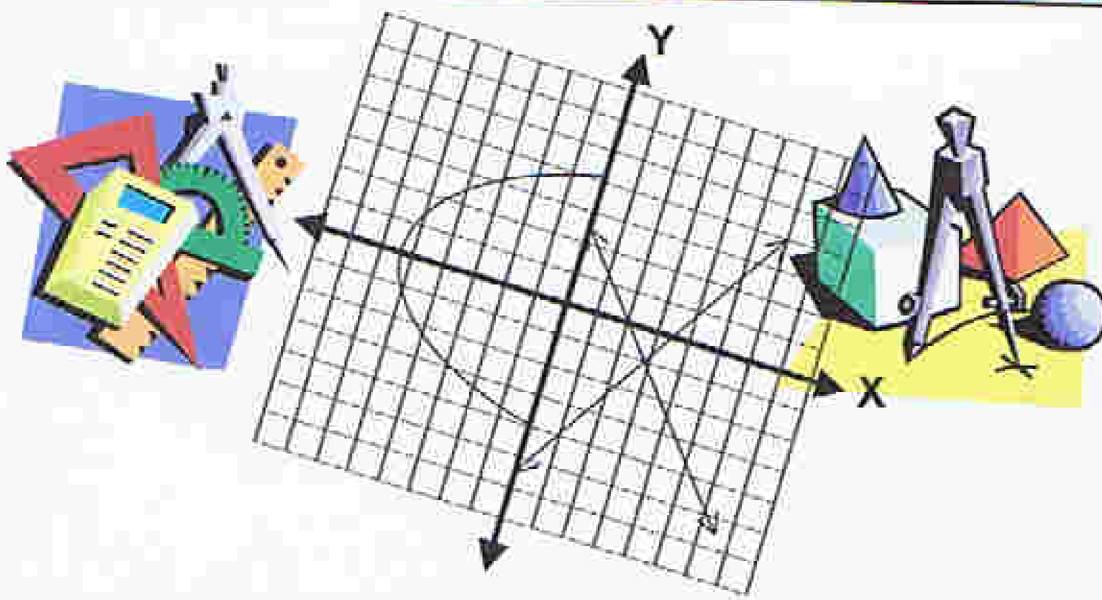
Project EASE

(Effective and Alternative Secondary Education)

MATHEMATICS I



MODULE 11

The way to XYZ

BUREAU OF SECONDARY EDUCATION
Department of Education
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Module 11
The Way to XYZ

What this module is all about

This module discusses the different properties of equality and inequality and its application in solving first degree equations and inequalities in one variable. As mentioned in Module 10, equations and inequalities are useful in industry and other fields like sciences. To illustrate, a certain investor might be interested in the set of values where the operational expenses are minimized while the benefits are maximized. Your previous knowledge in the properties of real numbers and skills on the four fundamental operations on monomials will help much in learning this module.

This module has four lessons:

Lesson 1 Properties of Real Numbers
Lesson 2 Properties of Equality
Lesson 3 Solving Linear Equations in One Variable
Lesson 4 Different Properties of Inequality and the Solutions of Inequalities in One Variable

What you are expected to learn

After going through this module, you are expected to:

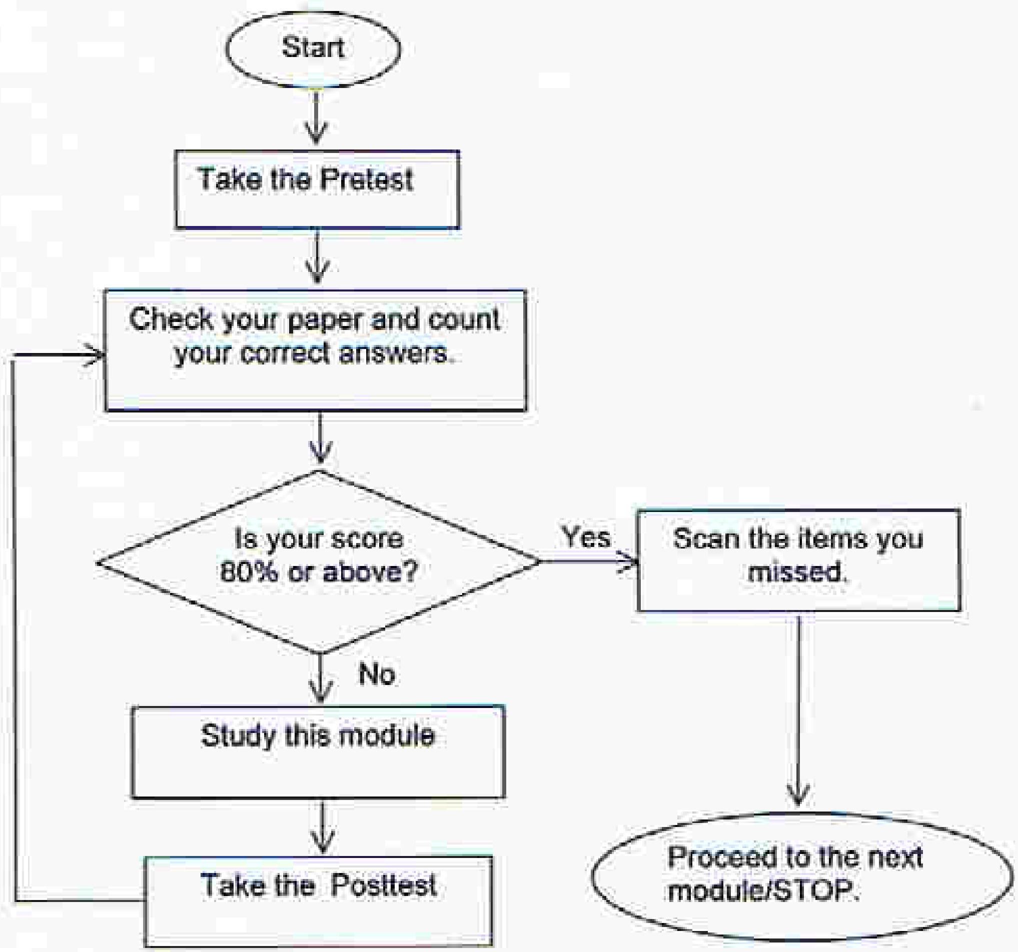
- review basic properties of real numbers;
- state and illustrate the different properties of equality;
- determine the solution set of first degree equations in one variable by applying the properties of equality;
- determine the solution set of first degree inequalities in one variable; and,
- visualize solutions of simple mathematical inequalities on a number line.

How to learn from this module

This is your guide for the proper use of the module:

1. Read the items in the module carefully.
2. Follow the directions as you read the materials.
3. Answer all the questions that you encounter. As you go through the module, you will find help to answer these questions. Sometimes, the answers are found at the end of the module for immediate feedback.
4. To be successful in undertaking this module, you must be patient and industrious in doing the suggested tasks.
5. Take your time to study and learn. **Happy learning!**

The following flowchart serves as your quick guide in using this module.



What to do before (Pretest)

A. Matching type: Match the number sentence in Column A to the property of real numbers it demonstrates found in Column B.

Column A	*Column B*
1) $(2+3)+7 = 2 + (3+7)$	a) Associative Property
2) $(5 \bullet 12) \bullet 7 = 60 \bullet 7$	b) Commutative Property
3) $(8 + 10) + 15 = 15 + (8 + 10)$	c) Closure Property
4) $9 \bullet (11 + 23) = (9 \bullet 11) + (9 \bullet 23)$	d) Identity Property
5) $8 \bullet (12 \bullet 1/12) = 8 \bullet 1$	e) Inverse Property
	f) Distributive Property

B. Matching type: Match the number sentence in Column C to the property of equality or inequality it demonstrates found in Column D.

Column C	*Column D*
6) If $8 + 2 < 14$ and $14 < 20$, then $8 + 2 < 20$.	g) Addition Property of Equality
7) If $(m-n) < (p+q)$ and $r > 0$, then $(m-n)r < (p+q)r$.	h) Multiplication Property of Equality
8) If $m=n$, then $m + p = n + p$.	i) Multiplication Property of Inequality
9) If $q + r = 15$, then $15 = q + r$.	j) Reflexive Property
10) If $15y = 75$, then $3y = 15$.	k) Symmetric Property
	l) Transitive Property

11) What is the solution of the equation $2(m - 3) = 4(3 + m)$?

a. -9   c. -3

b. -6   d. 0

12) What is/are the value/s of n that will make the equation true in $(3n - 7) = 1/5\,(21n + 3)$?

a. $n \geq 8$ c. 8

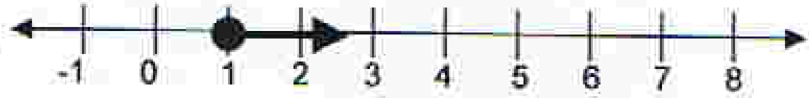
b. $n \leq -8$ d. -8

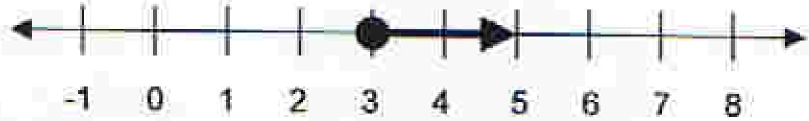
13) Which of the following is the solution set of $10 + 5 < 3 - 2z$?

a. $z < -5$ c. $z < 6$

b. $z < -6$ d. $z < 5$

14) Which of the following graphs represents the solution set of $8y - 5 \geq 10 + 3y$?

a. 

b. 

c. 

d. 

15) Which of the following is the solution set of $3q - 19 > 16 - 2q$?

a. $q > 7$
b. $q < 7$
c. $q > 35$
d. $q < 35$

Answer Key on page 29

What you will do

Read the following lessons carefully. Then do the suggested activities patiently.

Lesson 1 *Properties of Real Numbers*

In the previous module, you were taught how to find solutions to first-degree equations and inequalities in one variable using a given set of values and by inspection.

In this module, you will learn another way of solving first-degree equations and inequalities in one variable.

Let's take a moment to review the properties of real number that you learned before.

Properties of Real Number

1. *Closure Property*

a) Closure Property for Addition: The sum of any pair of real numbers is also a real number. In notation, we have :

If $a, b \in R$, then $a + b \in R$.

("$\in$ reads as element")

Ex. 1. 2 + 8 = 10 ; 2. 5 + 18 = 23

b) Closure Property for Multiplication: The product of any pair of real numbers is also a real number. In notation, we have:

If $a, b \in R$, then $ab \in R$.

Ex. 1. 2(6) = 12 ; 2. 8(7) = 56

2. *Commutative Property*

a) Commutative Property for Addition: The sum of two real numbers is the same, no matter what order the numbers are added. In notation, we have:

$a + b = b + a$

1. 8 + 5 ? 5 + 8;
   13 = 13

2. 12.5 + 3.8 ? 3.8 +12.5
   16.3 = 16.3

b). Commutative Property for Multiplication: The product of two real numbers is the same, no matter what order the numbers are multiplied. In notation, we have:

$ab = ba$

This means that multiplying two real numbers will give the same product regardless of the order in which the numbers are multiplied.

Ex. 1. 3 • 7 ? 7 • 3
   21 = 21

2. 2/3 • 1/5 ? 1/5 • 2/3
   2/15 = 2/15

3. *Associative Property*

a) Associative Property for Addition. The sum of three or more real numbers is the same, no matter how the numbers are grouped. In notation, we have:

$$a + b + c = (a + b) + c$$

Ex. 1. (6 + 5) + 9 ? 6 + (5 + 9)
11 + 9 □ 6 + 14
20 = 20

b) Associative Property for Multiplication. The product of three or more real numbers is the same, no matter how the numbers are grouped. In notation,

$$(ab)c = a(bc)$$

Ex. (20 • ¼) • 2 ? 20 • (¼ • 2)
5 • 2 □ 20 • ½
10 = 10

4. *Identity Property*

a) *Identity Property for Addition.* The sum of any real number and zero is equal to the given number. The number 0 is called the additive identity.

$$a + 0 = a \text{ and } 0 + a = a$$

Ex. 1. 0 + 8 = 8 2. 25m + 0 = 25m

b) *Identity Property for Multiplication.* The product of any real number and 1 is equal to the given number. The number 1 is called the multiplicative identity.

$$a \bullet 1 = a \text{ and } 1 \bullet a = a$$

Ex. 1. 37 • 1 = 37 2. 1 • 19n = 19n

5. *Inverse Property*

a) Inverse Property for Addition. The sum of a real number and its opposite is 0. The number opposite the given real number is called its **additive inverse**.

$$a + (-a) = 0 \text{ and } (-a) + (a) = 0$$

Ex. 1. 2/3 + (-2/3) = 0 2. –5q + (5q) = 0 *b) Inverse Property for Multiplication.* The product of a real number and its reciprocal is 1. The reciprocal of any given real number is called its **multiplicative inverse**. Note also that zero does not have a multiplicative inverse.

$a \bullet 1/a = 1$ and $1/a \bullet a = 1$

Ex. 1. $2/3 \bullet 3/2 = 1$ 2. $(-5) \bullet (-1/5) = 1$

15) *Distributive Property. Multiplication* is distributive with respect to addition.

$a(b + c) = ac$ and $(b + c)a = ba + ca$

Ex. 1. $2(3 + 5) = 2(3) + 2(5)$; 2. $(7n + 6)5 = 7n(5) + 6(5)$

16) *Multiplicative Property of Zero.* The product of a given real number and 0 is 0.

$a \cdot 0 = 0$ and $0 \cdot a = 0$

Ex. 1. $2(0) = 0$ 2. $0(9m + 2) = 0$

Before you proceed to take the self-Check, remember the following:

In each statement that follows, *a*, *b*, and *c*, are all real numbers.

A. Closure Property : If $a, b, c \in R$, then $a + b \in R$.
If $a, b \in R$, then $ab \in R$.

B. Commutative Property: $a + b = a + b$
$a(b) = b(a)$

C. Associative Property: $(a + b) + c = a + (b + c)$
$(ab)c = a(bc)$

D. Identity Property: $a + 0 = a$ and $0 + a = a$
$a(1) = a$ and $1(a) = a$

E. Inverse Property: $a + (-a) = 0$ and $(-a) + a = 0$
$a \cdot 1/a = 1$ and $1/a \cdot a = 1$

F. Distributive Property: $a(b + c) = ab + ac$
$(b + c)a = ba + ca$

G. Properties of Multiplication: $0(a) = 0$ and $a(0) = 0$.

Self-check 1

Name the property illustrated in each statement.

1. $2 + 3 = 5$ ____________________
2. $3 + 8 = 8 + 3$ ____________________
3. $5(7) = 35$ ____________________
4. $2(5 + 7) = 10 + 14$ ____________________
5. $2/5 + 3/7 = 3/7 + 2/5$ ____________________
6. $18 \times 1 = 18$ ____________________
7. $8 + (-8) = 0$ ____________________
8. $3/5 + (2/5 + 4/5) = (3/5 + 2/5) + 4/5$ ____________________
9. $1000(0) = 0$ ____________________
10. $1,000,000 + 0 = 1,000,000$ ____________________

Answer Key on page 29

Lesson 2 *Properties of Equality*

In Lesson 1, you have reviewed the different properties of real numbers. These properties are very helpful in solving first-degree equations and inequalities in one variable. Another set of properties, the **Properties of Equality** will be useful in solving equations.

Here are the properties of equality.

Properties of Equality

1. *Reflexive Property of Equality (RPE)*

Observe the following:

$$8 = 8$$
$$y + 4 = y + 4$$
$$5m + 3 = 5m + 3$$
$$20n - 7 = 20n - 7$$

These equations demonstrate the reflexive property of equality. How should we complete the equation below to show the reflexive property of equality?

$8p - 12 =$ ________?

If your answer is $8p - 12$, then you are correct. The above equations are examples that demonstrate **reflexive property of equality**. In your own words, explain the reflexive property. ______________________________

Then verify your notion with the statement that follows:

The reflexive property of equality means that any number is <u>equal to itself</u>.

In symbol, we write:

$a = a, a \in R$

2. *Symmetric Property of Equality (SPE)*

Observe the next set of expressions, and try to establish a pattern.

1. If $3 + 5 = 8$, then $8 = 3 + 5$.
2. If $20 = 4(5)$, then $4(5) = 20$.
3. If $15 = 2m + 3$, then $2m + 3 = 15$.
4. If $2w - 7y = 5z$, then $5z = 2w - 7y$.

Now, using the pattern that you have observed, what do you think should be written on the blanks so that the next set of expressions is written in a similar form as those of the expressions written above?

5. If $8p - 12 = 30$, then __________.
6. If $5c + 2d = 7f$, then __________.

If your answers are <u>$30 = 8p - 12$</u>, and <u>$7f = 5c + 2d$</u>, then your pattern is correct. The above examples demonstrate the symmetric property of equality. Try to generate your own notion of symmetric property, then, verify your notion with the statement that follows:

The symmetric property of equality means that <u>when two quantities are equal, the equality will hold true, no matter in what side of the equation is each of them is written</u>.

In symbol, we write:

If $a = b$, then $b = a$; $a, b \in R$.

3. *Transitive Property of Equality*

Observe the third set of expressions, and try to establish a pattern.

1. If 2 + 3 = 5 and 5 = 1 + 4, then 2 + 3 = 1 + 4.
2. If 4(8) = 32 and 32 = 2(16), then 4(8) = 2(16).
3. If 4m + 7 = 9n, and 9n = 45, then 4m + 7 = 45.
4. If 2w – 7y = 5z and 5z = 9y + 3, then 2w – 7y = 9y + 3.

Now, using the pattern that you have observed, what do you think should be written on the blanks so that the next set of expressions is written in a similar form as those of the expressions written above?

5. If 8p – 12 = 7q and 7q = 15 p, then ____________.
6. If 5c + 2d = 7f and 7f = 10 c -25, then ____________.

If your answers are <u>8p – 12 = 15p,</u> and <u>5c + 2d = 10c - 25</u>, then your answers are correct. The above examples demonstrate the transitive property of equality.

To further illustrate, supposed that the price of 5 t-shirts is the same as the price of two pairs of pants; and the price of these two pairs of pants is the same as the price of 7 pairs of shorts. Then, how do you compare the prices of the 5 t-shirts and the 7 pairs of shorts? ________________

If you were able to conclude that the two prices are equal, then you are correct!

Can you generate now your own notion of transitive property? Try it, then, verify your notion with the statement that follows:

The transitive property of equality means that <u>when the first two quantities are equal to the same quantity, then the first two given quantities are equal.</u>

In symbols, we write:

If $a = b$ and $b = c$, then $a = c$ where $a,b,c \in R$.

4. *Addition Property of Equality (APE)*

Observed the following figures below.

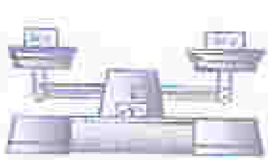


Figure 1

Figure 2

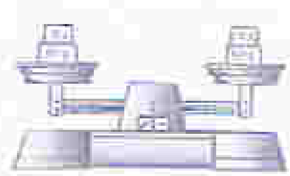


Figure 3

In Figure 1, the scale is in balance and each side holds 50 g each.

In the second figure, 20 g is added to only one side of the scales. What happened to the scale now? ___________

Yes, you are correct if you observed that the scale is no longer in balance. That will always happen, if the two sides do not hold equal mass.

In the third figure, 20 g were added to both sides. What happened to the scale? ________________________

Yes, you are correct if you observed that the scale maintains its balance. That is, adding equal amounts to both sides of an equation maintains the equality of both sides.

This time, observe the next set of expressions, and try to establish a pattern.

1. If $2 + 3 = 5$, then $(2 + 3) + 7 = 5 + 7$.
2. If $4(8) = 32$, then $4(8) + (-12) = 32 + (-12)$.
3. If $4m + 7 = 9n$, then $(4m + 7) + 3 = 9n + 3$.
4. If $2w - 7y = 5z$, then $(2w - 7y) + (-21) = 5z + (-21)$.

Now, using the pattern that you have formulated, what do you think should be written on the blanks so that the next set of expressions is written in a similar form as those of the expressions written above?

5. If $8p - 12 = 7q$, then $(8p - 12) + 12 =$ ____________.
6. If $5c + 2d = 7f$, then $(5c + 2d) + (-19) =$ ____________.

If your answers are <u>7q + 12</u>, and <u>7f + (-19)</u>, then your answers are correct. The above examples demonstrate the addition property of equality. Can you generate now your own notion of addition property of equality? Try it, then, verify your notion with the statement that follows:

The addition property of equality means that <u>when two quantities are equal and the same quantity is added to each of the two quantities, then, the sums are equal.</u>

In symbol, we write:

If $a = b$ then $a + c = b + c$, where $a, b, c \in R$.

5. Multiplication Property of Equality (MPE)

Observe the figures below.

Figure 4

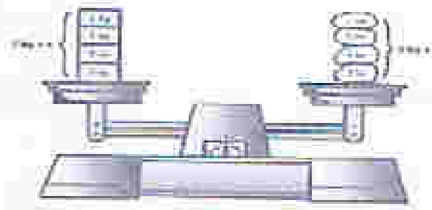


Figure 5

In Figure 4, the scale is in balance. Multiplying each weight by the same number does not tip the balance to one side as shown in Figure 5.

This principle illustrates the multiplication property of equality.

Furthermore, observe the fifth set of expressions, and try to establish a pattern.

1. If 2 + 3 = 5, then (2 + 3) • 7 = 5 • 7.
2. If 4(8) = 32, then 4(8) • (-12) = 32 • (-12).
3. If 4m + 7 = 9n, then (4m + 7) • 3 = 9n • 3.
4. If 2w – 7y = 5z, then (2w – 7y) • (-21) = 5z • (-21).

Now, using the pattern that you have formulated, what do you think should be written on the blanks so that the next set of expressions is written in a similar form as those of the expressions written above?

5. If 8p – 12 = 7q, then (8p –12) • 12 = ________________.
6. If 5c + 2d = 7f, then (5c + 2d) • (-19) = ________________.

If your answers are <u>7q • 12</u>, and <u>7f • (-19)</u>, then your answers are correct. The above examples demonstrate the multiplication property of equality. Can you generate now your own notion of multiplication property of equality? Try it, then, verify your notion with the statement that follows:

The multiplication property of equality means that <u>when two quantities are equal and the same quantity is multiplied to each of the two quantities, then, the products are equal.</u>

In symbol, we write:

If $a = b$ then $a \bullet c = b \bullet c$ where $a, b, c \in R$.

Remember the following: In each statement *a*, *b*, *c* are real numbers.

A. Reflexive Property of equality: $a = a$.

B. Symmetric Property of Equality: If $a = b$, then $b = a$.
C. Transitive Property of Equality: If $a = b$ and $b = c$, then $a = c$.
D. Addition Property of Equality: If $a = b$, then $a + c = b + c$.
E. Multiplication Property of Equality: If $a = b$, then $ac = bc$.

Self-check 2

Identify the property illustrated in each of the following:

1. If $6 + 2 = 8$ and $8 = 7 + 1$, then $6 + 2 = 7 + 1$ ____________
2. $16 - 5 = 16 - 5$ ____________
3. If $2a + 3 = a + 5$, then $a + 5 = 2a + 3$ ____________
4. If $3(5) = 15$, then $3(5)(1/5) = 15(1/5)$ ____________
5. If $3x - 5 = 4$, then $3x - 5 + 5 = 4 + 5$ ____________
6. $13m - 5n = 13m - 5n$ ____________
7. If $5 = 2 + 3$, then $2 + 3 = 5$ ____________
8. If $10 - 2 = 4x$, then $4x = 10 - 2$ ____________
9. $18(0) = 0$ ____________
10. $(25 + 8) + 0 = (25 + 8)$ ____________

Answer Key on page 29

Lesson 3 *Solving Linear Equations in One Variable*

In Lesson 1, we reviewed the properties of real numbers. In Lesson 2, we discussed the properties of equality. In this lesson, we will use these properties of real numbers and equality in solving first-degree equations.

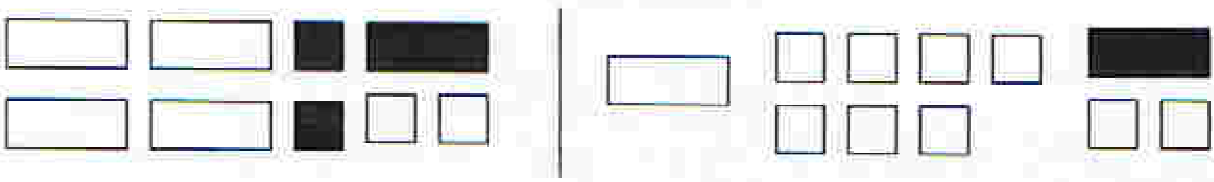
Suppose we are asked to solve for the value of x in the equation: $4x - 2 = x + 7$.

We will use the following algebraic tiles to solve this equation.

Step 1: Represent the equation using the algebraic tiles.

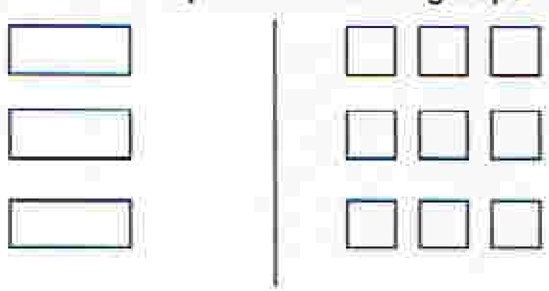
$4x - 2 \qquad = \qquad x + 7$

Step 2: Add one black rectangular tile and two white square tiles to both sides. (Recall: What would happen to a scale in balance if the same amount were added to both sides?)



Step 3: Simplify. A white and a black rectangle will cancel out. Similarly, a white and a black square will cancel out.

Step 4: Divide the number of squares into three groups.



How many white squares correspond to each rectangle? ____________________

There are three squares that correspond to each rectangle. Hence, $x = 3$.

To verify, replace x with 3 and check if the equation holds true.

Thus,

$$4x - 2 = x + 7$$
$$4(3) - 2 = 3 + 7$$
$$10 = 10$$

It's correct!

To summarize, we applied addition property of equality by adding (-x) and 2 to both sides. By closure property, the equation became 3x = 9. Then, we applied multiplication property of equality by multiplying 1/3 to both sides. By closure property, the equation ended up with x = 3.

Ex. 2. This time, we try to solve the equation without the algebraic tiles.

Suppose we are asked to solve for the value of x in the equation: $x + 15 = 37$. Our goal here is apply series of operations so that only the variable x will be left on one side of the equation. What do you think should be done so that the left side of the equation will only have the variable x? ____________________

Verify your solution with the steps that followed. If you were not able to generate your own solution, try to follow the discussion below. Steps are being given on the left column. Try to execute these steps on the space provided on the right column.

Step 1: Add (-15) to both sides of the equation. ____________________
Why can we do that? ____________

You are correct if your answer is
$[(x + 15)] + (-15) = 37 + (-15)$.
We can do this, because of <u>APE</u>.

Step 2: Regroup the addends on the left side of the equation. ____________________
Why can we do that? ____________

You are correct if your answer is
$x + [15 + (-15)] = 37 + (-15)$.
We can do this, because of <u>Associative Property of Addition.</u>

Step 3: Perform the indicated operation on both sides. ____________________
What property of real numbers is used in each operation? ____________

You are correct if your answer is $x + 0 = 22$.
The left side used the <u>Inverse Property of Addition</u> while the right side used <u>Closure Property for Addition.</u>

Step 4: Add x + 0. What property of real number Is used? ____________ ____________________

You are correct if your answer is $x = 22$.
That is because of identity property of addition.
Hence, the solution of the equation is 22. The solution set is {22}

To verify, replace x with 22 and check if the equation holds true.

Thus,

$$x + 15 = 37$$
$$22 + 15 = 37$$
$$37 = 37 \quad \textit{It's correct!}$$

In the next example, the steps to solve for the value of x in the equation $10 = -25 + x$, are being executed on the left column. Provide the reason for each step on the space provided on the right column.

Ex. 3. Solve for x in the equation $10 = -25 + x$

Solution:

$10 = -25 + x$	Given
$-25 + x = 10$	1) ____________
$x + (-25) = 10$	2) ____________
$[x + (-25)] + 25 = 10 + 25$	3) ____________
$x + [(-25) + 25] = 10 + 25$	4) ____________
$x + 0 = 10 + 25$	5) ____________
$x = 10 + 25$	6) ____________
$x = 35$	7) ____________

You are correct if what you have listed as the reasons are the same with what is listed here: (1) Symmetric Property of Equality, (2) Commutative Property of Addition, (3) Addition Property of Equality, (4) Associative Property of Addition, (5) Inverse Property of Addition, (6) Identity Property of Addition, and (7) Closure Property of Addition.

To check, we substitute 35 for the value of x, and verify if the equality holds true:

$$10 = -25 + x$$
$$10 = -25 + 35$$
$$10 = 10 \quad \textit{It's correct!}$$

Ex. 4. Solve for the value of x in $2x - 6 = 12$.

Solution: By addition property of equality, add 6 to both sides of the equation . So $2x - 6 = 12$ becomes ______________. Then, apply inverse property of equality so that the equation becomes ______. Next, by applying the identity property of addition, the equation becomes __________. By multiplication property of equality, multiply both sides of the equation by ½ (the reciprocal of 2). The equation now becomes ________. Then, apply the inverse property of multiplication so that the equation becomes ___________. Lastly, apply the identity property of multiplication to have the equation _____________.

Below is the summary of the processes involved in solving example # 3. You may verify your answers with the following:

$2x - 6 = 12$	
$2x - 6 + 6 = 12 + 6$	Addition Property of Equality
$2x + 0 = 18$	Inverse Property for Addition
$2x = 18$	Identity Property Addition
$½(2x) = ½(18)$	Multiplication Property of Equality
$(1)x = 18/2$	Inverse Property for Multiplication
$x = 9$	Identity Property for Multiplication

To check:

$2(9) - 6 = 12$

$18 - 6 = 12$

$12 = 12$ *It's correct!*

For example # 5, the reasons are being given to you. You need to figure out the resulting equation for every indicated reason.

Example 5: Solve for x in $12 - 4x = 21 - 7x$

Steps to be taken	Resulting equation	Reason
Add 7x to both sides	$(12 - 4x) + 7x = (21 - 7x) + 7x$	APE
Regroup the addends on both sides	$12 + (-4x + 7x) =$ ________	________
Add –7x and 7x on the right side of the equation	________________	________
Perform addition on the right side of the equation	________________	________
Add –4x + 7x	________________	________
Interchange the addends on the left side of the equation	________________	________
Add (-12) to both sides	________________	________
Regroup the addends on the left side of the equation	________________	________
Add 12 and (-12)	________________	________
Perform addition on the left side of the equation	________________	________
Perform addition on the right side of the equation	________________	________
Multiply both sides by 1/3	________________	________
Perform multiplication on the left side of the equation	________________	________

Perform multiplication on the right side of the equation ______________________ ________

You may verify your answers below:

Steps to be taken	Resulting equation	Reason
Add 7x to both sides	$(12 - 4x) + 7x = (21 - 7x) + 7x$	APE
Regroup the addends on both sides	$12 + (-4x + 7x) = 21 + (-7x + 7x)$	Associative P for +
Add –7x and 7x on the right side of the equation	$12 + (-4x + 7x) = 21 + 0$	Inverse P for +
Perform addition on the right side of the equation	$12 + (-4x + 7x) = 21$	Identity P for +
Add –4x + 7x	$12 + 3x = 21$	Closure P for +
Interchange the addends on the left side of the equation	$3x + 12 = 21$	Commutative P for +
Add (-12) to both sides	$(3x + 12) + (-12) = 21 + (-12)$	APE
Regroup the addends on the left side of the equation	$3x + [12 + (-12)] = 21 + (-12)$	Associative P for +
Add 12 and (-12)	$3x + 0 = 21 + (-12)$	Inverse P for +
Perform addition on the left side of the equation	$3x = 21 + (-12)$	Identity P for +
Perform addition on the right side of the equation	$3x = 9$	Closure P for +
Multiply both sides by 1/3	$(3x)(1/3) = (9)(1/3)$	MPE
Perform multiplication on the left side of the equation	$x = (9)(1/3)$	Inverse P of x
Perform multiplication on the right side of the equation	$x = 3$	Closure P for x

To check:

$$12 - 4x = 21 - 7x$$
$$12 - 4(3) = 4 - 2(2)$$
$$12 - 12 = 21 - 21$$
$$0 = 0$$

It's correct!

On your own, try to solve for x in $2(3x - 6) = 4 - 2x$.

______________________ ______________________

______________________ ______________________

______________________ ______________________

If your solution set is {2}, then your answer is correct!

Self-check 3

A. The solution 3(3 – 2) = 5(x + 12) is given. Just supply the missing part.

Solution: $3(x-2) = 5(x+12)$	
$3x - 6 = 5x + 60$	1. ______________
2. ______________	Apply commutative property on both sides of the equation. Then add -5x to both sides (APE).
$-2x - 6 = 60$	Combine 3x – 5x.
$-2x - 6 + 6 = 60 + 6$	3. ______________
$-2x + 0 = 66$	Inverse Property for Addition
$-2x = 66$	4. ________ and ________
5. ______________	Multiply both sides by -1/2 (MPE).
$(1)x = -33$	Inverse Property for Multiplication
$x = -33$	Identity Property for Multiplication

B. Solve the following equations. Write the letter corresponding to the equation on the box(es) containing its solution to reveal the message.

Message in the Boxes

H: $x + 5 = -3$
L: $x - 20 = -11$
G: $x - 18 = -5$
M: $x + 5 = 19$
E: $y + 54 = 81$
S: $y + 75 = 28$

N: $2y - 4 = y - 4$
T: $4 + 3y = 16$
R: $6z - 5 = 2z + 3$
I: $3(3z - 2) = 4z + 9$
A: $5(z - 2) = 4(2z + 5)$
Y: $9 + 5z = 3(z - 5)$

9	27	-10	2	0	3	0	13

14	-10	4	-8

3	-47

27	-10	-47	-12

!

Answer Key on page 29

Lesson 4 *Different Properties of Inequality and the Solutions of Inequalities in One Variable*

This lesson focuses on solving first-degree inequalities in one variable. It would be helpful for you to recall the different properties of real numbers and of equality as we discuss the different properties of inequality.

Properties of Inequalities

A. *Transitive Property of Inequality*

If Jose is younger than Celia, and Celia is younger than Minda, how will you compare the age of Jose and Minda? ____________________

Jose

Celia

Minda

Moreover, if Minda is taller than Celia, and Celia is taller than Jose, how will you compare the height of Minda and Jose? ________________________.

You are correct if your answers are "Jose is younger than Minda" and "Minda is taller than Jose" respectively.

The above examples illustrate the transitive property of inequality. In real numbers, we may have the following examples that would illustrate such property.

1) If $3n > 5p$, and $5p > 9r$, then $3n > 9r$.
2) If $(6b - 5c) > (9b + 1)$, and $(9b + 1) > (7c - 12)$, then $(6b - 5c) > (7c - 12)$.
3) If $5y < 7w$, and $7w < 2m$, then $5y < 2m$.
4) If $(21g - 13) < (2k + 5)$, and $(2k + 5) < (4f + 17)$, then $(21g - 13) < (4f + 17)$.

Following the pattern above, fill in the blanks to complete the statement.

5) If $(12p - 9) > 25m$, and $25m > (17n + 13)$, then __________________.
6) If $(17w + 3) < (2y - 1)$, and $(2y - 1) < (3d - 10)$, then _________________.

You are correct if your answers are $\underline{(12p-9) > (17n+13)}$ and $\underline{(17w+3) < (3d-10)}$ respectively.

In your own words, describe the transitive property of inequality.

____.

You may verify your answer with the statements below.

> Given $a, b, c \in R$.
> If $a > b$ and $b > c$, then $a > c$
> If $a < b$ and $b < c$, then $a < c$

B. Addition Property of Inequality (API)

The discussion here is very similar to the discussion of Addition Property of Equality. Recall that, for any real number a, b, c, when a = b, then a + c = b + c.

This time, we deal with inequality.

Suppose that Rodora has P7000 in a bank while Mavic has P5000. Who has a greater amount of deposit? ______________

Then, both of them deposit P2000 each. How much would be the deposit of each now? Who has a greater amount of deposit? ____________________

Then, suppose after two months, both of them withdrew P1000 each. How much would be the deposit of each now? Who has a greater amount of deposit? ________________

You are correct if you conclude that after both had deposited P2000 each, Rodora's deposit is greater than Mavic's. Similarly, after both had withdrawn P1000 each, Rodora's deposit is greater than Mavic's.

The above examples illustrate the addition property of inequality. The statements below illustrate this property of inequality as well.

1) If $(6b - 5c) > (9b + 1)$, then $(6b - 5c) - 5 > (9b + 1) - 5$.
2) If $5y < 7w$, then $5y + 3m < 7w + 3m$.
3) If $(21g - 13) < (2k + 5)$, then $(21g - 13) - 8f < (2k + 5) - 8f$.

Following the pattern above, fill in the blanks to complete the statement.

4) If $(12p - 9) > 25m$, then $(12p - 9) + 5y$ __ ____________.

5) If (17w + 3) < (2y – 1), then (17w + 3) – 10n ___ ____________.

You are correct if your answers are **> 25 m + 5y** and **< (2y - 1) – 10n** respectively.

In your own words, can you now describe the addition property of inequality?
____.

You may verify your answer with the statement below.

Given $a, b, c \in R$. If $a > b$ and $a + c > b + c$ If $a < b$ and $a + c < b + c$

C. *Multiplication Property of Inequality*

Suppose that we have the inequality 8 > 5.

a) Multiply both sides by any positive number, say 4.
8 (4) > 5 (4)
32 > 20 True

b) Multiply both sides by any negative number, say (-1).
8 (-1) > 5 (-1)
(-8) > (-5) False.

What did you observe? ______________________________

You are correct if you were able to observe that the direction of the inequality does not change when both sides are multiplied to a positive number. However, the direction of the inequality changes when both sides are multiplied to a negative number. In notations, we have:

If a > b, and c > 0, then ac > bc. Also, if a < b, and c > 0, then ac < bc.

If a > b, and c < 0, then ac < bc. Also, if a < b, and c < 0, then ac > bc.

Now, could you figure out if both sides of the inequality were multiplied to zero?
______________________________.

Yes, you are correct if your answer is: ***both products will be zero, and hence are equal to each other.***

At this point, you are now ready to solve first-degree inequalities in one variable. The steps involved are very similar to the steps we consider in solving equalities. However, instead of using the properties of equality, we now use the properties of inequality.

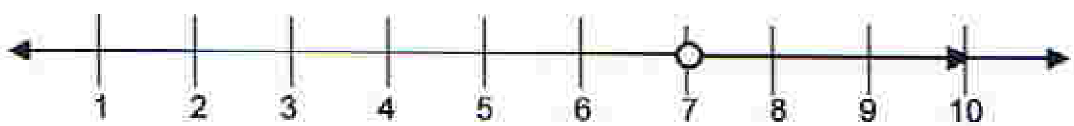
Example 1: Solve $x + 5 > 12$

$(x + 5) + (-5) > 12 + (-5)$	Add (-5) to both sides of the inequality (API)
$x + [5 + (-5)] > 7$	Associative Property, and Closure Property
$x + 0 > 7$	Inverse Property for Addition
$x > 7$	Identity Property for Addition

Therefore, the solutions are all real numbers greater than 7. To check, you take several values such as 8, 9, and 10 and substitute these in the original inequality. To illustrate:

If $x = 8 \rightarrow 8 + 5 > 12$; $13 > 12$. *Correct!* If $x = 9 \rightarrow 9 + 5 > 12$; $14 > 12$. *Correct!* If $x = 10 \rightarrow 10 + 5 > 12$ $15 > 12$. *Correct!*

You illustrate the solutions on the number line, thus,



The hollow dot or unshaded circle indicates that 7 is not included in the solution set.

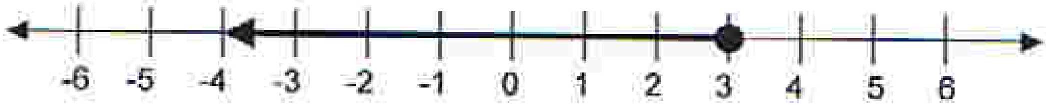
Example 2: Solve $4x - 3 \leq 9$

Solution: $4x - 3 \leq 9$

$(4x - 3) + 3 \leq 9 + 3$	Add 3 to both sides of the inequality (API)
$4x + [(-3) + 3] \leq 12$	Associative and Closure Property
$4x + 0 \leq 12$	Inverse Property for Addition
$4x \leq 12$	Identify Property for Addition
$\frac{1}{4}(4x) \leq \frac{1}{4}(12)$	Multiply both sides of the inequality by ¼ (MPI)
$1(x) \leq 3$	Associative and Inverse Property for x
$x \leq 3$	Identity Property for Multiplication

The solution set consist of all numbers less than or equal to 3.

The representation of the solution set on the number line is shown below:



The solid dot or shaded circle indicates that 3 is included in the solution set.

Example 3: Solve $3x - 6 > 5x + 4$

Solution:

$$3x - 6 > 5x + 4$$
$$(-6) + 3x > 4 + 5x$$
$$[(-6) + 3x) + (-5x) > [4 + 5x] + (-5x)$$
$$(-6) + [\, 3x + (-5x)] > 4 + [\, 5x + (-5x)]$$
$$(-6) + (-2x) > 4 + 0$$
$$(-2x) + (-6) > 4$$
$$[(-2x) + (-6)] + 6 > 4 + 6$$
$$-2x + [\, (-6) + 6] > 10$$
$$-2x + 0 > 10$$
$$-2x > 10$$

$(-2x)\,(-1/2) < (10)\,(-1/2)$ Why?

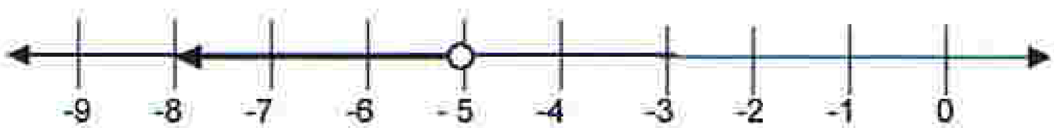
$X < -5$

To check, you take $x = 6$

$$3(-6) - 6 > 5(-6) + 4$$
$$18 - 6 > -30 + 4$$
$$-24 > -26$$

Correct!

The graph of the solution set is shown below.



Example: Solve $3(2x + 4) \leq 2(15 - 6x)$

Solution:

$$
\begin{gathered}
3(2x + 4) \leq 2(15 - 6x) \\
6x + 12 \leq 30 - 12x \\
6x + 12x + 12 \leq 30 - 12x + 12x \\
18x + 12 \leq 30 \\
18x + 12 + (-12) \leq 30 + (-12) \\
18x \leq 18 \\
1/8(18x) \leq 1/18(18) \\
x \leq 1
\end{gathered}
$$

Check and represent the solution set on the number line.

Self-check 3

Solve each of the following inequalities. Represent each solution set on the number line.

1. $x + 2 < 7$
2. $10 + x > 8$
3. $12 - y \leq -4 + 3y$
4. $y - 13 \geq 3 - 7y$
5. $8z + 13 > -3 + 10z$

Answer Key on page 29

Let's summarize

Look back!

- Properties of Real Numbers

A. Closure Property

If $a, b, c \in R$, then $a + b \square R$.
If $a, b \in R$, then $ab \in R$.

B. Commutative Property

$a + b = a + b$
$a(b) = b(a)$

C. Associative Property

$(a + b) + c = a + (b + c)$
$(ab)c = a(bc)$

D. Identity Property

$a + 0 = a$ and $0 + a = a$
$a(1) = a$ and $1(a) = a$

E. Inverse Property

$a + (-a) = 0$ and $(-a) + a = 0$
$a \cdot 1/a = 1$ and $1/a \cdot a = 1$

F. Distributive Property

$a(b + c) = ab + ac$
$(b + c)a = ba + ca$

G. Properties of Multiplication

$0(a) = 0$ and $a(0) = 0$

◉ Properties of Equality

A. Reflexive Property of equality

$a = a, a \in R$

B. Symmetric Property of Equality

If $a = b$, then $b = a$

C. Transitive Property of Equality

If $a = b$ and $b = c$, then $a = c$

D. Addition Property of Equality

If $a = b$, then $a + c = b + c$

E. Multiplication Property of Equality

If $a = b$, then $ac = bc$

- Properties of Inequalities

A. Transitive Property of Inequality

If $a > b$ and $b > c$, then $a > c$ for $a, b, c, \in R$.
If $a < b$ and $b < c$, then $a < c$ for $a, b, c, \in R$.

B. Addition Property of Inequality

If $a > b$, then $a + c > b + c$ for $a, b, c, \in R$.
If $a < b$, then $a + c < b + c$ for $a, b, c, \in R$.

C. Multiplication Property of Inequality

If $a > b$ and $c > 0$, then $ac > bc$.
If $a < b$ and $c > 0$, then $ac < bc$.
If $a > b$ and $c < 0$, then $ac < bc$.
If $a < b$ and $c < 0$, then $ac > bc$.

- To solve first degree equations and inequalities in one variable algebraically is to apply the properties of real number and the properties of equality and inequality.

What to do after (Posttest)

A. Matching type: For #1-5: Match the number sentence in Column A to the property of real numbers it demonstrates found in Column B.

Column A	*Column B*
1) $5 \bullet (2m + 7n) = (5 \bullet 2m) + (5 \bullet 7n)$	a) Associative Property
2) $(12p + 19q) + 0 = 12p + 19q$	b) Commutative Property
3) $[(-8) + 8) + 15y = 0 + 15y$	c) Closure Property
4) $9 \bullet (11 \bullet 23) = (11 \bullet 23) \bullet 9$	d) Identity Property
5) $(8 \bullet 12) \bullet 1/12 = 8 \bullet (12 \bullet 1/12)$	e) Inverse Property
	f) Distributive Property

B. Matching type: For # 6-10: Match the number sentence in Column C to the property of equality or inequality it demonstrates found in Column D.

Column C

6) If 3m + 2n = 14p, then 14p = 3m + 2n.
7) If (m-n)< (p+q) and (p+q) < 0, then (m-n)r < 0.
8) If m > 7n, then m + p > 7n + p.
9) If q+ r =15, then q + r –13m = 15 –13m.
10) If 15y < 75, then -3y > -15.

Column D

g) Addition Property of Inequality
h) Multiplication Property of Equality
i) Multiplication Property of Inequality
j) Reflexive Property
k) Symmetric Property
l) Transitive Property

C. Multiple Choice. Choose the letter of the correct answer.

11. What is the value of x in 2 (x + 3) = 5x + 7?
a. –1/3
b. 1/3
c. 1/3
d. 3

12. What is the solution of 3(2x + 4) = 2(24 – 6x)?
a. x = 0
b. x = 1
c. x = 2
d. x = 3

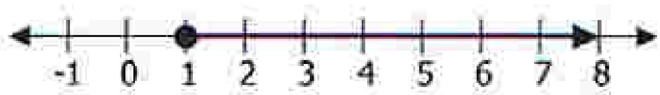
13. Which of the following inequality has {*y*/*y* > 1} the solution set?
a. -5 + 3*y* < 8
b. -5 – 3*y* < 8
c. -5 + 3*y* < -8
d. -5 – 3*y* < -8

14. Which of the following graphs represents the solution set of 5z – 1 ≤ 20 + 2z?

a.


b.


c.


d.


15. What is the solution set of 2 (3y +10) < 7 (2x –4)?
a. y = 6
b. y > 6
c. y = -6
d. y < 6

Answer Key on page 31

Answer Key

Pretest page 29

1. Distributive Property
2. Closure Property
3. Commutative Property
4. Distributive Property
5. Inverse Property
6. Transitive property
7. Multiplication Property of Inequality
8. Addition Property of Equality
9. Symmetric Property
10. Multiplication Property of Equality
11. a
12. d
13. c
14. b
15. a

Lesson 1 *Self-check 1* page 8

1. Closure property
2. Commutative Property
3. Closure Property
4. Distributive Property
5. Commutative Property
6. Identity Property for Multiplication
7. Inverse Property for Addition
8. Associative Property
9. Multiplication Property of Zero
10. Identity Property for Addition

Lesson 2 *Self-check 2* page 13

1. Transitive Property of Equality
2. Reflexive Property of Equality
3. Symmetric Property of Equality
4. Multiplication Property of Equality
5. Addition Property of Equality
6. Reflexive Property of Equality
7. Symmetric Property of Equality
8. Symmetric Property of Equality
9. Multiplication Property of Zero
10. Identity Property

Lesson 3 *Self-check 3* page 25

A.
1. Distributive Property
2. $(-6) + 3x + (-5x) = 60 + 5x + (-5x)$
3. Addition Property of Equality
4. Inverse Property and Identity Property for Addition
5. $-1/2(-2x) = -1/2(66)$

B.

H: $x + 5 = -3$
$x = -3 - 5$

G: $x - 18 = -5$
$x = 18 - 5$

E: $y + 54 = 81$
$y = 81 - 54$

$x = -8$

$x = 13$

$y = 27$

L: $x - 5 = -11$
$x = -11 + 20$
$x = 9$

N: $2y - 4 = y - 4$
$2y - 4 = 4 - 4$
$y = 0$

T: $4 + 3y = 16$
$3y = 16 - 4$
$3y = 12$
$y = 4$

R: $6z - 5 = 2z + 3$
$6z - 2z = 3 + 5$
$4z = 8$
$z = 2$

M: $x + 5 = 19$
$x = 19 - 5$
$x = 14$

S: $y + 75 = 28$
$y = 28 - 75$
$y = -47$

I: $3(3z - 2) = 4z + 9$
$6z - 6 = 4z + 9$
$9z - 4z = 9 + 6$
$5z = 15$
$z = 3$

A: $5(z - 2) = 4(2z + 5)$
$5z - 10 = 8z + 20$
$5z - 8z = 20 + 10$
$3z = 30$
$z = 10$

Y: $9 + 5z = 3(z - 5)$
$9 + 5z = 3z - 15$
$5z - 3z = -15 - 9$
$2z = -24$
$z = 12$

Message in the Boxes

L	E	A	R	N	I	N	G
9	27	-10	2	0	3	0	13

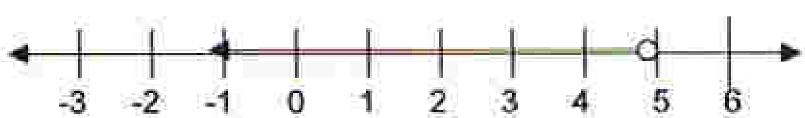
M	A	T	H
14	-10	4	-8

I	S
3	-47

E	A	S	Y
27	-10	-47	-12

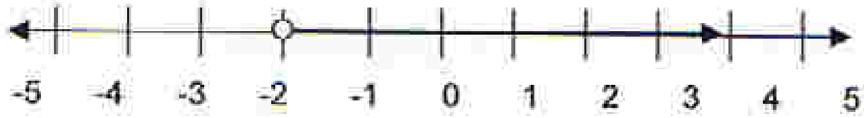
Exploration 4

1. $x + 2 < 7$
$x < 7 - 2$
$x < 5$

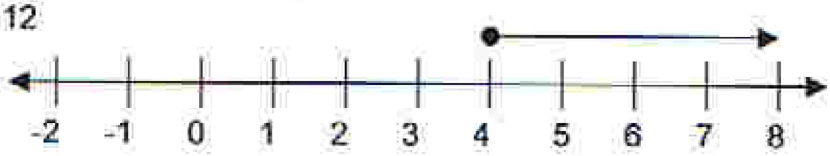


2. $10 + x > 8$
$x > 8 - 10$

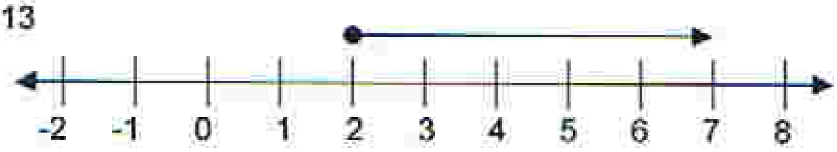
$x > -2$



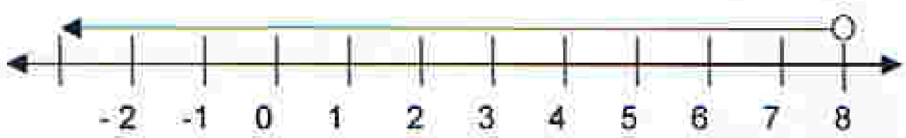
3. $12 - y \leq -4 + 3y$
   $-y - 3y \leq -4 - 12$
   $-4y \leq -16$
   $y \geq 4$



4. $y - 13 \geq 3 - 7y$
   $y + 7y \geq 3 + 13$
   $8y \geq 3 + 13$
   $8y \geq 16$
   $y \geq 2$



5. $8z + 13 > -3 + 10z$
   $8z - 10z > -3 - 13$
   $-2z > -16$
   $z < 8$



Posttest page 27

1. Distributive Property
2. Identity Property
3. Inverse Property
4. Commutative Property
5. Associative Property
6. Symmetric property
7. Transitive Property
8. Addition Property of Inequality
9. Addition Property of Equality
10. Multiplication Property of Equality
11. a
12. c
13. d
14. c
15. b

END OF MODULE