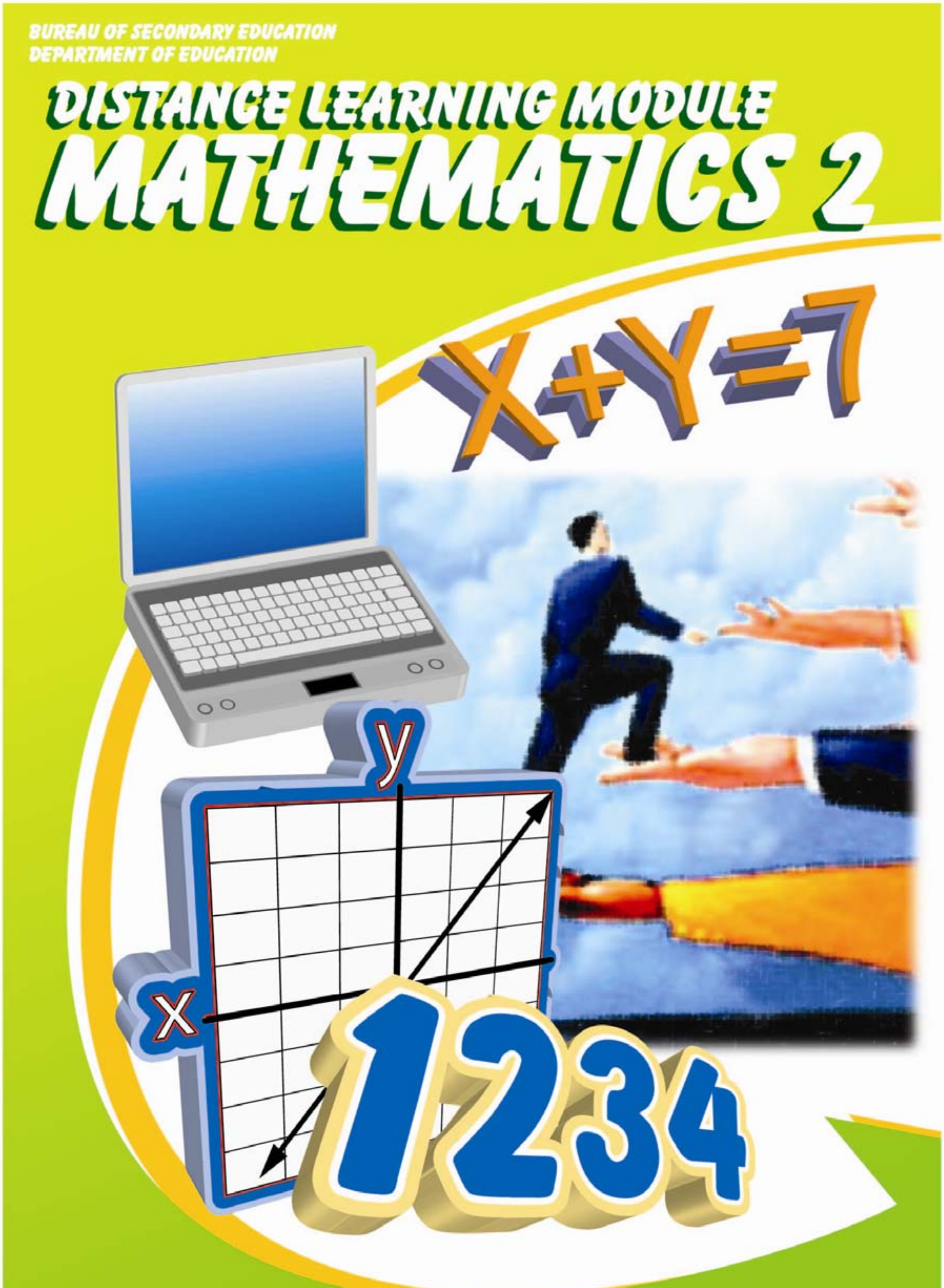

BUREAU OF SECONDARY EDUCATION
DEPARTMENT OF EDUCATION
DISTANCE LEARNING MODULE
MATHEMATICS 2
X+Y=7
y
x
1234

QUADRATIC EQUATIONS

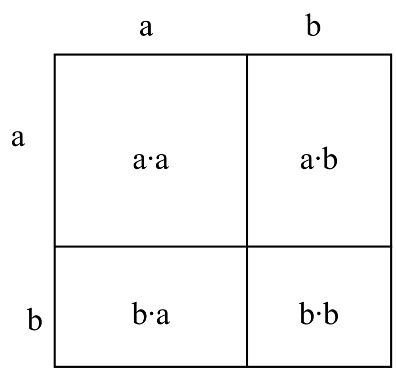
UNIT II

Mark and Rj are members of the baseball team who will compete for the championship games. Every morning they practice for about one hour. They usually do this by throwing the ball to each other as shown.

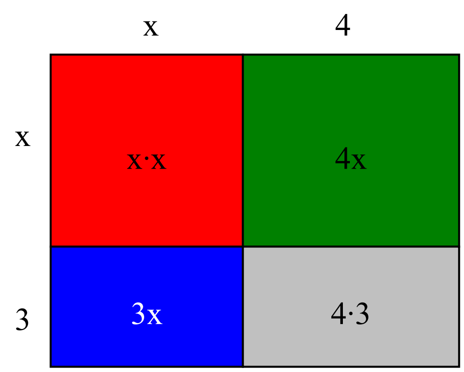
What have you noticed about the path made by the ball? How do you determine the maximum height reached by the ball? If both of them will release the ball at the same time, at what point, if ever, will the balls meet? When will the ball reach the ground?

This unit will help you answer the above questions and many more. Also, you will learn what a quadratic equation is and how its solution set is solved. You will also learn the different techniques of solving a quadratic equation. Have you ever asked yourself of the importance of this topic in real life? Of course, this topic will give you the extension of its applications to problem solving. Word and number problems, geometry, motion, interest, etc. are some of the applications of quadratic equations that you will study here.

Lesson 2.1 Special Products



$$(a+b)^2 = a^2 + 2ab + b^2$$



$$(x+3)(x+4) = x^2 + 3x + 4x + 12$$

A geometric model can help you recall the pattern for the square of a sum; or the product of two binomials.

In multiplying two binomials, you may find it helpful to use the FOIL method, which is a memory device for using the distributive property. Look at these examples:

Examples:

1. Use the FOIL method to multiply…

a. (x+3)(2x-5)

Solution:

(x+3) (2x-5)

F O I L

x(2x) + x(-5) + 3(2x) + 3(-5)

$2x^2$ + (-5x) + 6x + (– 15)

$2x^2$ + x – 15

b. (-2a+5)(3a-4)

Solution:

(-2a+5) (3a-4)

F O I L

(-2a)(3a) + (-2a)(-4) + 5(3a)+ 5(-4)

– $6a^2$ + 8a + 15a + (-20)

– $6a^2$ + 23a – 20

From the above example, follow the following steps:

1. Multiply the first terms of each binomial;
2. Multiply the outer terms;
3. Multiply the inner terms;
4. Get the middle term by adding the outer and inner terms;
5. Multiply the last terms of each binomial;
6. Combine the products of the first, middle and last terms.

Some products of binomials are used so frequently that they are given special names. One of these is the product that results when a binomial is squared. Look for the patterns in the following simplifications.

$(b+5)^2 = (b+5)(b+5)$
$= b^2+5b+5b+5^2$
$= b^2+2(5b)+5^2$
$= b^2+10b+25$

$(d-4)^2 = (d-4)(d-4)$
$= d^2+(-4d)+(-4d)+(-4)^2$
$= d^2+2(-4d)+(-4)^2$
$= d^2 - 8d + 16$

In each case, what have you noticed about the middle term in the product? What about the first term in the product? the last term? Notice that each product has three terms, thus, each is a trinomial. Since the trinomial results from squaring a binomial, it is called a *perfect square trinomial***.** These patterns are generalized as follows:

Squares of Sums and Differences

For any real numbers a and b;

$(a+b)^2 = a^2 + 2ab + b^2$ $\quad$ $(a-b)^2 = a^2 - 2ab + b^2$

2. Simplify each expression.

a. $(m + 8)^2$

Solution:

$(a + b)^2 = a^2 + 2ab + b^2$ use the square of a sum pattern

$(m + 8)^2 = m^2 + 2(m)(8) + 8^2$ by substitution

$= m^2 + 16m + 64$ simplifying

b. $(x-9)^2$

Solution:

$(a – b)^2 = a^2 – 2ab+b^2$ use the square of a difference pattern

$(x-9)^2 = x^2–2(x)(9)+9^2$ by substitution

$= x^2–18x+ 81$ simplifying

c. $(3w^2+7)^2$

Solution:

$(a+b)^2 = a^2+2ab+b^2$ use the square of a sum pattern

$(3w^2+7)^2 = (3w^2)^2+2(3w^2)(7)+7^2$ substituting

$= 9w^4+42w^2+49$ simplifying

d. $(5y-3z)^2$

Solution:

$(a – b)^2 = a^2–2ab+b^2$ use the square of a difference pattern

$(5y – 3z)^2 = (5y)^2 –2(5y)(3z)+(3z)^2$ substituting

$= 25y^2 – 30y^2+9z^2$ simplifying

Another pattern emerges when you multiply the sum of two terms by the difference of the same two terms. Study these two simplifications.

(x+9)(x–9)	(7a+2b)(7a–2b)
F O I L	F O I L
$= x^2 + (-9x) + 9x + 9(-9)$	$=(7a)^2 + 7a(-2b) + 2b(7a) + (2b)(-2b)$
$= x^2 + 0x + (-81)$	$= 49a^2 + (-14ab) + 14ab + (-4b^2)$
$= x^2 - 81$	$= 49a^2 + 0ab + (-4b^2)$
	$= 49a^2 - 4b^2$

In each case, the "outer" and "inner" terms of the FOIL multiplication are additive inverses, and so their sum is zero. The resulting expression is the square of the first term minus the square of the second term. This pattern is called the *difference of two squares* and can be generalized as follows:

> Difference of Two Squares:
>
> For any real numbers a and b,
>
> $$(a + b)(a - b) = a^2 - b^2$$

Example 3. Simplify each expression

a. $(5x+4y)(5x-4y)$

Solution:

$(a+b)(a-b) = a^2 - b^2$

$(5x+4y)(5x-4y) = (5x)^2 - (4y)^2$ a. Square the first term.

$= 25x^2 - 16y^2$ b. Square the second term.

c. Combine the 2 products

b. $(3 - 4m)(3 + 4m)$

Solution:

$(a - b)(a + b) = a^2 - b^2$

$(3 - 4m)(3 + 4m) = (3)^2 - (4m)^2$

$= 9 - 16m^2$

Let's Practice for Mastery 1.

A. Answer the following:

1. The expression $(r-6)^2$ is equivalent to?

   a. $r^2 + 36$ c. $r^2 + 12r + 36$

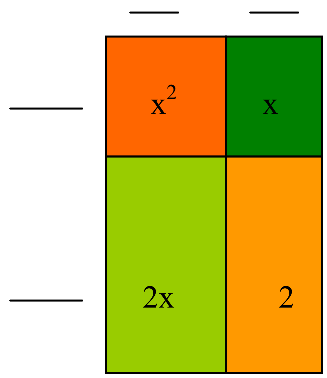
   b. $r^2 - 36$ d. $r^2 - 12r + 36$

2. What is the product of (3x+11) and(3x-11)?

   a. $9x^2 - 121$ c. $9x^2 + 22x - 121$

   b. $19x^2 + 121$ d. $9x^2 - 22x + 121$

3. Fill in the missing values on the edges of each rectangle diagram



B. Simplify each expression :

4. $(7y + 4x)(3y - 2x)$
5. $(3m + 2n)(3m - 2n)$
6. $(5x + 2y)^2$
7. $(8p - 7q)^2$
8. $(.5x + .2y)(.5x - .2y)$

C. Answer the following:

9. Ailene says that $(a + b)^2$ and $a^2 + b^2$ are equivalent expressions. Do you agree or disagree? Explain.
10. Ramon says $(x - y)^2$ and $(y - x)^2$ are equivalent expressions. Do you agree or disagree? Explain.

Let's Check Your Understanding 1.

Answer the following :

I. Simplify

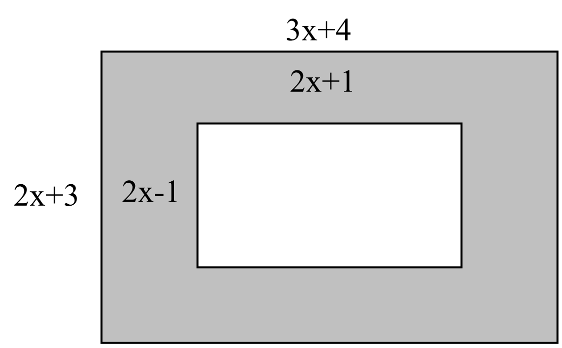
1. $(x + 10)^2$
2. $(3w - 4)^2$
3. $(7x + y)(7x - y)$
4. $(2x + 7)(2x + 7)$
5. $(2a - b)(2a + b)$
6. $(.3m^2+2)(.3m^2-2)$

II. Solve

7. One side of a square is (3x+2) cm long. Find its area in cm^2.
8. What is the area of a rectangular garden with length (7x-2) cm and width (x+5) cm?

For numbers 9-10, refer to the figure below.

9. What is the area of the big rectangle in terms of x? small rectangle?
10. What is the area of the shaded region in terms of x? Explain your answer.



Lesson 2.2 Special Factoring Techniques

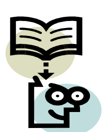
Some special binomials and trinomials can be factored as the product of two binomials. Let's recall the special product pattern $a^2 - b^2$, which is the result when the sum of two terms is multiplied by the difference of the same two terms. In other words, when the two binomials have the form (a + b) and (a − b), you can easily get the product as $a^2 - b^2$ which is the difference of 2 perfect squares. For example, $(x + 5)(x - 5) = x^2 - 25$. Therefore, whenever you encounter a binomial that has the form $a^2 - b^2$, you can do the reverse process where in the given terms are both perfect squares.

Say,

$$x^2 - 25 = (x)^2 - (5)^2$$
$$= (x + 5)(x - 5)$$

This pattern can be generalized as follows:

> A binomial that is the difference between two squares, $a^2 - b^2$, for any real numbers, a and b, can be factored as the product of the sum (a + b) and the difference (a − b) of the terms that are being squared:
>
> $$a^2 - b^2 = (a + b)(a - b)$$



Examples :

1. Factor the following:

a. $m^2 - 49$

Solution :

$a^2 - b^2 = (a + b)(a - b)$ use difference of two squares pattern

$m^2 - 49 = (m)^2 - (7)^2$ substituting

$= (m + 7)(m - 7)$

b. $4a^2 - 9b^2$

Solution :

$a^2 - b^2 = (a + b)(a - b)$ use difference of two squares pattern

$4a^2 - 9b^2 = (2a)^2 - (3b)^2$ substituting

$= (2a + 3b)(2a - 3b)$

c. Factor $0.16y^4 - .09$

Solution :

$a^2 - b^2 = (a + b)(a - b)$ use difference of two squares pattern

$0.16y^4 - .09 = (0.4y^2)^2 - (0.3)^2$ substituting

$= (0.4y^2 + 0.3)(0.4y^2 - 0.3)$

Some polynomials can be factored using patterns of special products. Look for the patterns in the following reversed process.

$$x^2 + 6x + 9 = (x)^2 + 2(3x) + (3)^2$$
$$= (x + 3)(x + 3)$$
$$= (x + 3)^2$$

$$x^2 - 8x + 16 = x^2 + 2(-4x) + (-4)^2$$
$$= (x - 4)(x - 4)$$
$$= (x - 4)^2$$

In each case, what have you noticed about the middle term in relation to the first term and the last term? Because the first term and the last term of the trinomial are perfect squares, it is a perfect square trinomial. The process of getting the factors can be generalized as follows:

Factoring special products:

For any real numbers a and b:

Square of a sum

$a^2 + 2ab + b^2 = (a + b)^2$

Square of a difference

$a^2 - 2ab + b^2 = (a - b)^2$

2. Factor each expression completely:

a. $x^2 + 10x + 25$

Solution :

$a^2 + 2ab + b^2 = (a + b)^2$ use square of a sum pattern

$x^2 + 10x + 25 = (x)^2 + 2(5x) + 5^2$ substituting

$= (x + 5)(x + 5)$ putting in factored form

$= (x + 5)^2$

b. $4x^2 - 32x + 64$

Solution :

$a^2 - 2ab + b^2 = (a - b)^2$ use square of the difference pattern

$4x^2 - 32x + 64 = (2x)^2 - 2(16x) + 8^2$ substituting

$= (2x - 8)(2x - 8)$ putting in factored form

$= (2x - 8)^2$

c. $25m^2 + 90mn + 81n^2$

Solution :

$a^2 + 2ab + b^2 = (a + b)^2$

$25m^2+90mn+81n^2 = (5m)^2 + 2(45mn) + (9n)^2$ what pattern is used?

$= (5m)^2 + 2(5m)(9n) + (9n)^2$ why?

$= (5m + 9n)(5m + 9n)$ putting in factored form

$= (5m + 9n)^2$

There are quadratic trinomials which are the results of multiplying two binomial factors. Study carefully the multiplications in the table below.

		Quadratic Trinomial						
Factors	F O I L	First term		Middle term		Last term	Factors	product	sum
(x+5)(x+3)	$x^2+3x+5x+15$	x^2	+	8x	+	15	5,3	15	8
(x+6)(x+4)	$x^2+4x+6x+24$	x^2	+	10x	+	24	4,6	24	10
(x-3)(x-8)	$x^2-8x-3x+24$	x^2	-	11x	+	24	-8,-3	24	-11
(x-5)(x-4)	$x^2-4x-5x+20$	x^2	-	9x	+	20	-4,-5	20	-9
(x+7)(x-5)	$x^2-5x+7x-35$	x^2	+	2x	-	35	-5,7	-35	2
(x-8)(x+4)	$x^2+4x-8x-32$	x^2	-	4x	-	32	4,-8	-32	-4

What have you observed about the terms of the quadratic trinomial? Are they related to the terms of their binomial factors? Notice that the constant term is the product of the last terms of the factors, and the numerical coefficient of the middle term, the sum of the last terms of the factors.

Let's look at these examples:

3. Factor $m^2 + 16m + 15$

Solution :

Look for factors of 15 whose sum is 16.

Factors of 15	Sum of factors of 15	
1, 15	1+ 15= 16	both factors must be positive
−1, −15	(−1) + (−15) = −16	
3, 5	3+5 = 8	

The numbers 1 and 15 have a product 15 and a sum of 16.

Hence, $m^2 + 16x + 15 = (m + 1) (m + 15)$.

4. Recognizing the correct factored form:

Which factored form yields a product of $x^2 - 5x + 6$?

a. $(x + 3)(x - 2)$ c. $(x + 3)(x + 2)$

b. $(x - 3)(x - 2)$ d. $(x - 3)(x + 2)$

Solution:

Examine the choices

(a) and (d)	These products will each have a negative constant term.
(c)	This product will have a positive constant term, but the numerical coefficient of the middle term will also be positive.
(b)	by the elimination technique option (b) must be the correct factorization. Check (b) by multiplying.

$$(x - 3)(x - 2) = x^2 + (-2x) + (-3x) + 6$$
$$x^2 - 5x + 6.$$

5. Factor $x^2 + 5x - 36$

Solution:

Look for the factors of −36 whose sum is 5

Factors of −36	Sum of factors
−12, 3	−9 , why?
6, −6	0 , why
−9, 4	−5 , why?
9, −4	5 , why?

Which of the factors satisfy the required condition?

Thus, the factors of $x^2 + 5x - 36$ are $(x + 9)(x - 4)$.

Hence, $x^2 + 5x - 36 = (x + 9)(x - 4)$.

6. Factor: $6x^2 + 11x + 3$

Solution:

Note: Since the middle and the last term are both positive, hence, the two binomial factors must be both positive.

Look for factors of $6x^2$ and 3.

Factors of $6x^2$	Factors of 3
6x and x	3 and 1
2x and 3x	

Factors of $6x^2$ will serve as first term of each binomial, while factors of 3 will serve as the last term. Choose from the above factors a combination that will give a middle term of 11x.

First trial:

$(6x + 3)(x + 1)$ by FOIL method, the middle term is 9x. Hence, $6x + 3$ and $x + 1$ are not the factors.

Second trial:

$(2x + 1)(3x + 3)$ by FOIL method, the middle term is 9x. Hence, $2x + 1$ and $3x + 3$ are not the factors.

Third trial:

$(2x + 3)(3x + 1)$ by FOIL method, the middle term is 11x.

Therefore, the factors $2x + 3$ and $3x + 1$ are the factors of $6x^2 + 11x + 3$.

7. Factor: $4x^2 - 5x - 6$

Solution:

Note: Since the last term is negative, therefore the last terms of the 2 binomial factors are of opposite signs.

Factors of $8x^2$: 2x and 2x, 4x and x.

Factors of – 6: 2 and – 3, 3 and –2, 1 and – 6, 6 and – 1.

First trial: $(4x - 2)(x + 3)$ by FOIL method, the middle term is 10x.

Second trial: $(4x + 3)(x - 2)$ what is the middle term? Is it – 5x? Yes.

Therefore, the Factors of $4x^2 - 5x - 6$ are $4x + 3$ and $x - 2$.

Let's Practice for Mastery 2.

Answer the following:

1.Which is a perfect square trinomial?

a. $x^2 - 8x + 16y^2$

b. $x^2 + 8x - 16$

c. $x^2 - 8x + 16$

d. $x^2 - 8x - 16$

2. Which expression is a factor of $x^2 - 7x + 12$?

a. $x - 2$

b. $x - 6$

c. $x - 3$

d. $x + 4$

3. Factor the following:

a. $m^2 - 64$

b. $x^2 + 2xy + y^2$

c. $16k^2 - 16k + 4$

d. $16y^2 + 48y + 36$

e. $c^2 + 14c + 45$

f. $2p^2 + 9p + 7$

4. For what value(s) of p is $x^2 + px + 81$ a perfect square trinomial?

5. Find the missing factor in

$3x^2 + 6x = (\_\_\_\_)(x + 2)$.

Let's Check Your Understanding 2.

Answer the following completely:

1. Which expression is equivalent to $9r^2 - 16s^2$?

a. $(3r + 4s)(3r - 4s)$

b. $(3r - 4s)^2$

c. $(9r + 16s)(9r - 16s)$

d. $(9r - 16s)^2$

2. Which is a factor of $y^2 + y - 30$?

a. $y - 6$

b. $y + 6$

c. $y - 3$

d. $y + 3$

3. Which is not a true statement?

a. $a^2 + 2ab + b^2 = (a + b)^2$

b. $a^2 - 2ab + b^2 = (a - b)^2$

c. $a^2 + 2ab - b^2 = (a - b)^2$

d. $a^2 - b^2 = (a + b)(a - b)$

4. For what value(s) of m is $x^2 + 2x + m$ a perfect square trinomial? Explain.

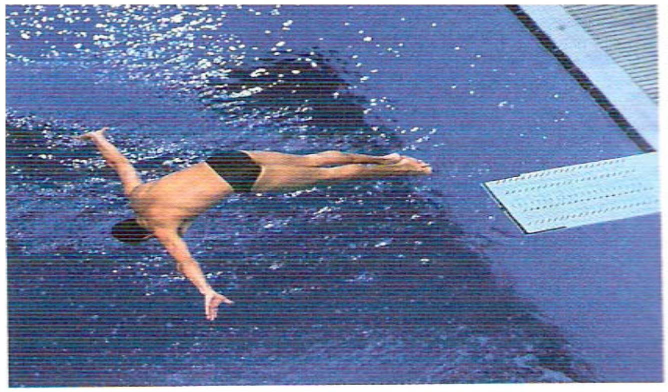
5. Factor the following:

a. $x^2 + 22x + 121$

b. $4x^2 - 36x + 81$

c. $3m^2 - 4m - 20$

Lesson 2.3. Solving Quadratic Equations by Extracting Square Roots



The path made by the diver is an inverted U-shaped curve. Can you figure out the diver's position when he reaches his maximum height.

A quadratic equation is any equation that can be written in the form $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$. This form is called the standard form of a quadratic equation in x.

Examine the following examples of equations.

1. $3x + 5 = 26$. Is it quadratic? No. The exponent of the variable x is 1. Hence, it is linear in x.
2. $n^2 = 3n + 4$. The highest exponent of the variable is 2, right? So, it is quadratic, but it is not written in standard form. The standard form is $n^2 - 3n - 4 = 0$.
3. $x^2 - 5x = 0$. Is it quadratic? Why? Is it written in standard form? Why?

Let's Practice for Mastery 3.

Answer the following:

A. Identify which equation is quadratic. Explain.

1. $x^2 - 7 = 0$
2. $5x = 16 + x$
3. $5x^2 = 25$
4. $12a^3 = 32$
5. $5x^2 + 6x = -1$

B. Which of the quadratic equations in A are written in standard form?

Let's Check Your Understanding 3.

Answer the following:

1.Which is not a quadratic equation ?

a. $x^2 = 16$ c. $3x = 2x + 5$

b. $3x^2 = 4x+1$ d. $x^2 = 7$

2. Which of the following is written in standard form?

a. $3x^2 = 7$ c. $x^2 = 25$

b. $2x^2 - 4x + 5 = 0$ d. $4x = x^2 + 5$

3. Write $6x^2 = 5x - 12$ in standard form.

4. Is $ax^2 = 3x + 5$ always quadratic? Explain.

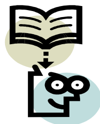
A solution to a quadratic equation in one variable is any number, x, that makes the equation true. For example, if $x^2 = 9$, then x is either $(-3)^2 = 9$, or $(3)^2 = 9$. Hence, both −3 and 3 are solutions. (A solution is also called a root.)
However, (− 5) and 5 are not roots of the equation, since $(-5)^2 \neq 9$ and $5^2 \neq 9$.
How about − 4) and 4 ? (−7) and 7? Are these numbers roots of the same equation ? Explain your answer. We can also use substitution, if $x^2 = 9$, $3^2 = 9$, and $(-3)^2 = 9$.

The above examples remind you that an equation of the form $x^2 = r$ or $x^2 - r = 0$, where $r > 0$, has two solutions.

This pattern can be generalized as follows:

Solving Quadratic Equations by Extracting Square Roots
If $x^2 = r$ and $r > 0$, then $x = \pm\sqrt{r}$.

Use Extracting Square Roots to solve each equation. Verify your answers:



Examples :

1. a. $x^2 = 16$ b. $y^2 = 15$

Solution: *Solution:*

$x^2 = 16$ $y^2 = 15$

$x = \pm 4$ $y = \pm\sqrt{15}$ Get the square root of both sides

The roots of example 1a are 4 and – 4, while example 1b has roots $\sqrt{15}\, and\ -\sqrt{15}$.

2. a. $x^2 - 81 = 0$

Solution:

$x^2 - 81 = 0$

$x^2 = 81$ Use the Addition Property of Equality.

$x = \pm 9$ Get the square root of both sides.

The roots are -9 and 9 .

b. $y^2 - 21 = 0$

Solution: $y^2 - 21 = 0$

$y^2 = 21$ why?

$y = \pm\sqrt{21}$ by getting the square root of both sides

3. $x^2 - 54 = 10$

Solution:

$x^2 - 54 = 10$

$x^2 = 64$ why ?

$x = \pm 8$ why?

Therefore, the roots are – 8 and 8.

4. $2m^2 - 72 = 0$

Solution:

$2m^2 - 72 = 0$

$2m^2 = 72$ why ?

$m^2 = 36$ why ?

$m = \pm 6$ why ?

Therefore, the roots are -6 and

5. $3m^2 + 8 = 35$

Solution :

$3m^2 + 8 = 35$

$3m^2 = 27$ why?

$m^2 = 9$ why?

$m = \pm\sqrt{9}$

$m = \pm 3$ why?

Therefore, $m = -3$ and $m = 3$.

6. $3(y + 5)^2 = 75$

Solution :

$\frac{1}{3}[\,3(y + 5)^2 = 75\,]$

$(y + 5)^2 = 25$ Multiplying each side by $\frac{1}{3}$

$y + 5 = \pm\sqrt{25}$

$y + 5 = \pm 5$ by extracting square roots

$y + 5 = 5$ or $y + 5 = -5$ by Addition Property of Equality

$y = 0$ or $y = -10$ by simplifying

Therefore, the roots are -10 and 0.

Let's Practice for Mastery 4.

A. Answer the following:

1. Which equation has the same solution as $2(b^2 - 5) = 18$?

   a. $b^2 = 14$ c. $2b^2 = 23$

   b. $b^2 = 8$ d. $b^2 = \frac{18}{2} - 5$

2. Which equation has exactly one solution? Explain

   a. $(x - 3)^2 = 0$ c. $(x - 3)^2 = -5$

   b. $(x - 3)^2 = 6$ d. $x - (2x^2 + 1) = -5$

B. Solve for the roots by extracting square roots

3. $x^2 = 49$
5. $3a^2 - 5 = 43$
4. $2m^2 - 98 = 0$

Let's Check Your Understanding 4.

Solve the following:

1. Find the roots of $3(x - 5)^2 = 147$.
2. Solve for the roots of $-4t^2 = -48$.
3. Find the value for c so that the equation $x^2 - c = 0$ has -11 and 11 as solutions.
4. Michael solved $x^2 + 25 = 0$ and found that the solutions - 5 and 5, was he correct? Explain the mistake that Michael made.
5. Anna looked for the roots of the equation $3x^2 - 27 = 0$.

   Verify if she got the correct roots. Explain.

   Here is Anna's solution:

   $3x^2 - 27 = 0$

   $3x^2 = 27$ By Addition Property of equality

   $x^2 = 81$ By the Multiplication Prop. of Equality.

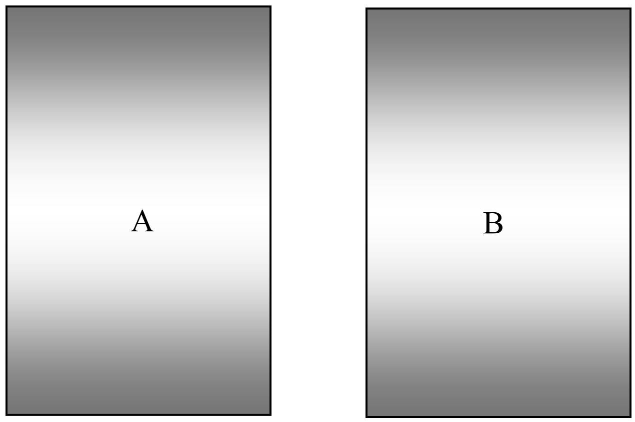
   $x = \pm 9$ By extracting square roots

   There fore, the roots are -9 and 9.

Lesson 2.4. Solving Quadratic Equation by Factoring

Zero – Product Property

Suppose that each card below has a number written on the reverse side.



cards

If you are told that the product ab of the numbers does not equal 0, what conclusion can you give regarding the two numbers? Why?

If you are told that $ab = 0$, then you must conclude that at least one of the cards has 0 written on it. For example, if card A has 4 written on it, then $4b = 0$. This equation says that b must equal 0.

The *Zero- Product Property* is a generalization of this discussion.

> Zero – Product Property
>
> If a and b are any real numbers, and $ab = 0$, then either $a = 0$ or $b = 0$.

Analyze the given statements below:

If $2x = 0$, then $x = 0$.

If $x(x - 2) = 0$, then $x = 0$ or $x - 2 = 0$.

If $-3(x - 5) = 0$, then $x - 5 = 0$.

If $5x(2x + 1) = 0$, then $5x = 0$ or $(2x+1) = 0$.

Using the *Zero- Product Property* study the examples below:

Examples :

1. Which is true given that $(n-5)(n+4)=0$?
   a. $(n-5)=0$ and $(n+4)=0$
   b. $(n-5)=0$ and $(n+4)\neq 0$
   c. $(n-5)=0$ or $(n+4)=0$
   d. $(n-5)\neq 0$ and $(n+4)=0$

Solution :

By the *Zero – Product Property*, one or the other factor in $(n-5)(n+4)$ must be 0. Therefore, $(n-5)=0$ or $(n+4)=0$.

Hence, the correct option is c.

2. Solve each equation
   a. $3x(x+7)=0$

   Solution:

   $3x(x+7)=0$

   $3x=0$ or $x+7=0$ Zero – Product Property

   $x=0$ or $x=-7$

   Therefore, the solutions are -7 and 0

   b. $(x+7)(x+4)=0$

   Solution:

   $(x+7)(x+4)=0$

   $x+7=0$ or $x+4=0$ why? Explain.

   Thus, $x+7=0$ $x+4=0$ why?

   $x=-7$ or $x=-4$

   Therefore, the solutions are -7 and -4 .

B. Factoring

What have you noticed about each quadratic equation below?

$b^2 = -3b$ $\qquad$ $m^2 + m - 6 = 0$ $\qquad$ $z^2 + 4z + 4 = 0$

None of the quadratic equations is given in factored form. Before getting the roots of such equations, you have to transform them first in standard forms before factoring.

Tips used in solving quadratic equations by factoring:

1. Write the given quadratic equation in standard form.
2. Factor the quadratic equation.
3. Use the Zero – Product Property to write a pair of linear equations.
4. Solve the linear equations.
5. The solution to the linear equations is the solution to the given equation.

Examples:

1. Solve the following by factoring

a. $x^2 + 5x = 0$

Solution:

$x^2 + 5x = 0$

$x(x + 5) = 0$ — Use the distributive property

$x = 0$ or $x = -5$ — Use the zero – product property

The roots are 0 and − 5.

b. $m^2 + 4m + 4 = 0$

Solution:

$m^2 + 4m + 4 = 0$ — Perfect square trinomial

$(m + 2)(m + 2) = 0$ — Use special factoring technique

$m + 2 = 0;\ m + 2 = 0$ — Use the zero – product property

$m = -2;\ m = -2$

Hence, the roots are − 2 and – 2

c. $x^2 + 4x = 12$

Solution:

$x^2 + 4x = 12$

$x^2 + 4x - 12 = 0$ why?

$(x + 6)(x - 2) = 0$ why?

$x + 6 = 0$ or $x - 2 = 0$ why?

$x = -6$ or $x = 2$

the roots are -6 and 2.

d. $3x^2 = 8 - 2x$

Solution :

$3x^2 = 8 - 2x$

$3x^2 + 2x - 8 = 0$ why?

$(3x - 4)(x + 2) = 0$ why?

$3x - 4 = 0$ $x + 2 = 0$

$x = \frac{4}{3}$ or $x = -2$ why?

The roots are $\frac{4}{3}$ and -2.

2. Find the error in the solution of $2x^2 + 5x = 0$.

Solution:

$2x^2 = -5x$ Subtract 5x from both sides

$2x = -5$ Divide both sides by x

$x = -\frac{5}{2}$ Multiply both sides by $\frac{1}{2}$

The root is $-\frac{5}{2}$ Is it the only root? Explain.

3. The area of a rectangular garden is 220 sq.m. The length of the rectangle is 12 m more than its width. What are the dimensions of the rectangle?

Solution:

Step 1: Represent the given information in an equation:

Let w represents the width of the rectangle

w + 12 represents the length

$(w + 12)\, w = 220$ why?

Step 2: Solve the equation

$w^2 + 12w - 220 = 0$ why?

$(w + 22)(w - 10) = 0$ why?

$w = -22$ or $w = 10$ why?

Step 3: Interpret the answer

The length cannot be – 22 m why?

The width of the rectangle is 10 m. why?

The length of the rectangle is 10 + 12 = 22 m why?

4. The product of two consecutive positive numbers is 56. Find the numbers.

Solution:

Step 1. Represent the given information in an equation

Let x = the smaller number

x + 1 = the bigger number why?

So, $x(x + 1) = 56$ why?

Step 2. Solve the equation

$x^2 + x = 56$ why?

$x^2 + x - 56 = 0$ why?

$(x + 8)(x - 7) = 0$ why?

$x = -8$ or $x = 7$ why?

Step 3. Interpret the answer

Can x be – 8 ? why?

Therefore, the numbers are 7 and 8. why?

Let's Practice for Mastery 5

A. Which is true given that

1. $(x + 3)(x – 7) = 0$?

a. $x + 3 = 0$ or $x – 7 = 0$

b. $x + 3 = 0$ and $x – 7 = 0$

c. $x + 3 = 0$ and $x – 7 \neq 0$

d. $x + 3 \neq 0$ or $x + 7 = 0$

2. $5y(x – 3) = 0$?

a. $5y \neq 0$ and $x – 3 = 0$

b. $5y = 0$ and $x – 3 \neq 0$

c. $5y = 0$ or $x – 3 = 0$

d. $5y = 0$ and $x – 3 = 0$

B. Solve each equation:

3. $2x(x – 5) = 0$

4. $(x – 3)(2x – 4) = 0$

C. Solve for the roots by factoring: Check all answers.

5. $a^2 + 12a + 35 = 0$

6. $5x^2 – 25x + 20 = 0$

7. $81 – 4x^2 = 0$

8. $36x^2 = – 12x – 1$

D. Solve the following. Show all work.

9. Find the two missing numbers of the binomial factors in $(2x + \_\_)(x + \_\_) = 0$, by finding the two integers whose product is – 15 and that make the sum of the outer and inner products equal to – 7x. (Hint: list the possible pairs of factors of – 15.)

10. Find two consecutive odd numbers whose product is 63.

a. If x is the first odd number, how do you represent the next?

b. How do you write their product ?

c. Find the numbers

Let's Check Your Understanding 5

A. Supply the missing numbers of the binomial factors.

1. $(3x + \_\_\_\_) (x + \_\_\_\_) = 0$

   6x

   12x

2. $x^2 + \_\_\_\_\_ x - 16 = 0$, and

   $(x + \_\_\_) (x - \_\_\_) = 0$

   8x

   -2x

B. Solve each equation

3. $3x(x + 6) = 0$
4. $(x + 2)(x – 5) = 0$

C. Factor the following completely, then solve for the roots:

5. $x^2 + 22x + 121 = 0$
6. $4x^2 – 81 = 0$
7. $16 x^2 – 40x + 25 = 0$
8. $3x^2 + 5x – 50 = 0$

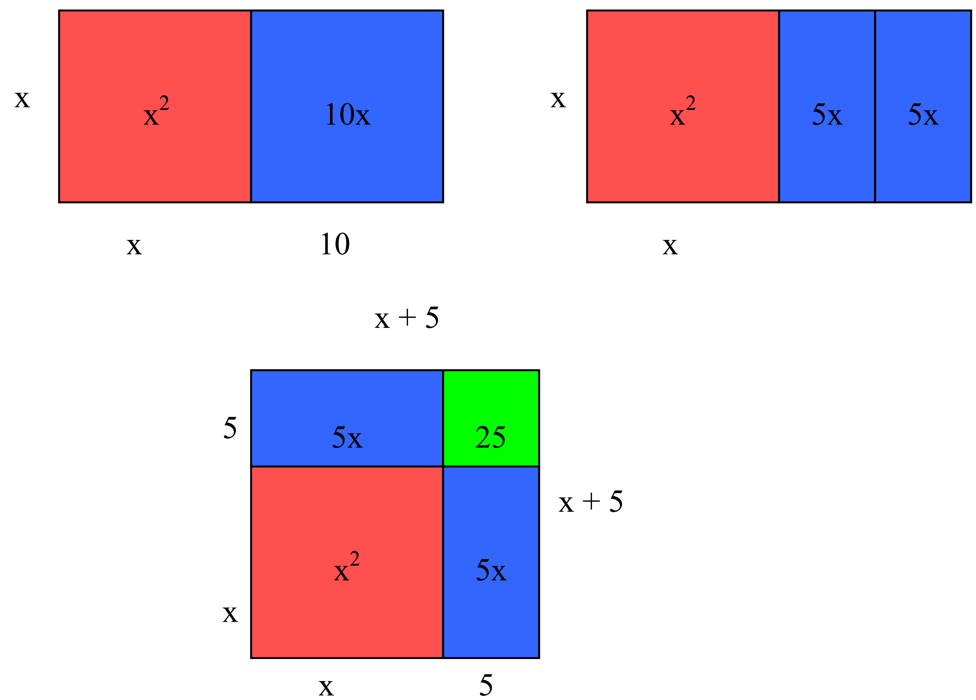
D. For numbers 9 – 10, solve :

The sum of two positive integers is 31. The sum of the squares of these numbers is 625.

9. Using the given conditions, find the resulting quadratic equation.
10. Find the smaller of the two numbers.

Lesson 2.5 Solving Quadratic Equations by Completing the Squares

A visual interpretation of completing the square



In all figures, the sum of the red and blue areas is $x^2 + 10x$. However, by splitting the blue area in half and rearranging, as shown above, you can "complete" a square by adding the green area. The green area is $5 \cdot 5$ or 25 square units. In effect, $(x+5)^2 - 25 = x^2 + 10x$

In the last lesson you learned that if a quadratic equation takes any of the forms

$$x^2 = k,$$

$$(x + c)^2 = k \quad \text{or}$$

$$(ax + c)^2 = k,$$

where k is a positive number, then the equation can be solved by taking the square root of both sides. But, when the polynomial can easily be factored, the best way to solve the equation is by factoring. However, when the polynomial is not factorable, you can use a method called completing the square to solve for its roots.

Completing the square refers to the process of "forcing" a trinomial to become a perfect square trinomial.

Consider the following quadratic equation:

$$x^2 + 10x = 4$$

What would you add to both sides of this equation to make the left side a perfect square trinomial? Recall that the square of the binomial (x + a) is

$(x + a)^2 = x^2 + 2ax + a^2$, where a^2 is the square of half the coefficient of x. Going back to the quadratic equation $x^2 + 10x = 4$, half the coefficient of x is 5, and $5^2 = 25$. This suggests that you should add 25 on both sides.

Thus,

$x^2 + 10x + 25 = 4 + 25$	Adding 25 on both sides
$(x + 5)(x + 5) = 29$	Factoring the trinomial
$(x + 5)^2 = 29$	
$x + 5 = \pm\sqrt{29}$	Using extracting square roots
$x + 5 = \sqrt{29}$ or $x + 5 = -\sqrt{29}$	Using zero – product property
$x = -5 + \sqrt{29}$ or $x = -5 - \sqrt{29}$	

So, the roots are $-5 \pm \sqrt{29}$

Remember that the key to solving $x^2 + 10x = 4$ was adding 25 to both sides of the equation, thus making the left side a perfect square trinomial.

Let's have a summary of the steps used in completing the square method.

To solve a quadratic equation $ax^2 + bx + c = 0$ by completing the square, we have the following steps :

1. If $a \neq 1$, multiply by $\frac{1}{a}$ on both sides so that the x^2 – coefficient is 1.
2. When the x^2 – coefficient is 1, rewrite the equation in the form

   $x^2 + bx = -c$, or $x^2 + \frac{b}{a}x = -\frac{c}{a}$ (if step (1) has been applied.)
3. Take half of the x – coefficient and square it. Add the result to both sides of the equation.
4. Factor the trinomial square and combine like terms.
5. Solve the resulting quadratic equation by extracting square roots.
6. Check each solution.

Examples:

1. Add the square of half of the coefficient of x to each expression below

a. $x^2 - 6x$ b. $x^2 - 5x$ c. $x^2 - \frac{4}{3}$

Solutions: a. $x^2 - 6x + [\frac{1}{2}(-6)]^2 = x^2 - 6x + (-3)^2$

$= x^2 - 6x + 9$

$= (x - 3)^2$

b. $x^2 - 5x + [\frac{1}{2}(-5)^2 = x^2 - 5x + (-\frac{5}{2})^2$

$= x^2 - 5x + \frac{25}{4}$

$= (x - \frac{5}{2})^2$

c. $x^2 - \frac{4}{3}x + [\frac{1}{2}(-\frac{4}{3})]^2 = x^2 - \frac{4}{3}x + (\frac{-4}{6})^2$

$= x^2 - \frac{4}{3}x + \left(-\frac{2}{3}\right)^2$

$= (x - \frac{2}{3})^2$

2. Complete the square in each:

a. $x^2 + 22x$

Solution:

$x^2 + 22x + (11)^2$ — Use $a^2 + 2ab + c$, where $c = \frac{1}{2}(b)$ or $\frac{1}{2}(22)$

$x^2 + 22x + 121$ — Why?

$(x + 11)^2$ — Why

Therefore, $x^2 + 22x + 121$ is a perfect square trinomial.

b. $x^2 + 4x$

Solution:

$x^2 + 4x + \left(\frac{4}{2}\right)^2$ — Why?

$x^2 + 4x + 4$ — Why?

Is $x^2 + 4x + 4 = (x + 2)^2$? — Explain.

Hence, $x^2 + 4x + 4$ is a perfect square trinomial.

3. Find the value of b such that each is a perfect square trinomial.

a. $x^2 + bx + 49$

Solution:

$x^2 + bx + 49 = x^2 + bx + (7)^2$

$b = 2(a)(c)$ — Use $a^2 + 2ab + b^2 = (a + b)^2$

$= 2(1)(7)$ — Substitute $a = 1, c = 7$

$b = 14$ — Why?

Therefore, $x^2 + bx + 49 = x^2 + 14x + 4$

b. $4x^2 + bx + 16$

Solution: $4x^2 + bx + 16 = (2x)^2 + bx + (4)^2$

$b = 2(a)(c)$ — Why?

$b = 2(2)(4)$ — Why?

$b = 16$ — Why?

Hence, the perfect square trinomial is $4x^2 + 16x + 16$.

4. Find the roots of the following. Show all work.

a. $x^2 - 10x + 14 = 0$

Solution: $x^2 - 10x + 14 = 0$

$x^2 - 10x = -14$ why?

$x^2 - 10x + 25 = -14 + 25$ why?

$(x - 5)^2 = 11$ why?

$x - 5 = \pm\sqrt{11}$ why?

$x - 5 = \sqrt{11}$ or $x - 5 = \pm\sqrt{11}$ why?

$x = \pm\sqrt{11}$

The roots are $5 + \sqrt{11}$ and $5 - \sqrt{11}$

b. $3x^2 + 6x = -15$

Solution:

$\frac{1}{3}(3x^2 + 6x) = (-15)\frac{1}{3}$ multiply $\frac{1}{3}$ to make the coefficient of $x^2 = 1$.

$x^2 + 2x = -5$

$x^2 + 2x + 1 = -5 + 1$ why?

$(x + 1)^2 = -4$ why?

$x + 1 = \sqrt{-4}$ why?

$x + 1 = \pm\sqrt{4(-1)}$ why?

$x + 1 = \pm 2\sqrt{-1}$ why?

$x + 1 = \pm 2\,\mathrm{i}$ substitute i for $\sqrt{-1}$

Hence, $x = -1 + 2i$ or $x = -1 - 2i$

The roots are $-1 \pm 2i$. Explain your answer.

Let's Practice for Mastery 6.

Answer the following:

1. Add the square of half the coefficient of x to each:

   a. $x^2 - 12x$ b. $m^2 + \frac{9}{2}m$

2. Complete the square

   a. $x^2 + 18x$ b. $y^2 - \frac{2}{5}y$

3. Find b such that (a) $4x^2 + bx + 16$ and (b) $x^2 + 8x + b$ is a perfect square trinomial.

4. Solve by completing the square..

   a. $x^2 - 10x = 22$

   b. $x^2 + 7x - 2 = 0$

   c. $4x^2 + 12x = 7$

5. Explain in your own words how completing the square enables us to solve equations.

Let's Check Your Understanding 6.

1. Make each of the following a perfect square:

   a. $x^2 - 20x +$ _____ c. $3x^2 - 2x +$ _____

   b. $x^2 + \frac{3}{4}x +$ _____ d. $6x^2 + 5x +$ _____

2. Find the roots of each by completing the square

   a. $x^2 - 8x + 12 = 0$ c. $3x^2 - 8x = 16$

   b. $x^2 + 18x + 72 = 0$

3. Find b such that the trinomial is a square

   a. $x^2 + bx + 55 = 0$ b. $4x^2 + bx + 16 = 0$

4. A student states that "since solving a quadratic equation by completing the square relies on the principle of the square roots, the solutions are always opposites of each other." Is the student correct? Why or why not?

Lesson 2.6 Solving Quadratic Equations by the Quadratic Formula

Consider a quadratic equation in standard form, that is, $ax^2 + bx + c$, where $a \neq 0$. By using completing the square method, you can determine the solution set or the roots of the equation.

Starting with the standard form, and following the steps used in completing the square, find the roots of the quadratic equation $ax^2 + bx + c = 0$.

Step 1. Divide each side by the coefficient of the x^2 term $\quad x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \quad$ why?

Step 2. Add the additive inverse of the constant term $\quad x^2 + \frac{b}{a}x = -\frac{c}{a} \quad$ why?

Step 3. Take $\left[\frac{1}{2}(coefficient\ of\ x)\right]^2 \quad \left[\frac{1}{2}\left(\frac{b}{a}\right)\right]^2 \quad$ why?

Step 4. Simplify $\quad \frac{b^2}{4a^2} \quad$ why?

Step 5. Add the result to both sides in step 2. $\quad x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2} \quad$ why?

Step 6. Simplify $\quad x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2 - 4ac}{4a^2}$

Step 7. Factor the trinomial $\quad \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$

Step 8. Use extracting square roots $\quad x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$

Step 9. Solve for x $\quad x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

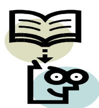
Step 10. Simplify $\quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Hence, the solution set is $\quad \frac{-b + \sqrt{b^2 - 4ac}}{2a}; \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

The solution set for the general equation is usually referred to as the quadratic formula.

The quadratic formula as what the name implies is a formula for solving any quadratic equation of the form $ax^2 + bx + c = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ where a, b, c are real numbers and } a \neq 0$$



Examples :

1. Find the values of a , b , and c in

   a. $3x^2 - 5x + 2 = 0$

   Solution:

   $3x^2 - 5x + 2 = 0$ use the standard equation $ax^2 + bx + c = 0$

   $a = 3$

   $b = -5$ why?

   $c = 2$

   b. $5x^2 = 7 - 2x$

   Solution:

   $5x^2 + 2x - 7 = 0$ use the standard equation $ax^2 + bx + c = 0$

   $a = 5$

   $b = 2$ why?

   $c = -7$

In some cases, there is a need for us to find the value of $b^2 - 4ac$ so that if its value is positive, then x is a real number.

2. For each equation, find the value of $b^2 - 4ac$

a. $2x^2 + 3x + 1 = 0$

Solution:

$2x^2 + 3x + 1 = 0$

$a = 2, b = 3, c = 1$ why?

$b^2 - 4ac = (3)^2 - 4\,(2)\,(1)$ why?

$= 9 - 8$

$b^2 - 4ac = 1$ why?

b. $5x^2 - 6x + 2 = 0$

Solution:

$5x^2 - 6x + 2 = 0$

$a = 5\ ,\ b = -6\ ,\ c = 2$ why?

$b^2 - 4ac = (-6)^2 - 4(5)(2)$ why?

$= 36 - 40$

$b^2 - 4ac = -4$ why?

3. Use the Quadratic formula to solve

a. $x^2 + 5x + 6 = 0$

Solution:

$x^2 + 5x + 6 = 0$

in this equation , a = 1 , b = 5 , and c = 6 why?

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ quadratic formula

$$x = \frac{-5 \pm \sqrt{(5^2) - 4(1)(6)}}{2(1)}$$ why?

$$x = \frac{-5 \pm \sqrt{25 - 24}}{2}$$ simplify

$x = \frac{-5 \pm 1}{2}$ why?

Thus, $x = \frac{-5+1}{2}$ *and* $x = \frac{-5-1}{2}$ why?

$x = \frac{-4}{2}$ *and* $x = \frac{-6}{2}$ simplifying

x = − 2 x = − 3 Are these two values both solutions? Explain.

b. $3x^2 = 2x + 4$

Solution:

$3x^2 - 2x - 4 = 0$ why?

a = 3, b = − 2, c = − 4 why?

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ quadratic formula

$x = \frac{-(-2) \pm \sqrt{(-2^2) - 4(3)(-4)}}{2(3)}$ why?

$x = \frac{2 \pm \sqrt{4+48}}{6}$ why?

$x = \frac{2 \pm \sqrt{52}}{6}$

$x = \frac{2 \pm \sqrt{4 \cdot 13}}{6}$ why?

$x = \frac{2 \pm 2\sqrt{13}}{6}$

$x = \frac{1 \pm \sqrt{13}}{3}$ why?

So, $x = \frac{1}{3} + \frac{\sqrt{13}}{3}$ *or* $x = \frac{1}{3} - \frac{\sqrt{13}}{3}$

Let's Practice for Mastery 7.

Answer the following.

1. In $2x^2 + 3x + 1 = 0$,
   a. Identify a, b, and c.
   b. Use the quadratic formula to solve for the roots.
2. Given $x^2 + 5x + 6 = 0$,
   a. Use the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to solve the roots.
   b. Verify your answers in (a) using factoring.
3. Using the same given in # 2,
   a. solve for x using completing the square
   b. compare your answers in 2a, 2b, and #3.
4. Cite a good warning when you are using the quadratic formula.

Let's Check Your Understanding 7.

Solve the following.

1. a. Find the values of a, b, and c in $2x^2 = 3 - x$.
   b. Using the values of a, b, c in no. 1, find the roots by using the quadratic formula.
   c. Did you get $\frac{-3}{2}$ and 1 as roots?
   If not check the values of your a, b, and c.
2. Using the quadratic formula, solve for the roots of $x^2 - 4x - 7 = 0$. Use a calculator to approximate the solutions to the nearest thousandths.
3. The hypotenuse of a right triangle is 6m long. One leg is 1m longer than the other. Find the lengths of the legs. Round to the nearest hundredths.

Lesson 2.7 Nature of Roots of a Quadratic Equation

The roots of any quadratic equation can be found by using the quadratic formula. Note that a quadratic equation may have one real number solution, two real number solutions, or no real number solution. The number and kind of solution of quadratic equations can be determined from a part of the quadratic formula, that is, from $b^2 - 4ac$.

This portion of the quadratic formula that determines the nature of roots of a quadratic equation is called the *discriminant*.

Let's recall example 2 from lesson 2.6, where you are asked to find the value of $b^2 - 4ac$ given the equation $2x^2 + 3x + 1 = 0$. Substituting a = 2, b = 3, and c = 1, gives $(3)^2 - 4(2)(1) = 9 - 8 = 1$. Thus, $b^2 - 4ac = 1$. Is 1 a perfect number? Why?

Use factoring to get the actual roots of $2x^2 + 3x + 1 = 0$. What answers did you get?

So, the roots are -1 and $-\frac{1}{2}$, which are rational and unequal.

How about the roots of $x^2 + 4x + 4$?

By factoring,

$(x + 2)(x + 2) = 0.$	why?
$x + 2 = 0$ or $x + 2 = 0$	why?
$x = 2$ or $x = 2$	why?

Thus, the roots are -2 and -2, which are equal.

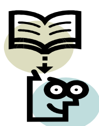
Let's verify the value of $b^2 - 4ac$, where a = 1, b = 4, and c = 4.

$(4)^2 - 4(1)(4) = 16 - 16 = 0$	why?
So, $b^2 - 4ac = 16 - 16 = 0$	why?

Hence, there is only one solution, since the roots are equal.

The next table summarizes the nature and the number of solutions of any quadratic equation. Analyze each kind and examine the next examples.

Nature and Number of roots of a Quadratic Equation		
Discriminant	**Nature of Roots**	**Number of Solution**
1. If $b^2 - 4ac = 0$	Real and Equal	One solution
2. If $b^2 - 4ac > 0$ and		
a. perfect square no.	Rational and Unequal	Two distinct solutions
b. not a perfect square	Irrational and unequal	
3. If $b^2 - 4ac < 0$	Imaginary	Two distinct solutions



Examples :

1. Use the discriminant to determine the nature of the roots of

$$3x^2 - 5x - 1 = 0,$$

Solution:

In $3x^2 - 5x - 1 = 0$,

$a = 3$, $b = -5$, $c = -1$ why ?

$b^2 - 4ac = (-5)^2 - 4(3)(-1)$ Substitute the values in the discriminant

$= 25 + 12$ why?

$= 37$

Since $37 > 0$ and not a perfect number, there are two distinct irrational roots. Verify by solving the actual roots using the quadratic formula.

2 . Find the number of solutions , if there are any of

$$x^2 - 6x + 9 = 0$$

Solution:

In $x^2 - 6x + 9 = 0$,

$a = 1, b = -6, c = 9$ why?

$b^2 - 4ac = (-6)^2 - 4($ \1 $)(9)$

$= 36 - 36$ why?

$= 0$

There is only one root. why?

Let's verify by getting the actual roots.

Using factoring,

$(x - 3)(x - 3) = 0$ why?

$x = 3$ or $x = 3$ why?

Thus, the roots are 3 and 3 why?

Hence, there is only one solution which is 3.

3. Describe the roots of $-x^2 + x - 5 = 0$.

Solution:

In $-x^2 + x - 5 = 0$,

$a = -1$, $b = 1$, $c = -5$ why ?

$b^2 - 4ac = (1)^2 - 4(-1)(-5)$

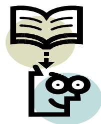
$= 1 - 20$ why?

$= -19$

There are no real number solutions. why?

Verify your answer by solving for the actual roots.

(Use the quadratic formula.)



4. For what values of k will the equation $x^2 + k x + 3 = 0$ have two real solutions ?

Solution:

For $x^2 + k x + 3 = 0$ to have two real solution ,

$$b^2 - 4 a c > 0.$$

$a = 1, \; b = k \; , \; c = 3$ why ?

$(k)^2 - 4 (1) (3) > 0$

$k^2 - 12 > 0$ why ?

$(k + 2\sqrt{3}) (k - 2\sqrt{3}) > 0$ Use factoring

$k < -2\sqrt{3}$ or $k > 2$

Since $a b > 0$, then Case 1. $a > 0$ and $b > 0$ or Case 2. $a < 0$ and $b < 0$

Hence, the complete solution is $k < -2\sqrt{3}$ or $k > 2\sqrt{3}$

5. Lester claimed that the discriminant of $2x^2 + 5x - 1 = 0$ had the value 17. Was he right? Solve it to verify. If not, what error did he make? Compare your solution with the given solution below.

Solution:

In $2 x^2 + 5 x - 1 = 0$,

$a = 2 , b = 5 , c = -1$ why ?

$b^2 - 4 a c = (5)^2 - 4 (2) (-1)$ substituting and

$= 25 + 8$ simplifying

$b^2 - 4 a c = 33$

So, the discriminant is 33. Lester was wrong. He got 17, because he did subtraction , $25 - 8 = 17$.

He forgot that the product of two negative numbers is (+).

Let' Practice for Mastery 8.

Answer the following questions.

1. If the roots of a quadratic equation are equal, what is the value of the discriminant?
2. What is the value of k if you want the roots of $2kx^2 + 6 = x^2 + 8x$ to be real and equal?
3. Describe the roots of $5x^2 - 4x + 1 = 0$.
4. Explain the graphical interpretation of the roots of any quadratic equation.

Let's Check Your Understanding 8.

Answer the following problems:

1. Find the discriminant of the quadratic equation $4x^2 + 3x + 1 = 0$.
   Explain your answer.
2. Determine the value of k for which $kx^2 + 3x - 7 = 0$ has no real roots.
3. Describe the nature of roots of the following quadratic equations :
   a. $4x^2 + 7x - 1 = 0$
   b. $x^2 - 10x + 25 = 0$

   Explain your answer.
4. If the discriminant of a quadratic equation is 0, and you want to add the roots, that is, $r_1 + r_2$, what answer will you get? Explain.

(Hint: Use $r_1 = \dfrac{-b+\sqrt{b^2-4ac}}{2a}$ and $r_2 = \dfrac{-b-\sqrt{b^2-4ac}}{2a}$)

Lesson 2.8 Finding the Quadratic Equation from the Roots

Suppose the roots are given, how do you find the equivalent quadratic equation? What strategy can you apply ?

There are some interesting relations between the sum and product of the roots of a quadratic equation. Recall from completing the square method that the standard equation $ax^2 + bx + c = 0$, where $a \neq 0$ can be written as

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \qquad \text{why?}$$

A. If r and s are the roots of the quadratic equation, then from the quadratic formula

$$r = \frac{-b+\sqrt{b^2-4ac}}{2a} \qquad \text{and} \qquad s = \frac{-b-\sqrt{b^2-4ac}}{2a} \qquad \text{why?}$$

Adding the roots, that is

$$r + s = \frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a} \qquad \text{why?}$$

$$= \frac{-2b}{2a} = \frac{-b}{a} \qquad \text{why?}$$

Hence, the sum of the roots is $\frac{-b}{a}$.

Multiplying the roots, that is

$$rs = \left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right) \qquad \text{why?}$$

$$= \frac{b^2-(b^2-4ac)}{4a^2} = \frac{c}{a} \qquad \text{why?}$$

Observe the coefficients in the quadratic form $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$.

How do they compare with the sum and the product of the roots?

B. An alternative way of arriving at these relations is as follows :

Consider r and s as the roots of $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$.

Then, $(x - r)(x - s) = 0$ why?

Expanding gives $x^2 - sx - rx + rs = 0$ why?

or $x^2 - x(r + s) + rs = 0$ why?

Compare the coefficients of the corresponding terms.

Did you get $r + s = \frac{-b}{a}$ and $rs = \frac{c}{a}$?

The above relations between the roots and the coefficients provide a fast and convenient way of checking the solutions of a quadratic equation. It is also a fast way of getting the quadratic equation from the given roots.

Examples :

1. Find a quadratic equation whose roots are − 3 and 7.

Solution:

$r + s = -3 + 7 = 4$, so $\frac{-b}{a} = -4$. why?

Also, $rs = (-3)(7) = -21$. why?

Thus, the quadratic equation is $x^2 - 4x - 21 = 0$.

Can you apply the alternative solution presented above to solve the problem? Try it and compare your answer with the answer obtained here.

2. Solve and check. $2x^2 + x - 6 = 0$.

Solution :

$2x^2 + x - 6 = (2x - 3)(x + 2) = 0$ why?

$x = \frac{3}{2}$ or $x = -2$ why?

So, the roots are -2 and $\frac{3}{2}$. why?

To check : Add the roots : $\frac{3}{2} + (-2) = -\frac{1}{2} = \frac{-b}{a}$ why?

Multiply the roots : $\frac{3}{2}(-2) = -3 = \frac{c}{a}$ why?

$$2\left[x^2 + \frac{1}{2}x - 3 = 0\right] \rightarrow 2x^2 + x - 6 = 0$$

3. Without actual solving, find the sum and product of the roots of

$3x^2 - 6x + 8 = 0$.

Solution :

The sum of the roots is $r + s = -\frac{b}{a}$

$= -\left(\frac{-6}{3}\right) = 2$ why?

and their product is $rs = \frac{c}{a}$

$= \frac{8}{3}$ why?

Verify the answers by a. solving for the roots

b. getting the sum of the roots

c. getting the product of the roots

Let’s Practice for Mastery 9.

Answer the following:

A. Choose the correct answer :

1. Which of the following are roots of $x^2 + 8x - 9 = 0$?

a. -1 and 8 c. -9 and 1

b. 1 and -8 d. 9 and -1

2. If one root of $2x^2 - 3x - 9 = 0$ is 3, what is the other?

a. $\frac{3}{2}$ c. $\frac{-3}{2}$

b. -3 d. 3

B. Without solving for the roots, find the sum and product of the roots of:

3. $x^2 + 8x + 16 = 0$

4. $2x^2 - 5x - 7 = 0$

5. $5x^2 + x - 1 = 0$

6. $\frac{2}{x} + x = 3$

C. Given the roots, find a quadratic equation : (2 points each)

7. 5 and -8

8. -3 and $\frac{1}{2}$

Let’s Check Your Understanding 9.

Answer the following completely:

1. Given the $x^2 - 8x - 3 = 0$, what is the value of r + s (sum of the roots)?

a. 8 b. -3 c. 8 d. 3

2. Using the equation in # 1, what is the value of rs (product of the roots) ?

a. 8 b. -3 c. 8 d. 3

3. Verify if 2 and -7 are roots of the quadratic equation $x^2 - 5x - 14 = 0$.

4. If the sum and product of the roots of a quadratic equation are $\frac{-5}{2}$ and 3, find its equation. (2 points)

Lesson 2.9. Applications of Quadratic Equations

When the conditions of a problem can be represented by a quadratic equation , the problem can be solved by any one of the methods you discussed earlier.

The quadratic formula for example is important in physics when finding the velocity of a freely falling body. When an object is dropped, thrown or launched either straight up or down , you can use the formula to find the height of the object at a certain unit of time.

Examples:

1. Mark is planning to enlarge his graduation picture. His original picture is 7 cm long by 3 cm wide. He asked the photographer to enlarge it by increasing its length and width by the same amount. If he wants the area of the enlarged picture to be 96 sq. cm , what are its new dimensions?

Solution:

Given: $l = 7$ cm, $w = 3$ cm $A = 7\,(3) = 21$ sq. cm .

new area = 96 sq cm.

Representation:

Let x be the number of cm by which the length and the width are increased.

new length : $(x + 7)$ cm

new width : $(x + 3)$ cm

Working equation:

$A = lw$	formula for area of rectangle
$(x + 7)(x + 3) = 96$	substitute the given
$x^2 + 3x + 7x + 21 = 96$	use Foil method
$x^2 + 10x – 75 = 0$	Simplify
$(x + 15)(x – 5) = 0$	factoring
$x = – 15$ or $x = 5$	

Since the only possible length is 5 cm, this means the new length is 5 + 7 = 12 cm and the new width is 5 + 3 = 8 cm.

2. Members of the science club launch a model rocket from ground level with starting velocity of 96 ft. per second. If the equation of motion of the rocket is $h = – 16t^2 + vt + s$, after how many seconds will the rocket have an altitude of 128 ft?

Solution:

Vertical motion formula: $h = -16t^2 + vt + s$

where h - is the height of the object

t - is the time it takes the object to rise or fall to a given height.

v - the starting velocity

s - the starting height

$h = – 16t^2 + vt + s$	use the vertical motion formula
$128 = – 16t^2 + 96t + 0$	substitute the given
$0 = – 16t^2 + 96t – 128$	Addition Property of Equality
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	The quadratic formula

$$t = \frac{(-96) \pm \sqrt{(96)^2 - 4(-16)(-128)}}{2(-16)}$$ substitute the given

$$t = \frac{-96 \pm \sqrt{9216 - 8192}}{-32}$$ simplify

$$t = \frac{-96 \pm \sqrt{1024}}{-32}$$

$$t = \frac{-96 + 32}{-32} \; or \; t = \frac{-96 - 32}{-32}$$

t = 2 or t = 4

The rocket is 128 ft. above the ground after 2 sec. and after 4 sec.

3. If twice the reciprocal of a number is added to the number , the result is $3\frac{2}{3}$.

Find the number.

Solution:

Let x represent the number.

$\frac{1}{x}$ is the reciprocal of the number

$2 \cdot \frac{1}{x} = \frac{2}{x}$ $\rightarrow$ twice the reciprocal of the number

Working equation:

$$\frac{2}{x} + x = 3\frac{2}{3}$$ translate into mathematical symbols

$$3x\left[\frac{2}{x} + x = 3\frac{2}{3}\right]$$ simplifying with LCD = 3x

$6 + 3x^2 = 11x$

$3x^2 - 11x + 6 = 0$ write in standard form

$(3x - 2)(x - 3)$ use factoring method

$3x - 2 = 0$ or $x - 3 = 0$ zero – product property

$x = \frac{2}{3}$ or $x = 3$

Check: If x = 3, its reciprocal is $\frac{1}{3}$

$$2\left(\frac{1}{3}\right)+3=3\frac{2}{3}$$

If $x=\frac{2}{3}$, its reciprocal is $\frac{3}{2}$.

$$2\left(\frac{3}{2}\right)+\frac{2}{3}=3\frac{2}{3}$$

$$\frac{6}{2}+\frac{2}{3}=3\frac{2}{3}$$

$$3+\frac{2}{3}=3\frac{2}{3}$$

$$3\frac{2}{3}=3\frac{2}{3}$$

4. One boat finished a race one hour sooner than another boat. The first boat traveled 5 kilometers per hour faster than the slower boat over the 40-kilometer course. Find the rate of each boat to the nearest tenth of a kilometer.

Solution: Let r be the rate of the slower boat in kph

r + 5 be the rate of the faster boat in kph

Working Equation: Distance = rate x time.

(time of boat(faster)) = (time of boat(slower)) – 1

$\frac{D}{R}=T$ since d = rt

Slower boat $40 \div r = \frac{40}{r}$ substitute the given

Faster boat $40 \div (r+5) = \frac{40}{r+5}$ substitute the given

$r(r+5)\left[\frac{40}{r+5} = \frac{40}{r} - 1\right]$ substitute the values from the table into the equation

$40r = 40(r + 5) - r(r + 5)$ multiply by LCD r(r+5)

$40r = 40r + 200 - r^2 - 5$

$r^2 + 5r - 200 = 0$ simplify and write the equation in standard form

$$r = \frac{-5 \pm \sqrt{5^2 - 4(1)(-200)}}{2(1)}$$ use the quadratic formula

$$r = \frac{-5 \pm \sqrt{25 + 800}}{2}$$

$$r = \frac{-5 \pm \sqrt{825}}{2}$$

$$r = \frac{-5 \pm \sqrt{25(33)}}{2} = \frac{-5 \pm 5\sqrt{33}}{2}$$

$$r = \frac{-5 + 5\sqrt{33}}{2} \text{ or } \frac{-5 - 5\sqrt{33}}{2}$$

$r \approx 11.9$ kilometer per hour Explain the answers.

$r + 5 \approx 16.9$ kilometer per hour

Let's Practice for Mastery 10.

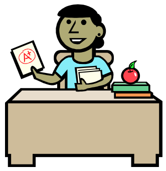
Solve the following:

1. Two vehicles traveling at the same speed depart from the same location at the same time. One travels due east and the other due south. After 57 minutes, they are 200 km apart. What distance has each vehicle traveled? (Hint: d = rt)
2. The length of a rectangle is 3 cm more than the width. The area of this rectangle is 25 sq. cm. Determine to the nearest hundredth of a cm the dimensions of the rectangle.
3. The number of diagonals d of a polygon of n sides is given by the formula $d = \frac{n^2 - 3n}{2}$. If a polygon has 27 diagonals, how many sides does it have?

Let's Check Your Understanding 10.

Solve the following problems:

1. Find the interest earned in one year on a deposit of Php 15000 at 6% annual interest. Find the amount in the bank after one year. (Hint : I = PRT)
2. The product of the first and third of three consecutive positive odd integers is 77. Find the integers.
   (Hint: let x represent the first odd integer. x + 2 represent the second odd integer._______ represent the third odd integer.)
3. If an object is thrown straight up with a velocity of 160 feet per second, its highest h(in feet) above the ground is given by the formula $h = 160t - 16t^2$, where t represents the time (in seconds) since it was thrown. How long will it take for the object to hit the ground.?



Let's Summarize

1. Squares of Sums and Differences

For any real numbers a and b;

$(a+b)^2 = a^2 + 2ab + b^2$ $(a-b)^2 = a^2 - 2ab + b^2$

2. Difference of Two Squares:

For any real numbers a and b,

$(a + b)(a - b) = a^2 - b^2$

3. A binomial that is the difference between two squares, $a^2 - b^2$, for any real numbers, a and b, can be factored as the product of the sum (a + b) and the difference (a − b) of the terms that are being squared:

$$a^2 - b^2 = (a + b)(a - b)$$

4. Factoring special products:

For any real numbers a and b:

Square of a sum: $a^2 + 2ab + b^2 = (a + b)^2$

Square of a difference: $a^2 - 2ab + b^2 = (a - b)^2$

5. Solving Quadratic Equations by Extracting Square Roots

If $x^2 = r$ and $r > 0$, then $x = \pm\sqrt{r}$.

6. Zero – Product Property

If a and b are any real numbers, and ab = 0, then either a = 0 or b =0.

7. Tips used in solving quadratic equations by factoring:

   1. Write the given quadratic equation in standard form.
   2. Factor the quadratic equation.
   3. Use the Zero – Product Property to write a pair of linear equations.
   4. Solve the linear equations.
   5. The solution to the linear equations is the solution to the given equation.

8. To solve a quadratic equation $ax^2 + bx + c = 0$ by completing the square, we have the following steps :

1. If $a \neq 1$, multiply by $\frac{1}{a}$ on both sides so that the x^2 – coefficient is 1.
2. When the x^2 – coefficient is 1, rewrite the equation in the form

   $x^2 + bx = -c$, or $x^2 + \frac{b}{a}x = -\frac{c}{a}$ (if step (1) has been applied.)
3. Take half of the x – coefficient and square it. Add the result to both sides of the equation.
4. Factor the trinomial square and combine like terms.
5. Solve the resulting quadratic equation by extracting square roots.
6. Check each solution.

9. The quadratic formula as what the name implies is a formula for solving any quadratic equation of the form $ax^2 + bx + c = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$, where a, b, c are real numbers and $a \neq 0$

10.

Nature and Number of roots of a Quadratic Equation		
Discriminant	**Nature of Roots**	**Number of Solution**
3. If $b^2 - 4ac = 0$	Real and Equal	One solution
4. If $b^2 - 4ac > 0$ and		
c. perfect square no.	Rational and Unequal	Two distinct solutions
d. not a perfect square	Irrational and unequal	
3. If $b^2 - 4ac < 0$	Imaginary	Two distinct solutions

Unit Test

I – Multiple Choice: On your answer sheet, write the letter of the correct answer.

1. The statement $2x + 2y = 2(x + y)$ uses which property of real numbers?

   a. distributive property
   b. associative property
   c. commutative property
   d. closure property

2. If one side of a square place mat is $(4x - 7)$ cm, which of the following represents the area covered by the place mat?

   a. $(16x^2 - 28x + 49)\ cm^2$
   b. $(16x^2 + 28x + 49)\ cm^2$
   c. $(16x^2 - 56x + 49)\ cm^2$
   d. $(16x^2 + 56x + 49)\ cm^2$

3. Raul solved the roots of the quadratic equation $2x^2 = 18$. In his solution, instead of dividing both sides by 2, he subtracted 2 from both sides. What wrong answers did he get?

   a. – 16 and 16
   b. – 3 and 3
   c. – 4 and 4
   d. – 20 and 20

4. Which one is the correct factored form of $x^2 + 3x - 10$?

   a. $(x - 5)(x - 2)$
   b. $(x - 5)(x + 2)$
   c. $(x + 5)(x + 2)$
   d. $(x + 5)(x - 2)$

5. Suppose that $2x^2 - 36 = x^2 - 49$. Which statement is correct ?

   The equation has

   a. two real solutions.
   b. exactly one real solution.
   c. no real solutions.
   d. solutions that cannot be determined.

6. The sum of two positive integers is 24. Which of the following represents their largest product?

   a. 140
   b. 240
   c. 154
   d. 144

II - Answer the following. Show all work. (2 points each)

1. Factor completely: $16m^2 - 81n^2$
2. Describe the nature of roots of the equation $2x^2 + 4x + 2 = 0$.
   Explain your answer. (Hint: Use $b^2 - 4ac$.)
3. Explain why there are no integer values of q for which $x^2 + 7x + q$ is a perfect square.
4. Find an equation with integer coefficients whose roots are
   a. -5 and 7 b. $-2 \pm \sqrt{3}$
5. What are the possible values of n that make $x^2 + nx - 10$ a factorable expression over the integers ?

III – Solve the following. Show all work. (3 points each)

1. A construction worker throws a bottled water toward a fellow worker who is 8 meters above the ground. The starting height of the bottled water is 2 m. Its starting velocity is 10 meters per second. Will the bottled water reach the second worker? Explain your answer.

2. The art staff at Central High School is determining the dimensions of the paper to be used in the senior year book. The area of each sheet is to be 432 cm^2. The staff has agreed on margins of 3 cm on each side and 4 cm on top and bottom. If the printed matter is to occupy192 cm on each page, what must be the overall length and width of the paper?

3. The mayor of a certain town is planning a circular duck pond for a new park. The depth of the pond will be 4 ft. Because of water resources the maximum volume will be $20{,}000 \text{ ft}^3$. Find the radius of the pond. (Hint : Use the equation $V = \pi r^2 h$, where V is the volume, r is the radius, and h is the depth. Use $\pi = 3.14$)

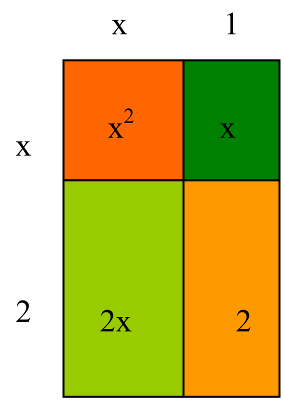
ANSWER KEY

Let's Practice for Mastery 1.

A. 1. d. $r^2 - 12r + 36.$

2. a. $9x^2 - 121$

3.



B. 4. $(7y+4x)(3y-2x) = 21y^2 - 14xy + 12xy - 8x^2$

$= 21y^2 - 2xy - 8x^2$

5. $(3m+2n)(3m-2n) = 9m^2 - 4n^2$

6. $(5x+2y)^2 = 25x^2 + 2(10xy) + 4y^2$

$= 25x^2 + 20xy + 4y^2$

7. $(8p-7q)^2 = 64p^2 - 2(56pq) + 49q^2$

$= 64p^2 - 112pq + 49q^2$

8. $(.5x+.2y)(.5x-.2y) = .25x^2 - .04y^2$

C. 9. Disagree.

$(a+b)^2 = (a+b)(a+b)$

$= a^2 + 2ab + b^2 \neq a^2 + b$

10. Agree

$(x-y)^2 = (x-y)(x-y)$

$= x^2 - 2xy + y^2$

$(y-x)^2 = y^2 - 2xy + x^2$

But Addition is commutative, which means 2 + 3 = 3 + 2 = 5

Hence $y^2 - 2xy + x^2 = x^2 - 2xy + y^2$

Therefore, $(x-y)^2 = (y-x)^2$

Let's Check Your Understanding 1.

I. 1. $(x+10)^2 = x^2 + 2(10x) + (10)^2$

$= x^2 + 20x + 100$

2. $(3w-4)^2 = (3w)^2 - 2(12w) + (-4)^2$

$= 9w^2 - 24w + 16w$

3. $(7x+y)(7x-y) = 49x^2 - y^2$

4. $(2x+7)(2x+7) = 4x^2 + 2(14x) + 49$

$= 4x^2 + 28x + 49$

5. $(2a-b)(2a+b) = 4a^2 + 2ab - 2ab - b^2$

$= 4a^2 - b^2$

6. $(.3m^2+2)(.3m^2-2) = .09m^4 - 4$

II 7. $A = s^2$

$= (3x+2)^2$

$= 9x^2 + 2(6)x + 4$

$A = 9x^2 + 12x + 4\ \ cm^2$

8. $A = lw$

$= (7x-2)(x+5)$

$= 7x^2 + 35x - 2x - 10$

$= 7x^2 + 33x - 10\ \ cm^2$

9. a. $A\ of\ big\ rectangle = lw$

$= (3x+4)(2x+3)$

$= 6x^2 + 9x + 8x + 12$

$= 6x^2 + 17x + 12$

b. $A\ of\ small\ rectangle = lw$

$= (2x+1)(2x-1)$

$= 4x^2 - 1\ \ cm^2$

10. c. *A of shaded region= Area of big rectangle – Area of small rectangle*

$= (6x^2 + 17x + 12) - (4x^2 - 1)$

$= (6x^2 + 17x + 12 - 4x^2 - 1)$

$= (2x^2 + 17x + 13)\ \ cm^2$

Let's Practice for Mastery 2.

1. c. $x^2 - 8x + 16$
2. c. $x - 3$
3. a. $(m+8)(m-8)$

   b. $(x+y)(x+y) or (x+y)^2$

   c. $(4k-2)(4k-2) or (4k-2)^2$

   d. $(4y+6)(4y+6) oi (4y+6)^2$

   e. $(c+9)(c+5)$

   f. $(2p+7)(p+1)$
4. $x^2 + pq + 81 = (x+9)(x+9)$

4. $x^2 + pq + 81 = (x+9)(x+9)$

$= x^2 + 2(9x) + 81$

$= x^2 + 18x + 81$

$\therefore p = 18$

5. $(3x^2 + 6x) = 3x(x+2)$

$= (3x)(x+2)$

Hence, the missing factor is 3x.

Let's Check your Understanding 2.

1. a. $(9r^2 - 16s^2) = (3r+4s)(3r-4s)$

2. $x^2 + 2x + m = x^2 + \frac{2}{2}x + (1)^2$

$= x^2 + 2x + 1 = (x+1)^2$

Therefore, m=1

3 a. $(x+11)(x+11) or (x+11)^2$

b. $(2x-9)(2x-9) or (2x-9)^2$

c. $(3m-10)(m+2)$

4. b.

5. c.

Let's Practice for Mastery 3.

A. 1. $x^2 - 7 = 0$. Quadratic. The highest exponent of the variable x is 2.

3. $5x^2 = 25$. Quadratic. The highest exponent of the variable x is 2.

5. $5x^2 + 6x = -1$. Quadratic.

B. 1. Standard form.

2. No. The Standard form is $5x^2 - 25 = 0$.

3. No. The Standard form is $5x^2 + 6x + 1 = 0$.

Let's Check Your Understanding 3.

A. 1. c. $3x = 2x + 5$

2. b. It is in $ax^2 + bx + c = 0$, which is in the standard form.

3. $6x^2 - 5x + 12 = 0$.

4. No. If a = 0, $3x + 5 = 0$ which is linear.

Let's Practice for Mastery 4.

1. a. $b^2 = 14$: $2(b^2 - 5) = 18$

$b^2 - 5 = 9$ Multiplication Property

$b^2 = 14$ Addition Property

2. a. $(x-3)^2 = 0$: $(x-3)^2 = 0$

$x - 3 = 0$ Extracting Square Roots

$x = 3$ Addition Property

3. $x^2 = 49$

$\sqrt{x^2} = \sqrt{49}$ The roots are -7 and 7.

$x = \pm 7$

4. $2m^2 - 98 = 0$

$2m^2 = 98$	Addition Property of Equation
$m^2 = 49$	Multiplication Property
$m = \pm 7$	Extracting Square roots

5. $3a^2 - 5 = 43$

$3a^2 = 48$	Addition Property of Equality
$a^2 = 16$	Multiplication Property
$a = \pm 4$	Extracting Square Roots

The roots are -4 and 4.

Let's Check Your Understanding 4.

1. $3(x-5)^2 = 147$

$(x-5)^2 = \frac{147}{3}$

$(x-5)^2 = 49$	Multiplication Property
$x - 5 = \pm 7$	Extracting Square Roots
$x = 12$	Addition Property of Equality
$x = -2$	

2. $-4t^2 = -48$

$t^2 = 12$	Multiplication Property
$t = \pm\sqrt{12}$	Extracting Square Roots
$t = \pm 2\sqrt{3}$	Simplifying

3. By working backward:

$x = -11$ or $x = 11$

$(x+11)(x-11) = 0$

$x^2 - 121 = 0$

$\therefore c = 121$

4. $x^2 + 25 = 0$

$x^2 = -25$

$x = \pm\sqrt{-25}$

$x = -5i$ and $5i$

Michael forgot that the square root of a negative number is imaginary.

5. $3x^2 - 27 = 0$

$3x^2 = 27$

$x^2 = 9$

$x = \pm 3$

Therefore, the correct roots are -3 and 3. Anna multiplied both sides of the equation by 3 instead of by $\frac{1}{3}$. That is why, she got $x^2 = 81$, instead of $x^2 = 9$.

Let's Practice for Mastery 5

A. 1. a. $x + 3 = 0$ or $x - 7 = 0$

2. c. $5y = 0$ or $x - 3 = 0$

B. 3. $2x\ (x - 5) = 0$

$2x = 0$ or $x - 5 = 0$

$x = 0$ $\quad$ $x = 5$

4. $(x - 3)(2x - 4) = 0$

$x - 3 = 0$ or $2x - 4 = 0$

$x = 3$ or $x = 2$

5. $a^2 + 12a + 35 = 0$

$(a+7)(a+5) = 0$

$a + 7 = 0$ or $a + 5 = 0$

$a = -7$ 0r $a = -5$

6. $5x^2 - 25x + 20 = 0$

$(5x - 5)(x - 4) = 0$

$5x - 5 = 0$ or $x - 4 = 0$

$x = 1$ or $x = 4$

7. $81 - 4x^2 = 0$

$4x^2 = 81$

$x^2 = \frac{81}{4} \rightarrow x = \pm \frac{9}{2}$

8. $36x^2 = -12x - 1$

$36x^2 + 12x + 1 = 0$ — reduce to standard form

$(6x + 1)(6x + 1) = 0$ — factoring

$6x + 1 = 0$ or $6x + 1 = 0$ — zero product property

$x = -\frac{1}{6}$ or $x = -\frac{1}{6}$

C. 9. (2x+__)(x+__) = 0

(2x + 3)(x + -5), use 3 and -5 as factors of -15.

10. a. Since x is the first odd numbers, x+2= the next odd numbers.

b. product: $x(x+2) = 63$

c. $x(x+2) = 63$

$x^2 + 2x - 63 = 0$

$(x + 9)(x - 7) = 0$

$x + 9 = 0$ or $x - 7 = 0$

$x = -9$ or $x = 7$

But $x \neq -9$, therefore, the consecutive positive odd numbers are 7 and 9.

Let's Check your Understanding 5.

A. 1. $(3x + \underline{6})(x + \underline{4}) = 0$

2. $x^2 + \underline{6}x - 16 = 0$

$(x + \underline{8})(x - \underline{2}) = 0$

B. 3. $3x(x + 6) = 0$

$3x = 0$ or $x + 6 = 0$

$x = 0$ or $x = -6$

4. $(x + 2)(x - 5)$

$x + 2 = 0$ or $x - 5 = 0$

$x = -2$ or $x = 5$

C. 5. $x^2 + 22x + 121 = 0$

$(x + 11)(x + 11) = 0$

$x + 11 = 0$ or $x + 11 = 0$

$x = -11$ or $x = -11$

6. $4x^2 - 81 = 0$

$(2x + 9)(2x - 9) = 0$

$2x + 9 = 0$ or $2x - 9 = 0$

$x = \frac{-9}{2}$ or $x = \frac{9}{2}$

7. $16x^2 - 40x + 25 = 0$

$(4x - 5)(4x - 5) = 0$

$4x - 5 = 0$ or $4x - 5 = 0$

$x = \frac{5}{4}$ or $x = \frac{5}{4}$

8. $3x^2 + 5x - 50 = 0$

$(3x - 10)(x + 5) = 0$

$3x - 10 = 0$ or $x + 5 = 0$

$x = \frac{10}{3}$ or $x = -5$

D. 9. Let x = one positive integer

$31 - x$ = other positive integer

$x^2 + (31 - x)^2 = 625$

$x^2 + (961 - 62x + x^2) = 625$

$2x^2 - 62x + 961 - 625 = 0$

$$\frac{2x^2 - 62x + 336}{2} = 0$$

$x^2 - 31x + 168 = 0$

$(x - 7)(x - 24) = 0$

$x - 7 = 0$ or $x - 24 = 0$

$x = 7$ or $x = 24$

10. The smaller of the two numbers is 7.

Let's Practice for Mastery 6.

1. a. $x^2 - 12x + (6)^2$

$x^2 - 12x + 36$

b. $m^2 + \frac{9}{2}m + \left(\frac{9}{4}\right)^2$

$m^2 + \frac{9}{2}m + \left(\frac{81}{16}\right)$

2. a. $x^2 + 18x + \left(\frac{18}{2}\right)^2$

$x^2 + 18x + (9)^2$

$x^2 + 18x + 81$

b. $y^2 - \frac{2}{5}y + \left(\frac{-2}{10}\right)^2$

$y^2 - \frac{2}{5}y + \frac{4}{100}$

3. a. $4x^2 - 10x = 24$

$4x^2 + 2((2 \cdot 4)x + 16$

b. $x^2 + 8x + b$

$x^2 + 8x + \left(\frac{8}{2}\right)^2$

$4x^2 + 16x + 16$ $\qquad$ $x^2 + 8x + (4)^2$

Hence, b = 16 $\qquad$ $x^2 + 8x + 16$, thus, b = 16

4. a. $x^2 - 10x = 24$

$$x^2 - 10x + \left(\frac{-10}{2}\right)^2 = 24 + (-5)^2$$

$$(x - 5)^2 = 24 + 25$$

$$\sqrt{(x-5)^2} = \sqrt{49}$$

$$x - 5 = \pm 7$$

$$x - 5 = 7 \text{ or } x - 5 = -7$$

$$x = 12 \text{ or } x = -2$$

b. $x^2 + 7x - 2 = 0$

$$x^2 + 7x + \left(\frac{7}{2}\right)^2 = 2 + \frac{7}{2}$$

$$x^2 + 7x + \frac{49}{4} = \frac{8+49}{4}$$

$$\sqrt{\left(x+\frac{7}{2}\right)^2} = \sqrt{\frac{57}{4}}$$

$$x + \frac{7}{2} = \frac{\pm\sqrt{57}}{2}$$

$$x = \frac{-7 \pm \sqrt{57}}{2}$$

c. $4x^2 + 12x = 7$

$$x^2 + \frac{12}{4}x = 7$$

$$x^2 + 3x + \left(\frac{3}{2}\right)^2 = 7 + \frac{9}{4}$$

$$\left(x+\frac{3}{2}\right)^2 = \frac{28+9}{4}$$

$$\sqrt{\left(x+\frac{3}{2}\right)^2} = \sqrt{\frac{37}{4}}$$

$$x + \frac{3}{2} = \frac{\pm\sqrt{37}}{2}$$

$$x = \frac{-3 \pm \sqrt{37}}{2}$$

5. Completing the square method works even if the given quadratic equation is not factorable because I am reducing the left side into an expression which is factorable by adding a constant to both sides, thereby, making the left a perfect square.

Let's Check Your Understanding 6.

1. a. $x^2 - 20x +$ ___ $\qquad$ b. $x^2 + \frac{3}{4}x +$ ___

$x^2 - 20x + (-10)^2$ $\qquad$ $x^2 + \frac{3}{4}x + \left(\frac{3}{8}\right)^2$

$x^2 - 20x + 100$

$x^2 + \frac{3}{4}x + \frac{9}{64}$

c. $3x^2 - 2x +$ ___

$\frac{3x^2}{3} - \frac{2x}{2 \cdot 3} + \left(\frac{-1}{3}\right)^2$

$x^2 - \frac{1}{3}x + \frac{1}{9}$

d. $6x^2 + 5x +$ ___

$x^2 + \frac{5}{6}x + \left(\frac{5}{12}\right)^2$

$x^2 + \frac{5}{6}x + \frac{25}{144}$

2. a. $x^2 - 8x = 16$

$x^2 - \frac{8}{2}x + (-4)^2 = -12 + 16$

$x^2 - 8x + 16 = 4$

$\sqrt{(x-4)^2} = \sqrt{4}$

$x - 4 = \pm 2$

$x - 4 = 2$ or $x - 4 = -2$

$x = 6$ or $x = 2$

b. $x^2 + 18x + 72 = 0$

$x^2 + 18x = -72$

$x^2 + 18x + (9)^2 = -72 + 81$

$\sqrt{(x+9)^2} = \sqrt{9}$

$x + 9 = \pm 3$

$x + 9 = 3$ or $x + 9 = -3$

$x = -6$ or $x = -12$

c. $3x^2 - 8x = 16$

$x^2 - \frac{8}{3}x = \frac{16}{3}$

$x^2 - \frac{8}{3}x + \left(\frac{-8}{6}\right)^2 = \frac{16}{3} + \frac{64}{36}$

$\left(x - \frac{8}{6}\right)^2 = \frac{192 + 64}{36}$

$\sqrt{\left(x - \frac{8}{6}\right)^2} = \sqrt{\frac{256}{36}}$

$x - \frac{8}{6} = \pm\frac{16}{6}$

$x - \frac{8}{6} = \frac{16}{6}$ or $x - \frac{8}{6} = -\frac{16}{6}$

$x = \frac{16+8}{6}$ or $x = \frac{-16+8}{6}$

$x = 4$ or $x = \frac{-4}{3}$

3. a. $x^2 + bx + 55 = 0$

$b = 2\ (1)\ \left(\sqrt{55}\right)$

$b = 2\sqrt{55}$

b. $4x^2 + bx + 16 = 0$

$b = 2\ (2)\ (4)$

$b = 16$

4. The student is wrong. The solutions or the roots of a quadratic equation are additive inverse if the equation is of the form $x^2 - c = 0$ or $ax^2 - c = 0$

Let's Practice for Mastery 7.

1. a = 2 $\quad x = \dfrac{-3 \pm \sqrt{3^2 - 4(2)(1)}}{2(2)}$

   b = 3 $\quad x = \dfrac{-3 \pm \sqrt{9-8}}{4}$

   c = 1 $\quad x = \dfrac{-3 \pm 1}{4}$

   $x = -1$ or $x = \dfrac{-1}{2}$

2. $x^2 + 5x + 6 = 0$

   $x = \dfrac{-5 \pm \sqrt{25 - 4(1)(6)}}{2(1)}$

   $x = \dfrac{-5 \pm \sqrt{1}}{2}$

   $x = \dfrac{-4}{2}$ or $x = \dfrac{-6}{2}$

   x = -2 or x = -3

   By Factoring:

   (x + 2) (x +3) = 0

   x + 2 = 0 or x + 3 = 0

   x = –2 or x = –3

3. a. $x^2 + 5x + 6 = 0$

   $x^2 + 5x = -6$

   $x^2 + 5x + \left(\dfrac{5}{2}\right)^2 = -6 + \left(\dfrac{5}{2}\right)^2$

   $\sqrt{\left(x + \dfrac{5}{2}\right)^2} = \sqrt{\dfrac{-24 + 25}{4}}$

   $x + \dfrac{5}{2} = \pm \dfrac{1}{2}$

   $x = \dfrac{-5 \pm 1}{2}$; x = -2 or x = -3

   b. They have the same answer, which means that even if you use the quadratic formulas, factoring, or completing the squares, you will get the same answers.

4. If the given quadratic equation is not factorable, use the quadratic formula.

Let's Check Your Understanding 7.

1. a. $2x^2 = 3 - x$ therefore, a = 2, b = 1 and c = – 3

   $2x^2 + x - 3 = 0$

b. $x = \frac{-1 \pm \sqrt{1-4(2)(-3)}}{2(2)}$ $\quad x = \frac{4}{4}$ or $x = \frac{-6}{4}$

$x = \frac{-1 \pm \sqrt{1+24}}{4}$ $\quad x = 1$ or $x = \frac{-3}{2}$

$x = \frac{-1 \pm \sqrt{25}}{4}$

c. Yes

2. $x^2 - 4x - 7 = 0$ $\quad x = \frac{4 \pm 2\sqrt{11}}{2}$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-7)}}{2(1)}$ $\quad x = 2 \pm \sqrt{11}$

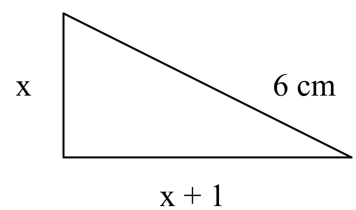
$x = \frac{4 \pm \sqrt{16+28}}{2}$ $\quad x \approx 2 \pm 3.317$

$x = \frac{4 \pm \sqrt{44}}{2}$ $\quad x \approx -1.317$ or $x \approx 5.317$

3. $x^2 + (x+1)^2 = 36$

$x^2 + x^2 + 2x + 1 = 36$

$2x^2 + 2x - 35 = 0$



$x = \frac{-2 \pm \sqrt{2^2 - 4(2)(-35)}}{2(2)}$ $\quad x = \frac{-1 \pm \sqrt{61}}{2}$

$x = \frac{-2 \pm \sqrt{244}}{4}$ $\quad x = \frac{-1 \pm 7.81}{2}$

$x = \frac{-2 \pm 2\sqrt{61}}{4}$ $\quad x = -4.41$ or $x = 3.41$

A leg cannot be -4.41, therefore, the shorter leg is about 3.71 m and the longer leg is about 4.71 m.

Let's Practice for Mastery 8.

1. 0
2. $2kx^2 + 6 = x^2 + 8x$

$2kx^2 - x^2 - 8x + 6 = 0$

$(2k - 1)x^2 - 8x + 6 = 0$

$a = 2k - 1, \quad b = -8, \quad c = 6$

$b^2 - 4ac = 0$, for the roots to be equal

$(-8)^2 - 4(2k - 1)(6) = 0$ substitute the values of a, b, and c

$64 - 24(2k - 1) = 0$

$64 - 48k + 24 = 0$

$-48k = -88$

$k = \dfrac{-88}{-48}$

$k = \dfrac{11}{6}$

3. $5x^2 - 4x + 1 = 0$

$a = 5, \quad b = -4, \quad c = 1,$ use $b^2 - 4ac$

$(-4)^2 - 4(5)(1) = 0$ substitute a, b, and c

$16 - 20 = -4$ evaluate

Since $b^2 - 4ac = -4$ which is less than 0, the roots are imaginary.

4. The roots of any quadratic equation, $ax^2 + bx + c = 0$, are the points on the x – axis where the graph of the quadratic function $y = ax^2 + bx + c$ crosses.

Let's Check Your Understanding 8.

1. $4x^2 + 3x + 1 = 0$

Using $b^2 - 4ac$, where a = 4, b = 3, and c = 1, the discriminant is

$3^2 - 4(4)(1) = 9 - 16$

$= -7$.

The discriminant, –7, means the roots of the given equation are imaginary.

2. $kx^2 + 3x - 7 = 0$

For the roots to be non real, $b^2 - 4ac < 0$.

So, $3^2 - 4(k)(-7) < 0$

$9 + 28k < 0$

$k < -\frac{9}{28}$

3. a. $4x^2 + 7x - 1 = 0$

Solving for $b^2 - 4ac$, $7^2 - 4(4)(-1)$

$49 + 16 = 65$

Since $65 > 0$, then the roots are irrational and unequal.

b. $x^2 - 10x + 25 = 0$

$b^2 - 4ac = (-10)^2 - 4(1)(25)$

$= 100 - 100$

$= 0$

The roots are real and equal.

4. Adding , $r_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$

$+ \quad r_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

$r_1 + r_2 = \frac{-b}{2a} + \frac{-b}{2a}$, the radical part becomes 0 since they have opposite signs.

Hence, $r_1 + r_2 = -\frac{2b}{2a}$ or $-\frac{b}{a}$

Let's Practice for Mastery 9.

A. 1. c 2. c

B. 3. sum : $-\frac{b}{a} = -8$ product : $\frac{c}{a} = 16$

4. sum: $-\frac{b}{a} = \frac{-(-5)}{2} = \frac{5}{2}$ product : $\frac{c}{a} = -\frac{7}{2}$

5. sum : $-\frac{b}{a} = -\frac{1}{5}$ product : $\frac{c}{a} = -\frac{1}{5}$

6. $\frac{2}{x} + x = 3$

$2 + x^2 - 3x = 0$ sum : $-\frac{b}{a} = \frac{-(-3)}{1} = 3$

$x^2 - 3x + 2 = 0$ product : $\frac{c}{a} = 2$

C. Let the roots be

7. r = 5 and s = – 8

(r + s) sum : $5 + (-8) = -3 = -\frac{b}{a}$

r • s product : $5(-8) = -40 = \frac{c}{a}$

So the quadratic equation is : $x^2 + 3x + -40 = 0$ or $x^2 + 3x - 40 = 0$

8. Let the roots be

$x = -3$ and $x = \frac{1}{2}$

$x + 3 = 0$ and $x - \frac{1}{2} = 0$ by working backward

$(x + 3)(2x - 1) = 0$ factored from FOIL Method

$2x^2 + 5x - 3 = 0$

Let's Check Your Understanding 9.

1. c 2. b

3. $x^2 - 5x - 14 = 0$

$(x - 7)(x + 2) = 0$

$x = 7$ or $x = -2$ No.

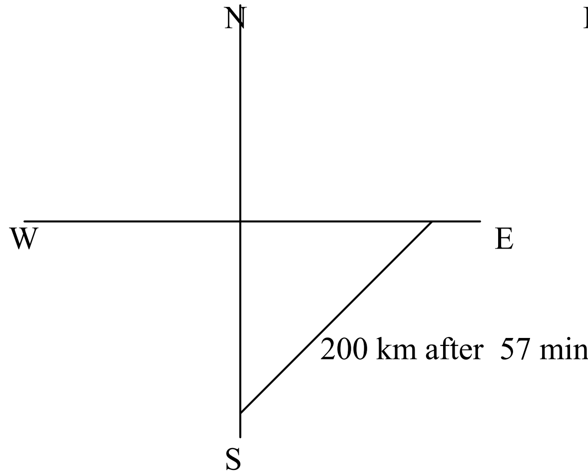
4. sum : $-\frac{b}{a} = \frac{-(-5)}{2} = \frac{5}{2}$

product : $\frac{c}{a} = 3$. Therefore the equation is $x^2 + \frac{5}{2}x + 3 = 0$

or $2x^2 + 5x + 6 = 0$

Let's Practice for Mastery 10.

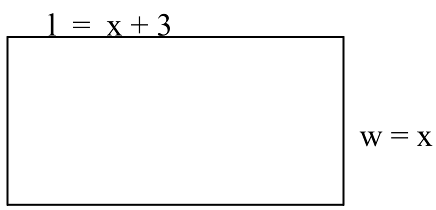
1.



Let x be the distance traveled by the vehicle.

$$x^2 + x^2 = 200^2$$
$$2x^2 = 40\ 000$$
$$x^2 = 20\ 000$$
$$x = 141.42$$

2.



$$A = lw$$
$$A = x(x+3) = 25$$
$$x^2 + 3x - 25 = 0$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-25)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9+100}}{2}$$

$$x = \frac{-3 \pm \sqrt{109}}{2}$$

$$x = \frac{-3 \pm 10.44}{2}$$

$$x = \frac{10.44 - 3}{2} \approx 3.72 \quad \text{or} \quad \frac{-13.44}{2} \approx -6.72$$

The width is about 3.72 cm and the length is about 6.72 cm.

Note : – 6.72 is rejected.

3. Number of diagonals :

$d = \frac{n^2 - 3n}{2}$; 27 is the number of diagonals

So, $27 = \frac{n^2 - 3n}{2}$

$n^2 - 3n = 54$ multiplying by2 to clear the fraction

$n^2 - 3n - 54 = 0$

$(n - 9)(n + 6) = 0$ Factoring

$n = 9$ or $n = -6$

Since the number of sides can not be negative, hence, the number of sides is 9. It is a nonagon.

Let's Check Your Understanding 10.

1. P = Php 15 000

Rate of interest = 6% or 0.06 per annum

Amount after 1 year

Formula : I = Prt

= 15 000 (0.06 (1)

I = Php 900

A = P + I

A = 15 000 + 900

A = Php 15 900

2. Let x = be the first integer

x + 2 = be the second integer

x + 4 = be the third integer

$x(x + 4) = 77$

$x^2 + 4x - 77 = 0$

$(x + 11)(x - 7) = 0$

x + 11 = 0 or x – 7 = 0

x = –11 or x = 7

The first odd integer is 7, the second is 9, the third is 11.

– 11 is rejected since numbers are positive.

3. Formula : $h = 160t - 16t^2$

Substituting : $160t - 16t^2 = 0$

$16t\ (10 - t) = 0$

$16t = 0$ or $10 - t = 0$

t = 0 sec t = 10 sec

The object will hit the ground after 10 seconds. When t = 0, the ball is still on the ground.

Let's Ponder II

I 1. a 2. c 3. c 4. d

5. $2x^2 - 36 = x^2 - 49$

$x^2 = -13$

$x = \pm\sqrt{13}i$ Correct answer: c

6. d. The numbers are 12 and 12

II 1. $16m^2 - 81n^2 = (4m + 9n)(4m - 9n)$

2. $2x^2 + 4x + 2 = 0$.

Using $b^2 - 4ac$, where $a = 2$, $b = 4$, $c = 2$

$(4)^2 - 4(2)(2) = 16 - 16$

$= 0.$

Since the discriminant, $b^2 - 4ac$, is equal to 0, then the roots are real and equal.

3. Using completing the square:

$x^2 + \frac{7}{2}x + \left[\frac{7}{2}\right]^2$

$x^2 + 7x + \frac{49}{4}$. Thus, q is $\frac{49}{4}$ which is not an integer.

4. Using working backward ;

a. $x = -5$ and $x = 7$

$(x + 5)(x - 7) = 0$

$x^2 - 2x - 35 = 0$

b. $x = -2 \pm \sqrt{3}$

$(x + 2)^2 = (\pm \sqrt{3})^2$

$x^2 + 4x + 4 = 3$ or

$x^2 + 4x + 1 = 0$

5. $x^2 + nx - 10 = 0$

Using the different factors of –10; –1 and 10, –2 and 5, 2 and –5, -10 and 1

a. $(x + 1)(x - 10) = 0$

$x^2 - 9x - 10 = 0$

b. $(x - 1)(x + 10) = 0$

$x^2 + 9x - 10 = 0$

c. $(x + 2)(x - 5) = 0$

$x^2 - 3x - 10 = 0$

d. $(x - 2)(x + 5) = 0$

$x^2 + 3x - 10 = 0$

Hence, the different values of n are { –9, 9, –3, 3 }.

III. 1. $h = -16t^2 + vt + s$ — equation of motion

$8 = -16t^2 + 10t + 2$ — substitute the given

$-16t^2 + 10t - 6 = 0$ — write in standard form

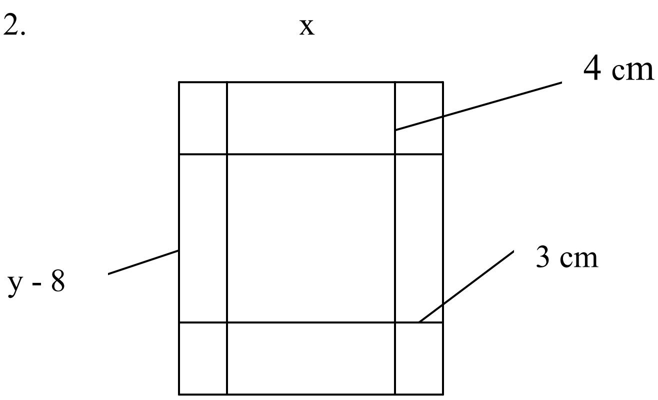
$b^2 - 4ac = (10)^2 - 4(-16)(-6)$ — Evaluate the discriminant

$= 100 - 384$

$= -284$

Since the discriminant is negative, the bottled water will not reach the second worker.

2.



Let x = width

y = length

y – 8 = reduced length

y – 6 = reduced width

Eq. 1. $(x - 6)(y - 8) = 192$; Eq. 2. $xy = 432$

From eq 1 : $xy - 8x - 6y + 48 = 192$; From eq 2: $x = \frac{432}{y}$

but $xy = 432$

hence, $432 - 6y - 8x + 48 = 192$

$$-6y - 8x = -288$$

$$6y + 8x = 288$$

Substitute $x = \frac{432}{y}$ from the above equation,

$$6y + 8\frac{432}{y} = 288$$

Solving the equation results to 18 and 32 as the width and the length of the paper respectively in cm.

Common Errors

1. $(-3)^2 = 6$ or sometimes –9. $(-3)^2$ is supposed to be $(-3)(-3) = 9$.
2. That $x^2 + y^2 = (x + y)^2$ like for example,

   $(x + 5)^2 = x^2 + 25$ which is supposed to be $(x + 5)(x + 5) = x^2 + 10x + 25$ which is the result of FOIL Method.
3. The product of two binomials like $(3y + 8)(2y - 3) = 6y^2 - 7y + 24$ or sometimes $6y^2 - 7y + 24$. Errors are either in getting the middle term or in the signs of the last term. Get the middle term by adding the products of outer and inner terms as discussed using the foil method. Hence, the correct product is $6y^2 + 16y - 9y - 24$ and is simplified as $6y^2 + 7y - 24$.
4. Solving quadratic equation by completing the square method.

   $x^2 + 9x + \frac{1}{2}(9)^2 = x^2 + 9x + \frac{81}{2}$ → they forgot to get the square of the denominator.

$$x^2 + 9x + \frac{1}{2}(9)^2 = x^2 + 9x + \frac{81}{4}$$

5. Solving $2x^2 = 18$ mistakenly subtract 2 from the right side which gives

$$x^2 = 18 - 2$$

$$x^2 = 16$$

hence, $x = \pm 4$

Solution should be $2x^2 = 18$ by multiplying both sides by $\frac{1}{2}$

$$x^2 = \frac{18}{2}$$

$$x = \pm 3$$

6. $3x^4$ is mistakenly equated to $(3x)^4$. The two are not equal since $(3x)^4$ means $(3x)(3x)(3x)(3x)$ and that $3x^4$ is $3(x)(x)(x)(x)$.

7. That $2x^0$ and $(2x)^0$ are the same.
But $2x^0 = 2$ while $(2x)^0 = 1$.