} = \frac{5}{\sqrt{4}\sqrt{3}} = \frac{5}{2\sqrt{3}} = \frac{5}{2\sqrt{3}}$$

Now we can rationalize the denominator to completely simplify.

$$\frac{5}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{2\sqrt{9}} = \frac{5\sqrt{3}}{2(3)} = \frac{5\sqrt{3}}{6}$$

There is another way to do this.

$$\frac{5}{\sqrt{12}} = \frac{5}{\sqrt{12}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{\sqrt{36}} = \frac{5\sqrt{3}}{6}$$

Notice that both numerator and denominator are multiplied by $\sqrt{3}$ instead of $\sqrt{12}$. What happens if we multiply both numerator and denominator by $\sqrt{12}$?

$$\frac{5}{\sqrt{12}} \cdot \frac{\sqrt{12}}{\sqrt{12}} = \frac{5\sqrt{12}}{\sqrt{144}} = \frac{5\sqrt{12}}{12}$$ it is not yet in simplest form.

$$\frac{5\sqrt{12}}{12} = \frac{5\sqrt{4 \cdot 3}}{12} = \frac{5\sqrt{4}\sqrt{3}}{12} = \frac{5(2)\sqrt{3}}{12} = \frac{10\sqrt{3}}{12} = \frac{5\sqrt{3}}{6}$$

Comparing the 3 methods of simplifying $\frac{5}{\sqrt{12}}$, the second method seems to be the easiest.

d. This example involves a variable in the denominator.

$$\frac{10}{\sqrt{x}} = \frac{10}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{10\sqrt{x}}{\sqrt{x^2}} = \frac{10\sqrt{x}}{x}$$

e. $$\frac{\sqrt{2x}}{\sqrt{5y^3}} = \frac{\sqrt{x}}{\sqrt{5y^3}} \cdot \frac{\sqrt{5y}}{\sqrt{5y}} = \frac{\sqrt{5xy}}{\sqrt{25y^4}} = \frac{\sqrt{5xy}}{5y^2}$$

Let's Practice For Mastery! 10

A. What expression should be multiplied to the following to make each a perfect square?

1. 3
2. 8
3. 12
4. 24
5. 32
6. x
7. $2x^3$
8. xy^5
9. 5x
10. $72a^9$

B. Simplify each of the following. Then, write the letter corresponding to the numerator of the simplified answer.

11. $\frac{2}{\sqrt{3}}$	D	12. $\sqrt{\frac{7}{5}}$	I
13. $\frac{5}{\sqrt{6}}$	M	14. $\sqrt{\frac{3}{8}}$	U
15. $\sqrt{\frac{27x^3}{5y}}$	S	16. $\sqrt{\frac{12x^5}{7y}}$	B
17. $\sqrt{\frac{75x^3y^2}{2z}}$	S	18. $\sqrt{\frac{50x^2y^3}{3z}}$	S

Who is the author of the book?

LONG WALK

By

____	____	____	____	____	.	____	____	____
$5\sqrt{6}$	$\sqrt{35}$	$3x\sqrt{15xy}$	$5xy\sqrt{6yz}$	$2\sqrt{3}$		$2x^2\sqrt{21xy}$	$\sqrt{6}$	$5xy\sqrt{6xz}$

Let's Check Your Understanding! 10

Rationalize the denominator:

1. $\sqrt{\frac{1}{3}}$
2. $\frac{\sqrt{5}}{\sqrt{2}}$
3. $\frac{4}{\sqrt{8}}$
4. $\frac{\sqrt{9}}{\sqrt{2x^5}}$
5. $\frac{5\sqrt{6x}}{\sqrt{50}}$

The next example illustrates simplifying radicals when the denominator involves a cube root. This is done by multiplying a radical that will produce a perfect cube in the radicand of the denominator.

Example 7 Rationalize the denominator

a. $\dfrac{7}{\sqrt[3]{4}}$ b. $\dfrac{2}{\sqrt[3]{x}}$

Solution:

a. $\dfrac{7}{\sqrt[3]{4}} = \dfrac{7}{\sqrt[3]{4}} \cdot \dfrac{\sqrt[3]{2}}{\sqrt[3]{2}}$ Note that we did not multiply by $\sqrt[3]{4}$ since $\sqrt[3]{4} \cdot \sqrt[3]{4} = \sqrt[3]{16}$ which is not a perfect cube.

$= \dfrac{7\sqrt[3]{2}}{\sqrt[3]{8}}$

$= \dfrac{7\sqrt[3]{2}}{2}$

b. $\dfrac{2}{\sqrt[3]{x}} = \dfrac{2}{\sqrt[3]{x}} \cdot \dfrac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}}$ Multiplying by $\sqrt[3]{x^2}$ gives a denominator w/c is a perfect cube

$= \dfrac{2\sqrt[3]{x^2}}{\sqrt[3]{x^3}}$

$= \dfrac{2\sqrt[3]{x^2}}{x}$

What about simplifying radicals when the denominators involve roots higher than 3? The same process is done except that what is multiplied is a radical that will produce perfect *n*th power in the radicand of the denominator.

Example 8 Rationalize the denominator.

a. $\dfrac{10}{\sqrt[4]{8}}$ b. $\dfrac{7}{\sqrt[5]{27}}$

Solution:

a. $\dfrac{10}{\sqrt[4]{8}} = \dfrac{10}{\sqrt[4]{8}} \cdot \dfrac{\sqrt[4]{2}}{\sqrt[4]{2}}$ why?

$= \dfrac{10\sqrt[4]{2}}{\sqrt[4]{16}}$ why?

$$= \frac{10\sqrt[4]{2}}{2} = \frac{10\sqrt[4]{2}}{2} = 5\sqrt[4]{2}$$

b. $\frac{7}{\sqrt[5]{27}} = \frac{7}{\sqrt[5]{27}} \cdot \frac{\sqrt[5]{9}}{\sqrt[5]{9}}$ why?

$$= \frac{7\sqrt[5]{9}}{\sqrt[5]{243}} = \frac{7\sqrt[5]{9}}{3}$$

Let's Practice For Mastery! 11

A. Rationalize the denominator.

1. $\frac{4}{\sqrt[3]{2}}$

2. $\frac{5}{\sqrt[3]{3}}$

3. $\sqrt[4]{\frac{8}{y}}$

4. $\frac{3}{\sqrt[3]{5}}$

5. $\sqrt[3]{\frac{4x}{3y}}$

6. $\sqrt[4]{\frac{27}{y}}$

7. $\sqrt[3]{\frac{2x}{9y}}$

8. $\sqrt[4]{\frac{8}{9x^3}}$

B. Answer the following questions.

9. What should be multiplied to 5 to make it a perfect cube?

10. What is wrong with the following?

$$\frac{3}{\sqrt[3]{5}} = \frac{3}{\sqrt[3]{5}} \cdot \frac{1}{\sqrt[3]{25}} = \frac{3}{\sqrt[3]{125}} = \frac{3}{5}$$

Let's Check Your Understanding! 11

Simplify the following.

1. $\frac{2}{\sqrt[3]{9}}$

2. $\frac{\sqrt[3]{3}}{\sqrt[3]{4}}$

3. $\sqrt[4]{\frac{3}{2x^2}}$

4. $\sqrt[3]{\frac{7x}{6y}}$

5. $\sqrt[3]{\frac{16a^4}{9c}}$

Lesson 5.3 Addition / Subtraction of Radical Expressions

Radical expressions are added or subtracted in much the same way as polynomials. Let us review how these operations are done with polynomials.

Example 1 Find the sums or differences.

a. $4a + 6b + 2a - 3b$

b. $\left(-3x^2 + 7xy - 6y^2\right) - \left(5xy + 3y^2 - 4x^2\right)$

Solution:

a. $4a + 6b + 2a - 3b = 4a + 2a + 6b + -3b$

$= 6a + 3b$

b. $\left(-3x^2 + 7xy - 6y^2\right) - \left(5xy + 3y^2 - 4x^2\right)$

$= -3x^2 + 7xy - 6y^2 - 5xy - 3y^2 + 4x^2$ (Taking off the parenthesis which is preceded by a negative sign changes the sign of the terms)

$= -3x^2 + 4x^2 + 7xy - 5xy - 6y^2 - 3y^2$

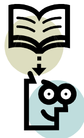
$= x^2 + 2xy - 9y^2$

Remember that addition and subtraction are allowed only when the terms are like or similar. Terms such as $-5x^3y^2$ and $3x^3y^2$ are like terms because their variables, x and y are the same, and they have the same exponents.

Similarly, radicals can be added or subtracted only when the radicals are like or similar.

Like radicals	Unlike radicals
$\sqrt{5}, 6\sqrt{5}$	$\sqrt{3}, \sqrt{5}$
$3\sqrt[3]{a^2}, -2\sqrt[3]{a^2}$	$3\sqrt[3]{a^2}, -2\sqrt[3]{b^2}$
$4\sqrt{2}, -3\sqrt{2}$	$4\sqrt{a}, 3\sqrt[3]{a}$

Like radicals, therefore, are radicals whose terms have the same variables with the same exponents and have the same radicand with the same index.



Example 2 Add or subtract as indicated:

a. $3\sqrt{5}+2\sqrt{2}-\sqrt{5}$ b. $6\sqrt[3]{3}-4\sqrt[4]{3}-8\sqrt[5]{3}$

Solution:

a. $3\sqrt{5}+2\sqrt{2}-\sqrt{5}=3\sqrt{5}-\sqrt{5}+2\sqrt{2}$

$=2\sqrt{5}+2\sqrt{2}$

b. Notice that the terms of $6\sqrt[3]{3}-4\sqrt[4]{3}-8\sqrt[5]{3}$ cannot be combined further since all 3 terms have different indices.

Let's Practice For Mastery! 12

A. Which are like terms?

1. $\sqrt{2}, 2\sqrt{2}, 3\sqrt{5}$
2. $5x\sqrt[3]{2}, -5y\sqrt[3]{2}, 10x\sqrt{2}$
3. $3\sqrt{7}, 2\sqrt[3]{7}, 8\sqrt[3]{7}$
4. $5\sqrt[3]{x^2}, 7\sqrt[3]{x^2}, -9\sqrt[3]{x^2}$

B. Add or subtract.

5. $3\sqrt{2}+2\sqrt{2}-4\sqrt{2}$
6. $5\sqrt{7}+6\sqrt{7}-11\sqrt{7}$
7. $10\sqrt{2}+7\sqrt{3}-4\sqrt{4}-5\sqrt{3}$
8. $5\sqrt[3]{10}-4\sqrt[3]{10}$
9. $3x\sqrt{2}-4x\sqrt{2}+x\sqrt{2}$
10. $6y\sqrt{a}-2x\sqrt{a}+7y\sqrt{a}$

Let's Check Your Understanding! 12

Simplify the following.

1. $3\sqrt{5}+8\sqrt{5}$
2. $4\sqrt{2}-11\sqrt{2}$
3. $6\sqrt[4]{2}-9\sqrt[4]{2}$
4. $6y\sqrt{a}+15y\sqrt{a}$
5. $3x\sqrt{2}-4x\sqrt{2}+x\sqrt{2}$

There are terms which may not look like they are similar or like radicals, but when some simplifications are done, they are actually like radicals and they can be combined.

Example 3 Add or subtract.

a. $\sqrt{8}-7\sqrt{2}$ b. $\sqrt{48}+6\sqrt{27}-7\sqrt{3}$

c. $10\sqrt{8}-\sqrt{72}+3\sqrt{98}$ d. $7\sqrt{75xy^3}-5y\sqrt{12xy}$

Solution: For each, our plan is to simplify first to find out whether we can combine the simplified terms or not.

a. Since $\sqrt{8}$ simplifies to $2\sqrt{2}$, we can thus add the like radicals

$$\sqrt{8}-7\sqrt{2}=\sqrt{4\cdot 2}-7\sqrt{2}$$
$$=2\sqrt{2}-7\sqrt{2}$$
$$=-5\sqrt{2}$$

b. $\sqrt{48}+6\sqrt{27}-7\sqrt{3}=\sqrt{16\cdot 3}+6\sqrt{9\cdot 3}-7\sqrt{3}$ why?

$=\sqrt{16}\cdot\sqrt{3}+6\left(\sqrt{9}\right)\sqrt{3}-7\sqrt{3}$ why?

$=4\sqrt{3}+6(3)\sqrt{3}-7\sqrt{3}$ They are like terms after all, so we can combine them.

$$=4\sqrt{3}+18\sqrt{3}-7\sqrt{3}$$
$$=15\sqrt{3}$$

c. $10\sqrt{8}-\sqrt{72}+3\sqrt{98}=10\sqrt{4\cdot 2}-\sqrt{36\cdot 2}+\sqrt{49\cdot 2}$ why?

$$=10\left(\sqrt{4}\right)\left(\sqrt{2}\right)-\sqrt{36}\cdot\sqrt{2}+3\sqrt{49}\cdot\sqrt{2}$$
$$=10(2)\sqrt{2}-6\sqrt{2}+3(7)\sqrt{2}$$
$$=20\sqrt{2}-6\sqrt{2}+21\sqrt{2}$$
$$=14\sqrt{2}+21\sqrt{2}$$
$$=35\sqrt{2}$$

d. $7\sqrt{75xy^3}-5y\sqrt{12xy}=7\sqrt{25y^2}\cdot\sqrt{3xy}-5y\sqrt{4}\cdot\sqrt{3xy}$ why?

$$=7(5y)\sqrt{3xy}-5y(2)\sqrt{3xy}$$
$$=35y\sqrt{3xy}-10y\sqrt{3xy}$$
$$=25y\sqrt{3xy}$$

Let's Practice For Mastery! 13

A. Add or subtract.

1. $\sqrt{18}+\sqrt{2}$
2. $\sqrt{12}+\sqrt{3}$
3. $\sqrt{20}-\sqrt{80}+\sqrt{45}$
4. $5\sqrt[3]{16}-4\sqrt[3]{54}$
5. $b\sqrt{25a^5}-a^2\sqrt{6ab^2}$

B. Each of the following statement is false. Correct the right side of each to make it true.

6. $3\sqrt{2x}+5\sqrt{2x}=8\sqrt{4x}$
7. $5\sqrt{3}-7\sqrt{3}=-2\sqrt{9}$
8. $\sqrt{16-9}=4-3$

Let's Check Your Understanding! 13

Find the sum or difference.

1. $\sqrt{8}-\sqrt{32}-\sqrt{18}$
2. $\sqrt{48}-3.\sqrt{27}$
3. $2\sqrt{50x^2}-8x\sqrt{18}$
4. $5\sqrt{3x}+\sqrt{12x}$
5. $a\sqrt{20a^3b^2}+b\sqrt{45a^5}$

Lesson 5.4 Multiplication and Division of Radical Expressions

In this lesson, we will do the multiplication and division of expressions that contain radicals. Multiplying radical expressions works the same way as with multiplying polynomials. The division problems in this lesson are just an extension of the work we did previously when we rationalized denominators. The two properties of radicals are listed below for easy reference.

Property 1 $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ Property 2 $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$, $b \neq 0$

It is easy to see that from the two properties, we have

$$\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab} \quad \text{and} \quad \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

Multiplication of Radical Expressions

As implied, the product of the *n*th roots of two numbers is equal to the *n*th root of the product of these numbers.

Example 1 Find the product.

a. $\sqrt{2} \cdot \sqrt{8}$ b. $\sqrt{3} \cdot \sqrt{6}$ c. $\sqrt{15} \cdot \sqrt{12}$

d. $\sqrt{x} \cdot \sqrt{x^3}$ e. $\sqrt[3]{4} \cdot \sqrt[3]{16}$

Solution:

a. $\sqrt{2} \cdot \sqrt{8} = \sqrt{2(8)} = \sqrt{16} = 4$

b. $\sqrt{3} \cdot \sqrt{6} = \sqrt{3(6)} = \sqrt{18} = \sqrt{9(2)} = 3\sqrt{2}$

c. $\sqrt{15} \cdot \sqrt{12} = \sqrt{15(12)} = \sqrt{180} = \sqrt{36(5)} = 6\sqrt{5}$

d. $\sqrt{x} \cdot \sqrt{x^3} = \sqrt{x \cdot x^3} = \sqrt{x^4} = x^2$

e. $\sqrt[3]{4} \cdot \sqrt[3]{16} = \sqrt[3]{4(16)} = \sqrt[3]{64} = 4$

Let's Practice For Mastery! 14

Multiply the following.

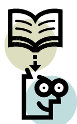
1. $\sqrt{3}\sqrt{3}$
2. $\sqrt{16}\sqrt{4}$
3. $\sqrt[3]{8}\sqrt[3]{8}$
4. $\sqrt{27}\sqrt{3}$
5. $\sqrt[3]{4}\sqrt[3]{250}$
6. $\sqrt{a^7}\sqrt{a^3}$
7. $\sqrt[3]{b^7}\sqrt[3]{b^5}$
8. $\sqrt{32}\sqrt{2}$
9. $\sqrt{18}\sqrt{14}$
10. $\sqrt{45}\sqrt{60}$

Let's Check Your Understanding! 14

Find the products.

1. $\sqrt[3]{7}\sqrt[3]{2}$
2. $\sqrt[4]{8}\sqrt[4]{9}$
3. $\sqrt[3]{16}\sqrt[3]{4}$
4. $-\sqrt{50}\sqrt{2}$
5. $\sqrt{20}\sqrt{15}$

To multiply monomials containing radicals, we multiply the coefficients and multiply the radicals separately and simplify the result, when possible.



Example 2 Multiply:

a. $2\sqrt{6}$ by $5\sqrt{3}$ b. $-2\sqrt[3]{7}$ by $8\sqrt[3]{4}$

Solution: The commutative and associative properties enable us to multiply the integers and radicals separately.

a. $2\sqrt{6}\cdot 5\sqrt{3} = 2(5)\sqrt{6}\sqrt{3}$

$= 2(5)\sqrt{6(3)}$ why?

$= 10\sqrt{18}$

$= 10\sqrt{9(2)} = 10\sqrt{9}\sqrt{2}$ why?

$= 10(3)\sqrt{2} = 30\sqrt{2}$

b. $-2\sqrt[3]{7}\cdot 8\sqrt[3]{4} = -2(8)\sqrt[3]{7}\sqrt[3]{4}$

$= -2(8)\sqrt[3]{7(4)}$

$= -16\sqrt[3]{28}$

To multiply a polynomial by a monomial, we can use the distributive property to remove parentheses and combine like terms.

Example 3 Multiply:

a. $\sqrt{7}\left(2+\sqrt{7}\right)$ b. $\sqrt{3}\left(2\sqrt{5}-7\sqrt{6}\right)$

Solution:

a. $\sqrt{7}\left(2+\sqrt{7}\right)=\sqrt{7}\cdot(2)-\sqrt{7}\cdot\sqrt{7}$ why?

$= 2\sqrt{7}-\sqrt{49}$ why?

$= 2\sqrt{7}-7$

b. $\sqrt{3}\left(2\sqrt{5}-7\sqrt{6}\right)=\sqrt{3}\left(2\sqrt{5}\right)-\sqrt{3}\left(7\sqrt{6}\right)$ why?

$= 2\sqrt{3}\cdot\sqrt{5}-7\sqrt{3}\cdot\sqrt{6}$ why?

$= 2\sqrt{15}-7\sqrt{18}$ why?

$= 2\sqrt{15}-7\sqrt{9(2)}$ why?

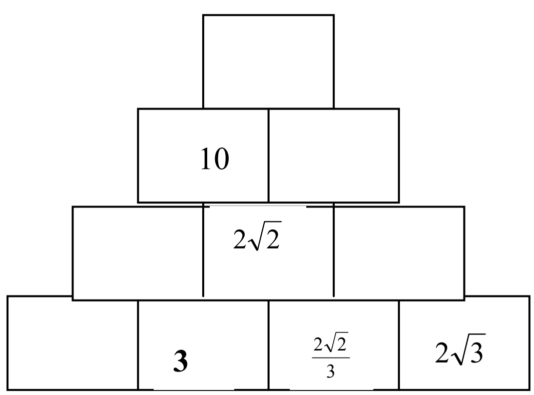
$= 2\sqrt{15}-7\sqrt{9}\cdot\sqrt{2}$ why?

$= 2\sqrt{15}-7(3)\sqrt{2}$ why?

$= 2\sqrt{15}-21\sqrt{2}$

Let's Practice For Mastery! 15

A. Each brick on this wall should have a number on it. Each number is the result of multiplying the two numbers on the bricks below. Can you fill in all the numbers? Example: $2\sqrt{2}$ is the product of 3 and $\frac{2\sqrt{2}}{3}$, the numbers on the bricks below $2\sqrt{2}$.



B. Multiply each.

6. $\sqrt{3}\left(\sqrt{2}-3\sqrt{2}\right)$

7. $\sqrt{2}\left(5\sqrt{3}+4\sqrt{2}\right)$

8. $6\sqrt{6}\left(2\sqrt{2}+1\right)$

9. $7\sqrt{5}\left(3\sqrt{15}-2\right)$

Let's Check Your Understanding! 15

Find the products in simplified form.

1. $2\sqrt{3}\left(6\sqrt{7}\right)$
2. $3\sqrt{5}\left(2\sqrt{6}\right)$
3. $3\sqrt[3]{2}\left(5\sqrt[3]{4}\right)$
4. $\sqrt{3}\left(4\sqrt{3}-7\right)$
5. $2\sqrt{5}\left(3\sqrt{2}+4\sqrt{3}\right)$

We can use the FOIL method to multiply one binomial by another.

Example 4 Multiply the following.

a. $\left(4\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-5\sqrt{2}\right)$

b. $\left(\sqrt{3}-2x\right)^2$

c. $\left(2-\sqrt{7}\right)\left(2+\sqrt{7}\right)$

d. $\left(\sqrt{6}+\sqrt{2}\right)\left(\sqrt{6}-\sqrt{2}\right)$

Solution:

a. $\left(4\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-5\sqrt{2}\right)$

F O I L

$$= 4\sqrt{3}\cdot\sqrt{3}-\left(4\sqrt{3}\right)\left(5\sqrt{2}\right)+\sqrt{2}\cdot\sqrt{3}-5\left(\sqrt{2}\right)\cdot\sqrt{2}$$

$$= 4\sqrt{3}\cdot\sqrt{3}-4(5)\sqrt{3}\cdot\sqrt{2}+\sqrt{2}\cdot\sqrt{3}-5\sqrt{2}\cdot\sqrt{2}$$

$$= 4\sqrt{9}-20\sqrt{6}+\sqrt{6}-5\sqrt{4}$$ why?

$$= 4(3)-19\sqrt{6}-5(2)$$ why?

$$= 12-19\sqrt{6}-10 = 2-19\sqrt{6}$$

b. This example shows application of special product called the square of a binomial. Remember that $(a+b)^2 = a^2 + 2ab + b^2$. Thus,

$$\left(\sqrt{3}-2x\right)^2 = \left(\sqrt{3}\right)^2 - 2\left(\sqrt{3}\right)(2x) + \left(2x^2\right)$$
$$= 3 - 2(2)\sqrt{3}x + 4x^2$$
$$= 3 - 4\sqrt{3}x + 4x^2$$

c. This is also a special product. It is the sum and difference of a binomial. Remember that $(a+b)(a-b)$ always gives the difference of two squares, $a^2 - b^2$.

$$\left(2-\sqrt{7}\right)\left(2+\sqrt{7}\right) = (2)^2 - \left(\sqrt{7}\right)^2$$
$$= 4 - 7 = -3$$

d. This is again the sum and difference of a binomial.

$$\left(\sqrt{6}+\sqrt{2}\right)\left(\sqrt{6}-\sqrt{2}\right) = \left(\sqrt{6}\right)^2 - \left(\sqrt{2}\right)^2$$
$$= 6 - 2 = 4$$

In Examples 4c and 4d, the two expressions $\left(2-\sqrt{7}\right)\left(2+\sqrt{7}\right)$ and $\left(\sqrt{6}+\sqrt{2}\right)\left(\sqrt{6}-\sqrt{2}\right)$ are called ***conjugates***. In general, the conjugate of $\sqrt{a}+\sqrt{b}$ is $\sqrt{a}-\sqrt{b}$. Multiplying conjugates of this form always produce a rational number. Why?

Let's Practice For Mastery! 16

A. Multiply and simplify all answers.

1. $\left(\sqrt{11}+3\right)\left(\sqrt{11}-6\right)$
2. $\left(3\sqrt{2}-2\sqrt{5}\right)^2$
3. $\left(\sqrt{a}+\sqrt{5}\right)\left(\sqrt{a}-\sqrt{5}\right)$
4. $\left(4+5\sqrt{3}\right)^2$
5. $\left(3\sqrt{2}-5\right)\left(3\sqrt{2}+5\right)$
6. $\left(5-2\sqrt{3}\right)\left(4+\sqrt{3}\right)$

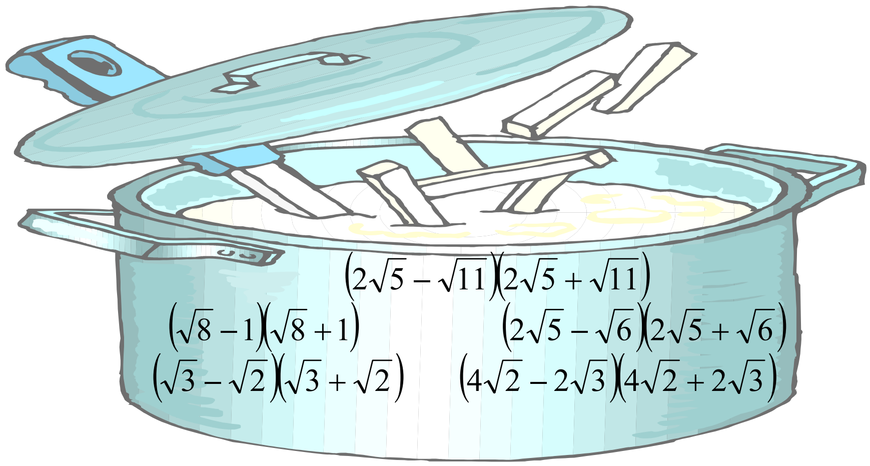
B. Write the conjugate of each.

7. $3\sqrt{x}-4$
8. $\sqrt{2y}+5$
9. $\sqrt{x}+5y$

C. Do what is asked.

10. 3 is a solution of $x^2 - 5x + 6 = 0$, since it makes the equation true. Verify that $2+\sqrt{10}$ is a solution to the equation $x^2 - 4x - 6 = 0$.

D. Multiply the conjugates. Change the answer (number) into letters, based on the order of the alphabet. Example, the number 5 is E in the alphabet. Unscramble the letters to find out what's cooking!



___ ___ ___ ___ ___ ___ ___

Let's Check Your Understanding! 16

Multiply.

1. $\left(2-3\sqrt{3}\right)\left(5-2\sqrt{7}\right)$
2. $\left(\sqrt{13}+2\right)\left(\sqrt{13}-2\right)$
3. $\left(\sqrt{3}-\sqrt{5}\right)^2$
4. $\left(2\sqrt{5}+\sqrt{7}\right)\left(2\sqrt{5}-\sqrt{7}\right)$
5. $\left(\sqrt{7}+2\sqrt{3}\right)\left(\sqrt{2}-\sqrt{3}\right)$

Division of Radical Expressions

To divide radical expressions divide the coefficients and radicands separately, then simplify when necessary.

Example 5 Divide the following. Simplify the result.

a. $\sqrt{20}\div\sqrt{5}$ b. $15\sqrt{4}\div 3\sqrt{2}$ c. $6\sqrt{27x^5}\div 2\sqrt{3x^3}$

Solution:

a. $\dfrac{\sqrt{20}}{\sqrt{5}} = \sqrt{\dfrac{20}{5}} = \sqrt{4} = 2$

b. $\dfrac{15\sqrt{14}}{3\sqrt{2}} = \left(\dfrac{15}{3}\right)\left(\sqrt{\dfrac{14}{2}}\right) = 5\sqrt{7}$

c. $\dfrac{6\sqrt{27x^5}}{2\sqrt{3x^2}} = \left(\dfrac{6}{2}\right)\left(\sqrt{\dfrac{27x^5}{3x^3}}\right)$

$= 3\sqrt{9x^2}$
$= 3(3x)$
$= 9x$

Let's Practice For Mastery! 17

Divide.

1. $\dfrac{4\sqrt{18}}{2\sqrt{2}}$

2. $\dfrac{3\sqrt{24}}{\sqrt{2}}$

3. $\dfrac{2\sqrt{54}}{6\sqrt{3}}$

4. $\dfrac{\sqrt{12xy^2}}{\sqrt{3xy}}$

5. $\dfrac{\sqrt{54}}{6\sqrt{6}}$

Let's Check Your Understanding! 17

Divide the following. Then simplify.

1. $\dfrac{4\sqrt{32}}{\sqrt{2}}$

2. $\dfrac{\sqrt{54}}{6\sqrt{6}}$

3. $\dfrac{5\sqrt{40}}{2\sqrt{5}}$

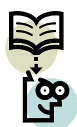
4. $\dfrac{\sqrt{8xy_3}}{\sqrt{2y}}$

5. $\dfrac{\sqrt{45x^3y}}{\sqrt{15xy}}$

Suppose we encounter a division problem like $7 \div \left(\sqrt{2}+\sqrt{3}\right)$? Here the divisor is a binomial. Writing it as a fraction gives us $\frac{7}{\sqrt{2}+\sqrt{3}}$. Multiplying the numerator and denominator by $\sqrt{3}$ or by $\sqrt{2}$ will not make the denominator a rational number. To solve this, we can multiply the numerator and denominator by the conjugate of the denominator, $\sqrt{2}+\sqrt{3}$, which is $\sqrt{2}-\sqrt{3}$. You can do this because as you have noted in the last lesson, the product of conjugates is a rational number.

Thus,

$$\frac{7}{\sqrt{2}+\sqrt{3}} = \frac{7}{\sqrt{2}+\sqrt{3}} \cdot \frac{\left(\sqrt{2}-\sqrt{3}\right)}{\left(\sqrt{2}-\sqrt{3}\right)} = \frac{7\left(\sqrt{2}-\sqrt{3}\right)}{\left(\sqrt{2}\right)^2-\left(\sqrt{3}\right)^2} = \frac{7\sqrt{2}-7\sqrt{3}}{-1} = -7\sqrt{2}+7\sqrt{3}$$



Example 6 Divide $\left(\sqrt{5}-2\right) \div \sqrt{5}+\sqrt{2}$

Solution: To divide, we rationalize by multiplying the numerator and denominator by the conjugate of the denominator.

$$\frac{\sqrt{5}-2}{\sqrt{5}+\sqrt{2}} = \frac{\sqrt{5}-2}{\sqrt{5}+\sqrt{2}} \cdot \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$= \frac{\left(\sqrt{5}-2\right)\left(\sqrt{5}-\sqrt{2}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}$$

$$= \frac{\left(\sqrt{5}\right)^2-\sqrt{2}\sqrt{5}-2\sqrt{5}+2\sqrt{2}}{5-2}$$

$$= \frac{5-\sqrt{10}-2\sqrt{5}+2\sqrt{2}}{3}$$

Let's Practice For Mastery! 18

Divide:

1. $\frac{2}{\sqrt{3}+2}$
2. $\frac{3}{\sqrt{5}-\sqrt{2}}$
3. $\frac{\sqrt{5}}{\sqrt{3}+\sqrt{3}}$
4. $\frac{\sqrt{5}}{\sqrt{5}+1}$
5. $\frac{2}{\sqrt{x}+\sqrt{7}}$
6. Show that the product $\left(\sqrt[3]{2}+\sqrt[3]{3}\right)\left(\sqrt[3]{4}-\sqrt[3]{6}+\sqrt[3]{9}\right)$ is 5.

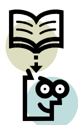
Let's Check Your Understanding! 18

Rationalize the denominator.

1. $\dfrac{8}{\sqrt{x}-3}$ 2. $\dfrac{-\sqrt{3}}{\sqrt{3}+1}$ 3. $\dfrac{\sqrt{8}}{\sqrt{5}-\sqrt{3}}$ 4. $\dfrac{\sqrt{3x}-1}{\sqrt{3x}+1}$

Products and Quotients of Radicals with Different Indices

It is important to remember that the product and quotient properties for radicals can be used only when radicals have the same index. When indices differ, rational exponents can be useful. Study the steps in the following example carefully.



Example 7 Multiply a. $\sqrt[3]{5}\cdot\sqrt{2}$ b. $\sqrt{x^3}\cdot\sqrt[3]{x}$

Solution:

a. $\sqrt[3]{5}\left(\sqrt{2}\right)=5^{\frac{1}{3}}\left(2^{\frac{1}{2}}\right)$ writing radicals into expressions with rational exponents

$=5^{\frac{2}{6}}\left(2^{\frac{3}{6}}\right)$ changing to equivalent fractions

$=\left(5^2\cdot 2^3\right)^{\frac{1}{6}}$ applying power of a product rule for exponents

$=\sqrt[6]{5^2\cdot 2^3}$ rewriting to radicals

$=\sqrt[6]{200}$

b. $\sqrt{x^3}\cdot\sqrt[3]{x}=x^{\frac{3}{2}}\cdot x^{\frac{1}{3}}$ why?

$=x^{\frac{9}{6}}\cdot x^{\frac{2}{6}}$ why?

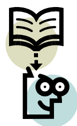
$=x^{\frac{11}{6}}$ why?

$=\sqrt[6]{x^{11}}$ why?

$=\sqrt[6]{x^6\left(x^5\right)}$ why?

$=\sqrt[6]{x^6}\cdot\sqrt[6]{x^5}$ why?

$=x\sqrt[6]{x^5}$



Example 8 Divide : $\sqrt{6} \div \sqrt[3]{2}$

Solution:
$$\frac{\sqrt{6}}{\sqrt[3]{2}} = \frac{6^{\frac{1}{2}}}{2^{\frac{1}{3}}}$$
$$= \frac{6^{\frac{3}{6}}}{2^{\frac{2}{6}}}$$
$$= \left(\frac{6^3}{2^2}\right)^{\frac{1}{6}}$$
$$= \sqrt[6]{\frac{216}{4}}$$
$$= \sqrt[6]{18}$$

Let's Practice For Mastery! 19

A. Do as indicated.

1. $\sqrt{x} \cdot \sqrt[3]{x}$
2. $\sqrt{2} \cdot \sqrt[4]{2}$
3. $\dfrac{\sqrt[4]{p^3}}{\sqrt[3]{p}}$

2. 4. $\sqrt[5]{a^3} \cdot \sqrt{a}$
5. $\sqrt[3]{4} \cdot \sqrt{2}$

B. Answer each of the following questions.

6. Which is greater, 2^8 or 2^{10}?
7. Which is lesser, $\left(\frac{1}{2}\right)^8$ or $\left(\frac{1}{2}\right)^{10}$

Let's Check Your Understanding! 19

Find the product or quotient.

1. $\sqrt{3} \cdot \sqrt[3]{9}$
2. $\sqrt{3} \cdot \sqrt[3]{3}$
3. $\dfrac{\sqrt[3]{b}}{\sqrt{b}}$
4. $\dfrac{\sqrt[3]{a}}{\sqrt{a}}$
5. $\sqrt[4]{p} \cdot \sqrt{p}$

Lesson 5.6 Radical Equations

Radical equations are equations in which the unknown is part of the radicand. The following are radical equations,

$\sqrt{x} = 4 \qquad \sqrt{x+1} = 6 \qquad 3\sqrt{2x+1} = 14 \qquad \sqrt[3]{x} = 27$

since x is found in the radicand.

For this lesson, we will discuss radical equations involving square roots. To solve such equations for the unknown, we need to transform it to a linear equation or to a factorable quadratic equation that we already know how to solve. The main property needed to solve these equations is the squaring property of equality.

If $a = b$, then $a^2 = b^2$.

This squaring property of equality can be used with radical equations involving square roots to produce another equation without square roots.

Example 1 Solve each of the following:

a. $\sqrt{x} = 4$ b. $\sqrt{x-2} = 3$

Solution:

a. $\sqrt{x} = 4$

$\left(\sqrt{x}\right)^2 = (4)^2$ squaring both sides

$x = 16$

Check:

$\sqrt{x} = 4$

$\sqrt{16} \, ? \, 4$

$4 = 4$

16 is a solution of the equation.

b. $\sqrt{x-2} = 3$

$\left(\sqrt{x-2}\right)^2 = (3)^2$ squaring both sides

$x - 2 = 9$

$x = 9 + 2$ using addition property of equality

$x = 11$

Check:

$\sqrt{x-2} = 3$

$\sqrt{11-2} \, ? \, 3$

$\sqrt{9} \, ? \, 3$

$3 = 3$

11 is a solution of the equation.

Let's Practice For Mastery! 20

A. Write the letter corresponding to the solution of each equation to find the code below.

R	$\sqrt{x}=5$	S	$5\sqrt{x}=2$
Q	$\sqrt{2x}=3$	O	$\sqrt{-3x}=9$
E	$\sqrt{x-4}=10$	A	$\sqrt{5x-5}=5$
U	$\sqrt{4x+17}=3$	T	$\sqrt{3x+7}=0$

What happens to a plant when it is put in a Math Classroom?

It develops a

$\frac{4}{25}$	$\frac{9}{2}$	-2	6	25	104		25	-27	-27	$-\frac{7}{3}$

Let's Check Your Understanding! 20

Solve each equation.

1. $\sqrt{x}=\frac{3}{4}$
2. $8=\sqrt{x-9}$
3. $\sqrt{\frac{n}{5}}=2$
4. $\sqrt{x+5}=4$
5. $2\sqrt{x}=5$

Notice that the radicals in Example 1 are placed on one side of the equation so that squaring both sides is easier done. What happens when the radical is not isolated on one side of the equation?

Given the equation, $\sqrt{x}-2=5$, where the radical is not isolated. See what happens when the squaring property is used right away.

$$\sqrt{x}-2=5$$

$$\left(\sqrt{x}-2\right)^2=(5)^2$$

$$\left(\sqrt{x}\right)^2-2(2)\sqrt{x}+4=25$$

$$x-4\sqrt{x}+4=25$$

Note that the simple equation becomes more complicated! Why?

Isolating the radical on one side of the equation gives us the following:

$\sqrt{x} = 5 + 2$ using addition property of equality

$\sqrt{x} = 7$

What is the solution of the equation?

The next example shows equations whose radicals are on both sides of the equation.

Example 2 Solve:

a. $\sqrt{3y+1} = \sqrt{2y+6}$ b. $\sqrt{x+4} - \sqrt{2x+1} = 0$

Solution:

a. $\sqrt{3y+1} = \sqrt{2y+6}$

$\left(\sqrt{3y+1}\right)^2 = \left(\sqrt{2y+6}\right)^2$ Using the squaring property

$3y + 1 = 2y + 6$

$3y - 2y = 6 - 1$ Solving the equation

$y = 5$

Check: $\sqrt{3y+1} \; ? \; \sqrt{2y+6}$ $\sqrt{16} \; ? \; \sqrt{16}$

$\sqrt{3(5)+1} \; ? \; \sqrt{2(5)+6}$ $4 = 4$

5 is a solution of the equation.

b. Rewriting the given equation, we have

$\sqrt{x+4} = \sqrt{2x+1}$

$\left(\sqrt{x+4}\right)^2 = \left(\sqrt{2x+1}\right)^2$ Why?

$x + 4 = 2x + 1$ Why?

$x - 2x = 1 - 4$

$-x = -3$

$x = 3$

Check: $\sqrt{x+4}-\sqrt{2x+1}=0$ $\qquad\qquad$ $\sqrt{7}-\sqrt{7}\,?\,0$

$\sqrt{3+4}-\sqrt{2(3)+1}\,?\,0$ $\qquad\qquad$ $0=0$

3 is a solution of the equation.

Let's Practice For Mastery! 21

Solve and check your answer.

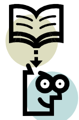
1. $\sqrt{m+23}=2\sqrt{m+11}$
2. $\sqrt{n}-4=10$
3. $\sqrt{1-3x}-2=5$
4. $\sqrt{y+2}=\sqrt{2y+5}$
5. $\sqrt{x+4}-\sqrt{4x+1}=0$

Let's Check Your Understanding! 21

Solve and check your answer.

1. $\sqrt{x}+6=13$
2. 2. $3+\sqrt{x-1}=5$
3. $\sqrt{a-1}=\sqrt{3}$
4. $\sqrt{3y+2}=2\sqrt{y}$
5. $\sqrt{3x+6}=2\sqrt{2x-11}$

It is important to note that in using the squaring property, there are solutions which may not really solve the equation as we will see in the next example.



Example 3 $\quad$ Solve $\sqrt{3y-1}=-4$

Solution:

$$\sqrt{3y-1}=-4$$

$$\left(\sqrt{3y+1}\right)^2=\left(-4\right)^2$$ Using the squaring property

$3y + 1 = 16$

$3y = 16 - 1$ Solving the equation

$3y = 15$

$y = 5$

Check: $\sqrt{3y+1} = -4$ $\sqrt{16} ? -4$

$\sqrt{3(5)+1} ? -4$ $4 \neq -4$

The solution does not check. The equation has no solution. Note that from the very start we could have stopped solving since the square root (principal) of any number is always positive.

Example 3 shows that when the squaring property is used, it may produce values that are not solutions of the equation. These solutions are called *extraneous solutions*.

> The following are steps in solving radical equations:
>
> 1. Rewrite the equation such that the radicals are on opposite sides of the equal sign.
> 2. Square both sides of the equation and solve the resulting equation
> 3. Check the solution to determine whether it is really a solution or an extraneous value.

Let's Practice For Mastery! 22

A. Solve:

1. $5 + \sqrt{x} = 3$
2. $\sqrt{7+3x} + 4 = 0$
3. $6 + \sqrt{x} = 13$
4. $2\sqrt{x} = 5$
5. $2\sqrt{4x+5} = 5\sqrt{x+4}$

B. Tell which solutions, if any, are extraneous for the given equation.

6. $x = \sqrt{2x}$ x = 0, x = 2

7. $\sqrt{12 - x} = x$ x = -4, x = 3

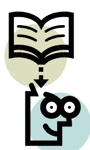
8. $-x = \sqrt{-x + 6}$ $x = -3, x = 2$

Let's Check Your Understanding! 22

Solve the following.

1. $\sqrt{9x + 25} - 2 = 3$
2. $\sqrt{7 - 5x} + 4 = 3$
3. $\sqrt{2x} = -4$
4. $\sqrt{n - 2} + \sqrt{2n - 5} = 0$
5. $4\sqrt{m - 1} = \sqrt{14m + 4}$

As has been mentioned, some radical equations produce quadratic equations after the squaring property is used.



Example 4 Solve.

a. $\sqrt{2x^2 - x - 2} = \sqrt{x^2 + 3x + 3}$ b. $\sqrt{2x + 10} - 1 = x$

Solution:

a.

$$\sqrt{2x^2 - x - 2} = \sqrt{x^2 + 3x + 3}$$

$$\left(\sqrt{2x^2 - x - 2}\right)^2 = \left(\sqrt{x^2 + 3x + 3}\right)^2$$

$$2x^2 - x - 2 = x^2 + 3x + 3 \qquad \text{Using the squaring property}$$

$$2x^2 - x^2 - x - 3x - 2 - 3 = 0$$

$$x^2 - 4x - 5 = 0$$

$$(x - 5)(x + 1) = 0 \qquad \text{Factoring}$$

$$x - 5 = 0 \quad \text{or} \quad x + 1 = 0 \qquad \text{Using the zero product property}$$

$$x = 5 \qquad x = -1$$

Check: For x = 5 | For x = -1

$$\sqrt{2(5)^2-5-2} \,?\, \sqrt{5^2+3(5)+3} \qquad \sqrt{2(-1)^2-(-1)-2} \,?\, \sqrt{(-1)^2+3(-1)+3}$$

$$\sqrt{2(25)-7} \,?\, \sqrt{25+15+3} \qquad \sqrt{2+1-2} \,?\, \sqrt{1-3+3}$$

$$\sqrt{43}=\sqrt{43} \qquad \sqrt{1} \,?\, \sqrt{1}$$

$$1 = 1$$

The solutions are 5 and -1.

b. $\sqrt{2x+10}-1=x$

$$\sqrt{2x+10}=x+1$$

$$\left(\sqrt{2x+10}\right)^2=(x+1)^2 \qquad \text{why?}$$

$$2x+10=x^2+2x+1$$

$$x^2+2x+1=2x+10 \qquad \text{why?}$$

$$x^2+2x-2x+1-10=0$$

$$x^2-9=0$$

$$(x-3)(x+3)=0 \qquad \text{why?}$$

x – 3 = 0 or x + 3 = 0 why?
x = 3 x = -3

Check: For x = 3 | For x = -3

$$\sqrt{2(3)+10}-1 \,?\, 3 \qquad \sqrt{2(-3)+10}-1 \,?\, 3$$

$$\sqrt{16}-1 \,?\, 3 \qquad \sqrt{-6+10}-1 \,?\, 3$$

$$4-1 \,?\, 3 \qquad \sqrt{4}-1 \,?\, 3$$

$$3=3 \qquad 2-1 \,?\, 3$$

$$1 \neq 3$$

The only solution is 3.

Let's Practice For Mastery! 23

Solve:

1. $\sqrt{4t-3}=t$
2. $\sqrt{2w+3}-w=0$
3. $\sqrt{x^2-3x}-2=0$
4. $\sqrt{b^2-7b+6}=(b-4)$

5. $\sqrt{3y^2 + 5y + 6} = y + 3$

6. Show that the statement $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$ is not true.

Let's Check Your Understanding! 23

Solve:

1. $\sqrt{5t - 4} = t$
2. $\sqrt{2w + 3} - 2 = 0$
3. $\sqrt{x^2 - 5x} - 6 = 0$
4. $\sqrt{c^2 - 4c + 7} + 1 = 0$
5. $\sqrt{2x^2 - 5x + 7} = x + 1$

Solving Equations Containing *n*th Roots for *n* > 2

You can use the power theorem to solve equations whose radicals have indices greater than 2.

The Power Theorem
Let n be a real number,
if $a = b$ then $a^n = b^n$

Note that the power theorem is a generalized statement of the squaring property.

Example 5 Solve the equations.

a. $\sqrt[3]{2x + 10} = 2$ b. $\sqrt[4]{n - 1} + \sqrt[4]{9 - n} = 0$

Solution:

a. Since the equation involves a cube root, we eliminate it by applying the power theorem. This time by raising both sides of the equation by 3. Thus,

$$\sqrt[3]{2x + 10} = 2$$

$$\left(\sqrt{2x + 10}\right)^3 = (2)^3 \qquad \text{why?}$$

$2x + 10 = 8$ why?

$2x = 8 - 10$

$2x = -2$

$x = -1$

Check: $\sqrt[3]{2x+10} = 2$ $\sqrt[3]{8} \,?\, 2$

$\sqrt[3]{2(-1)+10} \,?\, 2$ $2 = 2$

$\sqrt[3]{-2+10} \,?\, 2$

-1 is a solution of the equation.

b. The equation involves a fourth root. Raise both sides of the equation by 4.

$\sqrt[4]{n-1} + \sqrt[4]{9-n} = 0$

$\sqrt[4]{n-1} = -\sqrt[4]{9-n}$ why?

$\left(\sqrt[4]{n-1}\right)^4 = \left(-\sqrt[4]{9-n}\right)^4$ why?

$n - 1 = 9 - n$

$n + n = 9 + 1$

$2n = 10$

$n = 5$

The check is left for you.

What is the solution of the equation?

Note that the equation above has no solution since by checking the answer with the equation resulted to an inequality. Thus, the solution is extraneous.

Like the squaring property of equality, using the power theorem may also lead to extraneous solutions. It is therefore necessary to check the answer.

Let's Practice For Mastery! 24

Solve the following.

1. $\sqrt[3]{x-1} = 2$
2. $\sqrt[3]{x+1} = 2$
3. $\sqrt[3]{2x+5} - 3 = 0$
4. $\sqrt[4]{5x-8} = \sqrt[4]{4x-1}$
5. $\sqrt[4]{3x+1} = 2$

Let's Check Your Understanding! 24

Solve the following.

1. $\sqrt[3]{5x+7} = 2$
2. $\sqrt[3]{\frac{1}{2}(x-3)} = 2$
3. $\sqrt[4]{x+4} = 1$
4. $\sqrt[3]{3x+5} = \sqrt[3]{5-2x}$
5. $\sqrt[5]{x^2 - 3x} - 2 = 0$

Lesson 5.6 Applications of Radicals

Now that we have already learned all the skills concerning radicals, we are then ready to apply these skills in problem solving that involves such expressions.

Let us review the four steps in problem solving.

1. Familiarize. (Read and understand the problem. Identify what is given and what is needed to find.)
2. Plan. (Decide how to solve the problem. Translate the conditions of the problem into an equation.)
3. Solve. (Carry out the plan by solving the equation.)
4. Check the solutions. (Verify that the answer is correct by making sure it works in the original question.)

Many number problems use equations that involve radicals.

Example 1 Five times the square root of a number is 2.

Solution:

Familiarize Five times the square root of a number is 2

Plan $5 \cdot \sqrt{x} = 2$

Solve $5\sqrt{x} = 2$

$$\left(5\sqrt{x}\right)^2 = 2^2$$

$$25x = 4$$

$$x = \frac{4}{25}$$

Check $5\sqrt{x} = 2$

$$5\left(\sqrt{\frac{4}{25}}\right) ? 2$$

$$5\left(\frac{2}{5}\right) ? 2$$

$$2 = 2$$

The number is $\frac{4}{25}$.

Example 2 A number is increased by 3. The square root of this sum is 5. Find the number.

Solution:

Familiarize Let x be the number

$x + 3$ a number is increased by 3. (this is the sum)

$\sqrt{x+3}$ the square root of this su,

Plan $\sqrt{x+3} = 5$

Solve $\sqrt{x+3} = 5$

$$\left(\sqrt{x+3}\right)^2 = (5)^2$$

$$x + 3 = 25$$

$$x = 25 - 3 = 22$$

The check is left for you.

The number is 22.

Let's Practice For Mastery! 25

Solve the following.

1. Three more than the square root of a number is 7. What is the number?
2. Twice a number is decreased by 3. The square root of this difference equals 6. Find the number.

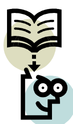
3. Four times the square root of three more than a number is six more than this number.

Let's Check Your Understanding! 25

Solve the following.

1. Eight less than the square root of a number is 0. What is the number?
2. Five times the square root of one more than a number is seven more than this number. Find the number.

The next examples show the applications of radicals.



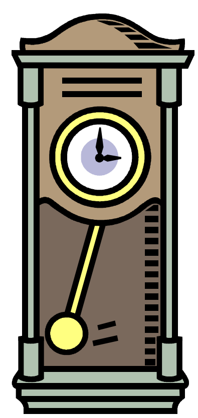
Example 3 The formula $T = 2\pi\sqrt{\frac{L}{9.8}}$ relates the time, T, in seconds, that it takes for a pendulum to swing back and forth once with the length of a pendulum in meters. What is the length of a pendulum that takes 3 seconds to swing back and forth once? Use $\pi = 3.14$.

Solution:

Familiarize Substitute the given in the formula

T = 3 sec

L = ?



Plan $T = 2\pi\sqrt{\frac{L}{9.8}}$

Solve $3 = 2(3.14)\sqrt{\frac{L}{9.8}}$

$$3 = 6.28\sqrt{\frac{l}{9.8}}$$

$$\frac{3}{6.28} = \sqrt{\frac{L}{9.8}}$$

$$(0.48)^2 = \left(\sqrt{\frac{L}{9.8}}\right)^2$$

$$0.23 = \frac{L}{9.8}$$

L = 0.23(9.8) = 2.25 m

Check The check is left for you.

Example 4 The cost of producing *n* computer chips per day at a manufacturing plant is given by $C = 1440\sqrt{n+45}$.

i. Find the overhead cost (the cost of producing 0 chips).

ii. Find the number of chips produced when the cost is Php7,280 per day?

Solution:

Familiarize Substitute the given in the formula. There are 2 things to answer.

i. n = 0 n = ?

ii. c = ? c = Php17,280

Plan and solve i. . $C = 1440\sqrt{n+45}$

$$= 1440\sqrt{0+45}$$

$$= 1440(6.71) = 9{,}662$$

ii. $C = 1440\sqrt{n+45}$

$$17{,}280 = 1440\sqrt{n+45}$$

$$\frac{17{,}280}{1440} = \sqrt{n+45}$$

$$(12)^2 = \left(\sqrt{n+45}\right)^2$$

144 = n + 45

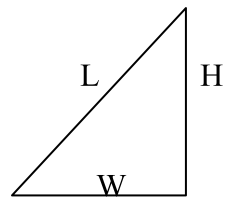
n = 99 chips

The check is left for you to do.

Let's Practice For Mastery! 26

Solve the following.

1. Rey is making a book frame and he wants to stabilize his book with a diagonal brace as shown in the figure below. The length (L) of the brace is given by $L = \sqrt{H^2 + W^2}$ where H is the height and W is the width. If the bottom of the base is attached 9cm from the corner and the brace is 12cm long, how far up the corner post should he nail it?

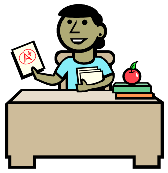


2. The velocity of a tsunami as it approaches land is approximated by the equation $V = 1.66\sqrt{d}$, where v is the velocity in meter per second, and d is the depth of the water in meter. Find the depth of the water when the velocity of a tsunami is 10 m/s.
3. The formula $S = 10\sqrt{d}$ can be used to estimate the speed (s) in km/hr that a car was traveling when it skidded d meters on a wet concrete road after the brakes were applied. Find the distance a car will travel on a wet road if the car is traveling 80 km/hr when the brakes are applied.

Let's Check Your Understanding! 26

Solve the following problems.

1. The formula $t = \sqrt{\dfrac{d^2}{560}}$ relates the time (t) in hours a storm with a diameter (d) in km will last. What is the diameter of a storm that last 1 hour?
2. The distance d, in meters, that an object will fall in t seconds is given by the formula $t = \dfrac{\sqrt{d}}{2.21}$. How high is a bridge above sea level if a stone is dropped down and lists the water in 4 seconds?



LET'S SUMMARIZE!

Square Roots

The *square root* of a positive number, x, is a number whose square is x.

The *principal square root* of a number is the positive square root.

The symbol $\sqrt{x}$ is called a *radical* and is used to indicate the principal square root of a number, x.

The *square root* of a negative number does not exist.

The *radicand* is the expression under the radical sign.

The square of an integer is a *perfect square*.

Cube Roots

The *cube root* of a positive number is also positive.

The *cube root* of a negative number is also a negative number.

The symbol $\sqrt[3]{x}$ is used to indicate the *cube root* of a number, x.

The square of an integer is a *perfect cube*.

Even and Odd Roots

The *nth root* of a number, x, is indicated by $\sqrt[n]{x}$.

The *nth root* of a positive number is positive.

The *nth root* of a negative number is negative when n is odd. The *nth root* of a negative number is not a real number when n is even.

Radicals are simplified when

a. the radicand has no perfect square, cube, or nth power factors;

b. no fraction appears in the radicand; and

c. no radical expression appears in the denominator.

Conjugates are binomial expressions that differ only in the sign of the second term. (The expressions $a + b$ and $a - b$ are conjugates.)

Rationalizing the denominator is the procedure used to remove a radical from the denominator of a fraction.

A ***radical equation*** is an equation that contains a variable expression in a radicand.

When a solution does not check with the equation the solution is an *extraneous solution*.

The Product Property of Radicals

If a and b are positive real numbers, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$.

The Quotient Property of Radicals

If a and b are positive real numbers, then $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$.

The Squaring Property of Equality

If a and b are real numbers and $a = b$, then $a^2 = b^2$.

The Power Theorem

If a and b are real numbers and $a = b$, then $a^n = b^n$.

UNIT TEST

A. Simplify:

1. $\sqrt{121x^8y^2}$
2. $5\sqrt{8}-3\sqrt{50}$
3. $\sqrt{3x^2y}\sqrt{6x^2}\sqrt{2x}$
4. $\sqrt{45}$
5. $\sqrt{72x^7y^2}$
6. $3\sqrt{8y}-2\sqrt{72x}+5\sqrt{18x}$
7. $\dfrac{\sqrt{162}}{\sqrt{2}}$
8. $\sqrt{192x^{13}y^5}$
9. $\left(\sqrt{a}-2\right)\left(\sqrt{a}+2\right)$
10. $\dfrac{3}{2-\sqrt{5}}$

B. Solve each equation.

1. $\sqrt{5x-6}=7$
2. $2=8-\sqrt{5x}$
3. $\sqrt{9x}+3=18$
4. $\sqrt{x}-\sqrt{x+3}=1$
5. $0=\sqrt{10x+4}-8$

C. Solve the following problems.

1. The square root of the sum of two consecutive integers is equal to 9. Find the smaller integer.

2. Find the length of a pendulum that makes one swing in 1.5s. The equation for the time of one swing of a pendulum is $T = 2\pi\sqrt{\frac{L}{32}}$, where T is the time in seconds, and L is the length in feet. Round to the nearest hundredth.

3. A bicycle will overturn if it rounds a corner too sharply or too quickly. The equation for the maximum velocity at which a cyclist can turn a corner without tipping over is given by the equation $v = 4\sqrt{r}$, where v is the velocity of the bicycle in miles per hour, and r is the radius of the corner in feet. Find the radius of the sharpest corner that a cyclist can safely turn when riding at a speed of 20 mph.

ANSWER KEY

Let's Practice For Mastery! 1

A. 1. $\sqrt{7}$ 2. $\sqrt[6]{v}$ 3. $\sqrt[4]{x^3}$ 4. $\sqrt[5]{2^2}$ 5. $\sqrt[3]{2x}$

B. 6. $11^{\frac{1}{3}}$ 7. $5^{\frac{1}{4}}$ 8. $w^{\frac{2}{3}}$ 9. $x^{\frac{3}{2}}$ 10. $\left(\frac{25}{16}\right)^{\frac{1}{2}}$

Let's Check Your Understanding! 1

A. 1. $\sqrt[4]{3x^3}$ 2. $2\sqrt{x^3}$ 3. $\sqrt[7]{64^3 a^3}$ 4. $2\sqrt{16p^3}$ 5. $\frac{4\sqrt[3]{4}}{7\sqrt[3]{7}}$

B. 6. $(40a)^{\frac{1}{2}}$ 7. $2^{\frac{5}{3}}$ 8. $8^{\frac{1}{5}}$ 9. $(2x)^{\frac{1}{4}}$ 10. $2^{\frac{3}{7}}$

Let's Practice For Mastery! 2

R A D I C A N D

Let's Check Your Understanding! 2

1. ± 13 2. -16 3. ± 18 4. not real 5. ± 0.15

Let's Practice For Mastery! 3

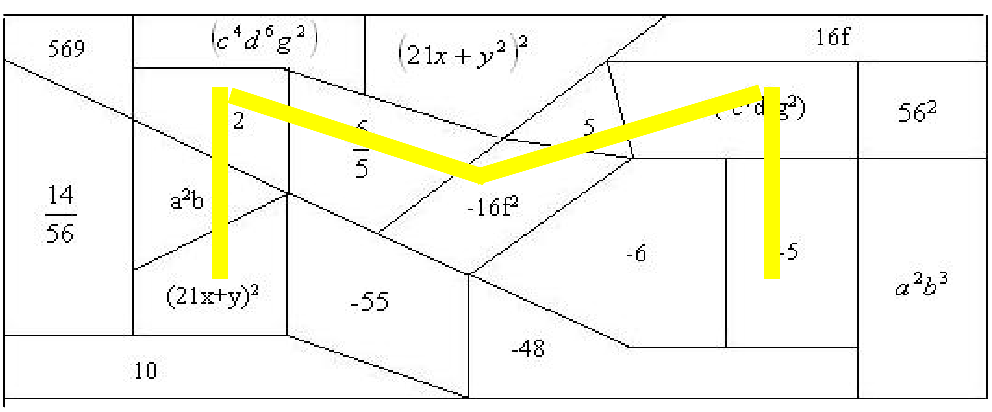
1. $-(f^5t^2)$ 2. $9t^4$ 3. $-(8c^2)$ 4. $\pm 5d^2$ 5. 25

6. $\frac{13p^5}{7}$ 7. $\pm 8b^6$ 8. $\frac{12}{13c^3}$ 9. $-(h^4s^6)$ 10. 3a

Let's Check Your Understanding! 3

1. $\pm d^3$ 2. r^6t^4 3. $-(3ab^7)$ 4. $\frac{81b}{100}$ 5. $2.5t^5$

Let's Practice For Mastery! 4



Let's Check Your Understanding! 4

1. -3 2. 3p 3. r^3st^2 4. $23d^4$ 5. $2x^3y^{12}$

Let's Practice For Mastery! 5

1. 6 2. 5 3. not real 4. $14b^2$ 5. -2

6. -2 7. 3 8. $-\left(ab^{12}c^5\right)$ 9. $\frac{x^2}{3}$ 10. $-\left(8a^2\right)$

Let's Check Your Understanding! 5

1. not real 2. 3x 3. $-2a^3$ 4. $\frac{-a^2b^3}{c^5}$ 5. $81x^{12}$

Let's Practice For Mastery! 6

A. 1. $2\sqrt{6}$ 2. $-3\sqrt{2}$ 3. $10\sqrt{2}$ 4. $5\sqrt{3}$ 5. $4\sqrt{5}$

6. $\sqrt{18}\cdot\sqrt{10}=6\sqrt{5}$

$\sqrt{180}\overset{?}{=}6\sqrt{5}$

$6\sqrt{5}=6\sqrt{5}$

B. BIG HANDS

Let's Check Your Understanding! 6

1. $4\sqrt{2}$ 2. $-3\sqrt{3}$ 3. $12\sqrt{2}$ 4. $4\sqrt{3}$ 5. $10\sqrt{5}$

Let's Practice For Mastery! 7

A STICK

1. $x\sqrt{x}$ 2. $5x^4\sqrt{x}$ 3. $3x^2\sqrt{2x}$ 4. $2x^2y^3\sqrt{3y}$

5. $4x^4y^6\sqrt{2xy}$ 6. $2x^7\sqrt{6x}$

Let's Check Your Understanding! 7

1. $a^5\sqrt{a}$ 2. $6p^8\sqrt{p}$ 3. $4xy^4\sqrt{3y}$ 4. $6\sqrt{3x}$

5. $5x^2\sqrt{x}$

Let's Practice For Mastery! 8

1. $2x^2\sqrt[3]{10x^2}$ 2. $2a^2\sqrt[4]{6}$ 3. $-ab^2c^3\sqrt[5]{-abc^2}$ 4. $-2x^2\sqrt[3]{-2}$

5. $3x^2\sqrt[3]{2x}$ 6. $3cd\sqrt[4]{2d^2}$ 7. $(x+y)\sqrt[3]{(x+y)}$ 8. $x^2yz^4\sqrt[5]{x^3y^3z^2}$

9. $x+3$ 10. $x-5$

Let's Check Your Understanding! 8

1. $2x\sqrt[3]{100x}$ 2. $-2a^2\sqrt[3]{4}$ 3. $2a^2\sqrt[4]{10b}$

4. $(a-b)\sqrt[3]{(a-b)^2}$ 5. $b^2c^3\sqrt[5]{a^3c^3}$

Let's Practice For Mastery! 9

1. $\frac{\sqrt{5}}{2}$ 2. $\frac{5\sqrt{2}}{7}$ 3. $\frac{2\sqrt{2}}{7}$ 4. $\frac{\sqrt{30}}{9x}$

5. $\frac{x^2\sqrt{7x}}{10}$ 6. $\frac{x^2yz\sqrt{xz}}{5}$ 7. $\frac{\sqrt{19}}{8xy}$ 8. $\frac{2x\sqrt{15}}{11}$

Let's Check Your Understanding! 9

1. $\frac{2\sqrt{2}}{3}$ 2. $\frac{\sqrt{7}}{12x^3}$ 3. $\frac{xy^2}{13}$ 4. $\frac{7x}{9y}$ 5. $\frac{5\sqrt{3y}}{9x^2}$

Let's Practice For Mastery! 10

A. 1. 3 2. 2 3. 3 4. 6 5. 2
6. x 7. 2x 8. xy 9. 5x 10. 2a

B. MISS D. BUS

11. $\frac{2\sqrt{3}}{3}$ 12. $\frac{\sqrt{35}}{5}$ 13. $\frac{5\sqrt{6}}{6}$ 14. $\frac{\sqrt{6}}{4}$

15. $\frac{3x\sqrt{15xy}}{5y}$ 16. $\frac{2x^2\sqrt{21xy}}{7y}$ 17. $\frac{5xy\sqrt{6x}}{2z}$ 18. $\frac{5xy\sqrt{6yz}}{3z}$

Let's Check Your Understanding! 10

1. $\frac{\sqrt{3}}{3}$ 2. $\frac{\sqrt{10}}{2}$ 3. $\sqrt{2}$ 4. $\frac{3\sqrt{2}}{2x^3}$ 5. $\sqrt{3x}$

Let's Practice For Mastery! 11

1. $2\sqrt[3]{4}$ 2. $\frac{5\sqrt[3]{9}}{3}$ 3. $\frac{\sqrt[4]{8y^3}}{y}$ 4. $\frac{3\sqrt[3]{25}}{5}$

5. $\frac{\sqrt[3]{36xy^2}}{3y}$ 6. $\frac{\sqrt[4]{27y^3}}{y}$ 7. $\frac{\sqrt[3]{6xy^2}}{3y}$ 8. $\frac{\sqrt[4]{72x}}{3x}$

9. 25 since 5(25) = 125 which is a perfect cube.

10. The fraction $\frac{3}{\sqrt[3]{5}}$ is multiplied to $\frac{1}{\sqrt[3]{25}}$. It should have been multiplied by $\frac{\sqrt[3]{25}}{\sqrt[3]{25}}$, so that the value of the fraction is not changed.

Let's Check Your Understanding! 11

1. $\frac{2\sqrt[3]{3}}{3}$ 2. $\frac{\sqrt[3]{6}}{2}$ 3. $\frac{\sqrt[4]{24x^2}}{2x}$ 4. $\frac{\sqrt[3]{252xy^2}}{6y}$ 5. $\frac{2a\sqrt[3]{6ac^2}}{3c}$

Let's Practice For Mastery! 12

A. 1. $\sqrt{2}$ & $2\sqrt{2}$ 2. none 3. $2\sqrt[3]{7}$ & $8\sqrt[3]{7}$ 4. all

B. 5. $\sqrt{2}$ 6. 0 7. $6\sqrt{2}+2\sqrt{3}$ 8. $\sqrt[3]{10}$

9. 0 10. $13y\sqrt{a}-2x\sqrt{a}$

Let's Check Your Understanding! 12

1. $11\sqrt{5}$ 2. $-7\sqrt{2}$ 3. $-3\sqrt[4]{2}$ 4. $21y\sqrt{a}$ 5. 0

Let's Practice For Mastery! 13

A. 1. $4\sqrt{2}$ 2. $3\sqrt{2}$ 3. $\sqrt{5}$ 4. $-2\sqrt[3]{2}$ 5. $a^2b\sqrt{6a}$

B. 6. $8\sqrt{2x}$ 7. $-2\sqrt{3}$ 8. $\sqrt{7}$

Let's Check Your Understanding! 13

1. $-5\sqrt{2}$ 2. -5 $\sqrt{3}$ 3. -14x $\sqrt{2}$ 4. 7 $\sqrt{3x}$ 5. 5 $a^2b\sqrt{5a}$

Let's Practice For Mastery! 14

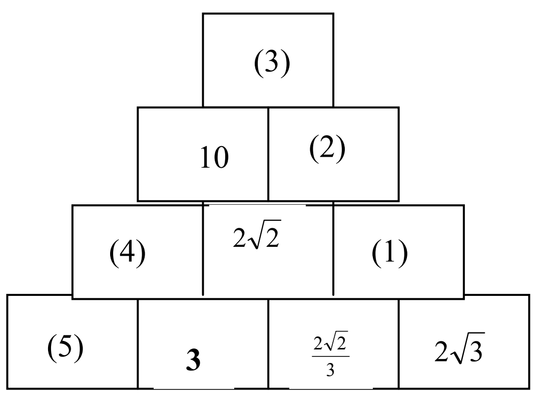
1. 3 2. 8 3. 4 4. 9 5. 10

6. a^5 7. b^4 8. 8 9. $b\sqrt{7}$ 10. 30 $\sqrt{3}$

Let's Check Your Understanding! 14

1. $\sqrt[3]{14}$ 2. $\sqrt[4]{72}$ 3. 2 4. -10 5. 10 $\sqrt{3}$

Let's Practice For Mastery! 15



1. $\frac{4\sqrt{6}}{3}$ 2. $\frac{16\sqrt{3}}{3}$ 3. $\frac{160\sqrt{3}}{3}$ 4. $\frac{5\sqrt{2}}{2}$ 5. $\frac{5\sqrt{2}}{6}$

6. $-2\sqrt{6}$ 7. $5+5\sqrt{6}$ 8. $24\sqrt{3}+6\sqrt{6}$ 9. $105\sqrt{3}-14\sqrt{5}$

Let's Check Your Understanding! 15

1. $12\sqrt{21}$ 2. $6\sqrt{30}$ 3. 30 4. $12-7\sqrt{3}$ 5. $6\sqrt{10}+8\sqrt{15}$

Let's Practice For Mastery! 16

A. 1. $-7-3\sqrt{11}$ 2. $38-12\sqrt{102}$ 3. $a-5$ 4. $91+40\sqrt{3}$

5. -7 6. $14-3\sqrt{3}$

B. 7. $3\sqrt{x}+4$ 8. $\sqrt{2y}-5$ 9. $\sqrt{x}-5y$

10. $x^2-4x--6=0$

$$\left(2+\sqrt{10}\right)^2-4\left(2+\sqrt{10}\right)-6=0$$
$$14+4\sqrt{10}-8-4\sqrt{10}-6=0$$
$$0=0$$

C. GINATAN

11. $\left(\sqrt{8}-1\right)\left(\sqrt{8}+1\right)=8-1=7$ G
12. $\left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{3}+\sqrt{2}\right)=3-2=1$ A
13. $\left(4\sqrt{2}-2\sqrt{3}\right)\left(4\sqrt{2}+2\sqrt{3}\right)=16(2)-4(3)=20$ T
14. $\left(2\sqrt{5}-\sqrt{11}\right)\left(2\sqrt{5}+\sqrt{11}\right)=4(5)-11=9$ I
15. $\left(2\sqrt{5}-\sqrt{6}\right)\left(2\sqrt{5}+\sqrt{6}\right)=4(5)-6=14$ N

Let's Check Your Understanding! 16

1. $52-19\sqrt{7}$ 2. 9 3. $8-2\sqrt{15}$ 4. 13 5. $\sqrt{14}+2\sqrt{6}-\sqrt{21}-6$

Let's Practice For Mastery! 17

1. 6 2. $6\sqrt{3}$ 3. $\sqrt{2}$ 4. $2\sqrt{y}$ 5. $\frac{1}{2}$

Let's Check Your Understanding! 17

1. 16 2. $\frac{1}{2}$ 3. $5\sqrt{2}$ 4. $2y\sqrt{x}$ 5. $x\sqrt{3}$

Let's Practice For Mastery! 18

1. $4-2\sqrt{3}$ 2. $\sqrt{5}+\sqrt{2}$ 3. $\frac{5-\sqrt{15}}{2}$ 4. $\frac{5-\sqrt{5}}{4}$ 5. $\frac{2\left(\sqrt{x}-\sqrt{y}\right)}{x-y}$

6. $\left(\sqrt[3]{2}+\sqrt[3]{3}\right)\left(\sqrt[3]{4}-\sqrt[3]{6}\right)+\sqrt[3]{9}=5$

$$\sqrt[3]{8}+\sqrt[3]{27}=5$$
$$2+3=5$$
$$5=5$$

Let's Check Your Understanding! 18

1. $\frac{8\sqrt{x}+24}{x-9}$ 2. $\frac{\sqrt{3}-3}{2}$ 3. $\sqrt{10}+\sqrt{6}$ 4. $\frac{3x-2\sqrt{3x}+1}{3x-1}$

Let’s Practice For Mastery! 19

1. $\sqrt[6]{x^5}$ 2. $\sqrt[4]{8}$ 3. $\sqrt[12]{p^5}$

4. $a\sqrt[10]{a}$ 5. $2\sqrt[6]{2}$ 6. 2^{10} 7. $\left(\frac{1}{2}\right)^{10}$

Let’s Check Your Understanding! 19

1. $3\sqrt[3]{3}$ 2. $\sqrt[6]{3^5}$ 3. $\frac{\sqrt[6]{b^5}}{b}$ 4. $\frac{\sqrt[14]{a}}{a}$ 5. $\sqrt[4]{p^3}$

Let’s Practice For Mastery! 20

SQUARE ROOT

Let’s Check Your Understanding! 20

1. $x = \frac{9}{16}$ 2. $x = 73$ 3. $n = 20$ 4. $x = 11$ 5. $x = \frac{25}{4}$

Let’s Practice For Mastery! 21

1. m = -7 2. n = 196 3. x = -16 4. y = 7 5. x = 1

Let’s Check Your Understanding! 21

1. x = 49 2. x = 5 3. a = 4 4. y = 2 5. x = 10

Let’s Practice For Mastery! 22

1. extraneous 2. extraneous 3. x = 49 4. $x = \frac{25}{4}$ 5. extraneous

6. both are solutions 7. -4 is an extraneous sol. 8. 2 is an extraneous sol

Let’s Check Your Understanding! 22

1. x = 0 2. extraneous solution 3. extraneous solution

4. extraneous solution 5. m = 10

Let’s Practice For Mastery! 23

1. t = 3 or t = 1 2. w = 2 3. x = -1 or x = 4

4. b = 10 5. y = -1 or y = $\frac{3}{2}$

6. $\left(\sqrt{a+b}\right)^2 \neq \left(\sqrt{a}+\sqrt{b}\right)^2$

$a + b \neq a + 2\sqrt{ab} + b$

$\sqrt{a+b}$ is not equal to $\sqrt{a}+\sqrt{b}$ if we square both equations.

Let’s Check Your Understanding! 23

1. t = 4 or t = 1 2. w = 2 3. x = 9 or x = -4

4. extraneous solution 5. x = 1 or x = 6

Let's Practice For Mastery! 24

1. x = 9 2. x = -9 3. x = 11 4. x = 7 5. x = 5

Let's Check Your Understanding! 24

1. $x = \frac{1}{5}$ 2. $x = 22$ 3. $x = 3$ 4. $x = 0$ 5. no solution

Let's Practice For Mastery! 25

Rubrics for Checking.

Points	Criteria
3	Correct equation, solution and simplified answer.
2	Correct equation, correct solution but the radical answer is not simplified or is simplified but wrong.
1	Correct equation but wrong solution.
0	No attempt.

1. 100 2. $\frac{39}{2}$ 3. -2 or 6

Let's Check Your Understanding! 25

1. 64 2. $\frac{49}{25}$

Let's Practice For Mastery! 26

Rubrics for Checking.

Points	Criteria
3	Correct substitution, solution and simplified answer.
2	Correct substitution of the given in the formula, correct solution but the radical answer is not simplified or is simplified but wrong.
1	Correct substitution of the given in the formula but wrong solution.
0	No attempt.

1. $3\sqrt{7}cm$ 2. $\frac{(10\sqrt{5})}{\sqrt{83}}m$ or $\frac{(10\sqrt{415})}{83}m$ 3. 64 meters

Let's Check Your Understanding! 26

1. $4\sqrt{35}$ km 2. $2\sqrt{2.21}$ meters

UNIT TEST

A.

1. $11x^4y$	2. $-5\sqrt{2}$	3. $6x^2\sqrt{xy}$	4. $3\sqrt{5}$
5. $6x^3y\sqrt{2y}$	6. $6\sqrt{2y}+3\sqrt{2x}$	7. 9	8. $8x^6y^2\sqrt{3xy}$
9. a – 4	10. $-6 - 3\sqrt{5}$		

B.

1. 11	2. 5	3. 25	4. no solution	5. 6

C.

1. 40	2. 1.82 ft	3. 25 ft

Misconceptions and Common Errors

1. The radical symbol is mistaken to be a division symbol. $3\sqrt{4}$ as opposed to $3\overline{)4}$ are two different notations. The first means 3 times the square root of 4 while the other means 4 divided by 3.

2. $\sqrt{-4} \neq -2$ since there is no number such that when it is squared gives -4.

3. $\sqrt{16} \neq \sqrt{4}$ In extracting $\sqrt{16}$, some students write the radical sign and write 4 under the radical sign so that $\sqrt{16} = \sqrt{4}$ which is wrong since $\sqrt{16} = 4$.

4. $\sqrt{7} + \sqrt{7} \neq \sqrt{14}$ since adding radicals is just like adding polynomials, we add the coefficients and not the radicals.

5. $\sqrt[5]{32^2} \neq 32^{\frac{5}{2}}$ since $\sqrt[5]{32^2} = 32^{\frac{2}{5}} = (32^{\frac{1}{5}})^2 = 2^2 = 4$. The numerator is the exponent of the radical expression and the denominator is the index.