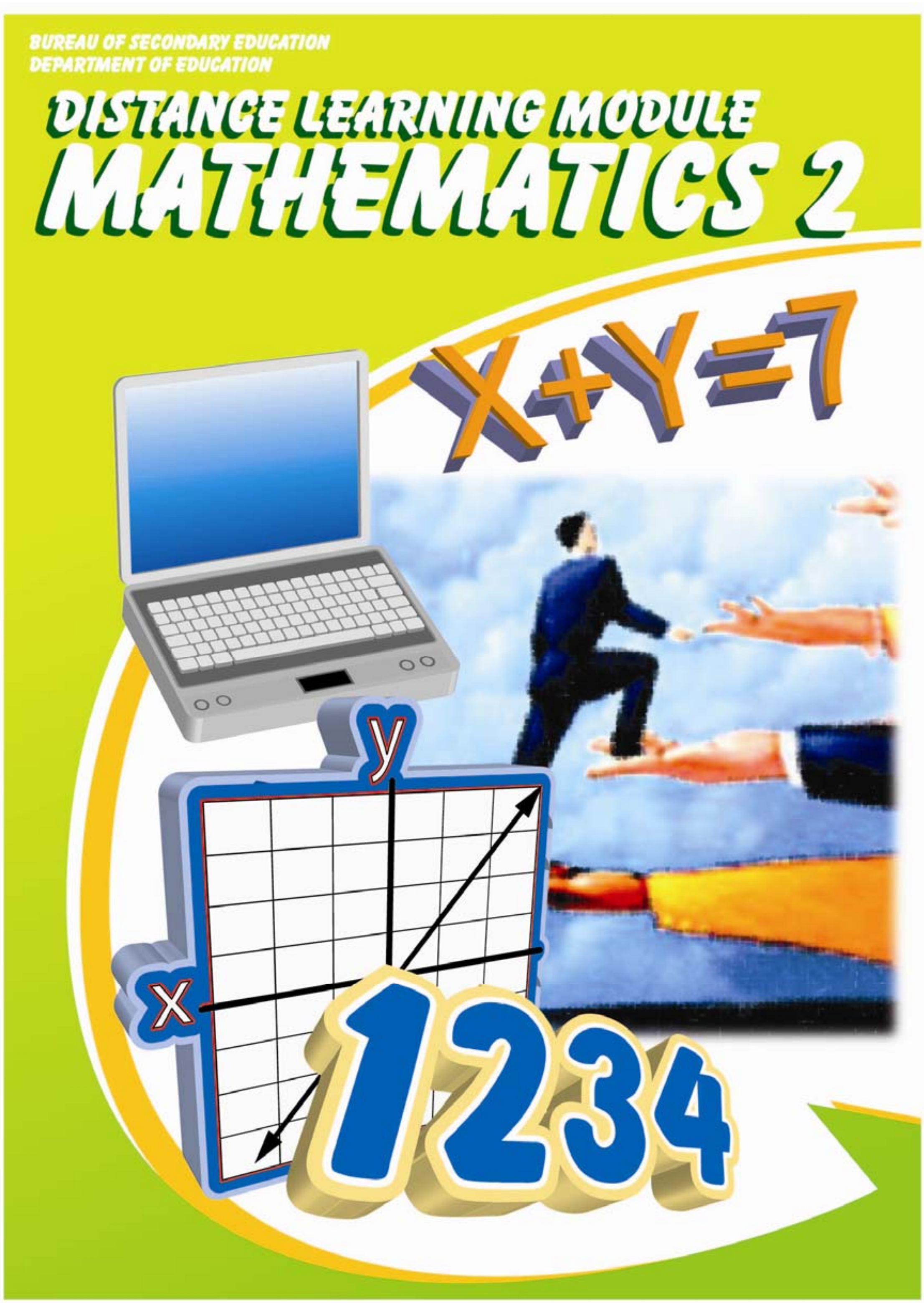

BUREAU OF SECONDARY EDUCATION
DEPARTMENT OF EDUCATION
DISTANCE LEARNING MODULE
MATHEMATICS 2
X+Y=7
y
x
1234

RATIONAL EXPONENTS

As Jose looks down at the river below, he wonders as to how high the bridge might be. What he does not know is that he can actually find the height of the bridge by throwing a stone into the river. He just needs to get the time it takes for the stone to reach the river and finally calculate the height using the formula $T = \frac{1}{2}h^{\frac{1}{2}}$.

The expression, $\frac{1}{2}h^{\frac{1}{2}}$. from the above equation is surely new to you. You will see that the laws of integer exponents which you studied in Mathematics I are extended to fractional exponents. You will then learn how to evaluate fractional exponents and how to simplify them.

UNIT IV

Lesson 4.1 Review of Integer Exponents

It is important to review the basic properties of exponents in order for us to deal with rational exponents.

First, remember that the expression a^n, is read as "*a to the nth power*". In this expression, a is the base and n is the exponent. The number represented by a^n is called "*the nth power of a*".

When n is a positive integer, a^n means

$$a^n = \underbrace{a \cdot a \cdot a \cdot a \cdots a}_{\text{n times}},$$

where a is taken as a factor n times.

Now observe how this definition can help you simplify products such as the following.

$x^5 \cdot x^3 = (x \cdot x \cdot x \cdot x \cdot x)(x \cdot x \cdot x) = x^8$ ← Notice that 5 + 3 = 8

$(p^2)^3 = (p^2)(p^2)(p^2) = p^6$ ← Notice that 2(3) = 6

$(rs)^4 = (rs)(rs)(rs)(rs) = r \cdot r \cdot r \cdot r \cdot s \cdot s \cdot s \cdot s = r^4 s^4$

These examples are generalized in the following properties of exponents.

Multiplication Properties of Exponents

Let m and n be integers and a and b are nonzero real numbers.

Product Property: $a^m \cdot a^n = a^{m+n}$

Power of a Power: $(a^m)^n = a^{mn}$

Power of a Product: $(ab)^m = a^m b^m$

Notice that these multiplication properties apply to all integer exponents, including zero and negative integers. Remember that:

For any real number a, $a^1 = a$

For any nonzero real number a, $a^0 = 1$

For any nonzero real number a, $a^{-n} = \dfrac{1}{a^n}$ and $\dfrac{1}{a^{-n}} = a^n$.

Example 1 Simplify each expression. Write answers with positive exponents.

a. 2^{-4} b. 4^0 c. $(3^2)^3$

d. $2^3 \cdot 2^4$ e. $\dfrac{1}{4^{-5}}$ f. $\dfrac{2^{-3}}{5^{-2}}$

Solution: Applying the laws of exponents and definitions just identified, we have

a. $2^{-4} = \dfrac{1}{2^4} = \dfrac{1}{16}$ by definition of negative exponents

b. $4^0 = 1$ by definition of zero exponents

c. $(3^2)^3 = 3^6 = 729$ using power of a power property of exponents

d. $2^3 \cdot 2^4 = 2^{3+7} = 2^7 = 128$ using product property of exponents

e. $\dfrac{1}{4^{-5}} = 4^5 = 1024$ by definition of negative exponents

f. $\dfrac{2^{-3}}{5^{-2}} = \dfrac{\frac{1}{2^3}}{\frac{1}{5^2}} = \dfrac{1}{2^3} \div \dfrac{1}{5^2} = \dfrac{1}{2^3} \cdot \dfrac{5^2}{1} = \dfrac{5^2}{2^3} = \dfrac{25}{8}$ why?

Example 2 Simplify each expression. Write answers using positive exponents.

a. $b^5 \cdot b^8$ b. $c^3 \cdot c^{-7}$ c. $(n^3)^3$

d. $(-2x)^3$ e. $(yz^4)^2$ f. $(v^{-2}w^3)^{-1}$

Solution: Applying the laws of exponents, we have

a. $b^5 \cdot b^8 = b^{5+8} = b^{13}$ why?

b. $c^3 \cdot c^{-7} = c^{3+(-7)} = c^{-4} = \dfrac{1}{c^4}$ why?

c. $(n^3)^3 = n^9$ why?

d. $(-2x)^3 = (-2)^3 x^3 = -8x^3$ why?

e. $(yz^4)^2 = (y^1z^4)^2 = (y^1)^2(z^4)^2 = y^2z^8$ why?

f. $(v^{-2}w^3)^{-1} = (v^{-2})^{-1}(w^3)^{-1} = v^{-2(-1)}w^{3(-1)} = v^2w^{-3} = \dfrac{v^2}{w^3}$ why?

Remember! We add exponents only when the bases are the same. For example, the expression, $x^5(y^3)$ cannot be simplified further. Why?

Now examine the following:

$$\frac{x^7}{x^2} = \frac{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}{x \cdot x} = x^5$$

$$\frac{x^3}{x^7} = \frac{x \cdot x \cdot x}{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x} = \frac{1}{x^4}, or, x^{-4}$$

$$\left(\frac{r}{s}\right)^5 = \left(\frac{r}{s}\right)\left(\frac{r}{s}\right)\left(\frac{r}{s}\right)\left(\frac{r}{s}\right)\left(\frac{r}{s}\right) = \frac{r^5}{s^5}$$

Results of these divisions are generalized in the following additional properties of exponents.

Division Properties of Exponents
Let *m* and *n* be integers and *a* and *b* represent nonzero real numbers.
Quotient Rule: $\dfrac{a^m}{a^n} = a^{m-n}$
Power of a Quotient: $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$

Example 3 Simplify each expression. Write answers using positive exponents

a. $\dfrac{5^2}{5^6}$ b. $\dfrac{k^3}{k^{-2}}$ c. $\left(\dfrac{2}{h^2}\right)^3$

Solution:

a. $\dfrac{5^2}{5^6} = 5^{2-6} = 5^{-4} = \dfrac{1}{5^4} = \dfrac{1}{625}$ why?

b. $\dfrac{k^3}{k^{-2}} = k^{3-(-2)} = k^5$ why?

c. $\left(\dfrac{2}{h^2}\right)^3 = \dfrac{2^3}{(h^2)^3} = \dfrac{8}{(h^2)^3} = \dfrac{8}{h^{2(3)}} = \dfrac{8}{h^6}$ why?

Remember! We subtract exponents only when the bases are the same. For example, the expression, $\dfrac{x^5}{y^3}$ cannot be simplified further. Why?

Let's Practice For Mastery! 1

A. Simplify each expression.

1. $3^3 \cdot 3^2$

2. $\left(2^{-3}\right)^4$

3. $(-3 \cdot 5)^{-2}$

4. $\dfrac{5^4}{5^7}$

5. $\dfrac{6^{-3}}{6^{-5}}$

6. $\left(-\dfrac{1}{2}\right)^3$

B. Try harder.

7. $g^{-3} \cdot g$

8. $\left(m^{-3}n^4\right)^{-4}$

9. $\dfrac{y^{-2}}{y^{-3}}$

10. $(-3b)^{-4}2$

11. $b^{-4} \cdot b^{-7}b^5$

12. $\dfrac{a^2b^2}{a^3b^2}$

13. $\left(\dfrac{x^3}{y^7}\right)^2$

14. $\left(\dfrac{s^3t^{-5}}{s^{-1}t^{-4}}\right)^2$

15. $\dfrac{3j^{-3}k^{-5}}{2j^{-5}k^{-3}}$

16. $\dfrac{15c^8d^3}{-3c^{10}d^5}$

C. Answer the following.

17. Is $(-3b)^4 = 12b^4$? Why or why not?

18. Which expression is equal to $\dfrac{1}{4}$?

a. 4^{-1} b. 2^{-2} c. -4^1 d. $\dfrac{1}{2^2}$ e. 1^4 f. -2^{-2}

19. When two numbers have a product of 1, they are called "reciprocals" of each other. For example, 5 and $\dfrac{1}{5}$ are reciprocals since $5 \cdot \dfrac{1}{5} = \dfrac{5}{5} = 1$. Show that a^n and a^{-n} are reciprocals of each other.

20. Explain why $x^3 \cdot y^5$ cannot be simplified.

Let's Check Your Understanding! 1

Write the letters of the correct answer.

_____1. Which is equivalent to r^4s^8?

a. $(rs)^{12}$ b. $(r^4s^4)^2$ c. $(rs^2)^4$ d. $r^4+s^4+s^4$

_____2. Which is a true statement?

a. $\frac{z^6}{z^2}=z^3$ b. $\frac{z^3}{z^5}=\frac{1}{z^{-2}}$ c. $\frac{z^0}{z^{-3}}=z^3$ d. $\frac{z^{-4}}{z^2}=z^{-2}$

_____3. All of the following are equivalent to $(-w)^2$ except one. Which is it?

a. $-w^2$ b. $w\cdot w$ c. w^2 d. $(-w)(-w)$

_____4. All of the following are equivalent to n^4 except one. Which one is it?

a. $n^2\cdot n^2$ b. $(n^2)^2$ c. $\frac{n^4}{n}$ d. $\frac{1}{n^{-4}}$

_____5. Which expression is equivalent to $\frac{a^2}{b^3}$?

a.) $\frac{a^6b^3}{a^3b^9}$ b.) $\frac{a^6b^6}{a^4b^5}$ c.) $\frac{a^2a}{b^{-1}b^4}$ d.) $\frac{a^8b^{-1}}{a^4b^{-2}}$

Lesson 4.2 Definition of Rational Exponents

Rational exponents are found in expressions such as $25^{\frac{1}{2}}, 8^{\frac{1}{3}}, (-8)^{\frac{-3}{4}}$, and $x^{\frac{-3}{4}}$.

One of the multiplication properties of integer exponents you just reviewed is $(a^m)^n=a^{mn}$. Recall how this property is used in the following:

$$(2^3)^2=2^6=64$$
$$(3^2)^4=3^8=6561$$
$$(a^3)^5=a^{15}$$

Now, study how the same property is used in the following:

a. $\left(4^{\frac{1}{2}}\right)^2=4$ b. $\left(25^{\frac{1}{2}}\right)^2=25$

c. $\left(8^{\frac{1}{3}}\right)^3 = 8$ d. $\left(81^{\frac{1}{4}}\right)^4 = 81$

In a, $4^{\frac{1}{2}}$ is the number which when squared gives 4 or $4^{\frac{1}{2}}$ is the square root of 4.

In b, $25^{\frac{1}{2}}$ is the number which when squared gives 25 or $25^{\frac{1}{2}}$ is the square root of 25.

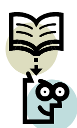
In c, $8^{\frac{1}{3}}$ is the number which when cubed gives 8 or $8^{\frac{1}{3}}$ is the cube root of 8.

And in d, $81^{\frac{1}{4}}$ is the number which when raised to the 4th power gives 81 or $81^{\frac{1}{4}}$ is the fourth root of 81.

Thus generally,

When a is a positive real number and n is a positive integer, then $\left(a^{\frac{1}{n}}\right)^n$ and $a^{\frac{1}{n}}$ is the nth root of a.

Note that for this unit, we limit our discussion when a is positive.



Example: Give the meaning of each:

Answer:

1. $3^{\frac{1}{2}}$ — $3^{\frac{1}{2}}$ is the square root of 3
2. $7^{\frac{1}{2}}$ — $7^{\frac{1}{2}}$ is the square root of 7
3. $64^{\frac{1}{3}}$ — $64^{\frac{1}{3}}$ is the cube root of 64
4. $6^{\frac{1}{3}}$ — $6^{\frac{1}{3}}$ is the cube root of 6
5. $9^{\frac{1}{4}}$ — $9^{\frac{1}{4}}$ is the fourth root of 9

Let's Practice For Mastery! 2

Give the meaning of each of the following.

1. $81^{\frac{1}{2}}$ 2. $125^{\frac{1}{3}}$ 3. $144^{\frac{1}{2}}$ 4. $36^{\frac{1}{2}}$ 5. $81^{\frac{1}{4}}$

Let's Check Your Understanding! 2

Match column A with column B. Write the letter of the correct match on the blank before each number.

	A	B
____1.	The cube root of 3	a. $18^{\frac{1}{2}}$
____2.	The square root of 49	b. $15^{\frac{1}{3}}$
____3.	The square root of 169	c. $3^{\frac{1}{3}}$
____4.	The cube root of 15	d. $169^{\frac{1}{2}}$
____5.	The fourth root of 18	e. $18^{\frac{1}{4}}$
		f. $49^{\frac{1}{2}}$

Lesson 4.3 Evaluating Rational Exponents

You can easily evaluate expressions with rational exponents using your knowledge on raising a number to a power and the definition of rational exponent.

Study the following:

1. $25^{\frac{1}{2}} = 5$ since $\left(25^{\frac{1}{2}}\right)^2 = 25$ and $(5)^2 = 25$

$25^{\frac{1}{2}}$ is the number which when squared is 25 but 5 is also the number which when squared is 25. This leads us to conclude that $25^{\frac{1}{2}}$ is equal to 5.

2. $49^{\frac{1}{2}} = 7$ since $\left(49^{\frac{1}{2}}\right)^2 = 49$ and $(7)^2 = 49$

$49^{\frac{1}{2}}$ is the number which when squared is 49 but 7 is also the number which when squared is 49. This leads us to conclude that $49^{\frac{1}{2}}$ is equal to 7.

3. $125^{\frac{1}{3}} = 5$ since $\left(125^{\frac{1}{3}}\right)^3 = 125$ and $(5)^3 = 125$

$125^{\frac{1}{3}}$ is the number which when cubed is 125 but 5 is also the number which when cubed is 125. This leads us to conclude that $125^{\frac{1}{3}}$ is equal to 5.

4. $81^{\frac{1}{4}} = 3$ since $\left(81^{\frac{1}{4}}\right)^4 = 81$ and $(3)^4 = 81$

$81^{\frac{1}{4}}$ is the number which when raised to the fourth power is 81 but 3 is also the number which when raised to the 4th power is 81. This leads us to conclude that $81^{\frac{1}{4}}$ equals 3.

Based from the above examples, we see that

$$\boxed{\text{If} \quad b^n = a \quad \text{then} \quad a^{\frac{1}{n}} = b}$$

In other words, finding the *n*th power of *a* is the reverse operation of finding the power *b*.

Let's Practice For Mastery! 3

A. Complete the statements by filling the blanks

1. $16^{\frac{1}{2}} =$ _____ since $\left(16^{\frac{1}{2}}\right)^2 =$ _____ and $4^2 = 16$

2. $27^{\frac{1}{3}} =$ _____ since $\left(27^{\frac{1}{3}}\right)^3 = 27$ and $(\_\_)^3 = 27$

3. $32^{\frac{1}{5}} =$ _____ since $\left(32^{\frac{1}{5}}\right)^5 =$ _____ and $(2)^2 = 32$

4. $121^{\frac{1}{4}} =$ _____ since $(\_\_)^2 = 121$ and $11^2 = 121$

5. $16^{\frac{1}{4}} =$ _____ since $\left(16^{\frac{1}{4}}\right)^4 = 16$ and $(\_\_\_)^4 = 16$

B. Find the following expressions without fractional exponents.

6. $16^{\frac{1}{2}}$ 7. $144^{\frac{1}{2}}$ 8. $4^{\frac{1}{2}}$

9. $9^{\frac{1}{2}}$ 10. $16^{\frac{1}{4}}$ 11. $27^{\frac{1}{3}}$

12. $36^{\frac{1}{2}}$ 13. $64^{\frac{1}{3}}$ 14. $64^{\frac{1}{6}}$

15. $121^{\frac{1}{2}}$

Let's Check Your Understanding! 3

Find the following expressions without fractional exponents.

1. $25^{\frac{1}{2}}$ 2. $36^{\frac{1}{2}}$ 3. $128^{\frac{1}{7}}$

4. $\left(\frac{1}{4}\right)^{\frac{1}{2}}$ 5. $\left(\frac{27}{8}\right)^{\frac{1}{3}}$

Expressions Involving Rational Exponents of the Form $a^{\frac{m}{n}}$

Notice that the fractional exponents you have been studying so far are unit fractions (that is when the numerator of the fraction is 1. What will you do if you encounter expressions like $8^{\frac{2}{3}}$? $64^{\frac{3}{5}}$, $81^{\frac{3}{2}}$? How will you evaluate these expressions?

To evaluate such expression you can use either of the following.

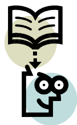
Take the root first then raise to the power. Example: $8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2 = 2^2 = 4$

Raise to a power first then take the root. Example: $8^{\frac{2}{3}} = \left(8^2\right)^{\frac{1}{3}} = 64^{\frac{1}{3}} = 4$

Notice that the result in each case is the same. Notice too that the 1st case is easier! Why?

Generally,

$$\boxed{a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = \left(a^m\right)^{\frac{1}{n}}}$$



Example: Evaluate the following:

1. $400^{\frac{1}{2}}$ 2. $100^{\frac{3}{2}}$ 3. $216^{\frac{2}{3}}$

Solution:

1. $400^{\frac{1}{2}} = 20$ since $20^2 = 400$

2. $100^{\frac{3}{2}} = \left(100^{\frac{1}{2}}\right)^3$ why?

$= 10^3$

$= 1000$

3. $216^{\frac{2}{3}} = \left(216^{\frac{1}{3}}\right)^2$ why?

$= 6^2$

$=36$

Let's Practice For Mastery! 4

Evaluate the following.

1. $16^{\frac{3}{4}}$
2. $64^{\frac{2}{3}}$
3. $\left(\frac{16}{25}\right)^{\frac{1}{2}}$
4. $\left(\frac{1}{32}\right)^{\frac{1}{5}}$
5. $9^{\frac{5}{2}}$
6. $1000^{\frac{1}{3}}$
7. $169^{\frac{1}{2}}$
8. $81^{\frac{3}{4}}$
9. $27^{\frac{2}{3}}$
10. $\left(\frac{8}{125}\right)^{\frac{1}{3}}$

Let's Check Your Understanding! 4

Evaluate the following.

1. $4^{\frac{1}{2}}$
2. $625^{\frac{1}{4}}$
3. $64^{\frac{1}{3}}$
4. $27^{\frac{4}{3}}$
5. $49^{\frac{3}{2}}$
6. $\left(\frac{27}{8}\right)^{\frac{2}{3}}$
7. $\left(\frac{4}{81}\right)^{\frac{1}{2}}$
8. $32^{\frac{3}{5}}$

Lesson 4.4 Simplifying Rational Exponents

Now that we know the meaning of rational exponents, we can then consider simplifying expressions such as, $x^{\frac{2}{3}} \cdot x^{\frac{1}{6}}, x^{-\frac{1}{4}} \cdot x^{\frac{3}{5}}$ and $\left(2x^{\frac{1}{4}}\right)^3$.

Negative Rational Exponents

An expression is considered simplified when all exponents involved are positive. So that whenever there are negative exponents, one must know how to get rid of them. Let us first review the basic definition of negative exponents as applied with rational exponents.

> When $\frac{m}{n}$ is a rational number and a is any number, not 0, then;
>
> $$a^{-\frac{m}{n}} \text{ means } \frac{1}{a^{\frac{m}{n}}} \text{ and } \frac{1}{a^{-\frac{m}{n}}} = a^{\frac{m}{n}}$$

Example 1 Rewrite with positive exponents.

a. $9^{-\frac{1}{2}}$ b. $(5xy)^{-\frac{4}{5}}$ c. $\frac{1}{16^{-\frac{1}{2}}}$

Solution:

a. $9^{-\frac{1}{2}} = \frac{1}{9^{\frac{1}{2}}}$ definition of negative exponents

$= \frac{1}{3}$ since $9^{\frac{1}{2}} = 3$

b. $(5xy)^{-\frac{4}{5}} = \frac{1}{(5xy)^{\frac{4}{5}}}$ definition of negative exponents

c. $\frac{1}{16^{-\frac{1}{2}}} = 16^{\frac{1}{2}}$ Why?

$= 4$ Why?

Remember! A negative exponent does not mean that the expression represents a negative quantity as we have seen in the example above.

Let's Practice For Mastery! 5

Rewrite with positive exponents.

1. $x^{-\frac{1}{3}}$ 2. $(5xy)^{-\frac{5}{6}}$ 3. $\dfrac{1}{x^{-\frac{2}{3}}}$

4. $\left(\dfrac{1}{10}\right)^{-\frac{2}{3}}$ 5. $(2rs)^{-\frac{3}{4}}$

Let's Check Your Understanding! 5

Rewrite with positive exponents.

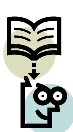
1. $y^{-\frac{1}{4}}$ 2. $\dfrac{1}{x^{-\frac{5}{6}}}$ 3. $\dfrac{3}{m^{-\frac{1}{2}}}$

4. $16^{-\frac{3}{4}}$ 5. $(7xy)^{-\frac{4}{5}}$

Multiplication Properties of Exponents

The properties of integer exponents are now extended to rational exponents.

If a and b are real numbers and m and n are rational numbers, then	
Product Rule	$a^m \cdot a^n = a^{mn}$
Power of a Power	$(a^m)^n = a^{mn}$
Power of a Product	$(ab)^m = a^m b^m$



Example 2 Simplify:

a. $x^{\frac{2}{3}} \cdot x^{\frac{5}{3}}$ b. $x^{\frac{1}{2}} \cdot x^{\frac{1}{3}}$ c. $(a^3)^{\frac{2}{3}}$

d. $(9x^4)^{\frac{1}{2}}$ e. $\left(2x^{-\frac{1}{4}}\right)^3$

Solution:

a. $x^{\frac{2}{3}} \cdot x^{\frac{5}{3}} = x^{\frac{2}{3}+\frac{5}{3}}$ Using product rule

$= x^{\frac{7}{3}}$

b. $x^{\frac{1}{2}} \cdot x^{\frac{1}{3}} = x^{\frac{1}{2}+\frac{1}{3}}$ Using product rule

$= x^{\frac{3+2}{6}}$ Adding dissimilar fractions

$= x^{\frac{5}{6}}$

c. $\left(a^3\right)^{\frac{2}{5}} = a^{3\left(\frac{2}{5}\right)}$ Using power of a power rule

$= a^{\frac{6}{5}}$

d. $\left(9x^4\right)^{\frac{1}{2}} = 9^{\frac{1}{2}} x^{4\left(\frac{1}{2}\right)}$ Using power of a product rule

$= 3x^{\frac{4}{2}}$

$= 3x^2$

e. $\left(2x^{-\frac{1}{4}}\right)^3 = 2^3 x^{\left(-\frac{1}{4}\right)(3)}$ why?

$= 8x^{-\frac{3}{4}}$

$= \dfrac{8}{x^{\frac{3}{4}}}$ why?

Let's Practice For Mastery! 6

Simplify the following.

1. $\left(9x^4\right)^{\frac{1}{2}}$

2. $8x^2 \cdot x^{\frac{1}{2}}$

3. $x^{\frac{3}{4}} \cdot x^{\frac{1}{8}}$

4. $\left(16x^{-\frac{1}{3}}\right)^{\frac{1}{4}}$

5. $5x^{\frac{2}{3}} \cdot x^{\frac{1}{2}}$

Let's Check Your Understanding! 6

Simplify the following.

1. $x^{\frac{1}{4}} \cdot x^{\frac{2}{4}}$

2. $\left(b^{\frac{2}{3}}\right)^{\frac{3}{2}}$

3. $\left(x^{\frac{3}{4}} y^{\frac{1}{8}} z^{\frac{5}{6}}\right)^{\frac{4}{5}}$

4. $5^{\frac{3}{4}} \cdot 5^{\frac{1}{8}}$

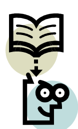
5. $\left(a^{-\frac{1}{3}} b^{\frac{2}{5}}\right)^{\frac{1}{2}}$

Division Properties of Exponents

The division properties of exponents also apply to rational exponents.

> If a and b are real numbers and m and n are rational numbers then
>
> Quotient Rule $\quad \dfrac{a^m}{a^n} = a^{m-n}$
>
> Power of a Quotient $\quad \left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$

See how the properties of exponents are used to simplify expressions.



Example 3 Simplify the following.

a. $\dfrac{x^3}{x^{\frac{1}{2}}}$ b. $\left(\dfrac{x^{\frac{1}{2}}}{y^{\frac{1}{3}}}\right)^6$ c. $\dfrac{16^{\frac{1}{4}}}{16^{\frac{1}{2}}}$

Solution:

a. $\dfrac{x^3}{x^{\frac{1}{2}}} = x^{3-\frac{1}{2}}$ Using the quotient rule

$= x^{\frac{6}{2}-\frac{1}{2}}$ Subtracting factors

$= x^{\frac{5}{2}}$

b. $\left(\dfrac{x^{\frac{1}{2}}}{y^{\frac{1}{3}}}\right)^6 = \dfrac{x^{\left(\frac{1}{2}\right)(6)}}{y^{\left(\frac{1}{3}\right)(6)}}$ Using the power of a quotient

$= \dfrac{x^{\frac{6}{2}}}{y^{\frac{6}{3}}}$ Multiplying Factors

$= \dfrac{x^3}{y^2}$

c. $\dfrac{16^{\frac{1}{4}}}{16^{\frac{1}{2}}} = 16^{\frac{1}{4}-\frac{1}{2}}$ why?

$= 16^{\frac{1}{4}-\frac{2}{4}}$ why?

$= 16^{-\frac{1}{4}}$

$= \dfrac{1}{16^{\frac{1}{4}}}$ why?

$= \dfrac{1}{2}$ why?

Let's Practice For Mastery! 7

A. Simplify the following:

1. $\dfrac{x^{\frac{2}{4}}}{x^{\frac{1}{5}}}$

2. $\dfrac{x^{\frac{2}{3}}}{x^{\frac{1}{9}}}$

3. $\left(\dfrac{27}{x^3}\right)^{-\frac{1}{3}}$

4. $\dfrac{x^2}{x^{\frac{2}{5}}}$

5. $\dfrac{x^{\frac{1}{8}}}{x^1}$

B. Answer the following.

6. Are $9^{\frac{1}{2}}$ and 9^2 reciprocals?

7. Ella wrote $5^{-\frac{1}{2}} = -5^{\frac{1}{2}}$ and Joni wrote $5^{-\frac{1}{2}} = \dfrac{1}{5^{\frac{1}{2}}}$. Whose work is correct?

Let's Check Your Understanding! 7

Simplify:

1. $\dfrac{x^{\frac{3}{4}}}{x^{\frac{1}{6}}}$

2. $\dfrac{x^{\frac{1}{2}}}{x^{-\frac{2}{3}}}$

3. $\left(\dfrac{49}{b^{-4}}\right)^{\frac{1}{2}}$

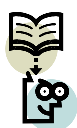
4. $\left(\dfrac{x^{3}}{x^{2}}\right)^{-\frac{1}{2}}$

5. $\left(\dfrac{x^{-6}}{xy^{-3}}\right)^{7}$

Applying Two or More of the Properties of Exponents

At this point you must already have learned how to use any of the properties of exponents in a given expression. There are even times when more than two of the properties need to be applied in order to simplify the expression.

Remember! Expressions are simplified when the bases appear as few times as possible, when all exponents are positive, when there are no power of powers and when fractions are in lowest terms.



Example 4 Simplify:

a. $\left(\dfrac{49x^{6}y^{-2}}{z^{-4}}\right)^{\frac{1}{2}}$

b. $\dfrac{\left(25a^{6}b^{4}\right)^{\frac{1}{2}}}{\left(8a^{-9}b^{3}\right)^{-\frac{1}{3}}}$

Solution:

a. $\left(\dfrac{49x^{6}y^{-2}}{z^{-4}}\right)^{\frac{1}{2}} = \dfrac{\left(49x^{6}y^{-2}\right)^{\frac{1}{2}}}{z^{(-4)\left(\frac{1}{2}\right)}}$ Using the power of a product and quotient rule

$= \dfrac{49^{\frac{1}{2}}x^{(6)\left(\frac{1}{2}\right)}y^{(-2)\left(\frac{1}{2}\right)}}{z^{(-4)\left(\frac{1}{2}\right)}}$ Using the power of a power rule

$$= \frac{7x^{\frac{6}{2}} y^{-\frac{2}{2}}}{z^{-2}}$$ Since $49^{\frac{1}{2}} = 7$, multiplying fractions

$$= \frac{7x^{3} y^{-1}}{z^{-2}}$$ Using the quotient rule

$$= \frac{7x^{3} z^{2}}{y}$$

b. $$\frac{\left(25a^{6}b^{4}\right)^{\frac{1}{2}}}{\left(8a^{-9}b^{3}\right)^{-\frac{1}{3}}} = \frac{25^{\frac{1}{2}} a^{(6)\left(\frac{1}{2}\right)} b^{(4)\left(\frac{1}{2}\right)}}{8^{-\frac{1}{3}} a^{(-9)\left(-\frac{1}{3}\right)} b^{(3)\left(-\frac{1}{3}\right)}}$$ why?

$$= \frac{5a^{3}b^{2}}{8^{-\frac{1}{3}} a^{3} b^{-1}}$$ why?

$$= \frac{5\left(8^{\frac{1}{3}}\right) a^{3}b^{2}}{a^{3}b^{-1}}$$ since $\frac{1}{8^{-\frac{1}{3}}} = \frac{8^{\frac{1}{3}}}{1}$

$$= 5(2)a^{3-3}b^{2--1}$$ why?

$$= 10a^{0}b^{3}$$

$$= 10(1)b^{3} = 10b^{3}$$ why?

Let's Practice For Mastery! 8

A. Simplify the following:

1. $\left(2x^{\frac{1}{2}} y^{\frac{1}{3}}\right)^{3}$

2. $\left(-x^{3}y^{6}z^{-6}\right)^{\frac{2}{3}}$

3. $\left(\frac{x^{2}y^{-3}}{z^{4}}\right)^{-\frac{1}{2}}$

4. $\left(\frac{49x^{4}}{100y^{-8}}\right)^{-\frac{1}{2}}$

5. $\frac{\left(27x^{4}y^{-1}\right)^{\frac{1}{2}}}{\left(2x^{\frac{1}{5}}y^{\frac{3}{5}}\right)^{3}}$

B. Answer the following.

6. Which of the following expressions is in simplest form?

a.) $\frac{\left(x^{3}\right)^{2}}{y^{7}}$ b.) $x^{-3}y^{2}$ c.) $\frac{a^{5}}{ab}$ d.) $\frac{2n^{3}m^{\frac{1}{2}}}{3p}$

7. If a = 4, b = 3 and c = 0, which expression has the greatest value?

a.) a^b b.) b^c c.) $\frac{1}{b^{-a}}$ d.) $\frac{a^c}{b^c}$ e.) $\frac{c}{b^{-a}}$

Let's Check Your Understanding! 8

Simplify.

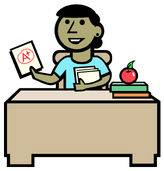
1. $\left(4x^{-34}y^{\frac{1}{5}}\right)^{-2}$

2. $\left(81x^{-8}y^{2}\right)^{-\frac{1}{4}}$

3. $\left(\frac{27x^3y^6}{z^9}\right)$

4. $\left(\frac{16x^{-4}y^3}{z^4}\right)^{\frac{3}{4}}$

5. $\frac{\left(8x^2y\right)^{\frac{1}{3}}}{\left(5x^{\frac{1}{3}}y^{-\frac{1}{2}}\right)^2}$



LET’S SUMMARIZE!

In $a^n = b$, a is the base, n is the exponent and b is the power.

When n is a positive integer,

a^n means $a^n = a \cdot a \cdot a \cdot a \cdots a$ where a is taken as a factor n times.

The Properties of Rational Exponents

If a and b are real numbers and m and n are rational numbers then

Product Rule	$a^m \cdot a^n = a^{mn}$
Power of a Power	$(a^m)^n = a^{mn}$
Power of a Product	$(ab)^m = a^m b^m$
Quotient Rule	$\frac{a^m}{a^n} = a^{m-n}$ a $\neq 0$
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ b $\neq 0$

Negative Rational Exponents

When $\frac{m}{n}$ is a rational number and a is any number, not 0, then;

$$a^{-\frac{m}{n}} \text{ means } \frac{1}{a^{\frac{m}{n}}} \text{ and } \frac{1}{a^{-\frac{m}{n}}} = a^{\frac{m}{n}}$$

UNIT TEST

A. Multiple Choice. Write the letter of the correct answer.

1. Which expression is equivalent to $\dfrac{4y^4}{xy^{-4}}$?

a.) $\dfrac{4xy^4}{m^2}$ b.) $\dfrac{xy^4}{4m^2}$ c.) $\dfrac{4y^4}{m^2x}$ d.) $\dfrac{4y^{-4}}{m^{-2}x}$

2. Which expression has the last value when b = 3 and c = -2?

a.) c^b b.) b^c c.) $\dfrac{5^{\frac{1}{2}}}{5^{\frac{1}{3}}}$ d.) $\left(b^c\right)^b$

3. Which expression is in simplest form and equivalent to $\left(3s^3\right)\left(2t^2\right)\left(4s^4\right)$?

a.) $24st^9$ b.) $24s^{12}t^2$ c.) $24s^7t^2$ d.) $\left(12s^7\right)\left(2t^2\right)$

4. Which is not equal to $5^{\frac{1}{2}} \cdot 5^{-\frac{1}{3}}$?

a.) $5^{\frac{1}{2}-\frac{1}{3}}$ b.) $5^{-\frac{1}{5}}$ c.) $\dfrac{5^{\frac{1}{2}}}{5^{\frac{1}{3}}}$ d.) $\left(5^{\frac{1}{2}}\right)^{-\frac{1}{3}}$

5. $25^{-\frac{1}{2}}$ is the same as

a.) $-25^{\frac{1}{2}}$ b.) $\dfrac{1}{25^{\frac{1}{2}}}$ c.) $\dfrac{1}{25^{-\frac{1}{2}}}$ d.) $25^{\frac{1}{2}}$

B. Evaluate the following.

6. $16^{\frac{1}{2}}$ 7. $81^{\frac{1}{4}}$ 8. $32^{\frac{3}{5}}$ 9. $9^{-\frac{1}{2}}$ 10. $64^{-\frac{2}{3}}$

C. Simplify each expression.

11. $\left[x^{\frac{1}{2}}\right]^2$

12. $y^{\frac{4}{7}}y^{\frac{10}{7}}$

13. $\left[x^{18}\right]^{\frac{1}{9}}$

14. $\left[2^{\frac{1}{5}}3^{\frac{2}{5}}\right]^5$

15. $\dfrac{8^{\frac{3}{2}}}{8^{\frac{1}{2}}}$

16. $\dfrac{x^{\frac{5}{6}}x^{\frac{5}{6}}}{x^{\frac{7}{6}}}$

17. $\left[7^{\frac{2}{5}}\right]^{\frac{5}{2}}$

18. $\left[\dfrac{x^{\frac{3}{5}}}{x^{\frac{2}{5}}}\right]^5$

19. $\left[\dfrac{z^{\frac{3}{5}}z^{\frac{6}{5}}}{z^{\frac{2}{5}}}\right]^5$

20. $\dfrac{12b^{-\frac{1}{3}}b^{-\frac{3}{4}}}{4b^{\frac{3}{5}}}$

ANSWER KEY

Let's Practice For Mastery! 1

A.

1. 243	2. $\frac{1}{4096}$	3. $\frac{1}{225}$	4. $\frac{1}{125}$
5. 36	6. $-\frac{1}{8}$	7. $\frac{1}{g^2}$	8. $\frac{m^{12}}{n^{16}}$
9. y	10. $\frac{1}{81b^4}$	11. $\frac{1}{b^6}$	12. $\frac{1}{a}$
13. $\frac{x^{16}}{y^{14}}$	14. s^8t^2	15. $\frac{3j^2}{2k^2}$	16. $\frac{-5}{c^2d^2}$

17. No, because $(-3b)^4 = 81b^4$

18. a, b, d, f

19. Since $a^{-n} = \frac{1}{a^n}$, then $a^n \cdot a^{-n} = a^n \cdot \frac{1}{a^n} = 1$

20. They have different bases.

Let's Check Your Understanding! 1

1. C	2. C	3. A	4. C	5. B

Let's Practice For Mastery! 2

A.

1. the square root of 81
2. the cube root of 125
3. the square root of 144
4. the square root of 36
5. the fourth root of 81

Let's Check Your Understanding! 2

1. C	2. F	3. D	4. B	5. E

Let's Practice For Mastery! 3

A.

1. 4	16
2. 3	3
3. 2	32
4. 11	$121^{\frac{1}{2}}$
5. 2	2

B.

6. 4	11. 3
7. 12	12. 6
8. 2	13. 4
9. 3	14. 2
10. 2	15. 11

Let's Check Your Understanding! 3

1. 5 2. 6 3. 2 4. $\frac{1}{2}$ 5. $\frac{3}{2}$

Let's Practice For Mastery! 4

A.

1. 8 2. 16 3. $\frac{4}{5}$ 4. $\frac{1}{2}$ 5. 13

6. 10 7. 27 8. 9 9. 243 10. $\frac{2}{5}$

Let's Check Your Understanding! 4

1. 2 2. 5 3. 4 4. 81

5. 343 6. $\frac{9}{4}$ 7. $\frac{2}{9}$ 8. 8

Let's Practice For Mastery! 5

1. $\frac{1}{x^{\frac{1}{3}}}$ 2. $\frac{1}{(5xy)^{\frac{5}{6}}}$ 3. $x^{\frac{2}{3}}$ 4. $10^{\frac{2}{3}}$ 5. $\frac{1}{(2rs)^{\frac{1}{2}}}$

Let's Check Your Understanding! 5

1. $\frac{1}{y^{\frac{1}{4}}}$ 2. $x^{\frac{5}{6}}$ 3. $3m^{\frac{1}{2}}$ 4. 8 5. $(7xy)^{\frac{4}{5}}$

Let's Practice For Mastery! 6

1. $3x^2$ 2. 8x 3. $x^{\frac{7}{8}}$ 4. $\frac{2}{x^{\frac{1}{12}}}$ 5. $5x^{\frac{7}{6}}$

Let's Check Your Understanding! 6

1. $x^{\frac{3}{4}}$ 2. b 3. $x^{\frac{3}{5}}y^{\frac{1}{10}}z^{\frac{2}{3}}$ 4. $5^{\frac{7}{8}}$ 5. $\frac{b^{\frac{1}{5}}}{4^{\frac{1}{3}}}$

Let's Practice For Mastery! 7

A

1. $x^{\frac{1}{5}}$ 2. $x^{\frac{5}{9}}$ 3. $\frac{x}{3}$ 4. $x^{\frac{8}{5}}$ 5. $\frac{1}{x^{\frac{7}{2}}}$

2. B. 6. No 7. Joni's work

Let's Check Your Understanding! 7

1. $x^{\frac{7}{12}}$ 2. $x^{\frac{7}{6}}$ 3. $7b^2$ 4. $\frac{1}{x^{\frac{1}{2}}}$ 5. $\frac{y^{21}}{x^{49}}$

Let's Practice For Mastery! 8

1. $8x^{\frac{3}{2}}y$ 2. $\frac{-x^2y^4}{z^4}$ 3. $\frac{z^2y^{\frac{3}{2}}}{x}$ 4. $\frac{10}{7x^2y^4}$ 5. $\frac{27^{\frac{1}{2}}x^{\frac{7}{5}}}{8y^{\frac{11}{5}}}$

6. D 7. C

Let's Check Your Understanding! 8

1. $\frac{x^{\frac{3}{2}}}{16y^{\frac{2}{5}}}$ 2. $\frac{x^2}{3y^{\frac{1}{2}}}$ 3. $\frac{z^3}{3x^2y^2}$ 4. $\frac{8y^{\frac{9}{4}}}{x^3z^3}$ 5. $\frac{2^{\frac{1}{3}}}{25}$

UNIT TEST

A. 1. C 2. A 3. C 4. A 5. B

B. 6. 4 7. 3 8. 8 9. $\frac{1}{3}$ 10. 1/8

11. x 12. y^2 13. x^2 14. 18 15. 8

16. $x^{\frac{1}{2}}$ 17. 7 18. x 19. z^7 20. $\frac{3}{b^{\frac{101}{60}}}$

Misconceptions and Common Errors

1. $4x^0 \neq 1$ since in $4x^0$, x is the base from which 0 is the exponent, so that $4x^0 = 4(1) = 4$.
2. $-3^4 \neq 3^4$ since $-3^4 = -(3)(3)(3)(3) = -81$ and $3^4 = (3)(3)(3)(3) = 81$.
3. $-3^2 \neq (-3)^2$ since $-3^2 = -(3)(3) = -9$ and $(-3)^2 = (-3)(-3) = 9$.
4. $-2^3 \neq -(-2)^3$ since $-2^3 = -(2)(2)(2) = -8$ and $-(-2)^3 = -(-2)(-2)(-2) = -(-8) = 8$.
5. $3^{-\frac{1}{2}} \neq -3^{\frac{1}{2}}$ since $3^{-\frac{1}{2}} = \frac{1}{3^{\frac{1}{2}}}$. $3^{-\frac{1}{2}}$ is not a negative number.
6. $9^{\frac{1}{2}} \neq \frac{1}{9^{\frac{1}{2}}}$ since $9^{\frac{1}{2}} = 3$ and $\frac{1}{9^{\frac{1}{2}}} = \frac{1}{3}$.