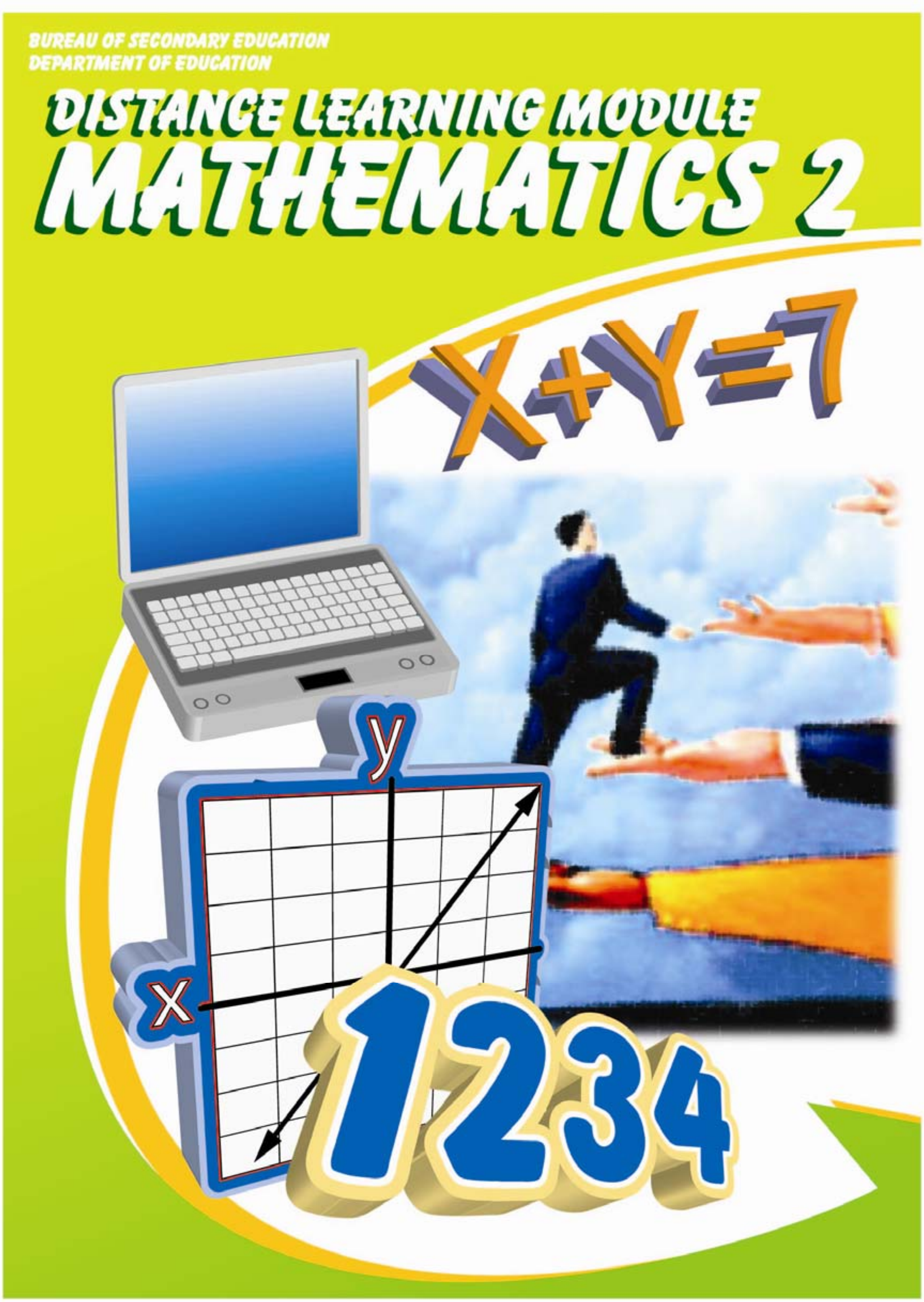
BUREAU OF SECONDARY EDUCATION
DEPARTMENT OF EDUCATION
DISTANCE LEARNING MODULE
MATHEMATICS 2
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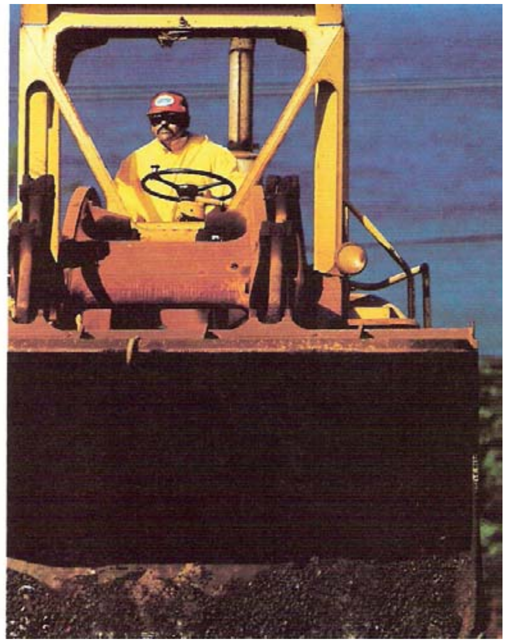
RATIONAL EXPRESSIONS AND RATIONAL EQUATIONS



UNIT III

Rey, one of the workers in a construction firm uses a bulldozer to excavate the foundation for a building.

If you were Rey, how would you determine the most efficient bulldozer that can be used to excavate the foundation for a building?

The study of rational expressions connects the work on exponents and factoring polynomials with operations on rational equations. The four operations with rational expressions are presented in an order of difficulty that builds confidence and ease with computational skills. With these pre-requisite topics, solving different types of problems would be easy.

This unit will start with a review on simplifying rational expressions. Applications of the concept will also be discussed. The main focus of the topic is the solution of rational equations, including its applications to real-life problems.

Lesson 3.1. Simplifying Rational Expressions

A rational expression is a ratio of two polynomials. The following are examples of rational expressions:

$$\frac{x}{5}, \qquad \frac{3}{x-2}, \qquad \frac{x^2+y^2}{4}, \qquad \frac{t^2-4t+5}{t+1}, \qquad \frac{m^2-n^2}{m+n}$$

Just like a rational number, a rational expression represents division of two polynomials, so the denominator cannot be 0. A rational expression is said to be undefined for any value of a variable that results in a denominator of zero. This leads to a clear definition.

> A *rational expression* is an expression of the form $\frac{p}{q}$, where p and q are polynomials and q $\neq$ 0.

On the other hand, the following cannot be considered as rational expressions:

$$\frac{2x^2}{x^{\frac{2}{3}}}, \qquad \frac{x+1}{\sqrt{x}}, \qquad \frac{3x-4x^3}{3x^{\frac{1}{3}}}.$$

Can you explain why?

The definition of a rational expression states that the denominator must not be equal to zero. In other words, the variable in a given rational expression may be replaced by any real number, except those that will make the denominator zero. As studied in your Mathematics I, the set of possible values for a variable is called its replacement set or domain.

Finding the domain of a rational expression is very important because you'll need it especially when you'll take functions later. Getting the domain of any expression or function is just the same thing as asking you this question: With what real numbers can I replace the variable(s) so that I get a real number as the result?

Examples :

A. Which of the following are rational expressions? Explain your answer.

1. $\dfrac{x+5}{7}$ 2. $\dfrac{15+x}{5x-1}$ 3. $\dfrac{x^3-2}{5-\sqrt{x}}$

Solutions :

1. Yes. It is a rational expression, because both the numerator and the denominator are polynomials.

2. Yes. Rational, since both numerator and denominator are polynomials.
3. No. Since the denominator does not define a polynomial.

B. Determine the replacement set or domain of each.

4. $\dfrac{2x-3}{x}$ 5. $\dfrac{x^2-2x+1}{6}$

Solutions:

4. The denominator , x, should not be zero. Thus, the domain is the set of real numbers except 0.
5. The denominator is a constant, so, the domain is the set of real numbers.

Let's Practice for Mastery 1.

1. Which of the numbers is not in the domain of the rational expression $\dfrac{5-4x}{x-3}$?

   a. 3 b. -3 c. -5 d. 4

2. Determine if each is an rational expression. Explain

   a. $\dfrac{4x-7}{9}$ b. $\dfrac{7x+6}{3-5x}$ c. $\dfrac{x^{\frac{1}{2}}}{5x-2}$

3. What value of x will make the rational expression $\dfrac{9-2x}{x^2-1}$ meaningless?

Explain.

Let's Check Your Understanding 1.

1. Which of the following are rational expressions ? Explain.

   a. $\dfrac{4+7x^2}{2x-9}$ b. $\dfrac{5}{3x}$ c. $\dfrac{6x}{3x^{\frac{2}{3}}}$

2. Determine the domain of the following rational expression:

   a. $\dfrac{x-7}{2x+4}$ b. $\dfrac{5x^3}{2x^2-8}$

3. Which of the numbers in the given set are in the domain of the rational expression $\dfrac{x+1}{x-7}$? Explain. $\{0,-2,-5,7,9\}$

A rational expression is in simplest form when the numerator and the denominator of the expression have no common factors other than 1 and -1.

Thus, to simplify a rational expression, you can use the following procedure:

Simplifying a Rational Expression

1. Factor both the numerator and denominator if, necessary.
2. Divide both numerator and denominator by their greatest common factor.

Examples : Answer the following :

1. Which of the following expressions is in lowest terms ? Explain.

a. $\frac{x}{5}$ b. $\frac{10x}{15y}$ c. $\frac{3x-5}{9}$ d. $\frac{2x-4}{8}$

Solution :

a. Yes. Can you explain why?

c. Yes, since the numerator and denominator do not have common factor.

Why is b not in lowest terms? What about d?

2. What is the greatest common factor of the numerator and denominator of the rational expression a. $\frac{7xy}{14xz}$? b. $\frac{4x^2+12xy}{20x^3y}$?

Solutions: a. Greatest common factor is 7x. Why ?

b. Greatest common factor is 4x. Why?

3. Simplify : $\dfrac{8x^3y}{16x^2y^2}$

Solution : Factor the numerator and denominator

$$\frac{8x^3y}{16x^2y^2} = \frac{8x^2y(x)}{8x^2y(2y)}$$ greatest common factor is $8x^2$.

$$= \frac{8x^2y}{8x^2y}\cdot\frac{x}{2y}$$ why?

$$= \frac{x}{2y}$$ simplest form

4. Simplify each expression. State any restrictions on the domain of the variable.

a. $$\frac{3x+6}{3x+12} = \frac{3(x+2)}{3(x+4)}$$ factor

$$= \frac{3}{3}\cdot\frac{x+2}{x+4}$$ $\dfrac{3}{3} = 1$

$$= \frac{x+2}{x+4}$$ To find the restrictions on the value x

solve: x + 4 = 0
x = − 4 .

so, the simplest form is

$\dfrac{x+2}{x+4}$, where x $\neq$ − 4.

b. $\frac{m^2n - mn}{m^2n} = \frac{mn(m-1)}{mn(m)}$ Factor Restrictions: $m^2 n = 0$

$m^2 = 0$ or $n = 0$

$= \frac{mn}{mn} \cdot \frac{(m-1)}{m}$ $\frac{mn}{mn} = 1$ $m = 0$ or $n = 0$

$= \frac{m-1}{m}$ why?

So, the simplest form is $\frac{m-1}{m}$, where $m \neq 0$, $n \neq 0$.

5. Simplify : $\frac{x^2 - 9}{x^2 + 5x + 6}$

Solution: $\frac{x^2 - 9}{x^2 + 5x + 6} = \frac{(x+3)(x-3)}{(x+3)(x+2)}$ common factor: x + 3

Restrictions:

$= \frac{x+3}{x+3} \cdot \frac{x-3}{x+2}$ $\frac{x+3}{x+3} = 1$ $x + 3 = 0$, or $x + 2 = 0$

$x \neq -3$ or $x \neq -2$

So, the simplest form is $\frac{x-3}{x+2}$ where $x \neq -3$, $x \neq -2$

6. Simplify: $\frac{a^2 - 9a + 18}{a^2 - 6a + 9}$ Restriction

Solution: $a - 3 = 0$

$\frac{a^2 - 9a + 18}{a^2 - 6a + 9} = \frac{(a-6)(a-3)}{(a-3)(a-3)}$ Factor. $a \neq 3$

$= \frac{a-3}{a-3} \cdot \frac{a-6}{a-3}$ But $\frac{a-3}{a-3} = 1$, $a \neq 3$

So, the simplest form is $\frac{a-6}{a-3}$, where $a \neq 3$.

7. Simplify: $\dfrac{3x^2 - 15xy + 18y^2}{9y^2 - x^2}$

Solution:

$$\frac{3x^2 - 15xy + 18y^2}{9y^2 - x^2} = \frac{3(x^2 - 5xy + 6y^2)}{9y^2 - x^2}$$

$$= \frac{3(x-3y)(x-2y)}{(3y-x)(3y+x)}$$ (x – 3y) & (3y – x) are opposites.

$$= \frac{3(x-3y)(x-2y)}{-(x-3y)(x+3y)}$$ (3y – x) = – (x – 3y)

$$= \frac{x-3y}{x-3y} \cdot \frac{3(x-2y)}{-(x+3y)}$$ common factor is (x – 3y), $\dfrac{x-3y}{x-3y} = 1$

$$= \frac{3(x-2y)}{-(x+3y)} \text{ or } -\frac{3(x-2y)}{x+3y}$$

Hence, the simplest form is $-\dfrac{3(x-2y)}{x+3y}$

Let's Practice for Mastery 2.

Answer the following:

1. What is the common factor in the numerator and denominator of $\dfrac{x^2 + x}{x+1}$?

2. Reduce each rational expression to lowest term. No denominator equals 0.

a. $\dfrac{6mn}{6m}$ b. $\dfrac{5x^3y + 15xy}{25}$

3. Simplify: $\dfrac{3x^2+4x+1}{x^2-1}$

4. Factor the numerator and the denominator, and then simplify:

a. $\dfrac{x^2-7x+10}{x^2+3x-10}$ b. $\dfrac{x^2+5x+4}{x^2+x}$

Let's Check Your Understanding 2.

Answer each problem completely.

1. Write the answer in simplest form:

a. $\dfrac{6x^3}{12x^5}$ b. $\dfrac{2x-5}{6x-15}$ c. $\dfrac{2y^2-y-3}{2y^2-5y+3}$

2. Factor the numerator and denominator. Simplify:

a. $\dfrac{3x+6}{4x+8}$ b. $\dfrac{x^2-3x-10}{x^2-7x-18}$

3. In the equation $\dfrac{8}{x-3}+\dfrac{9}{x-5}=0$, can the value of x be 3? 5? Why?

4. What is the simplest form of $\dfrac{4x^2-9}{4x^2-6x+9}$?

5. Determine the domain of $\dfrac{2x+1}{x^3-5x^2}$.

Lesson 3.2 Operations of Rational Expressions

A. Multiplication and Division

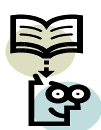
Simplifying rational expressions is basically the same as simplifying rational numbers, except that rational expressions involve not only numbers but also variables.

The same thing, the basic theorems that enabled us to perform multiplication and division of rational numbers hold for rational expressions.

Theorem:

1. If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$, where $b, d \neq 0$.
2. If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$, where b, d, c $\neq 0$.

In other words, the product of two fractions is obtained by multiplying the numerators and multiplying the denominators. If the result is not in simplest form, it must be simplified. This is true for rational expressions.



Examples:

1. Multiply: a. $\frac{4x}{3} \cdot \frac{9}{2x}$ b. $\frac{x^2+2x}{2x} \cdot \frac{4x}{x^2-4}$ c. $\frac{2x-1}{x+1} \cdot \frac{x^2-1}{1-2x}$

Solutions: Factor first any expression that is factorable. Simplify expression that can be simplified before multiplying.

a. $\frac{4x}{3} \cdot \frac{9}{2x} = 6$ common factors between the numerator and denominator reduces to 1.

b. $\frac{x^2+2x}{2x} \cdot \frac{4x}{x^2-4} = \frac{x(x+2)}{2x} \cdot \frac{4\cancel{x}}{(x+2)(x-2)} = \frac{2x}{x-2}$ why?

c. $\frac{2x-1}{x+1} \cdot \frac{x^2-1}{1-2x} = \frac{2x-1}{x+1} \cdot \frac{(x+1)(x-1)}{-(2x-1)} = \frac{x-1}{-1}$ or $1 - x$ why?

2. Divide: a. $\frac{x^2-1}{2} \div \frac{x^2-2x+1}{4x}$ b. $\frac{x^2-5x+6}{x^2-2x-3} \div \frac{x^2-4}{x^2-1}$

Solutions:

a. $\frac{x^2-1}{2} \div \frac{x^2-2x+1}{4x} = \frac{(x+1)(x-1)}{2} \cdot \frac{4x}{(x-1)(x-1)}$

$= \frac{2x(x+1)}{(x-1)}$ why?

b. $\frac{x^2-5x+6}{x^2-2x-3} \div \frac{x^2-4}{x^2-1} = \frac{(x-3)(x-2)}{(x+1)(x-3)} \cdot \frac{(x+1)(x-1)}{(x+2)(x-2)}$

$= \frac{x-1}{x+2}$ why?

3. Perform the indicated operations:

$= \frac{x^2-2x-3}{x+1} \cdot \frac{x-5}{x^2-x-6} \div \frac{x^2-6x+5}{x^2-3x-10}$

$= \frac{(x-3)(x+1)}{(x+1)} \cdot \frac{(x-5)}{(x-3)(x+2)} \cdot \frac{(x-5)(x+2)}{(x-5)(x-1)}$

$= \frac{x-5}{x-1}$ why?

Let's Practice for Mastery 3.

Answer the following:

A. 1. Which rational expression is in simplest form?

a. $\frac{x-1}{x^2-1}$ b. $\frac{x-1}{x^2-2x+1}$ c. $\frac{x+1}{x^2-1}$ d. $\frac{x+1}{x^2+1}$

2. Write the product $\frac{12y^2}{x^2+7x} \cdot \frac{x^2-49}{2y^5}$ in lowest terms.

3. Write the quotient $\frac{8m^2}{3} \div \frac{6m^3}{3m-12}$ in lowest terms.

B. Solve the following completely. Simplify the answers.

4. $\frac{(x^2-2x-8)}{(x^2-25)} \cdot \frac{(x-4)}{(x-5)} \div \frac{(x^2-4)}{(2x+10)}$

5. If an object is moving at an average rate of 18 $\frac{kilometer}{\min ute}$, what is its average rate of speed in meters per second?

(Hint: 1 km = 1000m ; 1 minute = 60 sec)

Let's Check Your Understanding 3.

Perform the indicated operations:

1. $\frac{3b}{4a} \cdot \frac{8a^2 - 4a}{9b^2}$

2. $\frac{2x+6}{8xy} \div \frac{x+3}{2y^2}$

3. $\frac{x^2+3x+2}{x^2-3x-10} \cdot \frac{x^2-6x+5}{x^2+8x+7}$

4. $\frac{x^2+6x+9}{x^2+2x-3} \div \frac{x^2-9}{x^2-x-6}$

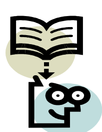
5. $\frac{(m-3)(m+3)}{(m-9)(m+10)} \cdot \frac{m^2-8m-9}{(m-3)(m-4)} \div \frac{m+4}{m+10}$

B. Addition and Subtraction

As for multiplication and division of algebraic expressions, the theorems for addition and subtraction of rational expressions are those that we have been using for addition and subtraction of rational numbers.

Theorem : If a, b, and c are any real numbers and b, d ± 0, then

a. $\frac{a}{b} \pm \frac{c}{b} = \frac{a \pm c}{b}$ b. $\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$



Examples:

1. Add: $\frac{3x}{x+2y} + \frac{4y}{x+2y}$

Solution : Since their denominators are the same, just add the numerator and copy the denominator.

$$\frac{3x}{x+2y} + \frac{4y}{x+2y} = \frac{3x+4y}{x+2y}$$

What if the denominators are not similar ? Yes, make first the denominators the same by getting the LCD.

2. Subtract: a + 1 − $\frac{1}{a-1}$.

Solution :

$$a+1-\frac{1}{a-1}=\frac{a+1}{1}-\frac{1}{a-1} \qquad \text{LCD : } a-1$$

$$=\frac{(a+1)(a-1)-1}{a-1} \qquad \text{why?}$$

$$=\frac{a^2-1-1}{a-1} \qquad \text{why ?}$$

$$=\frac{a^2-2}{a-1}$$

3. What is the LCD of $\frac{x+2}{x^2-4}-\frac{6}{x+2}+\frac{x-3}{x-2}$?

Solution:

$x + 5$	=	$x + 5$
$x^2 + 10x + 25$	=	$(x + 5)(x + 5)$
$x^2 - 25$	=	$(x + 5) \quad (x - 5)$
LCD	=	$(x+5)(x + 5)(x - 5)$

4. Perform the operations :

$$\frac{x+2}{x^2-4}-\frac{6}{x+2}+\frac{x-3}{x-2}$$

Get first the LCD : $x^2 - 4 = (x + 2)(x - 2)$

$x - 2$	=	$x - 2$
$x + 2$	=	$x + 2$
LCD	=	$(x + 2)(x - 2)$

Solution :

$$\frac{x+2}{x^2-4}-\frac{6}{x+2}+\frac{x-3}{x-2}=\frac{(x+2)-6(x-2)+(x-3)(x+2)}{(x+2)(x-2)}$$

$$=\frac{x^2-6x+8}{(x+2)(x-2)} \qquad \text{why?}$$

$$= \frac{(x-2)(x-4)}{(x+2)(x-2)}$$

$$= \frac{x-4}{x+2}$$

Let's Practice for Mastery 4.

Answer the following :

1. Add: $\dfrac{16c}{5} + \dfrac{9c-2}{5}$

2. Subtract : $\dfrac{x+1}{x} - \dfrac{x-1}{6x}$

3. Write a rational expression that, when subtracted from $\dfrac{3x-7}{2x+3}$, gives a difference of $\dfrac{x-5}{2x+3}$.

Let's Check Your Understanding 4.

1. Subtract $\dfrac{1}{x+2y}$ from the sum of $\dfrac{3x}{x+2y}$ and $\dfrac{4x}{x+2y}$.
2. Add: $\dfrac{x+2y}{2x-4} + \dfrac{3x-5y}{4-2x}$
3. Two sides of a triangle are described by $\dfrac{5x-3}{2x+6}$ *and* $\dfrac{3x}{x^2+2x-3}$. The perimeter is $\dfrac{6x^2-3x+2}{2x^2+4x-6}$. Find the third side.

Lesson 3. 3 Solving Rational Equations

A rational equation is an equation that contains one or more rational expressions.

Here are examples of rational equations:

$$\frac{x}{5} + \frac{1}{3} = \frac{3x+5}{15}, \qquad \frac{m}{9} = \frac{36}{m}, \qquad \frac{3}{a+1} = \frac{2}{a-3}, \qquad \frac{4}{b+1} = 1 + \frac{10}{w}$$

A method for solving rational equations is similar to the method you learned in solving equations with rational numbers.

Rational equations are generally easier to solve if the equation is cleared of fractions first. You can solve rational equations using the following procedure:

Solving a Rational Equation

1. Find the least common denominator (LCD) of all denominators.
2. Multiply each side of the equation by the LCD.
3. Simplify the resulting equation and solve for the variable.

Example 1. Solve: $\frac{n}{3} + \frac{n}{2} = 1$

Solution:

$\frac{n}{3} + \frac{n}{2} = 1$ The LCD = 6

$6\left[\frac{n}{3} + \frac{n}{2} = 1\right]$ multiply each side by 6

$6 \cdot \frac{n}{3} + 6 \cdot \frac{n}{2} = 6\ (1)$ distributive property

$2n + 3n = 6$ solve for n

$5n = 6$ multiplication property

$n = \frac{6}{5}$

Check: $\frac{n}{3} + \frac{n}{2} = 1$ substitute $n = \frac{6}{5}$

$$\frac{\frac{6}{5}}{3} + \frac{\frac{6}{5}}{2} \stackrel{?}{=} 1$$

$$\frac{6}{5} \cdot \frac{1}{3} + \frac{6}{5} \cdot \frac{1}{2} \stackrel{?}{=} 1$$

$$\frac{2}{5} + \frac{3}{5} \stackrel{?}{=} 1$$

$1 = 1$. Hence, the solution set is $\frac{6}{5}$.

Example 2. Solve: $\frac{5}{4} + \frac{1}{2y} = \frac{7}{6}$

Solution:

$\frac{5}{4} + \frac{1}{2y} = \frac{7}{6}$ the LCD is 12y

$12y \left[\frac{5}{4} + \frac{1}{2y} = \frac{7}{6} \right]$ multiply each side by 12y

$12y \cdot \frac{5}{4} + 12y \cdot \frac{1}{2y} = 12y \cdot \frac{7}{6}$ apply distributive property

$15y + 6 = 14y$ simplify each term

$y + 6 = 0$ solve

$y = -6$ addition property of equality

The solution set is $\{-6\}$.

Example 3. Solve: $\dfrac{4}{w+1} = 1 + \dfrac{10}{w}$

Solution: $\dfrac{4}{w+1} = 1 + \dfrac{10}{w}$ what is the LCD?

$$w(w+1) \cdot \frac{4}{w+1} = w(w+1)\left[1 + \frac{10}{w}\right] \qquad \text{why ?}$$

$$4w = w(w+1)(1) + w(w+1) \cdot \frac{10}{w} \qquad \text{why?}$$

$$4w = w^2 + w + 10w + 10 \qquad \text{why?}$$

$$4w = w^2 + 11w + 10$$

$$0 = w^2 + 7w + 10 \qquad \text{why?}$$

$$w = -2 \quad \text{or} \quad w = -5$$

Is the solution set {–5, –2}? Explain.

Example 4. Solve: $\dfrac{7}{x-4} - \dfrac{5}{x-2} = 0$

Solution: $\dfrac{7}{x-4} - \dfrac{5}{x-2} = 0$ the LCD is $(x-4)(x-2)$

$$(x-4)(x-2)\left[\frac{7}{x-4} - \frac{5}{x-2}\right] = (x-4)(x-2)(0) \qquad \text{why?}$$

$$7(x-2) - 5(x-4) = 0 \qquad \text{why?}$$

$$7x - 14 - 5x + 20 = 0 \qquad \text{why?}$$

$$2x + 6 = 0$$

$$x = -3 \qquad \text{why?}$$

Note that: $x \neq 2$ and $x \neq 4$. Is –3 the only solution? Why?

Sometimes there are values that are excluded from the domain of a rational expression. When you solve rational expressions, you must check your solutions to make sure that your answer satisfies the original equation.

Example 5. Solve: $\frac{x-4}{2(x-2)} + \frac{1}{3} = -\frac{1}{x-2}$

Solution: $\frac{x-4}{2(x-2)} + \frac{1}{3} = -\frac{1}{x-2}$

Here, $x \neq 2$. why?

$$6(x-2)\left[\frac{x-4}{2(x-2)} + \frac{1}{3}\right] = 6(x-2)\left[-\frac{1}{x-2}\right]$$ why?

What's the LCD?

$3(x-4) + 2(x-2) = -6$ why ?

$3x - 12 + 2x - 4 = -6$ why?

$5x - 16 = -6$

$5x = 10$ hence, $x = 2$ why?

Check: $\frac{x-4}{2(x-2)} + \frac{1}{3} = -\frac{1}{x-2}$

$$\frac{2-4}{2(2-2)} + \frac{1}{3} = -\frac{1}{2-2}$$

$$-\frac{2}{0} + \frac{1}{3} = -\frac{1}{0}$$ undefined? Why?

The equation has no solution because 2 makes the denominator equal to 0.

When each side of a rational equation is a single rational expression, you can solve the equation by using the property of a proportion.

Example 6. Solve: $\frac{k}{9} = \frac{36}{k}$

Solution: $\frac{k}{9} = \frac{36}{k}$ use the Property of Proportion

$k \cdot k = 9(36)$ write as cross product

$k^2 = 324$

$k = \pm 18$ use extracting square root strategy.

Verify if -18 and 18 are both solutions.

Example 7. Solve: $\dfrac{3}{b+1} = \dfrac{2}{b-3}$

Solution: 3 (b – 3) = 2 (b + 1) why?

3b – 9 = 2b + 2 why?

b = 11 why?

Check: $\dfrac{3}{b+1} = \dfrac{2}{b-3}$

$\dfrac{3}{11+1} = \dfrac{2}{11-3}$ $\qquad \dfrac{1}{4} = \dfrac{1}{4}$

Let' Practice for Mastery 5.

Solve the following.

1. $\dfrac{1}{x} = \dfrac{x-2}{24}$

2. x + 5 = $\dfrac{3}{x-5}$

3. $\dfrac{40}{x}$ -- $\dfrac{20}{x-3} = \dfrac{8}{7}$

4. Josef and Sally work together to encode a short story, and it takes them 4 hours to finish it. It would take Sally 6 hours more than Josef to encode the story alone. How long would each need to encode the story if each worked alone ?

Let's Check Your Understanding 5.

Solve each equation. What value of x will make the equation undefined?

1. $\dfrac{7}{x-4}=\dfrac{5}{x-2}$

2. $\dfrac{2x}{x-4}=\dfrac{4}{x+5}=2$

3. Two shirts and one jacket cost the same as two sweaters. One sweater and two shirts cost the same as one jacket. Which cost the least – a shirt, a jacket, or a sweater? Explain.

Lesson 3.4. Applications of Rational Equations

Many real life situations translate to rational equations. Sometimes, the rational equation is actually quadratic. If clearing the fractions produces a quadratic equation, solve that resulting equation in the usual way. Remember that possible solutions to the rational equations should be checked with the original equation.

In some problems, it is necessary to find how long it will take to complete a job when work is done at a uniform rate.

Example 1. One copy machine can complete a job in 25 minutes. This machine and a newer machine working together can complete the same job in 10 minutes. How long would it take the newer machine to complete the job working by itself?

Solution: Let m be the time in minutes for the newer machine to do the job alone.

$\frac{1}{25}$ - Part of the job done by the first machine in 1 min.

$\frac{1}{m}$ - Part of the job done by the newer machine in 1 min.

$\frac{1}{10}$ - Part of the job done by two machines in 1 min.

Step 1. Translate the situation into an equation

$$\frac{1}{25}+\frac{1}{m}=\frac{1}{10}$$

Step 2. Solve $\frac{1}{25}+\frac{1}{m}=\frac{1}{10}$ what is the LCD?

$$50\text{m}\left(\frac{1}{25}+\frac{1}{m}\right)=50\text{m}\left(\frac{1}{10}\right) \qquad \text{why?}$$

$$50\text{m}\left(\frac{1}{25}\right)+50\text{m}\left(\frac{1}{m}\right)=50\text{m}\left(\frac{1}{10}\right)$$

$$2\text{m} + 50 = 5\text{m} \qquad \text{why?}$$

$$50 = 3\text{m}$$

$$\text{m} = \frac{50}{3} \text{ or } 16\ \frac{2}{3}$$

Working alone, the newer machine can do the job in 16 $\frac{2}{3}$ minutes.

Example 2. When two – thirds is subtracted from the quotient of a number n and three, the result is equal to the ratio of one to n. What is n?

Solution:

Step 1. Translate the situation into an equation.

$$\frac{n}{3}-\frac{2}{3}=\frac{1}{n}$$

Step 2. Solve

$$\frac{n}{3} - \frac{2}{3} = \frac{1}{n}$$

$$\frac{n-2}{3} = \frac{1}{n} \qquad \text{why?}$$

$$(n-2)n = 3 \qquad \text{why?}$$

$$n^2 - 2n = 3$$

$$n^2 - 2n - 3 = 0 \qquad \text{why?}$$

$$(n - 3)(n + 1) = 0 \qquad \text{why?}$$

n = 3 or n = -1 Check if they are both solutions.

Example 3. One positive number is three times another. The difference of their reciprocals is $\frac{1}{6}$. Find the numbers.

Solution:

Let a be the smaller positive number

and 3a be the larger number

Larger number minus smaller number is $\frac{1}{6}$

$$\frac{1}{a} - \frac{1}{3a} = \frac{1}{6} \qquad \text{why?}$$

$$6a\left(\frac{1}{a} - \frac{1}{3a}\right) = 6a\left(\frac{1}{6}\right)$$

$$6 - 2 = a$$

$$a = 4$$

$$3a = 3(4)$$

$$= 12$$

Check: $\frac{1}{4} - \frac{1}{12} = \frac{1}{6}$

$$3 - \frac{1}{12} = \frac{1}{6}$$

$$\frac{1}{6} = \frac{1}{6}$$

Therefore, the numbers are 4 and 12.

Example 4. David spends 5 hours traveling to and from work each day. If his average rate of driving to work is 18 miles per hour and his average rate returning is 12 miles per hour how far is it from his home to his place of work?

Solution:

Let t = time going to work

5 – t = time returning

Organize the given information in a table

	Rate (r)	Time (t)	distance
Going	18mph	t	18t
Returning	12mph	5 - t	12 (5 –t)

Distance going = distance returning

Translate: $18t = 12(5 - t)$

Solve: $18t = 60 - 12t$

$30t = 600$

$t = 2$

Check: the distance from home to work is 2 · 18 = 36 miles.

Let's Practice for Mastery 6.

Solve the following problems

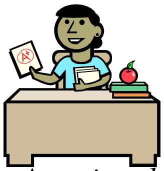
1. Miriam drove 400 km in the same amount of time that it took Ruth to drive 320 km. Miriam drive 15 km per hour faster than Ruth. Find the speed of each.
2. The sum of two fractions is $\frac{11}{12}$. one fraction is $\frac{3}{8}$ of the other. Find the fractions.

3. A band has 112 members. When they are arranged in a rectangular formation, the number of players in each row is 6 less than the number of rows. How many players are there in each row?
4. What number must be added to the numerator and subtracted to the denominator of $\frac{13}{38}$ to make a fraction equal to $\frac{7}{10}$?

Let's Check Your Understanding 6.

Solve the following:

1. Raul is one-third as old as his brother. In six years Raul will be one-half as old. What are their present ages?
2. Robert's motorcycle traveled 300 miles at a certain speed. He had gone 10 mph faster; the trip would take 1 hour less. Find the speed of the motorcycle.
3. What number must be subtracted from both the numerator and the denominator of the fraction 9/13 to make a fraction equal to $\frac{1}{3}$?
4. The 5 short pieces of chain shown below must be linked together to form one long chain. Explain how the pieces can be linked together by cutting only three of the rings.



Let's Summarize

1. A *rational expression* is an expression of the form $\frac{p}{q}$, where p and q are polynomials and q $\neq$ 0.
2. A rational expression is in simplest form when the numerator and the denominator of the expression have no common factors other than 1 and – 1.
3. Simplifying a Rational Expression

    1. Factor both the numerator and denominator if necessary.
    2. Divide both numerator and denominator by their greatest common factor.
4. Solving a Rational Equation
    a. Find the least common denominator (LCD) of all denominators.
    b. Multiply each side of the equation by the LCD.
    c. Simplify the resulting equation and solve for the variable.

Unit Test

I – Multiple Choice: On your answer sheet, write the letter of the correct answer

1. What is the simplest form of the expression $\dfrac{4(a+2b)^2}{4(a+2b)}$?

   a. 2   c. a + 2b

   b. a – 2b   d. 1

2. What number cannot be a value for x in $\dfrac{x^3-125}{x-5}$?

   a. – 5   c. 25

   b. 5   d. – 25

3. The number of single-cell organisms in a jar doubles every hour. The jar was full at exactly 6:00 PM. When was the jar half-full?

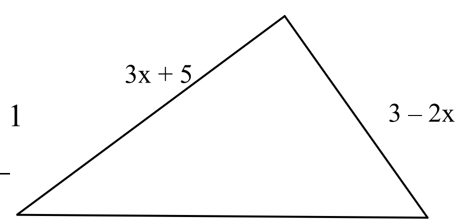
   a. 6:00 AM   c. 9:00 AM

   b. 4:00 PM   d. 5:00 PM

4. Find the perimeter of the given polygon:

   a. 9x + 1   c. 5x – 1

   b. 5x + 1   d. 9x –



1

2. To solve equations containing rational expressions, what must be multiplied by each side of the equation?

   a. least common denominator   c. greatest common factor

   b. least common factor   d. common integral factor

II – Answer the following. Show all work. (2 points each)

1. Solve and check : $\frac{6}{x-2} - \frac{4}{x-1}$

2. What number added to both numerator and denominator of the fraction $\frac{3}{7}$ results in a fraction equal to $\frac{3}{5}$?

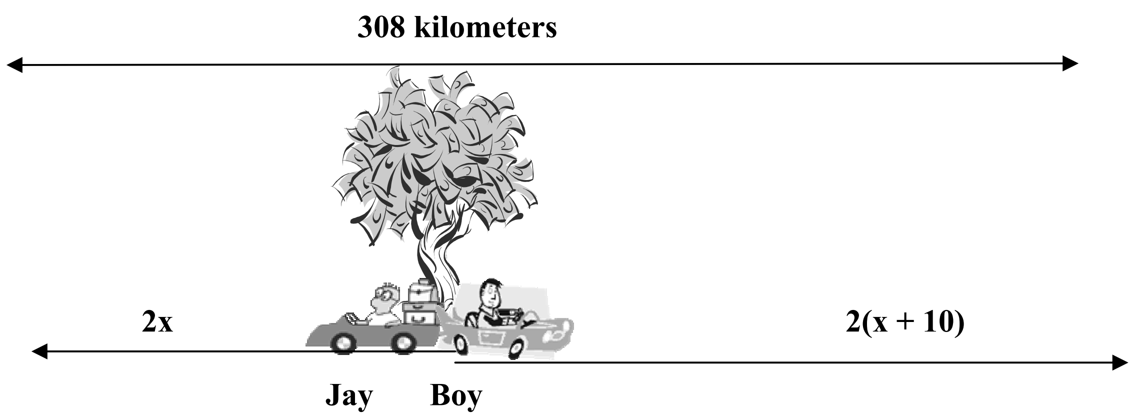
3. If $\frac{x}{8} = \frac{32}{x}$, can the value of x be 0? Explain.

4. How many values of x satisfy the rational equation given in number 3? Solve and explain your answers. (4 points)

III – Solve the following. Show all work.

1. The Mathematics Club of a certain high school is holding their annual car wash to raise fund for their projects. Ailene can wash and wax one car in 3 hours. Aljames can wash and wax one car in 4 hours. If they will work together, how long will it take them to wash and wax one car?

2. Boy and Jay leave the same point driving in opposite directions. Traffic conditions enable Boy to average 10 km/hr more than Jay. After two hours they are 308 km apart. Find the rate of each.



3. A certain company owns two electronic mail processors. The newer machine works three times as fast as the older one. Together, the two machines process 1000 pieces of mail in 25 minutes. How long does it take each machine, working alone, to process 1000 pieces of mail?

Answer Keys

Let's Practice for Mastery 1.

1. a

2. a. yes, both numerator and denominator are polynomials.

   b. yes, both numerator and denominator are polynomials.

   c. no, the denominator is not a polynomial.

3. $x^2 - 1 = 0$

   $x^2 = 1$ so, x cannot take the value

   $x = \pm 1$ -1 or 1, since when -1 or 1 is substituted to the denominator it will be 0.

Let's Check Your Understanding 1.

1. a. It is an algebraic expression because both the numerator and denominator are Polynomial

   b. both numerator and denominator are polynomial

2. a. set of real numbers except -2.

   b. set of real numbers except 2.

3. $\{0, -2, -5, 9\}$. If each is substituted to the variable x in $\frac{x+1}{x-7}$, the value is a real Number

Let's Practice for Mastery 2.

1. x + 1

2. a. $\frac{6mn}{6m} = \frac{6m}{6m} \cdot \frac{n}{1}$ factoring out a common factor

   $= n, \text{since } \frac{6m}{6m} = 1$

   b. $\frac{5x^3y + 15xy}{25} = \frac{5(x^3y + 3xy)}{5(5)}$ factoring

   $= \frac{x^3y + 3xy}{5}$, since $\frac{5}{5} = 1$

3. $\dfrac{3x^2+4x+1}{x^2-1}=\dfrac{(3x+1)(x+1)}{(x+1)(x-1)}=\dfrac{x+1}{x+1}\cdot\dfrac{3x+1}{x-1}=\dfrac{3x+1}{x-1}, \sin ce \dfrac{x+1}{x+1}=1$

4. a. $\dfrac{x^2-7x+10}{x^2+3x-10}=\dfrac{(x-2)(x-5)}{(x+5)(x-2)}=\dfrac{x-2}{x-2}\cdot\dfrac{x-5}{x+5}=\dfrac{x-5}{x+5}$

b. $\dfrac{x^2+5x+4}{x^2+x}=\dfrac{(x+4)(x+1)}{x(x+1)}=\dfrac{x+1}{x+1}\cdot\dfrac{x+4}{x}=\dfrac{x+4}{x}$

Let's Check Your Understanding 2.

1. a. $\dfrac{6x^3}{12x^5}=\dfrac{2(3x^3)}{4x^2(3x^3)} \quad = \quad \dfrac{1}{2x^2}$

b. $\dfrac{2x-5}{6x-15}=\dfrac{2x-5}{3(2x-5)} \quad = \quad \dfrac{1}{3}$

c. $\dfrac{2y^2-y-3}{2y^2-5y+3}=\dfrac{(2y-3)(y+1)}{x(x+1)} \quad = \quad \dfrac{2y-3}{2y-3}\cdot\dfrac{y+1}{y-1}=\dfrac{y+1}{y-1}$

2. a. $\dfrac{3x+6}{4x+8}=\dfrac{3(x+2)}{4(x+2)} \quad = \dfrac{3}{4}$

b. $\dfrac{x^2-3x-10}{x^2-7x-18}=\dfrac{(x-5)(x+2)}{(x-9)(x+2)} \quad =\dfrac{x-5}{x-9}$

3. No, If x = 3or x = 5, the fraction becomes undefined.

4. $\dfrac{4x^2-9}{4x^2-6x+9}=\dfrac{(2x+3)(2x-3)}{(2x-3)(2x-3)}$

$=\dfrac{2x+3}{2x-3}$

5. $\dfrac{2x+1}{x^2(x-5)}$. The domain is the set of real numbers except 0 and 5.

Let's Practice for Mastery 3.

A. 1. d

2. $\frac{12y^2}{x^2+7x}\cdot\frac{x^2-49}{2y^5}=\frac{2y^2\cdot 6}{x(x+7)}\cdot\frac{(x+7)(x-7)}{2y^2\cdot y^3}$

$$=\frac{6(x-7)}{xy^3}$$

3. $\frac{8m^2}{3}\div\frac{6m^3}{3m-12}=\frac{2m^2\cdot 4}{3}\cdot\frac{3(m-4)}{2m^2\cdot m}$

$$=\frac{4(m-4)}{m}$$

B.

4.
$$\frac{(x^2-2x-8)}{(x^2-25)}\cdot\frac{(x-4)}{x-5}\div\frac{(x^2-4)}{2x+10}=$$
$$\frac{(x-4)(x+2)}{(x+5)(x-5)}\cdot\frac{(x-4)}{x-5}\cdot\frac{2(x+5)}{(x+2)(x-2)}=\frac{2(x-4)^2}{(x-5)^2(x-2)}$$

5. $\frac{18km}{\min}=\frac{18km}{\min}\cdot\frac{1000m}{1km}\cdot\frac{1\min}{60\sec}$

$$=\frac{18\cdot 1000m}{60\sec}$$

$$=300\frac{m}{\sec}$$

Let's Check Your Understanding 3

1. $\frac{3b}{4a}\cdot\frac{8a^2-4a}{9b^2}=\frac{3b}{4a}\cdot\frac{4a(2a-1)}{(3b)(3b)}=\frac{2a-1}{3b}$

2. $\frac{2x+6}{8xy}\div\frac{x+3}{2y^2}=\frac{2(x+3)}{2\cdot 2y\cdot 2x}\cdot\frac{2y\cdot y}{(x+3)}=\frac{y}{2x}$

3. $\frac{x^2+3x+2}{x^2-3x-10}\cdot\frac{x^2-6x+5}{x^2+8x+7}=\frac{(x+2)(x+1)}{(x-5)(x+2)}\cdot\frac{(x-5)(x-1)}{(x+7)(x+1)}$

$$=\frac{x-1}{x+7}$$

4. $\frac{x^2+6x+9}{x^2+2x-3}\div\frac{x^2-9}{x^2-x-6}=\frac{(x+3)(x+3)}{(x+3)(x-1)}\cdot\frac{(x-3)(x+2}{(x+3)(x-3)}$

$$=\frac{x+2}{x-1}$$

5. $\frac{(m-3)(m+3)}{(m-9)(m+10)} \cdot \frac{m^2-8m-9}{(m-3)(m-4)} \div \frac{m+4}{m+10} =$

$$\frac{(m-3)(m+3)}{(m-9)(m+10)} \cdot \frac{(m-9)(m+1)}{(m-3)(m-4)} \cdot \frac{m+10}{m+4} = \frac{(m+3)(m+1)}{(m-4)(m+4)}$$

Let's Practice for Mastery 4.

1. $\frac{16c}{5} + \frac{9c-2}{5} = \frac{25c-2}{5}$

2. $\frac{x+1}{x} - \frac{x-1}{6x} = \frac{6(x+1)-(x-1)}{6x} = \frac{6x+6-x+1}{6x} = \frac{5x+7}{6x}$

3. $\frac{3x-7}{2x+3} - \frac{x-5}{2x+3} = \frac{3x-7-x+5}{2x+3} = \frac{2x-2}{2x+3}$

Let's Check Your Understanding 4

1. $\frac{3x}{x+2y} + \frac{4x}{x+2y} = \frac{7x}{x+2y}$

$$\frac{7x}{x+2y} - \frac{1}{x+2y} = \frac{7x-1}{x+2y}$$

2. $\frac{x+2y}{2x-4} + \frac{3x-5y}{4-2x} = \frac{x+2y}{2x-4} + \frac{3x-5y}{-(2x-4)}$

$$= \frac{x+2y}{2x-4} - \frac{3x-5y}{2x-4}$$

$$= \frac{x+2y-3x+5y}{2x-4} = \frac{-2x+7}{2x-4}$$

3. a. $\frac{5x-3}{2x+6} + \frac{3x}{x^2+2x-3} = \frac{5x-3}{2(x+3)} + \frac{3x}{(x+3)(x-1)}$

$$= \frac{(5x-3)(x-1)+6x}{2(x+3)(x-1)} = \frac{5x^2-8x+3+6x}{2(x+3)(x-1)}$$

$$= \frac{5x^2-2x+3}{2(x+3)(x-1)}$$

b. $\frac{6x^2-3x+2}{2x^2+4x-6} - \frac{5x^2-2x+3}{2(x+3)(x-1)} = \frac{6x^2-3x+2-5x^2+2x-3}{2(x-1)(x+3)}$

$$= \frac{x^2 - x - 1}{2x^2 + 4x - 6}$$

Let's Practice for Mastery 5.

1. $\frac{1}{x} = \frac{x-2}{24}$

x (x – 2) = 24

x^2 -2x -24 = 0

(x – 6) (x + 4) = 0

x = 6 or x = -4

2. $\frac{x+5}{1} = \frac{3}{x-5}$

(x + 5) (x – 5) = 3

x^2 – 25 = 3

x^2 = 25 + 3

x^2 = 28

$x = \pm\sqrt{28}$

$x = \pm\sqrt{4 \cdot 7}$

$x = \pm 2\sqrt{7}$

3. $\frac{40}{x} - \frac{20}{x-3} = \frac{8}{7}$

$$\frac{40(x-3) - 20(x)}{x(x-3)} = \frac{8}{7}$$

$$\frac{40x - 120 - 20x}{x(x-3)} = \frac{8}{7}$$

$7(20x - 120) = 8(x^2 - 3x)$

$140x - 840 = 8x^2 - 24x$

$8x^2 - 164x + 840 = 0$

$2x^2$ – 41x + 210 = 0

(2x – 21) (x – 10) = 0

2x - 21 = 0 or x – 10 = 0

x = $\frac{21}{2}$ or $x = 10$

4. Let x = number of hours for Josef to type the story alone.

x + 6 = number of hours Sally takes to type the story alone.

Solution: $4\left(\frac{1}{x}\right) + 4\left(\frac{1}{x+6}\right) = 1$

$$x(x+6)\left[4\left(\frac{1}{x}\right) + 4\left(\frac{1}{x+6}\right)\right] = x(x+6) \cdot 1$$

4 (x + 6) + 4x = x (x + 6)

4x + 24 + 4x = x^2 + 6x

x^2 – 2x – 24 = 0

(x – 6) (x + 4)

x = 6 or x = -4

Since there are no negative hours, -4 is not a solution. Therefore, by substitution, Josef can type in 6 hours and Sally can type in 12 hours.

Let's Check Your Understanding 5.

1. $\dfrac{7}{x-4} - \dfrac{5}{x-2}$

$2(x-2) = 5(x-4)$

$7x - 14 = 5x - 20$

$2x = -6$

$x = -3$

2. $\dfrac{2x}{x-4} - \dfrac{4}{x+5} = 2$

$\dfrac{2x(x+5) - 4(x-4)}{(x-4)(x+5)} = 2$

$\dfrac{2x^2 + 10x - 4x + 16}{x^2 + x - 20} = 2$

$2x^2 + 6x + 16 = 2x^2 + 2x - 40$

$4x = -56$ or $x = -14$

Let's Practice for Mastery 6.

1. Since d = rt d_1 = 400 km r_1 = x + 15; Miriam

Then t = $\dfrac{d}{r}$ d_2 = 320 km r_2 = x; Ruth

$\dfrac{d_1}{r_1} = \dfrac{d_2}{r_2} \quad = \quad \dfrac{400}{x+15} = \dfrac{320}{x}$

400x = 320x + 4800

80x = 4800

x = 60 – rate of Ruth

x + 15 = 75 - rate of Miriam

2. $\dfrac{11}{12} - \dfrac{3}{8} = \dfrac{22-9}{24} = \dfrac{13}{24}$

3. Let x = number of rows

x – 6 = number of players in a row

x (x – 6) = 112

x^2 – 6x -112 = 0

(x – 14) (x + 8) = 0

x = -8 there is no negative number for this problem. Therefore, x = -8 is not a solution.

x = 14 number of rows

x – 6 = 8 number of players per row.

4. Let x be the number

$$\frac{13+x}{38-x}=\frac{7}{10}$$

$$10(13+x)=7(38-x)$$

130+ 10x = 266−7x

17x = 266−130

17x = 136

x = 8

Let's Check Your Understanding 6.

1. let x = the present age of the brother

   1/3x = the present age of Raul

   x+6 = the age of the brother 6 years hence.

   1/3 + 6 = Raul's age 6 years hence.

   1/3 + 6 = ½ (x + 6) the required equation

   LCD = 6

   $$\left[\frac{x}{3}+6=\frac{x+6}{2}\right]6$$

   2x +36 = 3x +18

   2x – 3x = 18 – 36

   −x= −18

   x = 18 the present age of the brother

   $\frac{18}{3}=6$ the present age of Raul.

2.

Distance	Speed	Time
300	r	t
300	r + 10	T − 10

Equation 1. $r=\frac{300}{t}$ d = rt or $r=\frac{d}{t}$

Equation 2. $r+10=\frac{300}{t-1}$ (Substituting $\frac{300}{t}$ for r in 2.)

$$\frac{300}{t}+10=\frac{300}{t-1} \qquad t(t-1)\left[\frac{300}{t}+10\right]=t(t-1)\cdot\frac{300}{t-1} \qquad \text{(Multiply by LCD)}$$

$$300(t-1)+10(t^2-t)=300t$$

$$10t^2-10t-300=0$$
$$t^2-t-30=0$$
$$(t-6)(t+5)=0$$
$$t=6; t=-5$$

Therefore, t=6

$$r=\frac{300}{6}=50mph.$$

3. Let x = be the number to be subtracted from the numerator to the denominator

$$\frac{9-x}{13-x}=\frac{1}{3}$$

3 (9 – x) = 13 – x

27 -3x = 13 – x

-2x = 13 – 27

-2x = -14

x = 7

4. Cut each ring of the first chain, separate the rings. Insert one ring between each of the other four chains.

Unit Test

I. 1. c 2. b 3. d 4. b 5. a

II. 1. $\frac{6}{x-2} - \frac{4}{x-1} = 0 \rightarrow \frac{6(x-1)-4(x-2)}{(x-2)(x-1)} = 0$

6x – 6 – 4x + 8 = 0

2x = –2

x = –1

check: $\dfrac{6}{-1-2} - \dfrac{4}{-1-1} = 0$

$-2 - (-2) = 0$

$0 = 0$

2. Let x be the number to be added : $\dfrac{3+x}{7+x} = \dfrac{3}{5}$; Add x to both numerator and denominator

$5(3+x) = 3(7+x)$	Property of ratio and proportion
$15 + 5x = 21 + 3x$	distributive property
$2x = 6$	addition property
$x = 3$	multiplication property

check: $\dfrac{3+3}{7+3} = \dfrac{3}{5}$

$\dfrac{3}{5} = \dfrac{3}{5}$

3. For $\dfrac{x}{8} = \dfrac{32}{x}$, x can not be zero. Using the property of a proportion, 0 cannot be equal to 8(32).

4. Using the property of a proportion, $x^2 = 32(8)$

$x^2 = 256$

$x = \sqrt{256}$

x can only be equal to 16 or –16.

III. 1. $\dfrac{x}{3} + \dfrac{x}{4} = 1$

$\dfrac{4x+3x}{12} = 1$

$\dfrac{7x}{12} = 1$

$7x = 12$

$x = \frac{12}{7}$ hours, hence, Ailene and Al James can finish the work in $\frac{12}{7}$ hours together.

2. Let x = rate of Jay

$x + 10$ = rate of Boy

distance traveled by Jay = $2x$

distance traveled by Boy = $2(x + 10) = 2x + 20$

distance traveled by Jay + distance traveled by Boy = 308 km

$$2x + 2x + 20 = 308$$

$$4x = 308 - 20$$

$$4x = 288$$

$$x = \frac{288}{4}$$

x = 72 km per hour → rate of Jay

72 + 10 = 82 km per hour → rate of Boy

3. Let x = be the time for the new machine to finish the work alone.

3x = be the time for the old machine to finish the work alone.

$$\frac{1}{x} + \frac{1}{3x} = \frac{1}{25}$$

$$\frac{3+1}{3x} = \frac{1}{25}$$

$100 = 3x$ by the property of proportion

$x = \frac{100}{3}$ min or 33.3 min

Common Errors

1. Common factor of terms like $6ab^2$, $4a^2b$ and $8a^2b^2$ is mistakenly taken as $2a^2b^2$, instead of 2 ab.

2. Factors of $8x^3y - 12x^2y^2 + 4xy$ are mistakenly obtained as $4xy(2x^2 - 3xy)$ thinking that the last term becomes zero. To avoid this error always check the obtained factor by multiplying it back to obtain the original expression . The factors should be $4xy(2x^2 - 3xy + 1)$ which when multiplied by distributive property gives back the original $8x^3y - 12x^2y^2 + 4xy$.

3. LCD of $\frac{2}{x+1}$ and $\frac{3}{x^2-1}$ is $(x^2 - 1)(x + 1)$ thinking that LCD is the product of the denominators. But since $x^2 - 1 = (x+1 (x - 1)$, its LCD = $x^2 - 1$.

4. $\frac{3x+8}{x+16} = \frac{3x+1}{x+2}$ thinking that 8 and 16 may be reduced to $\frac{1}{2}$.

   But $\frac{3x+8}{x+16}$ is already in simplest form.

5. $12y\left[\frac{5}{4} + \frac{1}{3y} = 5\right]$ by distributive property

   $15y + 4 = 5$, forgot to multiply 12 y by 5.