BUREAU OF SECONDARY EDUCATION
DEPARTMENT OF EDUCATION

DISTANCE LEARNING MODULE
MATHEMATICS 2

SYSTEMS OF LINEAR EQUATIONS AND INEQUALITIES

What is the cost of starting a business like a beach resort ? How much profit will you get in a year?

In today's highly competitive world, businesses are looking for solutions that save money or conserve scarce resources. Skillful problem solvers use such tools as systems of equations and inequalities to find optimum solutions. You can be one of them so take an early start toward career success by going over this unit.

This unit deals with solving linear equalities and inequalities by graphing and by using algebraic method. Applications of the solutions will enable you to solve problems in daily life that will make you a good decision maker.

U N I T I

Direction for Use of the Module

1. Read the explanation in each lesson carefully, then study the examples given before doing the ***let's practice for mastery.***

2. Answer the ***let's check your understanding***. You can check your own responses with those on answer key page.

3. If you got at least 80% of the items correct , you may proceed to the next lesson. If you got less than 80% correct , go over the particular lesson again and prepare for a unit test .

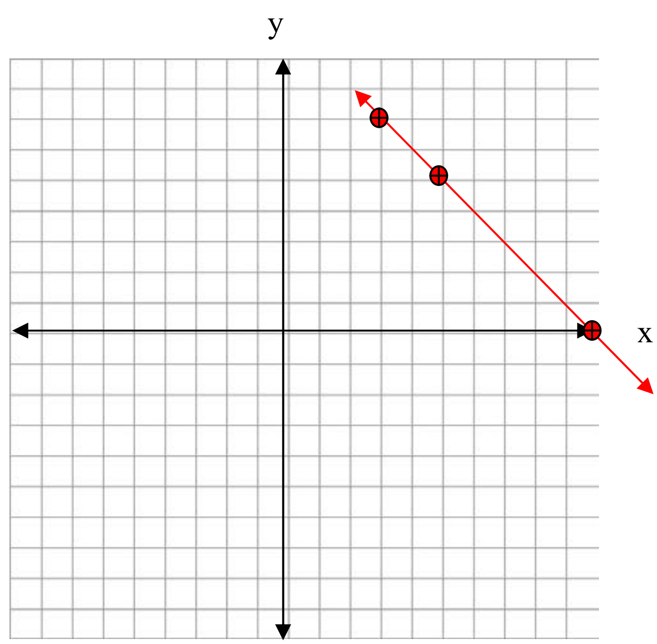
4. Take the unit test. If you got at least 80% of the items correct you can proceed to the next module. If you got a score less than 80% of the items, you should go over this particular module again and consult your teacher for clarification.

Lesson 1.1 Review on Graphing Linear Equations

Linear Equations can be graphed using table of values.

Example 1. Graph $x + y = 10$

x	3	5	10
y	7	5	0

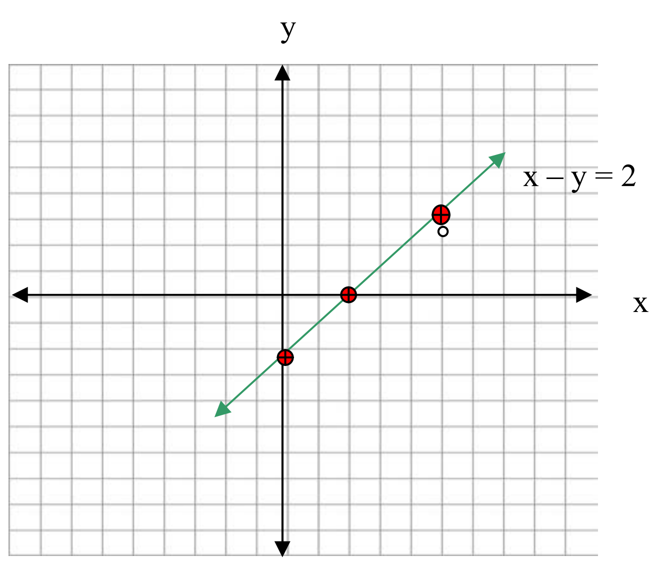


Plot the three points on the coordinate system and connect .

The graph of the linear equation is presented at the right.

Example 2 Graph $x – y = 2$

x	0	5	2
y	-2	3	0



Another way to graph linear equations in two variables is by using the slope-intercept method. Recall that slope is the steepness or inclination of a line. It is the ratio of the vertical change to the horizontal change, the rise over the run. In the form $y = mx + b$, m is the slope and b is the y-intercept. So, it is important that we transform the given equation into the y form in order to identify the slope and y-intercept. Remember, two points determine a line.

Examples: Transform the equations to y - form. Identify the slope and the y- intercept

1. $x + y = 24 \longrightarrow y = -x + 24$ $m = -1, b = 24$
2. $-4x - y = 5 \longrightarrow y = -4x - 5$ $m = -4, b = -5$
3. $-x + 2y = 8 \longrightarrow y = \frac{1}{2}x + 4$ $m = \frac{1}{2}, b = 4$

Let's Practice for Mastery 1

A. Transform the following equations to y-form.

1. $x + y = 8$	6. $2x + 3y = 0$
2. $x - y = -4$	7. $3x + y = 5$
3. $x + y = 2$	8. $5x - y = 3$
4. $2x - y = 8$	9.$2x + 3y = 12$
5. $3x + 2y = 5$	10. $4x - 3y = 6$

B. Identify the slope (m) and the y-intercept (b) in A.

Let's Check your Understanding 1

Identify the slope and the y-intercept

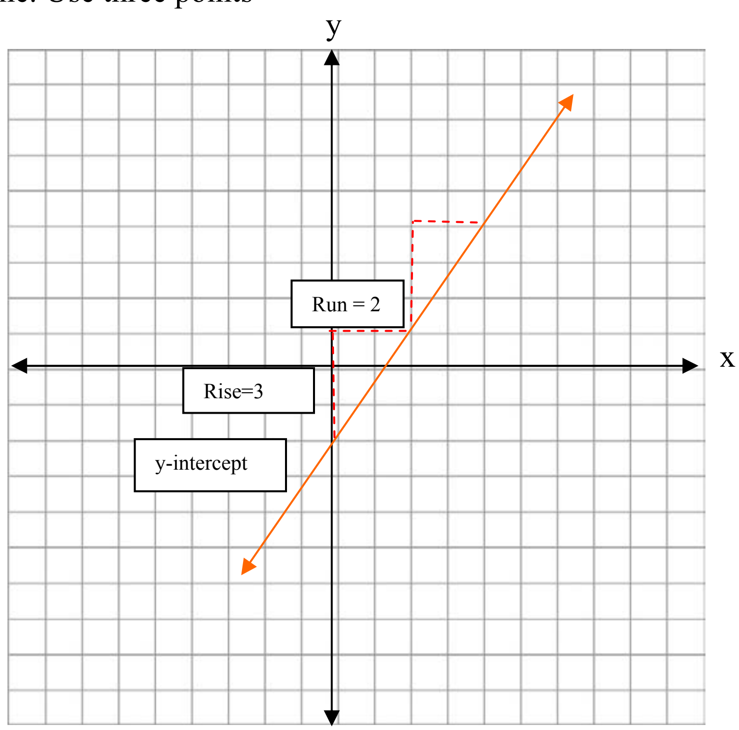
1. $y = -x + 3$	6. $-2x + 3y = -2$
2. $y = -3x + 12$	7. $2x + 5y + 5$
3. $y = -\frac{3}{2}x + 2$	8. $3y - 5x = 5\frac{21}{7}$
4. $-3x - 5y = 1$	9. $x - 3y = -2$
5 $x - 2y = 1$	10 $2x - 3y = 14$

Linear equations can be graphed using the slope-intercept form.

Let's graph the equation 3x -2y = 4.

Follow the steps.

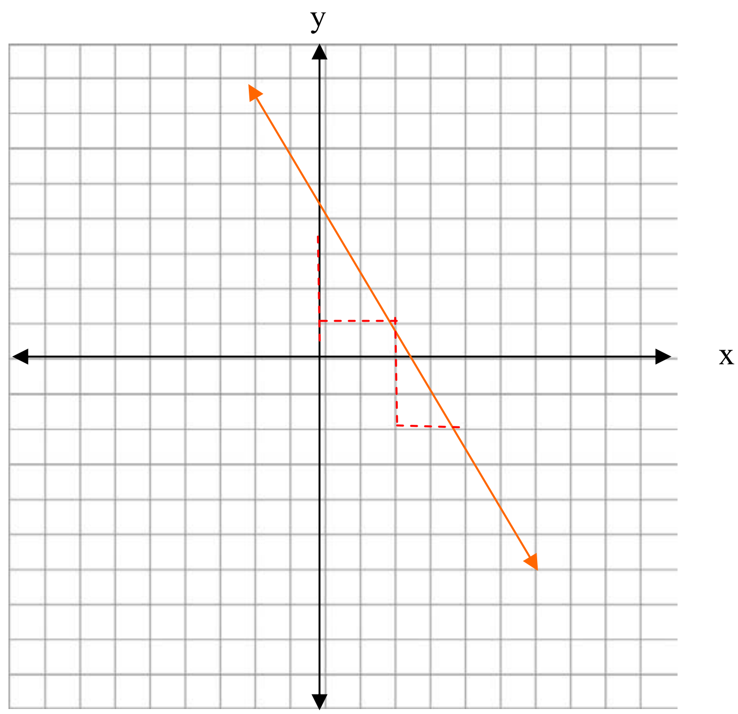
- Transform 3x -2y = 4 to y = *m*x + *b* $\longrightarrow$ $y = \frac{3}{2}x - 2$.
- The slope (*m*) of the equation is $\frac{3}{2}$ and the y- intercept (*b*) is -2.
- Locate the y-intercept, -2 at the y-axis. Mark the point associated with this.
- Starting from the y-intercept move 3 units upward (rise) and from this position move 2 units to the right (run) . Mark this point. (If the slope is negative move upward then to the left or move downward then to the right)
- Move again from this point using the ratio of the slope. Mark this point.
- Connect the points with a straight line. Use three points



This is now the graph of 3x – 2y = 4

Suppose our equation is -3x -2y = 4

Following the steps the graph now of -3x -2y = 4 is presented at the right. What have you noticed on the slope? Did you find any difference between the two equations and their graphs?



Let's Practice for Mastery 2

Graph the following linear equations using the slope- intercept method.

1. x – y = 8
2. -3x + 4y = 12
3. 2x – 3y = 6
4. 3x + 2y = 4
5. -x – 2y = -8

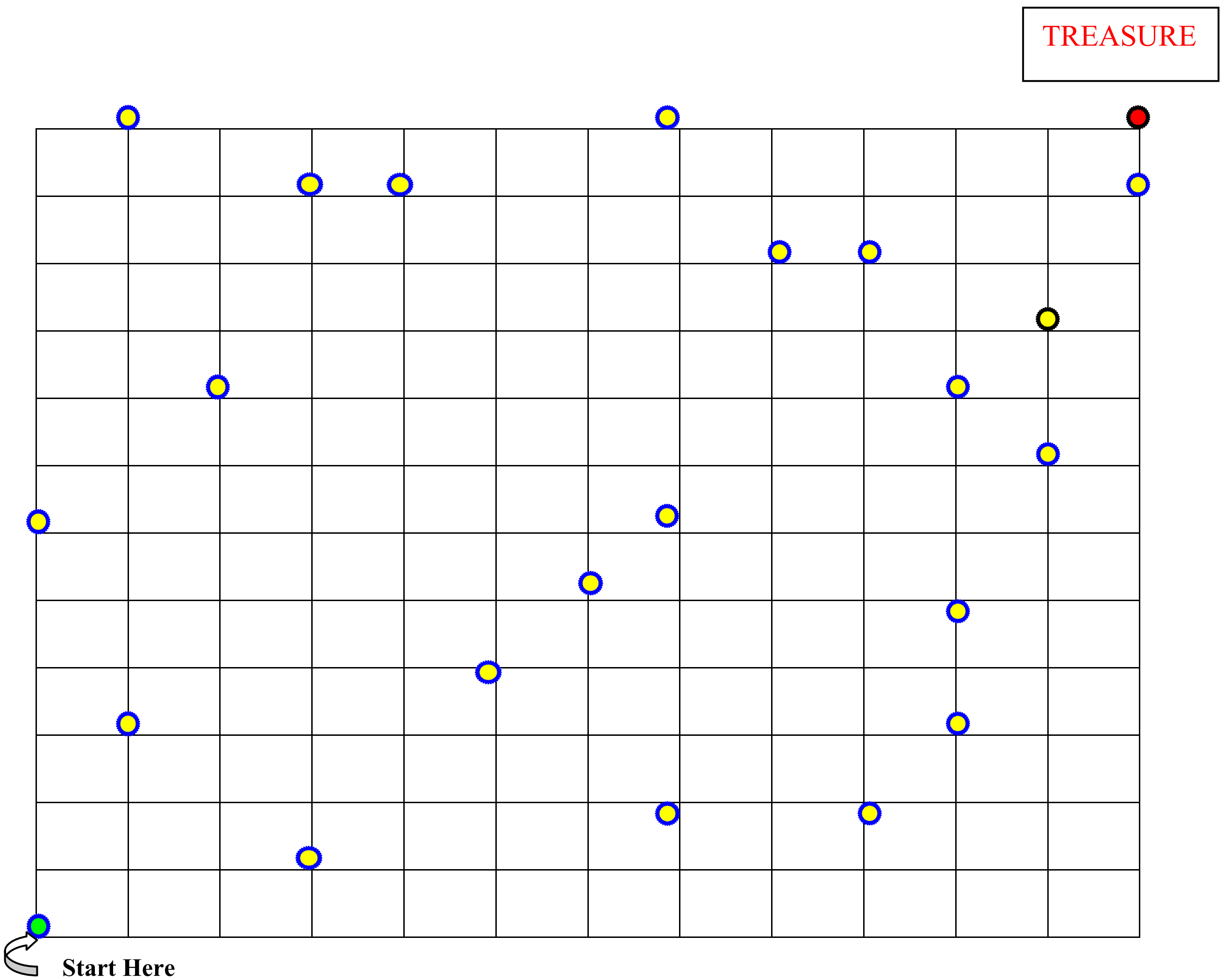
Let's Check Your Understanding 2

Graph the following linear equations using the slope- intercept method.

1. x + y = 3
2. 5x –y = 3
3. -x + y = 4
4. 2x + y = 3
5. -4x + y = - 6

Let's Do It. Treasure Hunt With Slopes

Using the definition of slope, draw lines with the slopes listed below. A correct solution will trace the route to the treasure.



1. 3
2. $\frac{1}{4}$
3. $-\frac{2}{5}$
4. 0
5. 1
6. -1

7. no slope   8. $\frac{2}{7}$ 9. $\frac{3}{2}$ 10. $\frac{1}{3}$ 11. $-\frac{3}{4}$ 12. 3

Lesson 1.2 Solving System of Linear Equations Graphically

Two or more equations such as $x + y = 10$ and $x – y = 2$ form a **system of linear equations**. To solve such a system, we find the ordered pair that makes both equations true. To solve a system of linear equations graphically, graph each equation on the same set of coordinate axes. For two lines that intersect at a point, the coordinates of that point are the **solution** of the system.

> To solve a system of linear equations graphically, graph each equation on the same set of coordinate axes. For two lines that intersect at a point, the coordinates of that point are the solution

Example 1.

Graph : $x + y = 10$

$x – y = 2$

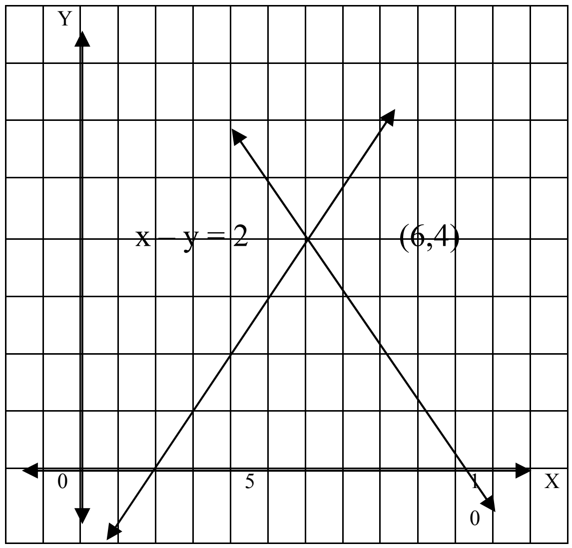
You can form table of values then graph.

$x + y = 10$

x	3	5	10
y	7	5	0

$x – y = 2$

x	0	5	2
y	-2	3	0



What is the intersection of the two lines? (6, 4)

(6, 4) is the solution set.

We can check that (6, 4) is the solution set by verifying that x = 6 and y = 4 makes both of the original equations true at the same time.

Example 3. Solve the system by graphing

$2x + y = 5$

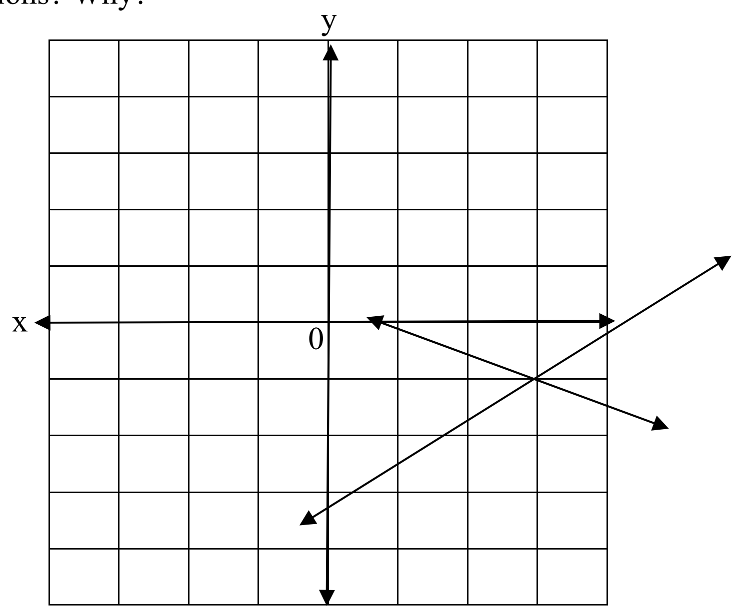
$y - x = -4$

Transform the equations into $y = mx + b$

$2x + y = 5 \longrightarrow y = -2x + 5$

$y - x = -4 \longrightarrow y = x - 4$

Each equation can be graphed using the slope – intercept method. As shown in the figure, what is the solution of the system of equations? Why?



Check: $x = 3$ $y = -1$

<u>2x + y = 5</u>

$2(3) + (-1) \overset{?}{=} 5$

$6 - 1 \overset{?}{=} 5$

$5 \overset{\surd}{=} 5$

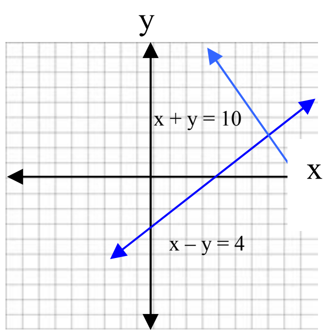
<u>y – x = -4</u>

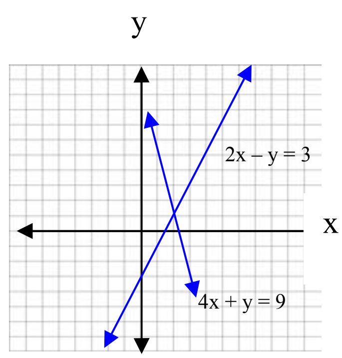
$-1 - 3 \overset{?}{=} -4$

$-4 \overset{\surd}{=} -4$

Let's Practice for Mastery 3

A. Read the solution set of each system of equations from its graph. Check by substituting in both equations.

1.


2.


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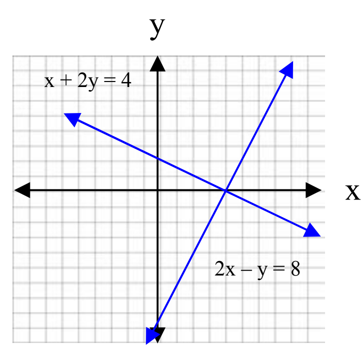

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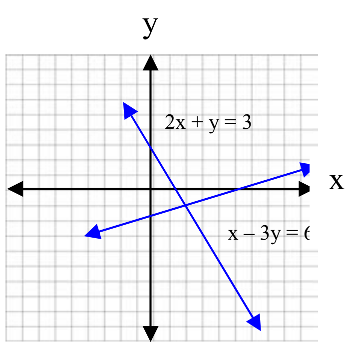
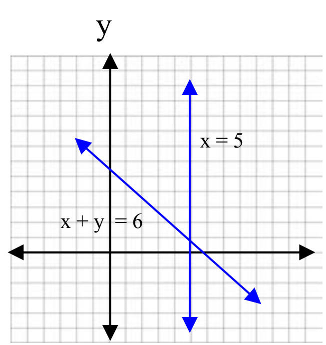
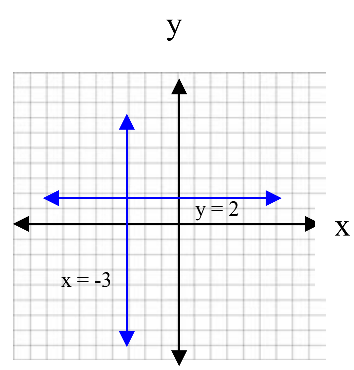


5.


6.


B. Solve each system by graphing.

7. x + y = 6
x – y = 4

8. x – y = 5
x + y = 7

9. x – y = 9
2x + y = -5

10. x + y = 2
2y - x =10

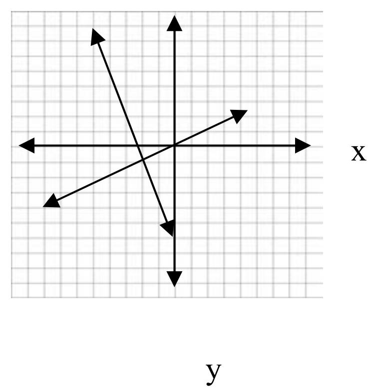
Let's Check Your Understanding 3

Match the graph of each system of equations . Identify the solution of the system.

1. $3x + y = -7$

   $y = \frac{1}{2}x$

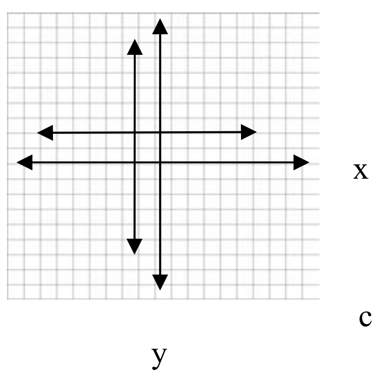
a.



2. $y = -x$

   $x - 4y = -5$

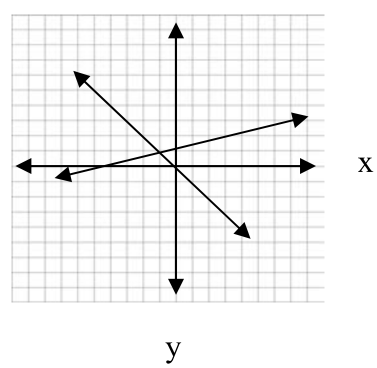
b.



3. $x = -1\frac{1}{2}$

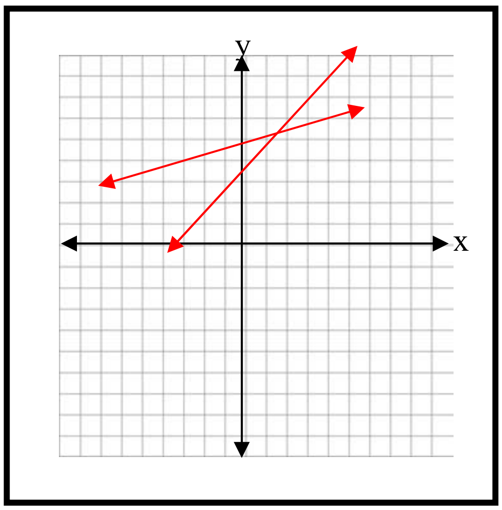
   $y = 2$

c.

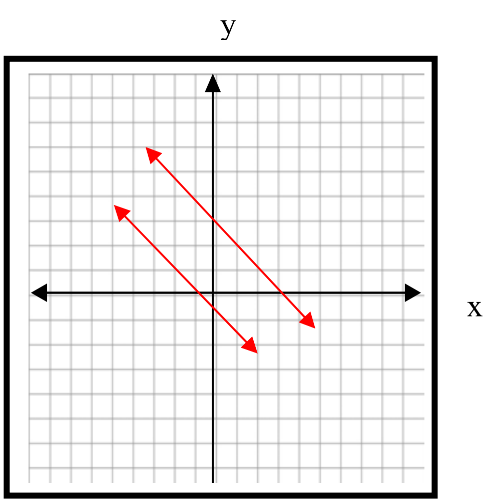


Lesson 1.3 Special Systems of Linear Equations

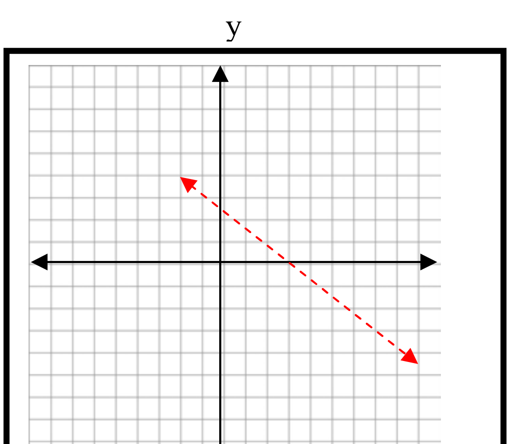
The figures below show how the graphs of two linear equations can be related.

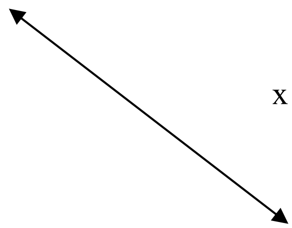


The graphs are intersecting lines. Thus, the system has one solution.



The graphs are parallel lines. Parallel lines are lines that lie in the same plane but have no point in common. Thus, the system has no solution.





The graphs are the same line. The lines coincide. Every point on the line is a solution; there are an infinite number of solutions.

Let's Do It 1 . Use your graphing paper.

Consider the following examples: Graph each system on the same set of coordinate axes.

Example 1

$2x - y = 1$ and $2x + y = 3$

Example 2

$-2x + y = -1$ and $-6x + 3y = 12$

Example 3

$4x + y = -2$ and $8x + 2y = -4$

The system of linear equations model many problems in agriculture, business, engineering industry and Science. Most of these linear systems have single solution. A system that has at least one solution is called **consistent**. If the system has only one solution, the system is **independent.** This is illustrated in Example 1.

In Example 2 the lines do not intersect, so the system has no solution. A system with no solution is called **inconsistent**. What have you noticed about the slopes of the lines? Let us have some more examples $y = -x + 6$ and $y = -x + 10$ can you identify the slope of each equation? How about this system : $-2x + y = -1$ and $-6x + 3y = 12$. What can you conclude about the slopes of parallel lines?

In Example 3 when each equation is written in slope – intercept form, the equations are identical. Every solution for one equation is also a solution for the other equation. The system is consistent because the system has at least one solution. In fact, the system has infinite solutions. The system is **dependent** because all the solutions are the same.

Let's Practice for Mastery 4

A. Think and Discuss.

1. Describe the graphs and slopes of a system of equations with no solution.
2. Describe the graphs, slopes and y – intercepts of a system of equations with an infinite number of solutions.
3. Describe the graphs of a system of equations that has one solution.
4. What is the difference between consistent and inconsistent systems of equations?
5. What do you mean when you say a system of equations is dependent or independent?

Let's Check Your Understanding 4

Fill the blanks with the correct word or group of words that will make the statement true.

1. The graphs of a system of equations with no solutions are ___________________.
2. A consistent system of linear equations has _______________ solution.
3. A system of linear equations in two variables which has an infinite number if solutions is called a/ an _________________ system.
4. The slope of parallel lines are ________________.
5. The graphs of a dependent system are _____________________ lines.

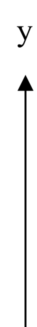
Let's Practice for Mastery 5

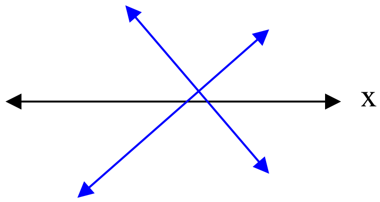
State whether the system is

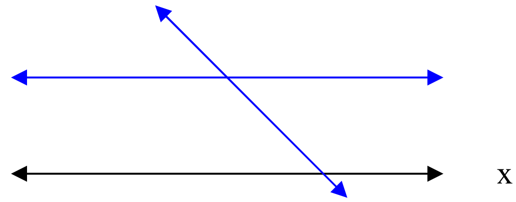
A. Consistent and Independent
B. Consistent and Dependent
C. Inconsistent

1.

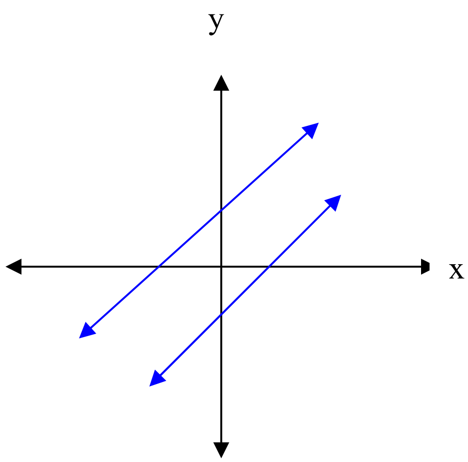
2.







3.



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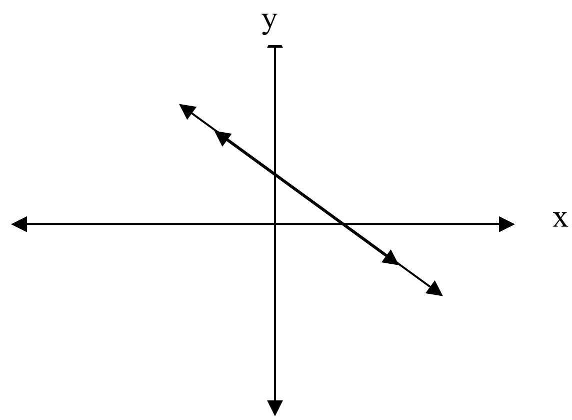

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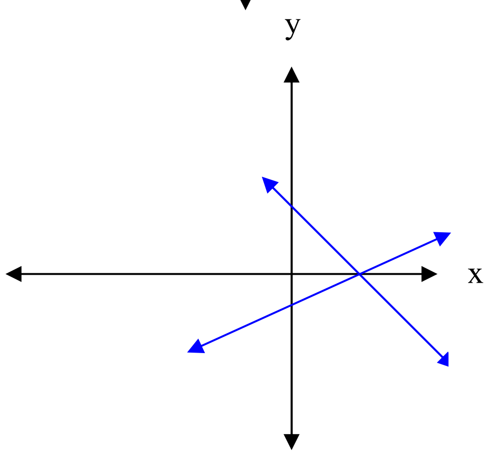

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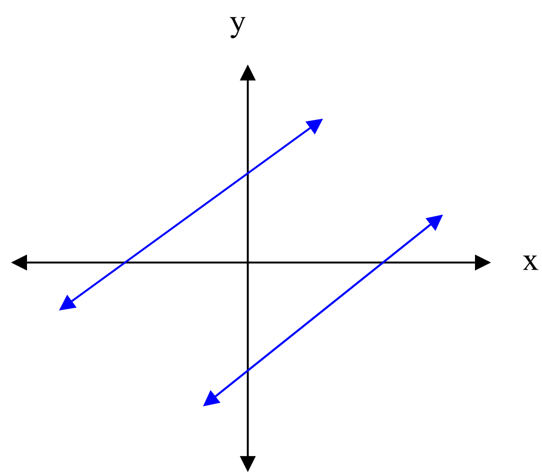


7. $x - y = 4$
$x - 3y = 4$

8. $2x - y = 4$
$2x - y = -2$

9. $3x + 2y = 2$
$6x + 4y = -3$

10. $5x + 2y - 3 = 0$
$10y + 25x = 15$

 **Let's Check Your Understanding 5**

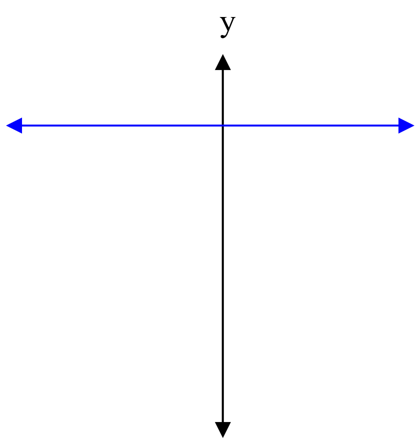
Tell whether the system is

A. Consistent and Independent

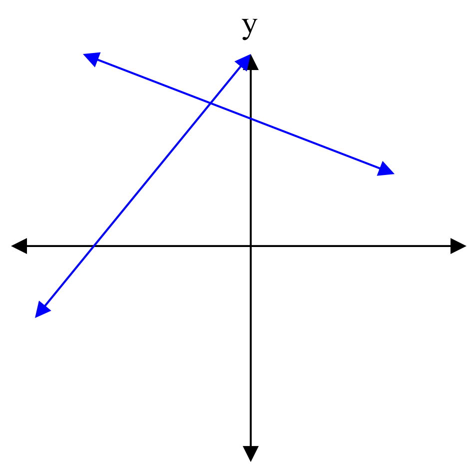
B. Consistent and Dependent

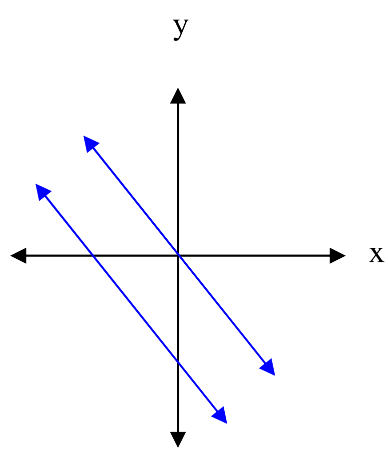
C. Inconsistent

1.



2.



3. 

4. $2x = y + 3$
$2y = 4x + 5$

5. $2x = 10y + 4$
$x - 5y = 2$

6. $x + y = 5$
$3x + 3y = 21$

Lesson 1.4 Solving System of Linear Equations Algebraically

To solve a system of linear equations algebraically, we must reduce the system to a single equation with only one variable.

Do you still remember the properties of equality?

Let us call the properties as the Golden Rule of Equations.

Whatever you do unto one side of an equation, do the same thing unto the other side of the equation. Then the equation will remain true. In particular, you can:

- Add the same number to both sides – A.P.E.
- Subtract the same number from both sides – S. P. E.
- Multiply both sides by the same number – M.P.E.
- Divide both sides by the same number –D.P.E.

Let us recall how to transform an equation with two variables to y = mx + b

Transform each of the following equations to y = mx + b

1. $x = 2y - 5 \longrightarrow -2y = -x - 5$

   $2y = x + 5$

   $y = \frac{x}{2} + \frac{5}{2}$

2. $4x + y = 5 \longrightarrow y = -4x + 5$

3. $4x + 8y = 16 \longrightarrow 8y = -4x + 16$

   $y = -\frac{1}{2}x + 2$

Now , are you ready to solve system of linear equations using the algebraic method?

1.4 A Using the **Substitution Method**

Example 1

Solve the following system of equations:

$x + 2y = 7$ E. 1

$y - 1 = 2x$ E. 2

Solution:

- Solve E. 2 for y:

  $y = 2x + 1$

- Eliminate y in E.1 by substituting 2x + 1 for y:

$$x + 2y = 7$$
$$x + 2(2x + 1) = 7$$
$$x + 4x + 2 = 7$$
$$5x + 2 = 7$$

$$5x = 7 - 2$$

$$5x = 5$$

$$x = \frac{5}{5} = 1$$

- Find the value of y when x = 1 by substituting 1 for x in either of the two original equations:

$$x + 2y = 7$$

$$1 + 2y = 7$$

$$2y = 7 - 1$$

$$2y = 6$$

$y = \frac{6}{2}$ $y = 3$ Solution(1, 3)

- Check that the solution (x = 1, y = 3) works in each of the two original equations:

$\underline{x + 2y = 7}$	$\underline{y - 1 = 2x}$
$1 + 2(3) \overset{?}{=} 7$	$3 - 1 \overset{?}{=} 2(1)$
$1 + 6 \overset{?}{=} 7$	$\sqrt{}$ $2 = 2$
$\sqrt{}$ $7 = 7$	

Example 2

Solve by substitution method:

$y = -x - 1$ E. 1

$4x - 3y = 24$ E. 2

Solution

$4x - 3y = 24$	E. 2
$4x - 3(-x - 1) = 24$	Substitute (-x-1) for y
$4x + 3x + 3 = 24$	Distributive Property
$7x + 3 = 24$	Combine like terms
$7x = 21$	APE
$x = \frac{21}{7}$	DPE

$\mathbf{x = 3}$

Solve for y when x = 3 by substituting 3 for x in either of the original equations

$y = -x - 1$

$y = -3 - 1$ **$y = -4$** **Solution (3, -4)**

Check:

$y = -x - 1$	$4x - 3y = 24$
$-4 \stackrel{?}{=} -3 - 1$	$4(3) - 3(-4) \stackrel{?}{=} 24$
$-4 \stackrel{?}{=} -4$	$12 + 12 \stackrel{?}{=} 24$
$-4 \stackrel{\surd}{=} -4$	$24 \stackrel{\surd}{=} 24$

Now let us summarize the steps used in the substitution method.

Steps in Solving Linear Equations by Substitution

1. Solve one of the equations for one of the variables.
2. Substitute the resulting expression in the other equation.
3. Solve the resulting equation.
4. Find the values of the variables.
5. Check the solution in both equations of the original system.

Let's Practice for Mastery 6

Solve by substitution

1. $x = 2y + 4$
   $3x - y = 26$

2. $x - y = 8$
   $3x + 3y = 24$

3. $x + y = 0$
   $y = x - 2$

4. $x - 5y = 6$
   $2x + 3y = 25$

5. $5x + 3y = 10$
   $2x + y = 3$

Let's Check Your Understanding 6

Solve each system by substitution

1. $x = 2y - 5$
   $x - 3y = 8$

2. $-4x + y = 5$
   $2x - 3y = 13$

3. $x - y = 4$
   $2x - 5y = 8$

4. $x + y = 0$
   $3x + 2y = 5$

5. $4x + 3y = 6$
   $2x + y = 4$

1.4 B Using the **Addition Method**

There are systems of linear equations which are difficult to solve using substitution, but can easily be solved by adding the equations. See the illustration below.

$$\begin{array}{rcll} 2x - 9y & = & 17 & (1) \\ \underline{5x + 9y} & \underline{=} & \underline{11} & (2) \\ 7x + 0 & = & 28 & \end{array}$$

$$x = \frac{28}{7} \qquad , \mathbf{x = 4}$$

The resulting equation does not contain variable y since the coefficients of y in the original equations are additive inverses, so their sum is 0. The process is *elimination by addition*.

Example 1

Solve by the addition method

$3x + y = 10$ E. 1

$2x - y = 5$ E. 2

- Since the equations are already in the form of Ax + By = C and the coefficients of y are additive inverses, add the corresponding sides of the equations to eliminate the terms containing y.

$3x + y = 10$

$\underline{2x - y = 5}$

$5x + 0 = 15$

$x = \frac{15}{5}$

$\mathbf{x = 3}$

- Substitute 3 for x in E.1 or E. 2 to find the value of y.

$3x + y = 10$

$3(3) + y = 10$

$y = 10 - 9$

$y = 1$

Solution (3, 1)

Check:

$3x + y = 10$	$2x - y = 5$
$3(3) + 1 \overset{?}{=} 10$	$2(3) - 1 \overset{?}{=} 5$
$9 + 1 \overset{?}{=} 10$	$6 - 1 \overset{?}{=} 5$
√	√
$10 = 10$	$5 = 5$

Example 2

Solve by Addition Method

$p + q = 4$ E. 1

$-p + q = 7$ E. 2

What term are you going to eliminate?

Eliminate p. Why?

$p + q = 4$

$\underline{-p + q = 7}$

$0 + 2q = 11$

$q = \frac{11}{2}$

$q = 5\frac{1}{2}$

Substitute $5\frac{1}{2}$ for q in the E.1 or E. 2

$p + q = 4$

$p + 5\frac{1}{2} = 4$

$p = 4 - 5\frac{1}{2}$

$p = -1\frac{1}{2}$

Solution: $(-1\frac{1}{2}, 5\frac{1}{2})$

Now, you do the checking.

Can you summarize the steps for solving a system of linear equations by using addition?

To solve a system of two linear equations by addition method

1. see to it that the equations are in the form of Ax + By = C.
2. add to eliminate one of the variables. Choose coefficients which are additive inverses. Solve the resulting equation,
3. substitute the known value of one variable in one of the original equations of the system. Solve for the other variable by substituting the known value of the variable.
4. Check the solution in both equations of the system.

Remember:

The variables in equations are not always x and y.

It is customary to give the solution(s) of the system in the alphabetical order of the variables involved. If the variables used were p and q. Thus, the solution is given as (p, q)

Let's Practice for Mastery 7

A. Determine which variable in each system would be easier to eliminate by the addition method.

1. $x + 2y = 3$
   $-x + y = -2$
2. $2x - 3y = -2$
   $x + 3y = 4$
3. $-2x + 3y = 7$
   $2x - 5y = -3$
4. $x + 4y = 7$
   $2x - 4y = -3$

B. Write the sum equation for each system

5. $x = -2$
   $y = 3$
6. $x + y = 7$
   $x - y = 2$
7. $p + q = \frac{1}{2}$
   $-p + q = -\frac{3}{2}$
8. $-r + s = 6$
   $r + s = 2$

Let's Check Your Understanding 7

A. Determine which variable in each system would be easier to eliminate and write the sum equation. (Remember to write the equations in the form Ax + By = C)

1. $x + y = 4$
   $x - y = 2$
2. $x + y = 6$
   $-x + y = 4$
3. $2x + y = 3$
   $x = 3 + y$
4. $12x = 15 - 3y$
5. $15a - 6b = 0$

$2x - 3y = 13$ $7a = -6b$

B. Use addition to solve each system.

1. $x - y = 4$
   $x + y = 2$
2. $2x + y = 7$
   $3x - y = 3$
3. $y - 4x = 8$
   $y + 4x = 0$
4. $a + 5b = 10$
   $-a + 4b = 8$
5. $y = 3x + 6$
   $2y = -3x + 3$

C. By using the Addition Method match the system with its solution.

1. $x + y = 10$, $x - y = 6$	a. (4, -1)
2. $2y + x = 2$, $3y - x = -7$	b. (8, 2)
3. $c + d = 12$, $- d = 8$	c. (20,-8)
4. $x - y = 8$, $y = 7$	d. (0,0)
5. $x - 2y = 2\frac{1}{2}$	e. $(2\frac{3}{4}, \frac{1}{8})$
$x + 2y = 3$	f. (15, 7)

1.4 C Using Subtraction Method

Example 1

Solve this system.

$$2x + 2y = 10 \longrightarrow 2x + 2y = 10$$
$$3x + 2y = 14 \longrightarrow \underline{-3x - 2y = -14}$$
$$-x \quad = -4$$
$$\mathbf{x = 4}$$

(Take note when you subtract , change the sign of the subtrahend and proceed to addition.)

To solve for y, substitute 4 for x in either of the two original equations.

Substitute 4 for x in E.1

$2(4) + 2y = 10$

$8 + 2y = 10$

$2y = 10 - 8$

$2y = 2$

$\mathbf{y = 1}$

The solution is (4, 1). Check this solution in both equations.

Example 2

Solve by subtraction $2x + y = -2 \longrightarrow 2x + y = -2$

$2x + 3y = 0 \longrightarrow -\underline{2x - 3y = 0}$

$-y = -2$

$\mathbf{y = 2}$

Solve for x then check.

What is the solution?

- When is the subtraction method best for solving a system of equations?
- What do you do with the sign of the subtrahend?
- How do you write an equation in standard form?
- Why should the two equations be written in standard form before a system is solved by addition or subtraction method?

Let's Practice for Mastery 8

Solve by subtraction.

1. $5y + 3x = 0$

2. $3x + y = 6.5$

-3y +3x =0 | x + y = 2.5

3. 2y -2x = 3 | 4. 2x + y = 2

2y – 5x = 9 | 2x + 3y = 14

5. x – y = -2

2x –y = 1

Let's Check Your Understanding 8

Use subtraction in solving the following systems:

1. 2x + 6y = 10
   2x + 10y = 6
2. x + 4y = 8
   x -2y = 10
3. 3x + 2y = 5
   3x – 5y = 16
4. 2x + y = 3
   -4x +y = -3
5. 2y = -2x + 10
   y = -2x + 5

1.4 D Using Multiplication with the Addition Method

Adding the corresponding sides of the two equations in a system of linear equations does not always eliminate one of the variables. To obtain a sum equation having a single variable in such cases, multiply each side of one or both equations of the system by a number or numbers, so that, after multiplying, the coefficients of one variable will be additive inverse.

Example 1

Solve for x and y

3y = 2x + 1 E. 1

5y – 2x = 7 E. 2

Solution:

- Transform the first equation to Ax + By = C, and write it above the second equation with similar terms aligned in the same columns:

$3y - 2x = 1$

$5y - 2x = 7$

- Multiply each term of the first (or second) equation by -1 so that the coefficients of the x – terms will be additive inverses:

$3y - 2x = 1 \longrightarrow -3y + 2x = -1$

$5y - 2x = 7 \longrightarrow 5y - 2x = 7$

- Add the two equations to eliminate the variable x:

$-3y + 2x = -1$

$\underline{5y - 2x = 7}$

$2y + 0 = 6$

$y = \frac{6}{2}$

y = 3

- Find the corresponding value of x by substituting 3 for y in either of the two original equations:

$5y - 2x = 7$

$5(3) - 2x = 7$

$15 - 2x = 7$

$-2x = 8$

$x = \frac{-8}{-2}$

x = 4

Solution is (4, 3)

You may do the checking yourself.

Example 2

Solve: $2x + 3y = -1$ E. 1

$5x - 2y = -12$ E. 2

- Multiply each side of Equation 1 by 2 and multiply each side of equation 2 by 3. Then the terms containing y will be additive inverses of each other.

$2(2x + 3y) = 2(-1) \rightarrow 4x + 6y = -2$

$3(5x - 2y) = 3(-12) \rightarrow 15x + 6y = -36$

- Add

$$\begin{array}{l} 4x + 6y = -2 \\ \underline{15x - 6y = -36} \\ 19x + 0 = -38 \end{array}$$

$$x = -\frac{38}{19}$$

$\mathbf{x = -2}$

- Substitute -2 for x in (1) to solve for y

$2x + 3y = -1$

$2(-2) + 3y = -1$

$-4 + 3y = -1$

$3y = 3$

$y = \frac{3}{3}$ $\mathbf{y = 1}$ Solution (-2, 1)

Checking is left as an exercise for you.

Can you enumerate the steps in solving systems of linear equations by using multiplication with addition?

To solve a system of two linear equations using multiplication with addition:

1. Rewrite each equation in the form Ax + By = C.
2. Multiply each side of either equation or both equations by appropriate non -zero numbers so that the coefficients of x or y will be additive inverses.
3. Use the addition method.
4. Check.

Let's Practice for Mastery 9

A. Determine the number by which you could multiply one equation or both equations in each pair in order to eliminate one of the variables.

1. $5x + 2y = 14$ E. 1
   $4x - y = 6$ E. 2

2. $2p + 3q = 1$ E. 1
   $3p - q = 18$ E. 2

3. $9a - 3b = 3$ E. 1
   $a + 5b = 11$ E. 2

4. $3x - 2y = -4$ E. 1
   $-5x + 3y = -1$ E. 2

5. $3x - 2y = -9$ E. 1
   $3x - 5y = -5$ E. 2

Let's Check Your Understanding 9

Determine the multiplier of each or both equations so that you can eliminate one variable.

1. $3x + 7y = 2$
   $2x - 8y = 2$

2. $5p + 3q = 17$
   $4p - 5q = 21$

3. $3x + y = 9$
   $2x + y = 1$

4. $3x + y = 10$
   $2x + y = 7$

5. $4x - 3y = 5$
   $-2x + 9y = 7$

Let's Practice For Mastery 10

Solve each system using multiplication with addition

1. $3x + 2y = -7$
   $5x - 2y = -1$

2. $x + 3y = 14$
   $x - 2y = -1$

3. $2x + 3y = 8$
   $3x + y = 5$

4. $2x + y = 3$
   $7x = 4y + 18$

5. $2x - 5y = 7$
   $3x - 2y = -17$

Let's Check Your Understanding 10

Use Multiplication with addition to solve for x and y.

1. $2x - 3y = 8$
   $3x - 7y = 7$

2. $4r + 3s = 7$
   $4r + 4s = 12$

3. $8x + 6y = 10$
   $-4x + 3y = -1$

4. $2a + 5b = 18$
   $-5a + b = -18$

5. $x + 3y = 17.3$
   $2x + 7y = 36.9$

Let's Practice For Mastery 11

Solve each system by the method indicated:

1. $2x - y = 8$
   $x + y = 4$
   (by graphing)

2. $x + y = 9$
   $x - y = 5$
   (by graphing)

3. $x + y = 10$
   $3x - 2y = 5$
   (by substitution)

4. $2x + y = 0$
   $4x - 3y = -10$
   (by substitution)

5. $x + 2y = -4$
   $-x + y = 1$
   (by addition or subtraction method)

6. $2a - 7b = 41$
   $a + 7b = -32$
   (by addition or subtraction method)

7. $2x + y = 7$

$-x + y = 4$

(by subtraction)

8. $4x - 7y = 18$

$x + y = -5$

(by multiplication with addition)

Let's Check Your Understanding 11

Multiple Choice

1.Which ordered pair is the solution to the following system of equations?

$2x - y = 10$

$x + y = 2$

a. (4, -2) b. (4, 2) c. (2, -4) d. (-4, 2)

2. At what point do the graphs of the equations $2x + y = 8$ and $x - y = 4$ intersect?

a. (0, 4) b. (4, 0) c. (-4, 0) d. (5, -2)

3.What is the solution for y in the following system of equations:

$4x + 3y = -6$ and $3x - 2y = 4$

a. -2 b. 2 c. $\frac{4}{2}$ d. $\frac{7}{3}$

For nos. 4 – 10 use any method to solve the following systems:

4. $2x + y = 10$

$3x - y = 15$

5. $2x + 3y = -6$

$5x + 2y = 7$

6. $2x = 5y + 8$

$3x + 2y = 31$

7. $3y = 2x - 6$

$x + y = 8$

8. $2x + 3y = 17$

$3x - 2y = -0.5$

9. $0.4a + 1.5b = -1$

$1.2a - b = 8$

10. $\frac{2}{3}x + y = 1$

$-x + 2y = 5$

Lesson 1.5 Solving Word Problems

When you solve word problems you have to translate from English into mathematical sentences and phrases. When a word problem involves two conditions, we may be able to translate each condition into an equation. The numbers that satisfy both equations at the same time represent the solution of the word problem.

Strategy for Solving Word Problems
1. **Read and understand the problem** . 2. **Plan**. Represent the unknown with a variable. Write the equations needed. 3. **Solve**. Solve for the value of the variable/s 4. **Check.** Check the answers in the original wording of the problem.

1.5 A. Number Relation Problems

Example 1

In the recent achievement test Jose has 6 fewer mistakes than Maria. Twice the number Maria's mistakes is 5 more than three times the number Jose has. How many mistakes does each have?

Let x = the number of mistakes Maria has

y = the number of mistakes Jose has

$x - y = 6$ E. 1 ⟶ $x - y = 6$

$2x = 3y + 5$ E. 2 ⟶ $2x - 3y = 5$

Use multiplication with addition

Multiply E. 1 by -3, $-3x + 3y = -18$

Write E. 2 as $\underline{2x - 3y = 5}$

$-x = -13$

$\mathbf{x = 13}$ number of mistakes Maria has

Solve for y

$x - y = 6$

$13 - y = 6$

$-y = -7$

$\mathbf{y = 7}$ number of mistakes Jose has

Check: Does Jose have 6 fewer mistakes than Maria? 13-7 = 6? Yes

Is twice the number of mistakes Maria has equal to 5 more than three times Jose's mistakes?

2 (13) = 3 (7) + 5 ? Yes

Maria has 13 mistakes and Jose has 7 mistakes.

Example 2

The sum of two numbers is 85. The difference of the same two numbers is 35. Find the numbers.

Let x = larger number

y = smaller number

$x + y = 85$ E. 1

$\underline{x - y = 35 \quad \text{E. 2}}$ Use addition method

$2x = 120$ Use DPE

$\mathbf{x = 60}$

$x + y = 85$ substitute 60 for x in E. 1

$60 + y = 85$

$y = 85 - 60$

$\mathbf{y = 25}$

Check: Is the sum of 60 + 25 = 85? Yes

Is the difference of $60 – 25 = 35$? Yes

The numbers are 60 and 25.

Example 3

The sum of the digits of a two-digit number is 6. The number is six times the units digit. Find the number.

Let t = the tens digit

u = the units digit

The number is $10t + u$

$t + u = 6$ E. 1

$10t + u = 6u$ or $10t – 5u = 0$ E. 2

Use multiplication with addition method

Multiply equation. 1 by 5

$5t + 5u = 30$

$\underline{10t – 5u = 0}$

$15t = 30$

t = 2

Solve for u

$t + u = 6$

$2 + u = 6$

$u = 6 – 2$

u = 4

The number is 10t + u = 10(2) + 4 = 24

Checking is left as an exercise for you.

Let's Practice for Mastery 12

Write a system of equations in two variables for each problem. Then solve the problem.

1. Find two numbers whose sum is 53 and whose difference is 15.
2. The sum of two positive integers is 51. The larger integer is 3 more than twice the smaller integer. Find the integers .
3. A class divides into two groups for a science experiment. The first group has five fewer students than the second group. If there are 27 students in the class , how many students are in each group?
4. The sum of the digits of a two-digit number is 12. The number is 12 times the tens digit. Find the number.
5. The sum of the digits of a two-digit number is 10. The tens digit is 4 more than the unit digit. Find the number.

Let's Check Your Understanding 12

Write a system of equations in two variables for each problem .Then solve the problem.

1. The band director must order 35 uniforms. There are usually five more boys than twice the number of girls in the band. How many uniforms of each type should the band director order?

2. In a basketball game, Juan made a total of 2-point shots and 3-point baskets totaling 26 points. How many 2-point shots and how many 3-point shots did Juan make?

3.Together Nina and Eric do 95 shows a week. If Nina does 16 fewer than twice as many as Eric, how many shows does each person do?

4 Find a two- digit number whose sum of the digits is 9.The number is 6 times the units digit.

5.Two numbers have a sum of 84. If twice the smaller number is 3 more than the larger number, what are the numbers ?

1.5 B. Business and Investment Problems

Example 1

Mr. Perez invested Php20,000 part of his earnings in a small business at 5% and the remaining amount at 7%. On these investments, he gets Php1,160 a year. How much did he invest at each rate?

Let x = the amount invested at 5%

y = the amount invested at 7%

	Amount Invested	Rate	Income
	x	5%	0.05x
	y	7%	0.07y
Total	Php20,000		1 160

$x + y = 20,000$ E. 1

$0.05x + 0.07y = 1,160$ E.2

Solve the system of equations yourself.

$x = 12\ 000$ $y = 8\ 000$

How much was invested at 5%?

How much was invested at 7%?

Do the checking.

1.5 C. Geometric Problems

Let us recall some geometric principles.

- A right angle is an angle whose measure is 90°.
- Complementary angles are two angles whose sum of their measures is 90°.
- Supplementary angles are two angles whose sum of their measures is 180°.
- The sum of the measures of the angles of a triangle is 180°.
- The perimeter of a rectangle is $P = 2l + 2w$, Perimeter of a square is $P = 4s$
- The area of a rectangle is $A = lw$, Area of a square is $A = s^2$

Example 1

The sum of the measures of two angles is 180°. Three times the measure of one angle is 24 less than the measure of the other angle. What is the measure of each angle?

Let x = measure of the smaller angle

y = the measure of the larger angle

$x + y = 180$ E. 1

$3x = y - 24$ E. 2

Solve for y in terms of x

$y = 180 - x$

Substitute 180 – x for y in E. 2

$3x = (180 - x) - 24$

$3x = 156 - x$

$4x = 156$

x = 39

Solve for y by substituting 39 for x in either equation.

$x + y = 180$

$39 + y = 180$

y = 141

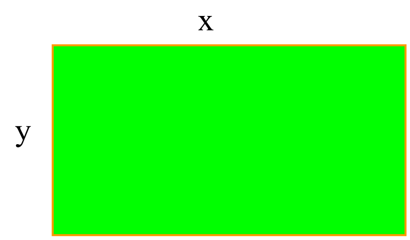
Solution is (39, 141)

The measure of the smaller angle is 39°. The measure of the larger angle is 141°.

The checking is left for you.

Example 2.

The perimeter of a rectangle is 48 cm, The width is 6 cm less than the length. Find the dimensions of the rectangle.



Let x = the length of the rectangle

y = the width of the rectangle

E. 1 $2x + 2y = 48$

E. 2 $y = x - 6$

By substitution solve the system

$x = 15$

$y = 9$

What is the length of the rectangle?

What is the width of the rectangle?

Do the checking.

1.5 D. Motion Problems

Can you recall the distance formula?

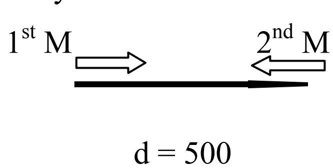
Distance = Rate x Time ($d = r\,t$)

Example 1

Two motorcycles travel toward each other from two points 500 kilometers apart. The two motorcycles meet in 4 hours. What is the average speed of each motorcycle if one motorcycle travels 15 kilometers per hour faster than the other.

Let x = rate of the first motorcycle

y = rate of the second motorcycle



	Rate	Time	Distance
1st Motorcycle	x	4	4x
2nd Motorcycle	y	4	4y
Total			500

4x + 4y = 500   E. 1

x   = y + 15   E. 2

You now solve for x and y.   Did you get (70, 55) as the solution of the system?

You do also the checking.

Some motion problems involve airplanes that fly with or against the wind, or boats that move with or move against the current. These problems contain two unknowns- the speed of the plane in still air and speed of the wind, or speed of the boat in still water and the speed of the current. These situations are summarized below.

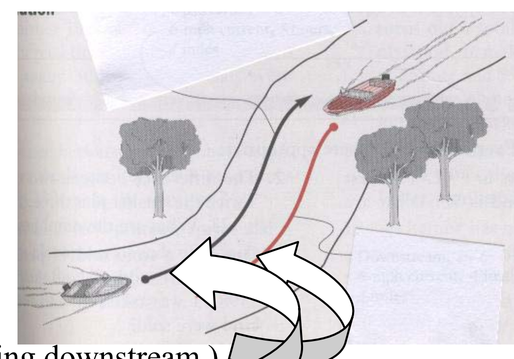
A plane flying with or against the wind

x = plane's speed

y = wind's speed

x + y   = plane's speed moving with the wind

x – y   = plane's speed moving against the wind



A boat moving with or against the current

x   = boat's speed in still water

y   = current's speed

x + y = boat's speed with the current (the boat is moving downstream)

x – y = boat's speed against the current (the boat is moving upstream)

Example 2

It takes 2 hours for a boat to travel 28 miles downstream. The same boat can travel 18 miles upstream in 3 hours. What is the speed of the boat in still water and the speed of the current of the river?

Let s = the speed of the boat in still water

y = the speed of the current

	Distance (d)	Rate (r)	Time (t)
Upstream	18	x – y	3
Downstream	28	x + y	2

Since d = rt the system of equations is: $18 = (x - y)\,3$ E. 1

: $28 = (x + y)\,2$ E. 2

Which is equivalent to

$$6 = x - y$$

$$14 = x + y$$

Use addition method, what is the value of x?

What is the value of y?

The speed of the boat in still water is 10 mph and the speed of the current is 4 mph.

How many kilometers are there in 10 miles? 4 miles?

Do the checking.

Let's Practice for Mastery 13

Complete the solution.

1. Suppose a person invests a total of Php10 000 in two accounts. One account earns 8% annually and the other earns 9% annually. If the total interest earned from both accounts in a year is Php860, how much is invested in each account?

If you let x equal the amount invested at 9% and y be the amount invested at 8%, how will you form the table to organize the data?

Write the system of equations: x + y = (1)_________.

(2) ___________ = 860

What is the amount of money invested t 9%? (3) ______________.

What is the amount of money invested at 8%? (4) ____ ________ .

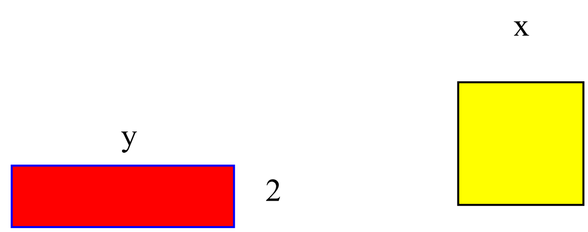
2. The perimeter of a rectangle is the same as the perimeter of a square. The unknown side of the rectangle is 3 units more than the side of the square.

Let x = a side of the square

y = length of the rectangle

Write a system of equations:

E.1 2y + 4 = 4x



E.2 (5) ______________

Solve the system

x = (6) ______

y = (7) ______

Find the area of each figure

Area of rectangle (8) _______ Area of square (9) _______

3. A patrol boat travels 4 hours downstream with a 6-mph current. Returning, against the current , takes 5 hours. Find the speed of the boat in still water.

	Distance	Rate	time
Downstream	d	r + 6	4
Upstream	d	r - 6	5

E.1 d =(r + 6) 4

E.2 (10) ____________

Solve the system of equations by substitution

What is the speed of the boat in still water? (11) ____

4. Nancy and Jaime are team leaders at the Bels Computer Chip Manufacturing Company. The production supervisor needs to report the number of computer chips each team made on Monday. The supervisor knows the total number of computer chips produced by both teams is 130. Nancy's team made 10 more chips than Jaime's team. If you were the supervisor what report are you going to present?
Solve using any method.

Let's Check Your Understanding 13

1. Two different routes between two towns differ by 24 kilometers. One car traveled the longer route at 65 kph and another car traveled the shorter route at 52 kph. If they made the trip between the two towns in exactly the same time, what is the length of each route?

Let x = the length of the shorter route

y = the length of the longer route

	Distance	Rate	Time
Car A	y	65 kph	(1)
Car B	x	(2)	$\frac{x}{52}$ h

Since the routes differ by 24 : E.1 y – x = (3) ________

Since the two cars made the trip in exactly the same time $\frac{y}{65}$ = (4)________.

: $y - x = 24$ E.1

$\frac{y}{65} - \frac{x}{52} = 0$ E.2

Solve by Substitution method x = 96 y = (5) ______.

. **Solve Completely;**

3. A teacher invested Php25 000 . Part of it was invested at 12% and the rest at 15%. If her total annual income from both investments is Php3 305 , how much did she invest at each rate?

1.5 E. Mixture Problems

Example 1

A coffee shop mixes two kinds of coffee to get the blend. They sell wholesale at Php80 a can. One of the kinds is worth Php50 a can, the other, Php90. How much of each kind must be mixed to get 100 cans of the blend?

Solution:

Step 1 Let x = no. of cans of coffee worth Php50 a can

y = no. of cans of coffee worth Php90 a can.

Step 2 Make a table to organize data.

	No. of cans	Amount/can	Total amount
Coffee A	x	Php50	50x
Coffee B	y	Php90	90y
mixture	100	Php80	80(100)

Form the equations from the given conditions in the problem.

There are 100 cans of the blend of two kinds of coffee:

$x + y = 100$ E. 1

The wholesale price of 100 cans of coffee is at Php80 a can.

$50x + 90y = 80\ (100)$ E. 2

Step 3 Solve for the value of the variables. Use substitution.

$x + y = 100$

$50x + 90y = 8000$

$x = 100 - y$ *Solve for x in terms of y*

$50(100-y) + 90y = 8000$ *Substitute 100-y for x in E. 2*

$500 - 50y + 90y = 8000$

$-50y + 90y = 8000 - 5000$

$40y = 3000$

$\mathbf{y = 75}$

$x + y = 100$ *Substitute 75 for y in E. 1*

$x + 75 = 100$

$x = 100 - 75$

$\mathbf{x = 25}$

There are 75 cans of coffee at Php90 a can and 25 cans of coffee at Php50 each.

Step 4 Check:

Are there 100 cans of coffee in the blend?

Is the amount spent on the two kinds of coffee the same as the product of the wholesale price Php80 and 100 cans of coffee blend?

If both answers are yes then our solution is correct.

Example 2.

A chemist needs to mix an 18% soap solution with a 45% soap solution to obtain a 12-liter mixture consisting of 36% soap. How many liters of each of the soap solution must be used?

Let x = the no. of liter of 18% soap solution to be used in the mixture.

y = the no. of liters of 45% soap solution to be used in the mixture.

	No. of liters	Percentage of soap	Amount of soap
18%soap solution	x	18%	0.18x
45% soap solution	Y	45%	0.45y
36%soap solution	12	36%	0.36(12)

$x + y = 12$ E. 1
$0.18x + 0.45y = 0.36(12)$ E. 2

Multiply E.2 by 100 to eliminate decimals

$x + y = 12$ E. 1
$18x + 45y = 432$ E. 2
Use multiplication with addition method

Multiply E. 1 by -18

$-18x – 18 y = -216$

$18x + 45y = 43$

$27y = 216$

$y = 216/27$

$\mathbf{y = 8}$

Substitute 8 for y in either equation

$x + y = 12$

$x + 8 = 12$

$\mathbf{x = 4}$

Checking is left for you.

Let's Practice for Mastery 14

A. Complete the solution .

1. An auditorium seats 2,500 people. How many balcony tickets must be sold for Php45 and how many orchestra tickets must be sold for Php52.50 in order to receive a total receipts of Php126,750 each time the auditorium is full.

Let x = no. of balcony tickets

y = no. of orchestra tickets

	Number	Price per ticket	Total sales
Balcony tickets	x	Php45	45x
Orchestra tickets	y	Php52.50	52.50y
Total	2,500		P126,750

E. 1 (1)_________________

E.2 (2)________________

Use substitution method.

x + y = 2,500

x = (3)______________

Substitute 2,500-y for x in E. 2

45(2,500 – y) + 52.50y = 126,750

112,500 – 45y + 52.50y = 126,750

7.5y = 14,250

y = (4) _______

Solve for x, do it yourself.

x = (5)________

B. Solve completely

2. A 12% salt solution is mixed with a 20% salt solution. How many kilograms of each solution are needed to obtain 24 kilograms of a 15% solution?

Let's Check Your Understanding 14

A. Complete the table, write the equations and solve for the value of x and y.

1. A bank teller received a savings deposit in Php50 and Php100 bills. Altogether there were 250 bills worth Php16,500. How many bills of each denomination were deposited?

Let x = number of Php50 bills

y = number of Php100 bills

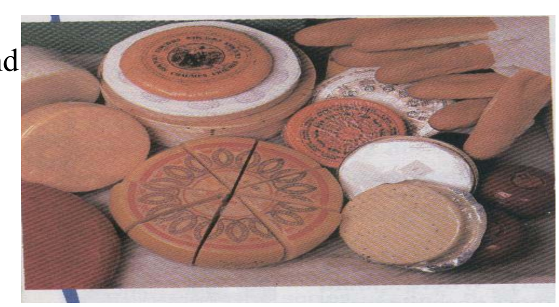
	No. of pieces	Amount
Php50	X	(1)
Php100	y	(2)

E. 1 (3)_______________ x = (5)_________

E. 2 (4)______________ y = (6)_________

2. Solve Completely.

A baker mixes cookies worth Php9.50 per pound with cookies worth Php17.00 per pound. How many of each kind must be used to produce a 45- pound mixture that sells for Php12.50 per pound?



1.5 F. Age Problems

If you are x years old now,

your age in 5 years or 5 years hence is $x + 5$.

Your age 5 years ago is represented as $x – 5$.

Example:

Alan is 20 years older than his cousin Chiz. In 3 years, Alan will be twice as old as Chiz. How old are Alan and Chiz now?

Let a = Allan's present age

c = Chiz ' present age

	Present age	Age in 3 years
Allan	a	a + 3
Chiz	c	c + 3

Now Alan is 20 years older than Chiz **E. 1**: $a = c + 20$

In 3 years Alan will be twice as old as Chiz **E. 2:** $a +3 = 2 (c+3)$

Solve for the value of a and c.

By substitution: In E. 2 $a + 3 = 2 (c + 3)$

$$c + 20 + 3 = 2c + 6$$

$$c – 2c = 6 – 23$$

$$-c = - 17$$

$$\mathbf{c = 17}$$

$$a = c + 20$$

$$a = 17 + 20$$

$$\mathbf{a = 37}$$

How old is Alan?__________________. How old is Chiz? __________________.

Check: Is Alan 20 years older than Chiz?

In 3 years is Alan's age twice Chiz' age?

Let's Practice for Mastery 15

The ages of Jancent and Miguel differ by 8 months. Twice Jancent's age increased by Miguel's age gives 53 months. If Miguel is older than Jancent , Find their present ages.

Let x = present age of Miguel

y = present age of Jancent

E. 1 $x - y = 8$

E. 2 $2x + y = 53$

What method is the best way to use? Why ?

How old is Miguel? . How old is Jancent ?

Check: Is the difference between ages of Miguel and Jancent 8?

Is twice Jancent's age increased by Miguel's age gives 53?

Let's Check Your Understanding 15

In 3 years, Patricia will be 3 times as old as his brother Kenneth. A year ago Patricia was 7 times as old as Kenneth. How old are they now?

Let x = the present age of Patricia

y = the present age of Kenneth

(1) Express the age of Patricia in 3 years

(2) Express the age of Kenneth I 3 years

(3) What was the age of Patricia a year ago?

(4) What is the age of Kenneth a year ago?

(5) If E.1 is $x + 3 = 3 (y + 3)$, how will you write E. 2 ?

(6) By any method solve for x and y.

(7) How old is Patricia? How old is Kenneth?

Lesson 1.6 Systems of Linear Inequalities in Two Variables

You have seen in the previous lessons how you could relate and solve daily life problems using equations. Yet not all of these problems require only one solution but may require a set of numbers which may be greater or less than a specific number. These mathematical statements which involve greater than and less than relations are what we call as **inequalities.** The symbols used in an inequality are as follows:

$>$ means is *greater than*

$\geq$ means is *greater than or equal to*

$<$ means is *less than*

$\leq$ means is *less than or equal to*

Here are some examples of inequalities, practice reading them.

$4 < 7$	4 is less than 7
$x > -4$	x is greater than -4
$m + 3 \geq 8$	the sum of m and 3 is greater than or equal to 8
$24 \leq 3y - 4$	24 is less than or equal to the difference of thrice y and 4
$x + 7 < 5 - x$	the sum of x and 7 is less than the difference of 5 and x
$x^2 + y^2 > 10$	the sum of the squares of x and y is greater than 10
$2x - 3y < 4$	the difference between twice x and thrice y is less than 4 .

A system of linear inequalities in two variables is usually solved by graphing.

A statement of inequality is used to indicate a set of real numbers, which we refer to as the solution set of the inequality.

These are the set of values or numbers which when substituted for the variable makes the inequality true.

Remember that the graph of an inequality that uses $>$ or $<$ <u>does not include</u> the graph of the related equation. To show this, the graph of the related equation is a dashed line.

The graph of an inequality that uses $\geq$ or $\leq$ includes the graph of the related equation. To show this, the graph of the related equation is a solid line.

Let us recall how to graph inequalities in two variables. Consider the following examples:

1. Graph $y - x \leq 3$

The graph of $y - x \leq 3$ includes either all the points above the line or below the line. To decide which, choose a test point not on the line $y - x = 3$. when it is not on the line, the origin (0, 0) is the simplest to work with. Substitute (0, 0) for x and y in the original inequality, to see if the resulting statement is true or false.

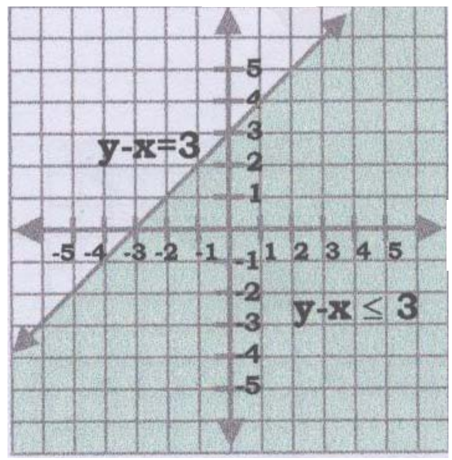
$y - x \leq 3$

$0 - 0 \leq 3$ Let x=0 an y=0

$0 \leq 3$ True

The statement is true. Therefore the graph of the inequality include the half plane containing (0,0).

y



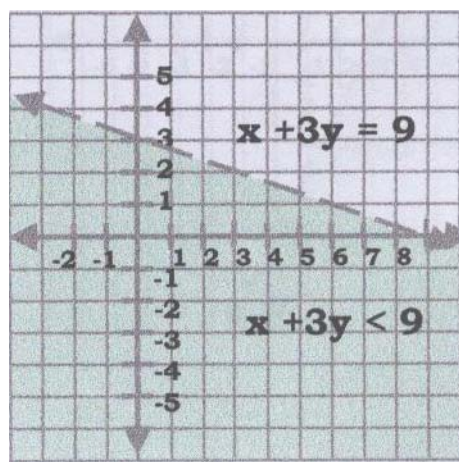
x

2. Graph $x + 3y < 9$

The inequality is “less than” so the points of the line $x + 3y = 9$ do not satisfy the inequality. We draw a broken line or dashed to show that it is not a part of the solution set for the inequality $x + 3y < 9$.

We use the intercepts to graph.

x	0	9
y	3	0



x

Use (0,0) as a test point to see which side of the line is to be shaded.

If you wish to find all the points that will satisfy two inequalities in a system, you must graph each inequality on the same plane and look for the shaded regions common to both of them.

Consider the following examples:

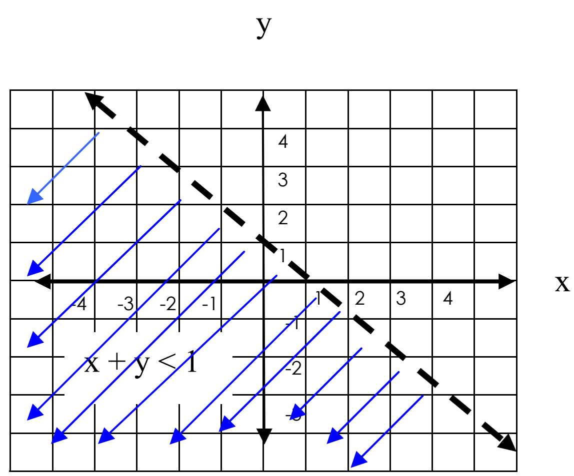
Example 1

Graph the solution of the system

$$x + y < 1$$
$$x - y < 5$$

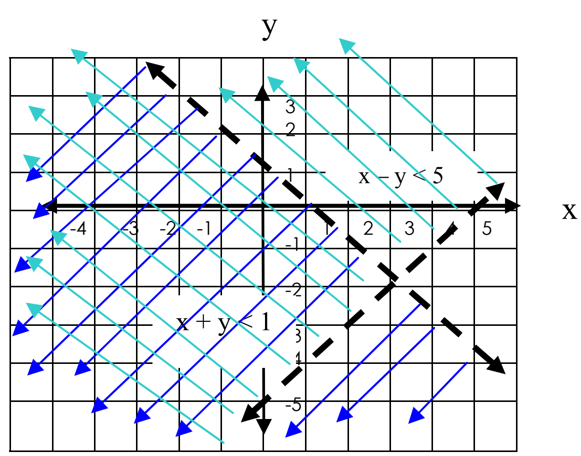
Solution:

Step 1: Draw the graph of $x + y < 1$ using the values that satisfy the equation $x + y = 1$. When drawing the graph, remember to use a dashed line, since the inequality is "less than". This is shown in the figure at the right



x	0	1
y	1	0

Step 2: On the same set of axes, draw the graph of $x - y < 5$ using the value that satisfy the equation $x - y = 5$. This is shown in the figure at the right.



x	0	5
y	-5	0

The double shaded region shows the ***feasible region*** or ***solution set*** of the system.

Step 3: To check, choose any point on the double shaded region and substitute it to the original inequalities. Say, we choose the point (1,-2).

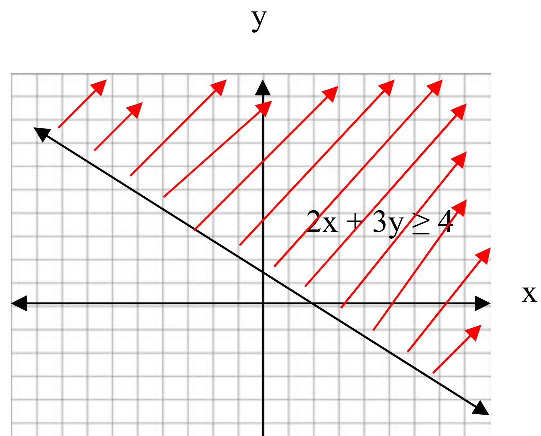
$$x + y < 1 \qquad\qquad x - y < 5$$
$$1 + (-2) < 1 \qquad\qquad 1 - (-2) < 5$$
$$-1 < 1 \quad \text{True} \qquad\qquad 3 < 5 \quad \text{True}$$

Since both inequalities were satisfied, we can say that we have shown the correct solution set of the system of inequalities.

Example 2 Find the feasible region of the system

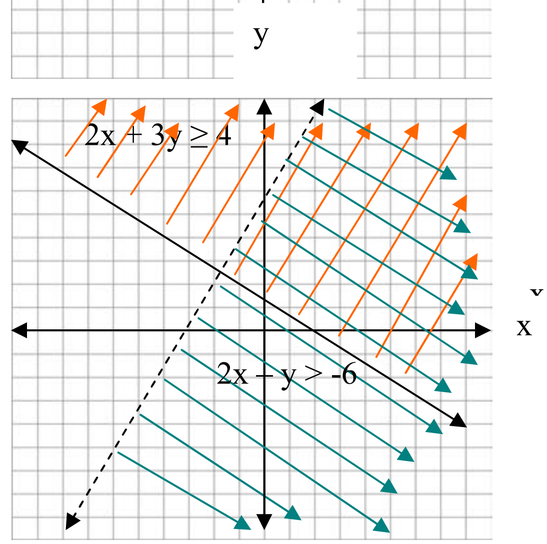
$$2x + 3y \geq 4$$
$$2x - y > -6$$

Solution:



Step1: Draw the graph of $2x + 3y \geq 4$ using the values that satisfy the equation $2x + 3y = 4$. Remember to use a solid line since the inequality is "greater than or equal to."

x	-1	2
y	2	0



Step2: On the same set of axes, draw the graph of $2x - y > -6$ using the values that satisfy the equation $2x - y = -6$. This is shown in the figure at the right.

x	0	-3
y	6	0

The double shaded region shows the *feasible region* of the system. Note that the point of intersection of the two lines is not a part of the feasible region.

Step3: To check, choose any point on the double shaded region and substitute it in the original inequalities. Say we choose the point (2, 2).

$$2x + 3y \geq 4 \qquad\qquad 2x - y > -6$$
$$2(2) + 3(2) \geq 4 \qquad\qquad 2(2) - 2 > -6$$
$$4 + 6 \geq 4 \quad 10 \geq 4 \ \text{True} \qquad\qquad 4 - 2 > -6 \quad 2 > -6 \ \text{True}$$

Since both inequalities were satisfied, we can say that we have shown the correct *feasible region* or *solution set* of the system of inequalities.

Example 3

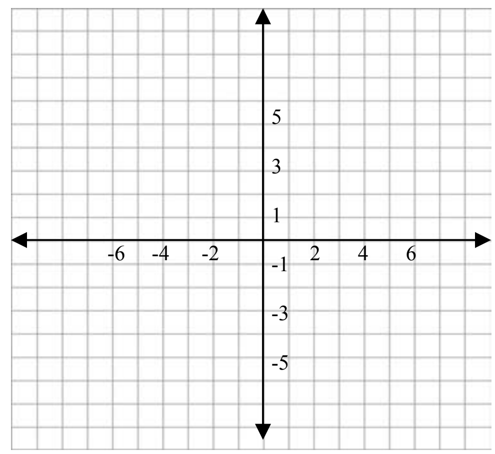
Find the feasible region of the system

$4x - 3y > 8$
$x < 2$

Solution:

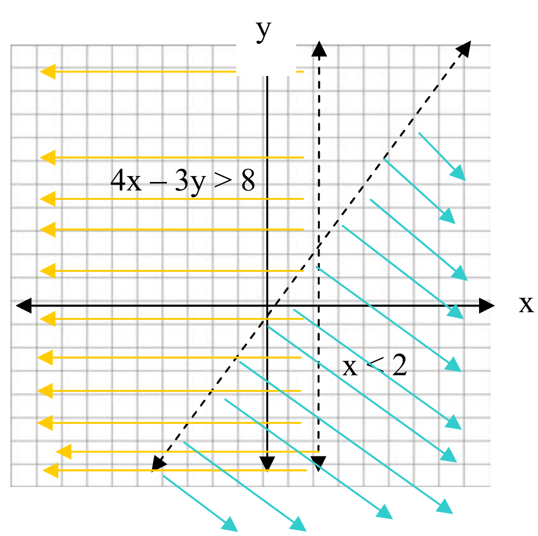
Step1: Draw the graph of the inequality $4x - 3y > 8$ using the values that satisfy the equation $4x - 3y = 8$. Use the values below to graph the inequality on the coordinate plane at the right.

x	-1	2
y	-4	0



Step2: On the same axes above, draw the graph of $x < 2$ using the line $x = 2$. The feasible region of the system is shown by the double shaded region and includes the portion of the two boundary lines. To check, choose any point in this region and substitute it to the original inequalities. If both inequalities are satisfied, then you have shaded the correct region for the solution of the inequality.

Your graph should look like this.



Be sure to remember this…

Procedure for Solving a System of Linear Inequalities

1. Select one of the inequalities. Replace the inequality symbol with an equal sign, and draw the graph of the equation. Draw the graph with a dashed line if the inequality is > or < and with a solid line if the inequality is ≤ of ≥.
2. Select a test point on one side of the line and determine whether the point is a solution to the inequality. If so, shade the area on the side containing the point. If the point is not a solution, shade the area on the other side of the line.
3. Repeat steps 1 and 2 for other inequality
4. The intersection of the two shaded areas and any solid line common to both inequalities form the *feasible region* or the *solution set* to the system of inequalities.

Let's Practice for Mastery 16

I. Tell which of the following points is/are solution(s) to the following system of inequalities.

1. $x - y \geq 0$
   $x - y - 4 \leq 0$
   i. (2, 0)
   ii. (1, -3)
   iii. (-1, -1)

   A. i only
   B. i and ii only
   C. i and iii only
   D. all of the above

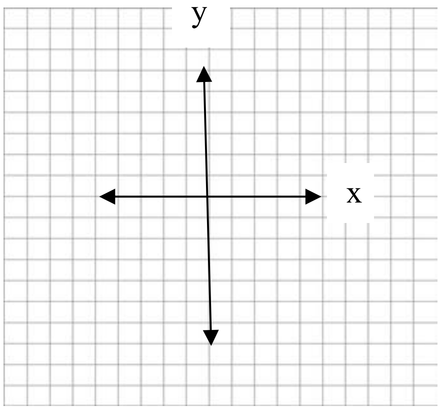
2. $x - 2y > 6$
   $2x > 4 - y$
   i. (3, 1)
   ii. (5, -1)
   iii. (-1, -2)

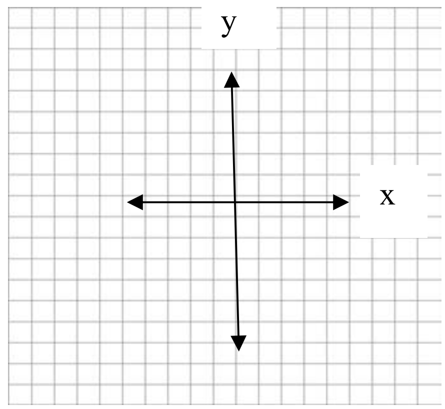
   A. i only
   B. ii only
   C. iii only
   D. all of the above

II. Graph the solution of each of the following systems of inequalities.

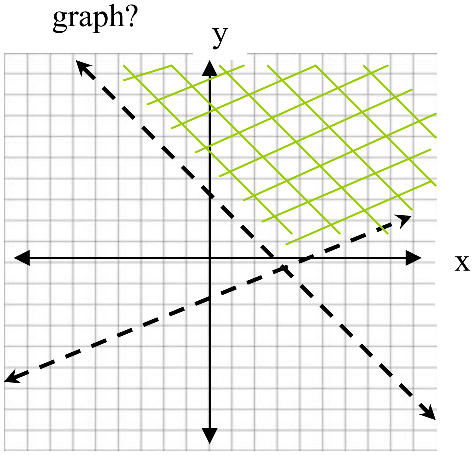
1. $x + y \leq 6$
   $x - y \leq 1$

2. $3x + y \leq -3$
   $x - y \geq 7$





III. Which system of linear inequalities is shown by the shaded by the shaded region in each graph?



A. $x + y > 3$
   $x - 2y < 4$

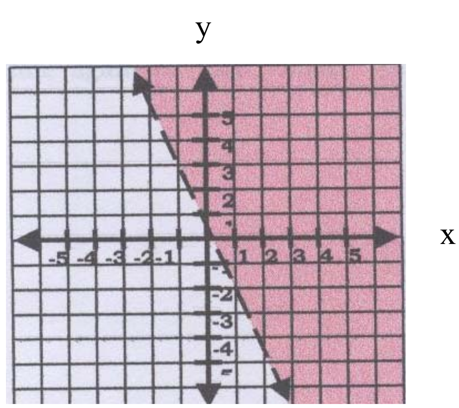
B. $x + y < 3$
   $x - 2y > 4$

C. $x + y \geq 3$
   $x - 2y \leq 4$

D. $x + y \leq 3$
   $x - 2y \geq 4$

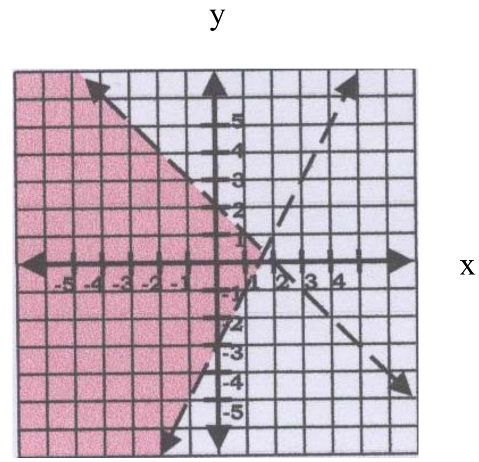
Let's Check Your Understanding 16

1. Which of the following inequalities describe the graph below?



a. $5x + 2y > 1$
b. $5x + 2y < 1$
c. $5x + 2y \geq 1$
d. $5x + 2y \leq 1$

2. Which of the following systems of inequalities describe the solution set in the graph below?



a. $x + y > 2$

   $2x - y < 3$

b. $x + y < 2$

   $2x - y < 3$

c. $x + y \geq 2$

   $2x - y > 3$

d. $x + y \leq 2$

   $2x - y > 3$

3. Find the solution of the system of inequalities by graphing. Use your graphing paper.

   $2x - 3y \geq 4$

   $8x + 2y \leq 2$

LET'S SUMMARIZE

- Linear Equations can be graphed in many ways, two of these methods are :
  a) using table of values
  b) using the slope-intercept method
- Two or more equations such as x+y = 10 and x – y = 2 form a system of liear equations.
- To solve a system of linear equations you find the ordered pair that makes both equations true.
- To solve a system of linear equations graphically, graph each equation on the same set of coordinate axes. The intersection of the two lines is the solution of the system.
- A system that has at least one solution is called *consistent.*
- A system with only one solution is called *independent* .The graphs of their equations intersect at one point.
- A system with no solution is called *inconsistent.* The graphs of their equations are parallel. Their slopes are equal and the intercepts are different.
- A system with infinite solutions is called *dependent.* The equations are identical and their graphs coincide.
- To solve a system of linear equations algebraically we can use substitution, addition, subtraction and multiplication with addition. To solve a system of equations by substitution, solve one of the equations for one of the variables, substitute the resulting expression in the other equation, solve the resulting equation, find the values of the variables and check.
- To solve a system of equations by addition, see to it that the equations are in the form Ax + By = C, add to eliminate one of the variables by choosing the coefficients which are additive inverses, solve the equation, substitute the known value of one variable of the original equations of the system, and check.
- To solve a system of equations by subtraction, see to it that the equations are in the form Ax + By = C, subtract following the rules of subtraction of integers(change the

sign of the subtrahend and proceed to addition), solve the equation, substitute the known variable in one of the original equations of the system and check.

- To solve a system of equations using multiplication with addition, rewrite each equation in the form $Ax + By = C$, multiply each side of either equation or both equations by appropriate non-zero numbers so that the coefficients of x or y will be additive inverses, use the addition method, and check.
- In solving word problems translate from English into mathematical phrases and sentences. When two conditions are given translate each condition to an equation to form a system. Remember, to read and understand the problem, plan, solve, and check.
- Statements that involve greater than and less than relations are called *inequalities,* the following symbols are used in inequalities: $>$ (is greater than), $<$ (is less than), $\geq$ is greater than or equal to, $\leq$ (is less than or equal to).
- To solve a system of linear inequalities by graphing, select one of the inequalities, replace the inequality symbol with an equal sign, and draw the graph of the equation, used dashed line if the inequality is $>$ or $<$ and with solid line if the inequality is $\geq$ or $\leq$., select a test point on one side of the line and determine whether the point is a solution to the inequality, if so, shade the area on the side containing the point, if the point is not a solution, shade the area on the other side of the line, repeat the procedure for the second inequality, the intersection of the two shaded areas and any solid line common to both inequalities form the solution set to the system of inequalities.

Unit Test

I. A. Solve the following systems of equations by graphing.

1. $x + y = 2$
   $x - y = 4$

2. $3x + 2y = 1$
   $x - 3y = -7$

B . Solve by any algebraic method the following systems of equations

3. $x - y = 8$
   $2x + y = 1$

4. $y - x = 1$
   $x + y = 7$

5. $x - 3y = 7$
   $2x + y = 7$

6. $x + 2y = 7$
   $x - y = -5$

C. Draw the graphs of the following systems of inequalities

7. $\mathbf{x + y} \le 6$
   $x - y \le 1$

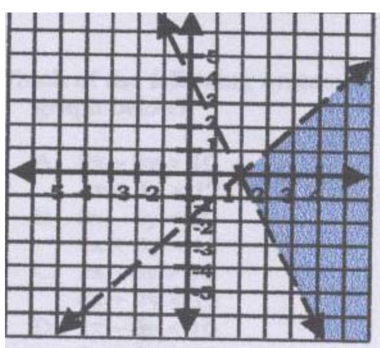
8. $3x + y \le -3$
   $x - y \ge 7$

II. 9. Which graph shows the solution of the following system of inequalities?

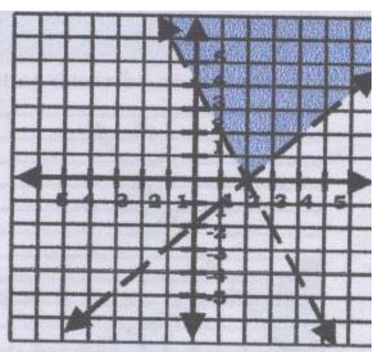
$x - y < 2$

$2x + y > 4$

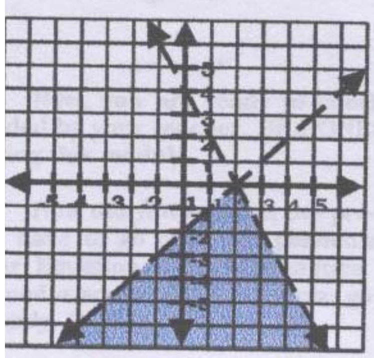
A.



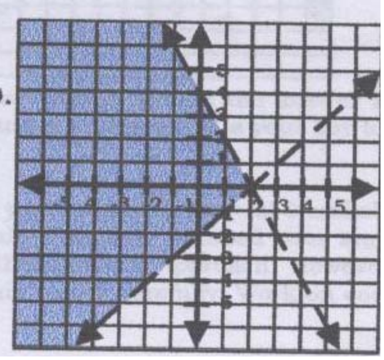
C.



B.



D.

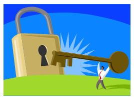


II. Solve the following problems completely.

10. The difference between two numbers is 10. The larger number is 6 less than twice the smaller number . Find the numbers.

11 . The ground floor of mall in the city is a rectangle whose perimeter is 860 meters. The length is 100 meter ore than the width. Find the length and the width?

12 . For Php100 , I can buy 4 banana cue and 12 camote cue,
or 5 camote cue and 10 banana cue. How much does one
of each kind cost?



ANSWER KEY

Let's Practice for Mastery 1

A.

1..$y = -x + 8$	6. $y = -2/3x + 0$
2. $y = x + 4$	7. $y = - 3x + 5$
3. $y = -x + 2$	8. $y = 5x -3$
4. $y = 2x -8$	9. $y = -2/3x + 4$
5. $y = -3/2 x + 5/2$	10. $y = 4/3 x - 2$

B.

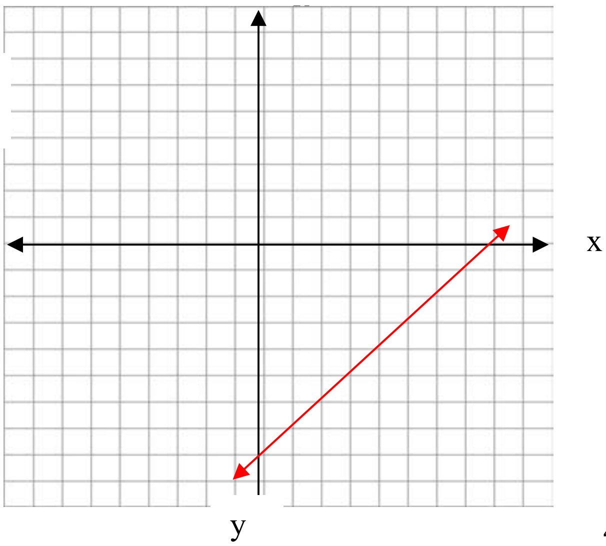
1. $m=-1$, $b= 8$	6. $m = -2/3$, $b = 0$
2. $m = 1$, $b = 4$	7. $m =-3$, $b =5$
3. $m = -1$, $b = 2$	8. $m =5$, $b =-3$
4. $m = 2$, $b = -8$	9. $m =-2/3$ $b =4$
5.$m = -3/2$, $b = 5$	10. $m =4/3$ $b = 2$

Let's Check Your Understanding 1

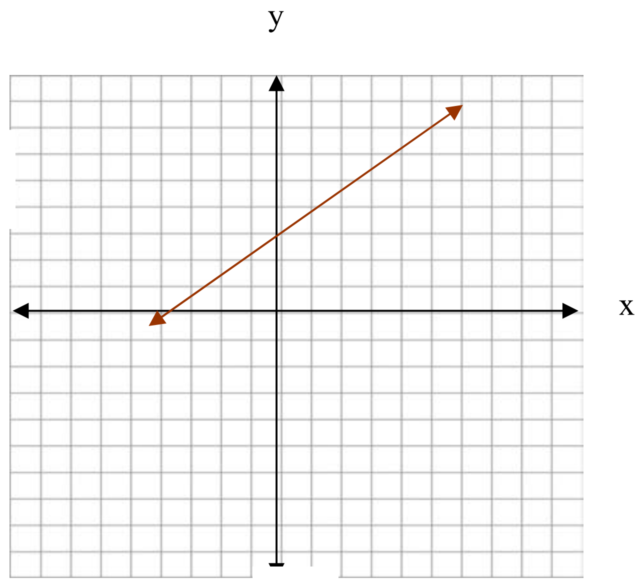
1. $m =-1$, $b=3$	6. $m =2/3$, $b= -2/3$
2. $m = -3$, $b = 12$	7. $m = -2/5$, $b =1$
3. $m = -3/2$, $b =2$	8. $m= 5/3$, $b = 5/3$
4. $m = -3/5$, $b= -1/5$	9. $m = 1/3$, $b = 2/3$
5. $m = ½$, $b = -1/2$	10. $m =2/3$, $b =-14/3$

Let's Practice for Mastery 2

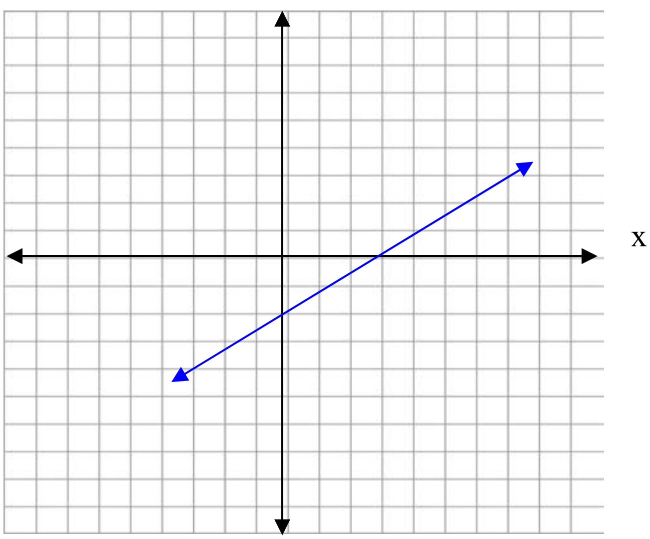
1.



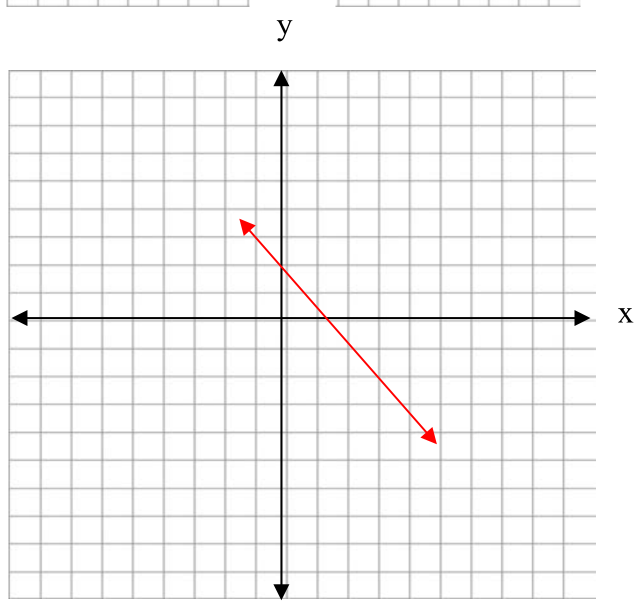
2.



3.



4.



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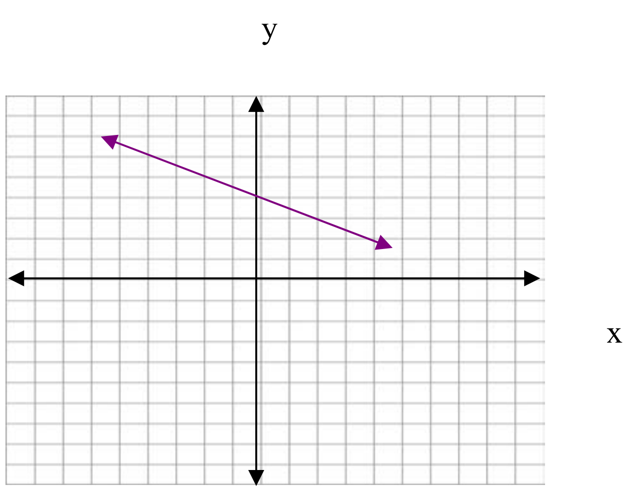
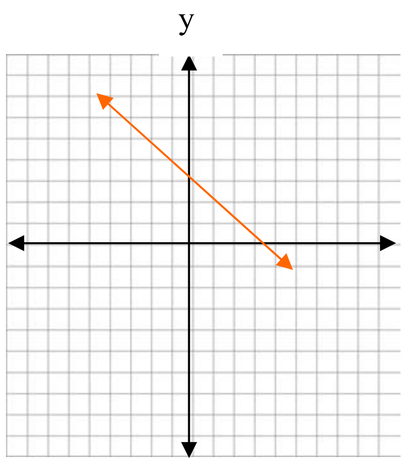


Let’s Check Your Understanding 2

1



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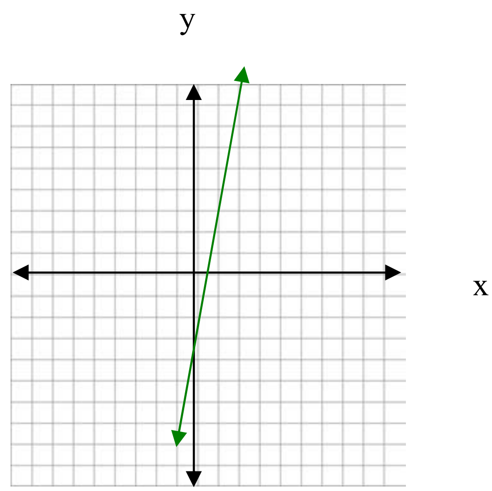

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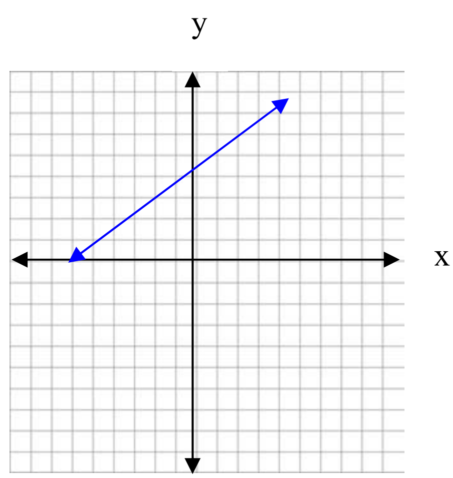

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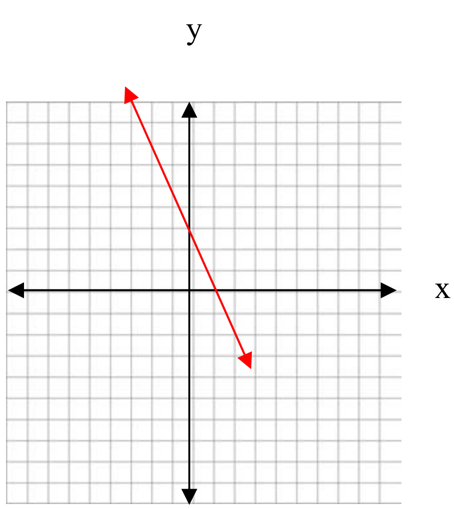

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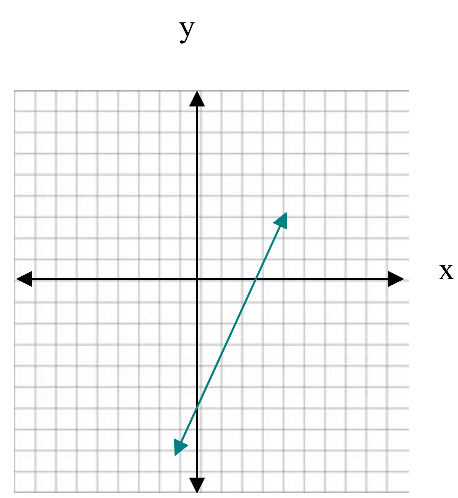
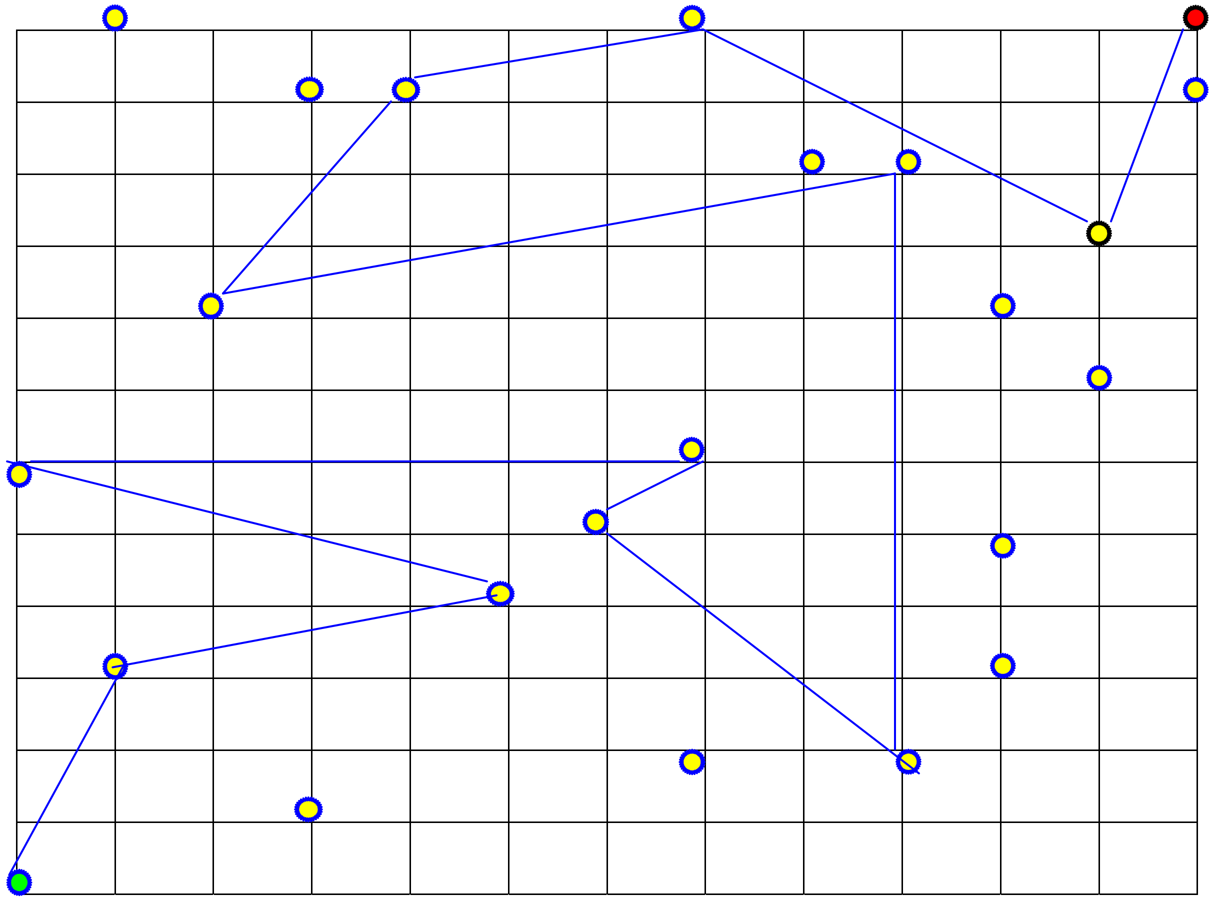


TREASURE



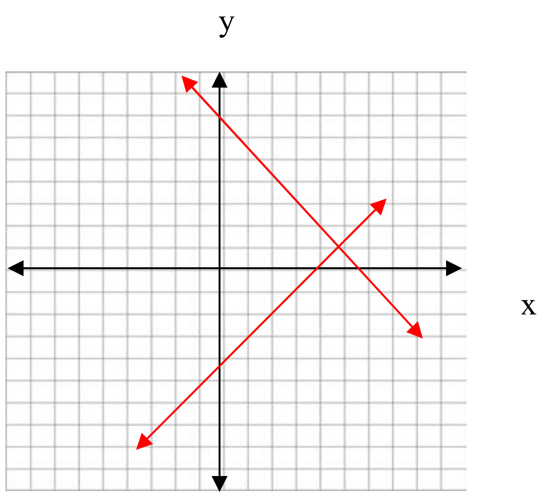
Key : TREASURE HUNT WITH SLOPES

Let's Practice for Mastery 3

A.   1. (7, 3)   2. (2, 1)   3. (4, 0)   4. (2,-1)   5. (5, 1)   6. (-3, 2)

B.

7



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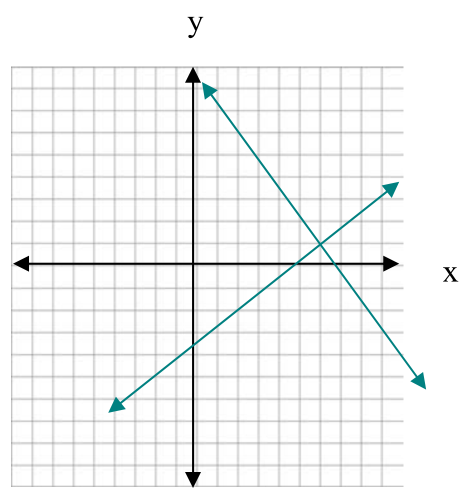

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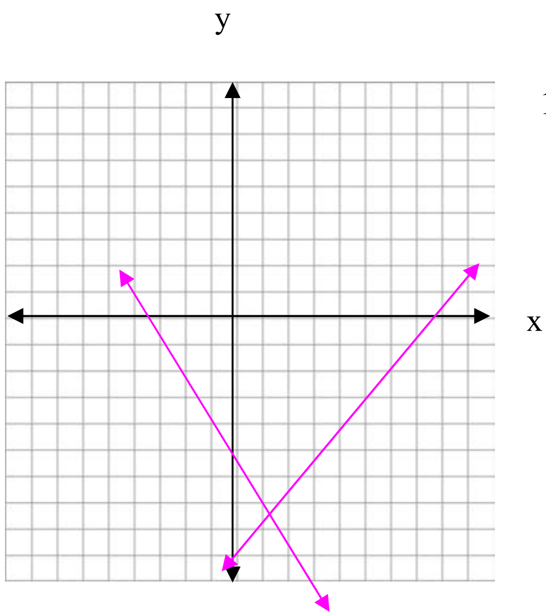

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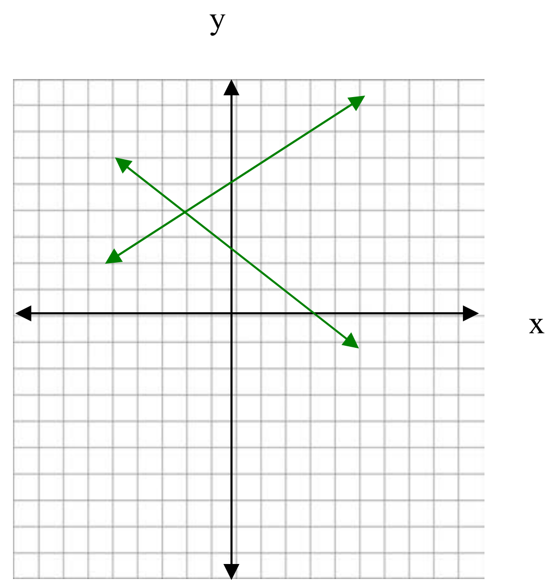


Let's Check Your Understanding 3

1\. a (-2, -1) 2. c (-1, 1) 3. b $(-1\frac{1}{2}, 2)$

Let's Practice for Mastery 4

1. The graphs are parallel and the slopes are equal.
2. the graphs coincide. Slopes are equal. Intercepts are equal.
3. The graphs intersect at one point.
4. Consistent system has at least one solution. Inconsistent system has no solution.
5. Dependent system has many solutions. Independent system has only one solution.

Let's Check Your Understanding 4

1\. parallel 2. at least one 3. inconsistent 4. equal 5. overlapping

Let's Practice for Mastery 5

1. A 2. A 3. C 4. B 5. A 6.C 7.A 8 C 9.C 10.B

Let's Check Your Understanding 5

1. C 2.A 3.C 4.C 5.B 6.C

Let's Practice for Mastery 6

1. (8,-2) 2. (8, 0) 3. (1,-1) 4. (11, 1) 5. (-1, 5)

Let's Check Your Understanding 6

1. (-31, -13) 2.(-2.8, -6.2) 3. (4, 0) 4. (5, 5) 5. (3,-2)

Let's Practice for Mastery 7

A. 1. x 2. y 3. x 4. y

B. 5. x +y = 1 6. 2x = 9 7. 2q = -1 8. 2s = 8

Let's Check Your Understanding 7

A.1. y , 2x = 6 2. x , 2y = 10 3. y , 3x = 6

4. y, 14x =28 5. b, 22a =0

B. 1. (3, -1) 2. (2, 3) 3. (-1, 4)

4. (0, 2) 5. (-1, 3)

C. 1. b 2. a 3. c

4. f 5. e

Let's Practice for Mastery 8

1. (0, 0) 2. (2, 0.5) 3. $(-3, -\frac{2}{3})$ 4. (-2, 6) 5. (3, 5)

Let's Check Your Understanding 8

1. (8, -1) 2. $(9\frac{1}{3}, -\frac{1}{3})$ 3. $(\frac{57}{21}, -\frac{11}{7})$ 4. (1, 1) 5. (0, 5)

Let's Practice for Mastery 9

1. E2 (2) 2. E2 (3) 3. E2 (-9) 4. E1 (3) , E2 (2) or E1 (5), E2 (3) 5. E2 (-1)

Let's Check Your Understanding 9

1. 3x + 7y = 2 (8)
   2x -8y =2 (7)

2. 5 p + 3q = 17 (5)
   4p – 5q = 21 (3)

3. 3x + + y = 9
   2x + y = 1 (-1)

4. 3x + y = 10 (-1)
   2x + y = 7

5. 4x – 3y = 5 (3)
   -2x + 9y = 7

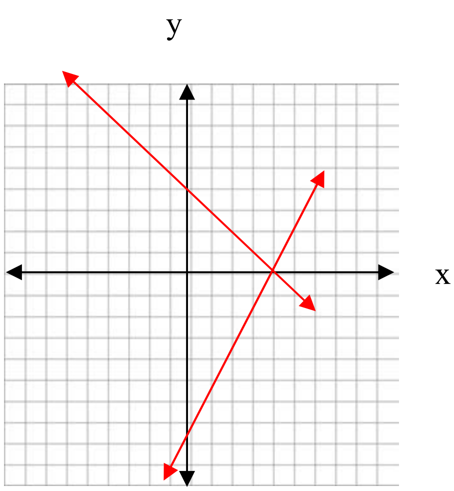
Let's Practice for Mastery 10

1. (-1, -2) 2. (5, 3) 3. (1, 2) 4. (4, 2) 5. (0.8, 5.5)

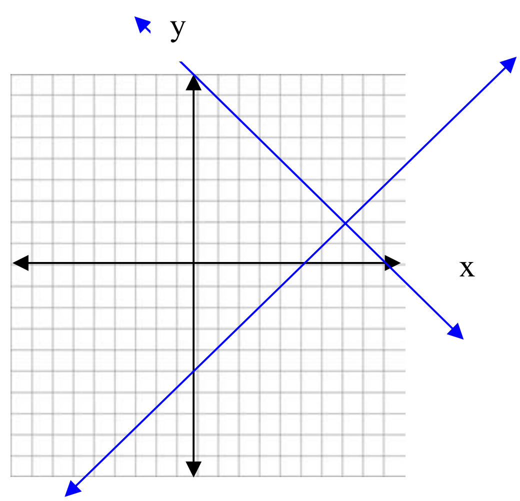
Let's Check Your Understanding 10

1. (7, 2) 2. (-2, 5) 3. ($\frac{3}{4}$, $\frac{2}{3}$) 4. (4, 2) 5. (10.4, 2.3)

Let's Practice for Mastery 11

1


(4, 0)

2


(7, 2)

3.(5 , 5) 4. (-1, 2) 5. (-2, -1) 6. (3, -5) 7. (3, 1) 8. ($\frac{17}{11}$,-$\frac{72}{11}$)

Let's Check Your Understanding 11

1. a 2. b 3. a 4. (5 , 0) 5.(3,-4) 6. (9, 2)

7. (6, 2) 8. (2.5, 4) 9. (5 , -2) 10. $(-\frac{9}{7}, \frac{13}{17})$

Let's Practice for Mastery 12

1. x + y = 53
   x – y = 15
   x = 34
   y = 19

2. x + y = 51
   y = 2x + 3
   x = 16
   y = 35

3. x + y = 27
   x = y + 5
   x = 16
   y = 11

4. t + u = 12
   10t + u = 12t
   t = 4
   u = 8 the number is 48.

5. t + u = 10
   t = u + 4
   u = 3
   t = 7
   the number is 73

Let's Check Your Understanding 12

1. Let x = the number of uniforms for girls
   y = the number of uniforms for boys

Equations;

x + y = 35

y = 2x + 5

x = 10 number of uniforms for girls

y = 25 number of uniforms for boys

2. Let t = ten's digit
   u = unit's digit

Equations:

t + u = 12

10t + u = 12 t

t = 4 ten's digit

u = 8 nit's digit , 48 is the number.

3. Let t = ten's digit
   u = unit's digit

Equations:

t + u = 9

10t +u = 6u

u = 6 unit's digit

t = 3 ten's digit, 3 is the number.

5. Let x = smaller number
   y = larger number

Equations;

x + y = 84

2x = y + 3

x = 29 the smaller number

y = 55 the larger number

Let's Practice for Mastery 13

1. 10 000 2. .08x +.09y =860 3. P6 000 4. P4 000

5. y = x + 3 6. x = 5 7. y =8 8. 16 sq units 9. 25 sq units

10. d= (r - 6) 5 11. r = 54

Let's Check Your Understanding 13

1. $\frac{y}{65}$h 2. 52 kph 3. 24 4. $\frac{x}{52}$ 5. 120

Solve completely:

Let x = amount invested at 12%

y = amount invested at 15 %

Equations: $x + y = 25\ 000$

$.12x + .15y = 3\ 305$

$x =$ P10 167

$y = 14\ 833$

Let's Practice for Mastery 14

A.

1. $x + y = 2\ 500$
2. $45x + 52.50y = 126\ 750$
3. $x = 2\ 500 - y$
4. 1 900 number of orchestra tickets
5. 600 number of balcony tickets

B.

Let x= number of kilograms for 12 % salt solution

y = number of kilograms for 20% salt solution

12%	x	12%	.12x
20%	y	20%	.20y
15%	24	15%	.15 (24)

Equations: $x + y = 24$

$.12x + .20y = .15\ (24)$

$x = 15, \quad y = 9$

Let's Check your Understanding

1. 50x
2. 100y
3. x + y = 250
4. 50x+ 100y = 16 500
5. 170
6. 80

Solve Completely:

Let x = number of cookies at Php9.50 per pound

y= number of cookies at Php17.00 per pound

	No' of cookies	Amount per pound	Amount
Php9.50	x	9.50	9.50x
Php17.00	y	17.00	17.00y
Php12.50	45		12.50(45)

Equations:

x + y = 45

9.50x + 17y = 12.50 (45)

x = 27 number of cookies at Php9.50 per pound

y = 18 umber of cookies at Php17.00 per pound

Let's Practice for Mastery 15

Addition

Coefficients of y are additive inverses

Miguel is 23 months old

Jancent is 15 months old

Yes

Yes

Let's check Your Understanding 15

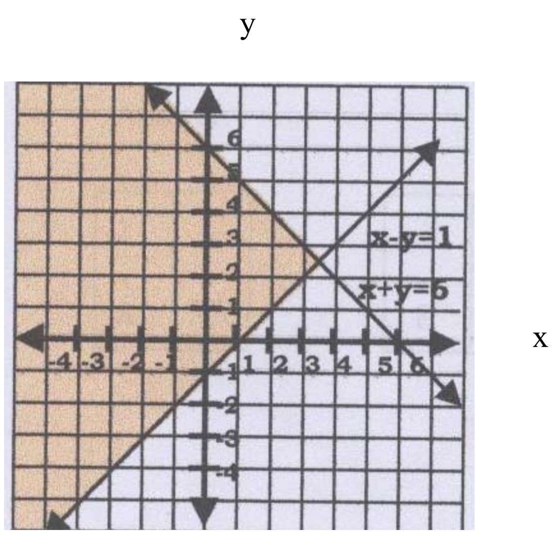
1. x + 3
2. y +3

3. x-1
4. y-1
5. x + 3 = 3 (y + 3)
6. x -1 = 7 (y – 1)
7. x = 15, y = 3
8. Kenneth is 3 years old, Patricia is 15 years old.

Lets Practice for Mastery 16

1. B 2. B

3.



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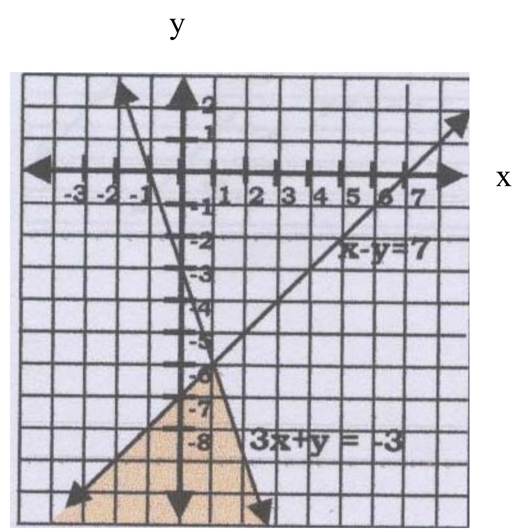
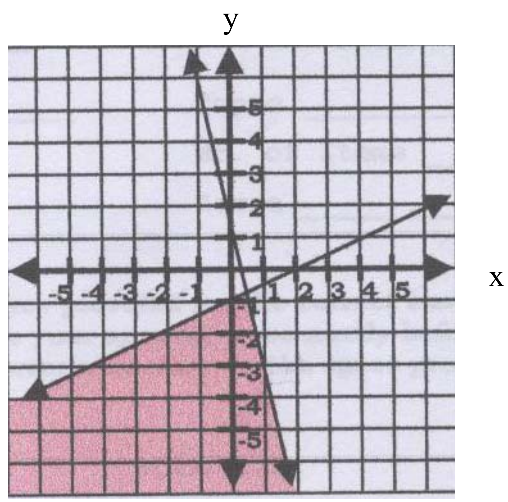


III. A

Let’s Check Your Understanding 16

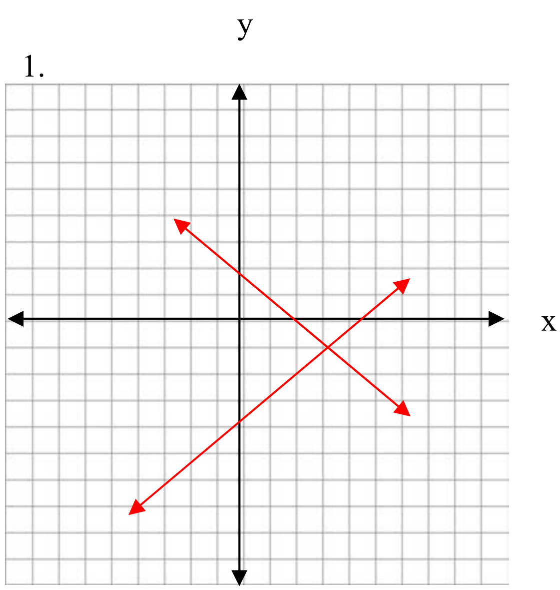
1.A. 2. B

3.

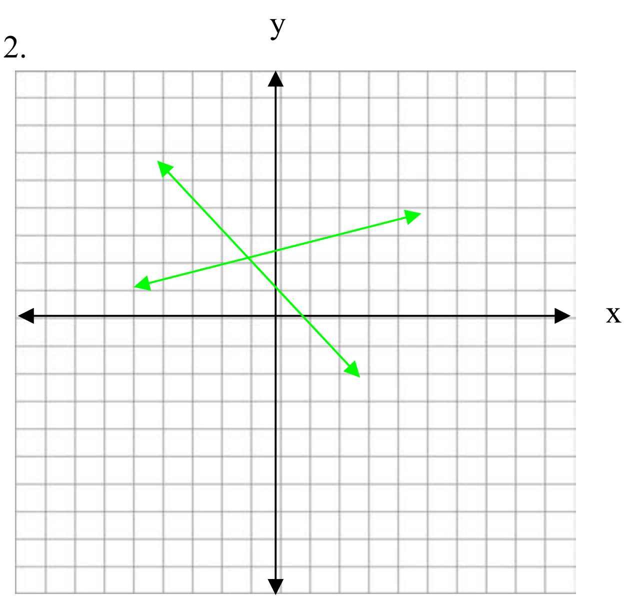


Unit Test

1.

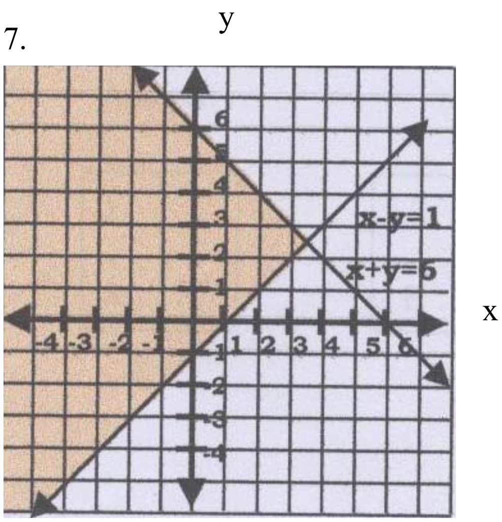


2.

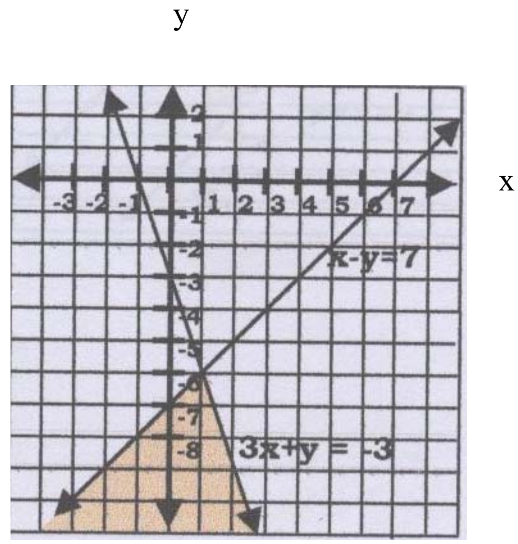


3. (3, 5) 4. (3, 4) 5. (4, -1) 6. (17, -5)

7.



8.



9. C

10. Let x = smaller number

y = larger number

Equations:

y –x = 10

y = 2x -6

x = 16 the smaller number

y = 26 the larger number

11. Let l = the length of the ground floor

w= the width of the ground floor

Equations:

2l + 2w = 860

l = 100 + w

l = 265 meters , the length of the ground floor

w = 165 meters, the width of the ground floor

12. Let x = the cost of one banana cue

y = the cost of one camote cue

Equations:

$$4x + 12y = 100$$

$$10x + 5y = 100$$

$$x = 7$$

$$y = 6$$

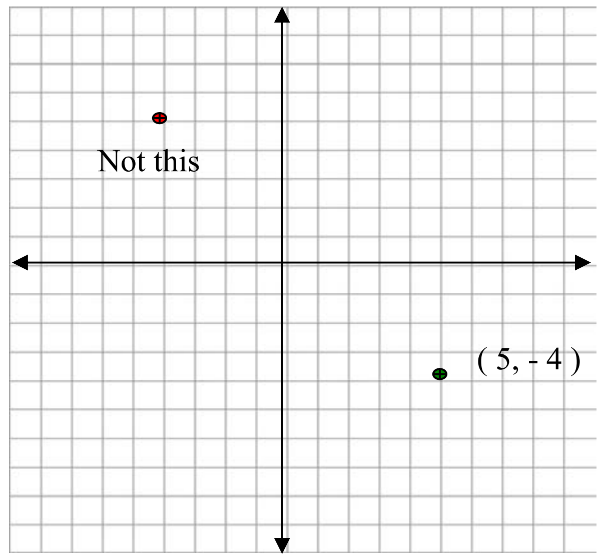
The required cost: One banana cue is Php7.00 and one camote cue is Php6.00.

Rubric for Problem Solving

Points	Criteria
3	Solves for the correct value of x and y (variables) by applying the method required to solve the system.
2	Applies the required method but fails to solve one of the values of the variables (x or y)
1	Writes few correct steps but fails to use the right properties to arrive at the correct answer.
0	No attempt done.

MISCONCEPTIONS AND COMMON ERRORS

1. In plotting of points sometimes we start reading the y coordinate instead of the x coordinate, Say plot (5 , -4)



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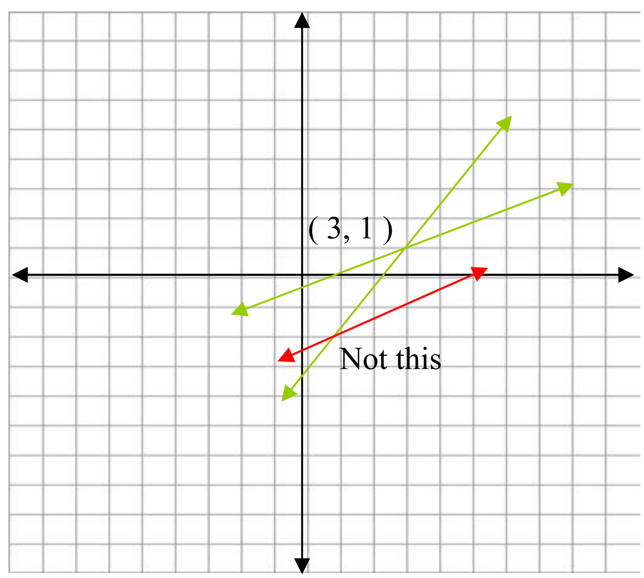
: 2. One line may be graphed incorrectly so the point of intersection is read incorrectly. So, it follows that the solution set is also incorrect. Instead of (3, 1) the correct solution Set, it will be read as (1 ,-2)



x

3.In solving system of equations by subtraction we sometimes forget to change the sign of the subtrahend before proceeding to addition.

Subtract	$5x + 2y = 14$		$5x + 2y = 14$
Not this	$-3x + 2y = -10$	But this	$3x - 2y = 10$
	$2x = 4$		$8x = 24$

4.In solving system of equations using the substitution method, we solve for y in terms of x and sometimes we substitute this value of y using the same equation.

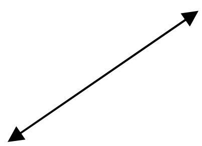
Solve by substitution; $x - y = 3$ $y = x - 3$

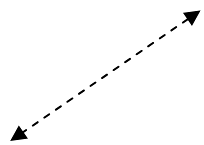
$2x + y = 3$

Solve for x: $x - (x - 3) = 3$ $2x + (x - 3) = 3$

Not this But this

5. In graphing inequalities such as > or < sometimes we used solid line instead of dashed line.





Not this But this