Module 10
Guess, Try and Check

What this module is all about

This module discusses the definition of the solution set of a first degree equation or inequality in one variable. It also deals with the representation of the solutions of equations and inequalities on a number line. Equations and inequalities are useful in industry and in other fields like science. Thus, it is important to know the different methods of solving equations and inequalities. Some of these methods are discussed in this module.

There are five lessons in this module.

Lesson 1	**Solving Linear Equations in One Variable**
Lesson 2	**Solving Linear Inequalities in One variable**
Lesson 3	**Solving Linear Equations and Inequalities on a Number Line**
Lesson 4	**Solutions of Linear Equations and Inequalities from Replacement Set**
Lesson 5	**Methods of Solving Linear Equations and Inequalities in One Variable**

What you are expected to learn

After working on this module, you are expected to:

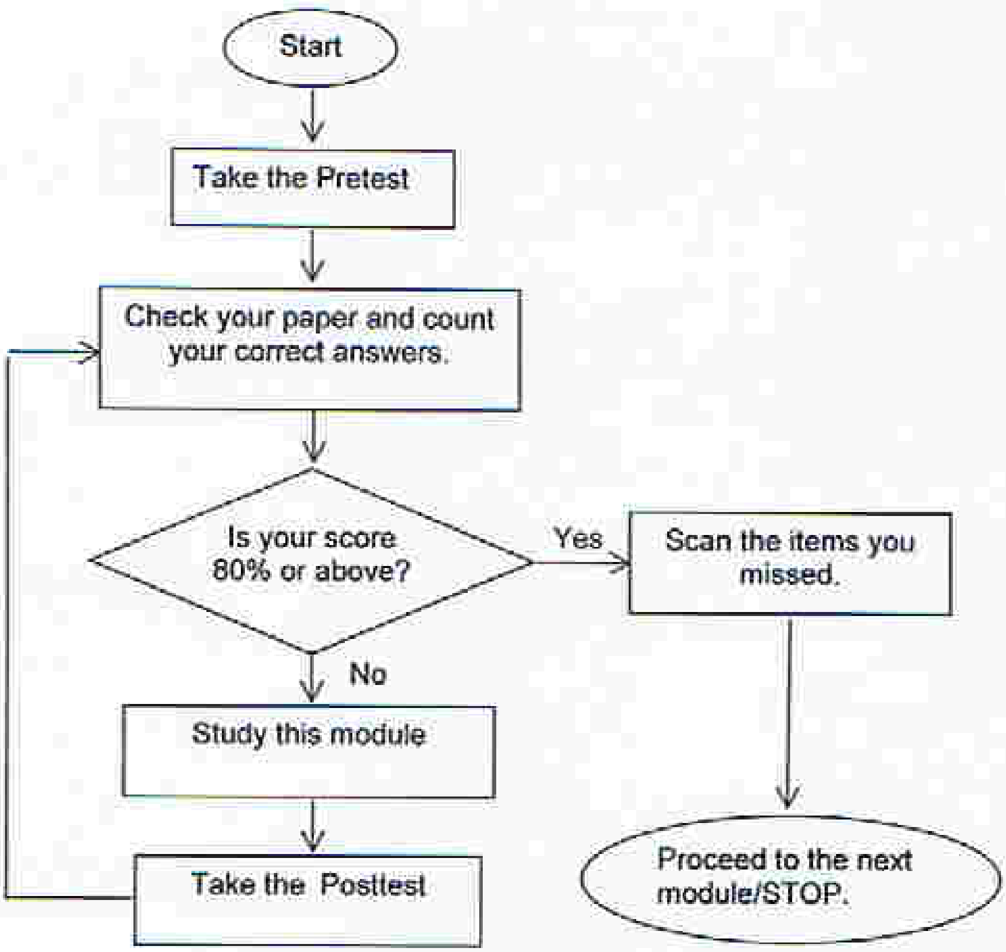
- define the solution set of a first degree equation or inequality in one variable
- illustrate the solution set of equations and inequalities in one variable on a number line
- find the solution set of simple equations and inequalities in one variable from a given replacement set
- find the solution set of simple equations and inequalities in one variable by inspection

How to learn from this module

This is your guide for the proper use of the module:

1. Read the items in the module carefully.
2. Follow the directions as you read the materials.
3. Answer all the questions that you encounter. As you go through the module, you will find help to answer these questions. Sometimes, the answers are found at the end of the module for immediate feedback.
4. To be successful in undertaking this module, you must be patient and industrious in doing the suggested tasks.
5. Take your time to study and learn. **Happy learning!**

The following flowchart serves as your quick guide in using this module.



What to do before (Pretest)

Multiple Choice. Choose the letter of the correct answer.

1. Which of the following statements is **false**?

   a. $3x = 3$, if $x = 1$
   b. $4 + x = -6$, if $x = -10$
   c. $2x = 10$, if $x = 8$
   d. $x > -3$, if $x = 3$

2. Which equation is true if b = 2?

   a. $3b = 6$
   c. $1 - 2b = 3$
   b. $4b - 1 = 0$
   d. $b = 3b - 1$

3. What equation has {0} as solution set?

   a. $y + 1 = 0$
   c. $y > 0$
   b. $y - 1 = -1$
   d. $y < 1$

4. What is the solution set of $3y + 1 = 10$?

   a. {2} b. {3} c. {4} d. {5}

5. Which of these open sentences has –1 as one of its solutions?

   a. $x > 1$
   c. $x < -2$
   b. $x < 1$
   d. $x > 0$

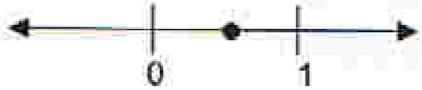
6. Which of the following equations represents the graph below?

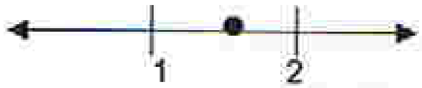
   a. $d - 1 = 3$
   c. $d - 1 > 3$
   b. $d + 1 = -1$
   d. $d + 1 < -1$

7. Illustrate the graph of $v = ½$.

   a.

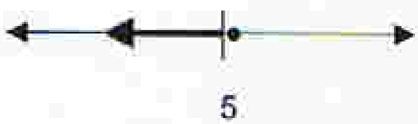

b. 

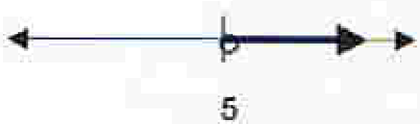
c. 

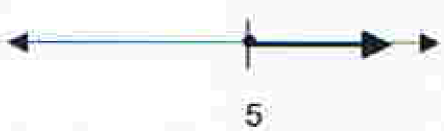
d. 

8. Which of the following graphs illustrates the solution set of $a > 5$?

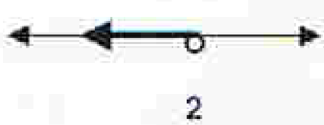
a. 

b. 

c. 

d. 

9. Which of the graphs drawn below represents the solution set of the inequality $s - 2 < 0$?

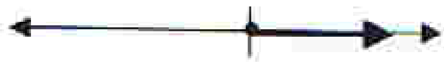
a. 

b. 

2

c. 

2

d. 

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10. Determine which sentence has 18 as the **only** solution.

a. $b + 2 = -16$
b. $h/2 = 9$
c. $r \geq 18$
d. $w \leq 18$

11. What is the solution set of $2z + 1 = 1$ taken from the replacement set $\{-2, -1, 0, 1\}$?

a. $\{-2\}$ b. $\{-1\}$ c. $\{0\}$ d. $\{1\}$

12. What sentence is satisfied by all elements of the set $\{-1, 0\}$?

a. $6m - 1 = 7$
b. $-2 > t - 3$
c. $5x + 3 = 4x + 3$
d. $b - 5 < -4$

13. Given the replacement set $\{1, 2, 3, 4\}$, determine the solution set of the inequality $2r + 3 < 8$.

a. $\{\ \}$ b. $\{1\}$ c. $\{1, 2\}$ d. $\{1, 2, 3\}$

14. Find the solution set of the inequality $3a + 1 \geq 4$

a. $\{ a/a \geq -1 \}$
b. $\{ a/a \geq 1 \}$
c. $\{ a/a \geq -5/3 \}$
d. $\{ a/a \geq 5/3 \}$

15. Taken from the replacement set $\{2, 4, 6\}$, what is the solution set of the inequality $3 - 2y \leq -5$?

a. $\{\ \}$ b. $\{2, 4\}$ c. $\{4\}$ d. $\{4, 6\}$

Answer Key on page 3

What you will do

Lesson 1 *Solution of Linear Equations in One Variable*

In the previous modules, you learned about open sentences, first degree equations in one variable and first degree inequalities in one variable. Let's recall their definitions.

> An **open sentence** is a sentence which becomes true or false when the variable is replaced by a given value.
>
> An open sentence of the form $ax + c = 0$ is called a **first degree equation in *x***.
>
> An open sentence of the form $ax + b > 0$, $ax + b \geq 0$, $ax + b < 0$, or $ax + b \leq 0$ is called a **first degree inequality in *x***.

Exploration

Let us consider the following first-degree equations.

Example 1. Given, $5 + x = 12$, solve for x.

Suppose we replace x by 2. Let us see what will happen.

	$5 + x = 12$
If $x = 2$,	$5 + 2 = 12$
This is false since	$7 \neq 12$

We say that 2 is **not a solution** of the given equation.

What value are you going to substitute for the variable x to make the equation true?

____

If your answer is 7, then you are right. Since $5 + 7 = 12$, it means that 7 must be replaced for the variable x to make the equation $5 + x = 12$ true.

Is that the only value that will make the equation true? Yes, 7 is the only value that will make the given equation true. We call 7 the **solution** of the first-degree equation $5 + x = 12$ and {7}, the **solution set**.

Example 2. Solve for y. $2y - 7 = 3$

What value are you going to replace for the variable y to make the sentence true? How will you check if your answer is correct? An example is done for you.

$2y - 7 = 3$
If $y = 6$, then $2(6) - 7 = 3$
This is false since $5 \neq 3$

We say that 6 is **not a solution** of the given equation.

Can you guess what will make the equation true? Complete the solution below.

$2y - 7 = 3$
If $y =$ ___, then $2(6) - 7 = 3$
___ $= 3$

If your answer is 5, then you are correct. The variable y must be replaced by 5 to make $2y - 7 = 3$ a true statement. Is 5 the only value that will make that equation true? Yes, 5 is the only value that will make the equation true. That means, 5 is the **solution** of the first degree equation $2y - 7 = 3$ and {5} is the **solution set** of that equation.

Example 3. Solve for z. $3z = 18$

Can you give a value that will make that equation true? Write your answer below and show how you check if your answer is correct.

$3z = 18$
If $z =$ _____, then $3(\_) = 18$
___ $= 18$

If your answer is 6, then you are correct. Is that the only value which will make the equation true? _____ Yes, 6 is the only value that will make $3z = 18$ a true sentence. Thus, 6 is the **solution** of $3z = 18$ and {6} is its **solution set**.

From examples 1 – 3 discussed above, how many solutions does a first degree equation in one variable have? ___

How do you define the solution of a first-degree equation in one variable?

____

What about the solution set of a first-degree equation in one variable?

____

Let's summarize

> A **first-degree equation in one variable** has **only one** solution.
>
> The **solution** of a first-degree equation in one variable is the value that makes the equation a **true** statement.
>
> The **solution set** of a first-degree equation in one variable is the **set of all its solution.**

Let's apply these ideas in the examples that follow.

Example 4. Solve for x in the equation $x - 2 = 10$.

What is the solution of the given equation? _________ Why? _________________
The **solution** of the given equation is 12 because if x is replaced by 12, the equation is true.

What is the solution set of the given equation? ______
If your answer is {12 }, then you are correct.

Example 5. $4a + 1 = 3$

What is the solution of the given equation? _____ What is its solution set? ____

If a is replaced by ½, the given equation is true. Thus, the solution of the given equation is ½ and its solution set is { ½ }.

Self-check 1

I. Determine if the given value of x is a solution of the first degree equation

$$14 = 3x + 2.$$

Write **YES** if it is a solution and **NO** if it is not.

1. $x = 2$ 2. $x = 4$ 3. $x = 5$

II. Use guess-and check method to identify the solution set of each of the following first-degree equations. Tell what number you choose as your first guess and why you chose it.

1. $x + 1 = 8$
2. $9a + 2 = 5$
3. $4z = 20$
4. $2y + 1 = 9$
5. $2b - 1 = 3$

Answer Key on page 27

Lesson 2 *Solutions of Linear Inequalities in one Variable*

We learned in the previous discussion that a first-degree equation in one variable has only one solution. How many solutions do first-degree inequalities in one variable have? Let us discover this by doing the following examples.

Exploration

Example 1. Solve for z. $z > 5$

If $z = 1$, is the inequality true? ______. No, the inequality is false if $z = 1$ because 1 is not greater than 5.

Can you give a value that will make the inequality true? _______.

Is that the only value that will make the equation true? Can you give other values? ____

If your answers are all greater than 5, say 5.1, 5.5, 6, 6.75, 6.9, 7, 8, etc., then you are correct. All values of the variables that make the inequality true are called **solutions** of the given inequality.

How many solutions does the inequality $z > 5$ have? ___________. You are correct. That inequality has **many solutions**. Some solutions of the given inequality are 5.1, 5.5, 6, 6.75, 6.9, 7, and 8.

Based on your observations in the discussions above, when is the inequality true? ______________________________ Correct, the inequality is true if the value of z is greater than 5. Thus, the **solution set** consists of all real numbers which are greater than 5. In symbols, the **solution set** can be written as $\{z \mid z > 5\}$ and is read as "the set of all z's such that z is greater than 5". This is the replacement set of $z > 5$ with the greatest number of elements.

Example 2. Solve for y. $y + 7 < 11$

What are some values of y that will make the inequality true? Write your answers below and check your answer.

If $y = 3$,
$y + 7 < 11$
(_) + 7 < 11
___ < 11

If $y = 1$,
$y + 7 < 11$
(_) + 7 < 11
___ < 11

If y = __,
$y + 7 < 11$
(_) + 7 < 11
___ < 11

If y = __,
$y + 7 < 11$
(_) + 7 < 11
___ < 11

Give some values that will make the given inequality a true statement. ____

Some solutions are -1, -2, -3.5, 0, ¼, 1/3, ½, 1, 2, 3, 3.2, 3.5 and 3.75.

In general, what values will make the inequality true? _______ If your answer is all real numbers less than 4, then you are right.

What is the **solution set** of the given inequality and how do you write that in symbols? The **solution set** is the set of all ______________________________. In symbols, this is written as $\{y \mid y < 4\}$

Example 3. Solve for *a*. $3a > 1$

Can you give some values of the variable *a* which will make the inequality true? Enumerate your answers below and show your checking.

$3a > 1$
If $a = ½$, $3(\_) > 1$
$\_\_\_ > 1$

$3a > 1$
If $a = \_\_$, $3(\_) > 1$
$\_\_\_ > 1$

$3a > 1$
If $a = \_\_$, $3(\_) > 1$
$\_\_\_ > 1$

$3a > 1$
If $a = \_\_$, $3(\_) > 1$
$\_\_\_ > 1$

If your answers are all greater than 1/3, then you are correct. What is the **solution set** of the given inequality? ______________________________

The solution set is the set of all real numbers greater than 1/3.

How do you write the solution set in symbols? ________________________
In symbols, the solution set is $\{a \mid a > 1/3\}$

Did you know

> A **first-degree inequality in one variable** has **many solutions**.
>
> A **solution** of a first-degree equation in one variable is a value that makes the inequality a true statement.
>
> The **solution set** of a first-degree equation in one variable is the set of values that makes the inequality a true statement.

Exploration

Example 4. Solve for h. $h + 2 > 5$.

What are some values that will make the inequality true? Write your answers and checking below.

If $h = 3.5$, $h + 2 > 5$
$(\_) + 2 > 5$
$\_\_\_ > 5$

If $h =$ __, $h + 2 > 5$
$(\_) + 2 > 5$
$\_\_\_ > 5$

If $h =$ __, $h + 2 > 5$
$(\_) + 2 > 5$
$\_\_\_ > 5$

If $h =$ __, $h + 2 > 5$
$(\_) + 2 > 5$
$\_\_\_ > 5$

What is the solution of the inequality? ________________________ The solution of the inequality consists of all real numbers greater than 3.

What is the solution set written in symbols? In symbols, the solution set is $\{h \mid h > 3\}$.

Example 5. Solve for r. $2r + 1 \leq 3$

Give some values that will make the inequality true. Write your answers and checking below.

If $r =$ __, $2r + 1 \leq 3$
$2(\_) + 1 \leq 3$
$\_\_\_ \leq 3$

If $r =$ __, $2r + 1 \leq 3$
$2(\_) + 1 \leq 3$
$\_\_\_ \leq 3$

If $r =$ __, $2r + 1 \leq 3$
$2(\_) + 1 \leq 3$
$\_\_\_ \leq 3$

What is the solution set of the given inequality? ________________
The solution set is $\{r \mid r \leq 1\}$

Self-check 2

I. Determine whether the inequality $3a < -6$ is true or false for the given value of a.

1. $a = -2$ 2. $a = -3$ 3. $a = -4$

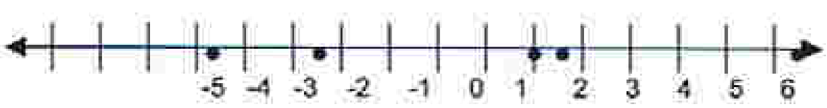
II. Give the solution set of each of the following inequalities and check for 3 values in the solution set.

1. $p + 3 < 0$
2. $4s < 0$
3. $m + 1 > 0$
4. $2y > -2$
5. $w + 2 \leq 12$
6. $-6z > 18$
7. $2c - 1 \leq 1$
8. $q - 5 \geq 11$
9. $2r + 1 \geq 5$
10. $2b - 1 > -3$

Answer Key on page 27

Lesson 3 *Linear Equations and Inequalities*

Consider the number line below and the position of the numbers –5, –3, 1, 1½ , and 6.

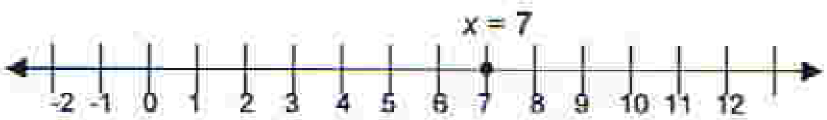


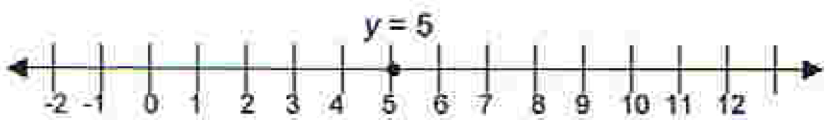
What do you notice with the points and the real numbers on the number line?
____

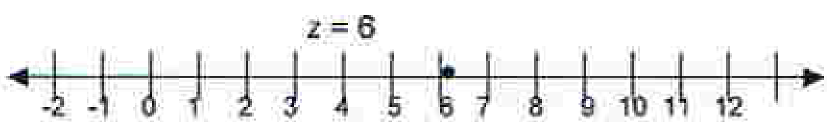
You are correct. Every real number corresponds to a point on the number line and every point on the number line corresponds to a real number. The real number corresponding to a point on the number line is called the **coordinate of the point**.

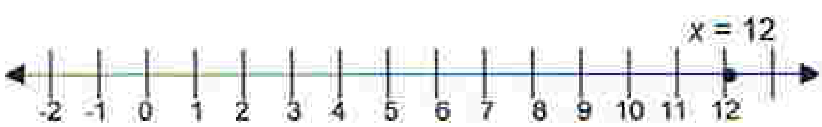
Let us represent the solution set of each equation that we solved in lesson 1 using a number line.

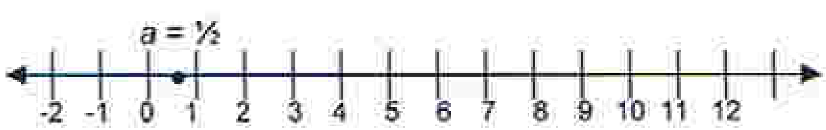
Equations	*Solution Set*
1. $5 + x = 12$	{7}
2. $2y - 7 = 3$	{5}
3. $3z = 18$	{6}
4. $x - 2 = 10$	{12}
5. $4a + 1 = 3$	{½}

1. 

2. 

3. 

4 

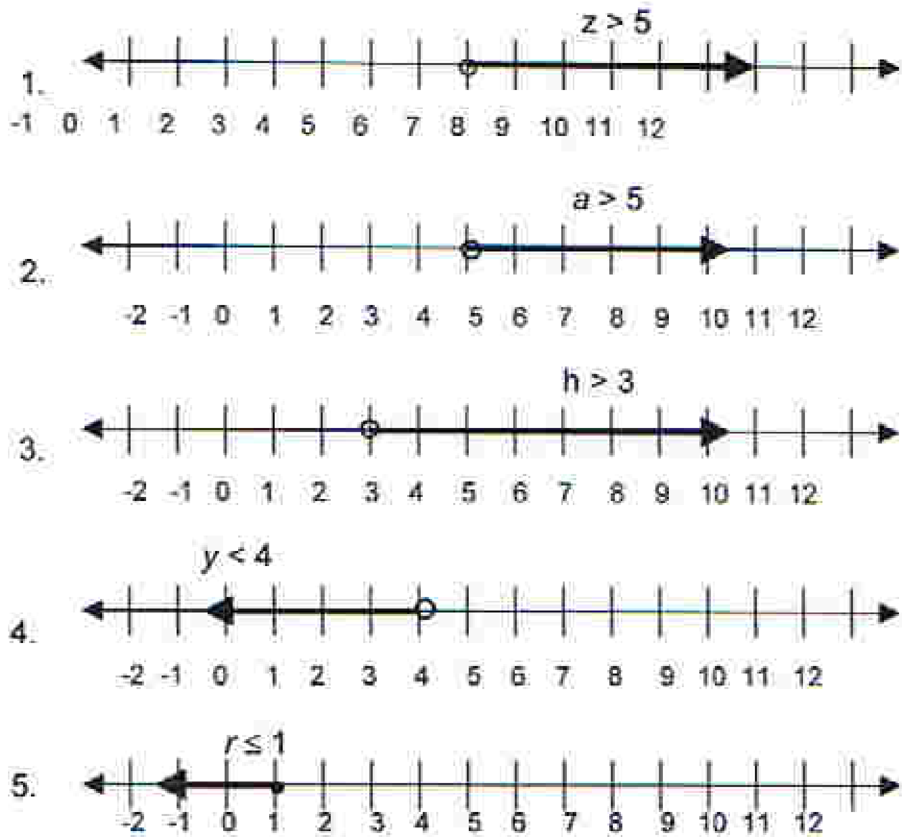
5. 

The representation of the solution of an equation on a number line is called the **graph** of the equation. What do you notice with the graph of the equations given above? ____

You are correct. The graph of a first-degree equation in one variable is just a point.

What if we are to graph the inequalities that we had in lesson 2?

Inequalities	*Solution Set*
$z > 5$	$\{z \mid z > 5\}$
$3a > 1$	$\{a \mid a > 5\}$
$h + 2 > 5$	$\{h \mid h > 3\}$
$y + 7 < 11$	$\{y \mid y < 4\}$
$2r + 1 \leq 3$	$\{r \mid r \leq 1\}$



The graph of a first-degree inequality in one variable is a ray. The ray can be extended to the right or to the left.

The ray is extended to the left if the solution set consists of real numbers which are less than a given value or if the solution set consists of real numbers which are less than or equal to a given value .

On the other hand, the ray is extended to the right if the solution set consists of real numbers which are greater than a given value or if the solution set consists of real numbers which are greater than or equal to a given value .

Again, let us take a look at the graphs of the given inequalities. In the first graph, 5 is called the **boundary point**. What is the **boundary point** of the second graph? ___ third graph? ___ fourth graph? ___ fifth graph? __

If your answers are 5, 4, 3 and 1, then you are right.

Let us relate the boundary points with the rays representing the solutions of the given inequalities.

How do you compare the ray that represents the solutions of the inequality $y + 7 < 11$ and the ray that represents the solutions of $2r + 1 \leq 3$? ______________
Yes, you are correct. The solutions of the inequality $y + 7 < 11$ is represented by a ray with an **open dot** as endpoint while the solutions of the inequality $2r + 1 \leq 3$ is represented by the ray with a **closed dot** as endpoint.

We use a **ray with an open dot** as endpoint if the **boundary point is not a solution** of the given inequality. On the other hand, we use a **ray with a closed dot** as endpoint if the **boundary point is a solution** of the given inequality.

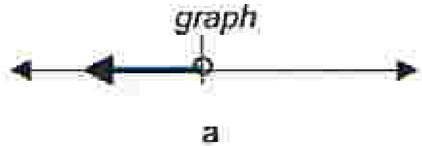
We remember the following.

The graph of a first degree equation in one variable is a point.

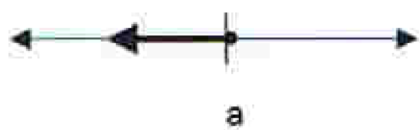
The graph of a first degree inequality in one variable is a ray as illustrated below.

inequality   *graph*

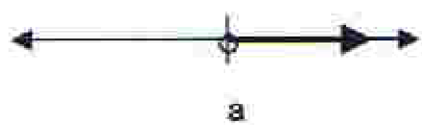
$x < a$



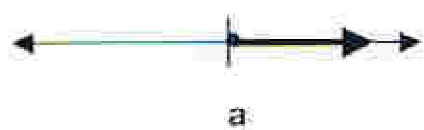
$x \leq a$



$x > a$



$x \geq a$



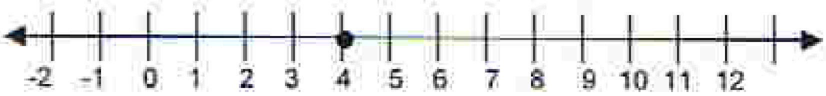
In the inequalities given above, **a** is called the boundary point. We use a **ray with an open dot** as endpoint if the **boundary point** is **not a solution** of the given inequality. On the other hand, we use a **ray with a closed dot** as endpoint if the **boundary point** is **a solution** of the given inequality.

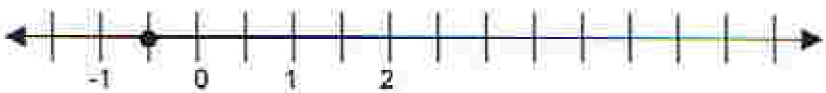
Self-check 3

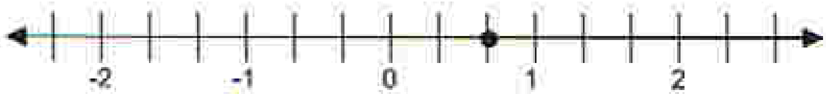
I. Represent the following solution sets on a number line.

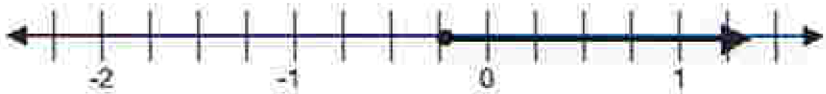
1. x = 3.5
2. y = -6
3. h < -2
4. r ≤ 5
5. z ≤ -8
6. a > 12.5
7. d > -9
8. k ≥ 0
9. m ≥ -4
10. n ≥ 10

II. Write the equation or inequality described by each graph. Use the variable *x*.

1. 

2. 

3. 

4. 

5. 

Answer Key on page 28

Lesson 4 *Solutions of Linear Equations and Inequalities on a Number Line*

In lessons 1 and 2, we learned about the solutions and the solution sets of equations and inequalities. In lesson 3, we studied how to represent an equation or inequality on a number line. This time, we will learn how to find the solution of a given equation or inequality from a given replacement set. The set of real numbers is considered the largest replacement set from which the solution set of a first-degree equation or a first-degree inequality may be obtained.

Recall that a first degree equation or inequality in one variable may be true or false when replaced by a specific value. The value(s) that would replace the variable will be taken from a **replacement set**. You should always **remember** that the solution set taken from a given replacement set is **not necessarily** the solution set of an equation or inequality.

We consider the following examples.

Example 1. Given the replacement set {1, 2, 3, 4, 5}, find the solution set of the equation $3x + 1 = 16$.

Solution:

If $x = 1 \rightarrow 3(1) + 1 = 16$
$4 = 16$ false

If $x = 2 \rightarrow 3(2) + 1 = 16$
$7 = 16$ false

If $x = 3 \rightarrow 3(3) + 1 = 16$
$10 = 16$ false

If $x = 4 \rightarrow 3(4) + 1 = 16$
$13 = 16$ false

If $x = 5 \rightarrow 3(5) + 1 = 16$
$16 = 16$ true

Thus, the solution of the equation $3x + 1 = 16$ is 5 and the solution set is {5}.

Example 2. Given the replacement set {-1, 0, 1}. Find solution(s) of the inequality
$a - 5 < -4$

Solution:

If $a = -1 \rightarrow -1 - 5 < -4$
$-6 < -4$ true

If $a = 0 \rightarrow 0 - 5 < -4$
$-5 < -4$ true

If $a = 1 \rightarrow 1 - 5 < -4$
$-4 < -4$ false

Thus, some solutions are –1 and 0. Finally, from the given replacement set, the solution set of $a - 5 < -4$ is {-1, 0}.

However, you should note that {-1, 0} is **not** the solution set of the inequality $a - 5 < -4$ when the replacement set is the set of all real numbers. {-1, 0} is just the **solution set taken from the given replacement set** which is {-1, 0, 1}.

Example 3. Given the replacement set {1, 1½, 2}, find the solution set of $-2 > t - 3$.

Solution:

If $t = 1$ → $-2 > 1 - 3$ Is this true or false? ________

You are correct. The resulting sentence is false, because $-2 = -2$.

If $t = 1½$ → $-2 > 1½ - 3$ Is this true or false? ______

The resulting sentence is false because $-2 < -1½$.

If $t = 2$ → $-2 > 2 - 3$ Is this true or false? ______

The resulting sentence is false because $-2 < -1$.

What do the above discussions imply?

The discussions above imply that no value from the replacement set makes the inequality true. Hence, from the given replacement set, the solution set is an empty set or ϕ (read as null set).

Again, it may be noted that when the replacement set is the set of all real numbers, the solution set of the inequality $-2 > t - 3$ is **not** the empty set.

What do you observe from examples 1 –3? How do we get the **solution set** of an equation or inequality **from a given replacement set?**

____

Let us now summarize what we discussed.

> To find the **solution set** of an equation or inequality **from a given replacement set**, we **substitute all the elements** of the given replacement set. The set of value(s) which makes the equation or inequality a **true** statement is the **solution set**. If **no** element from the **replacement set** makes the equation or inequality true, the solution set is an **empty set**.
>
> The **solution set** of an equation or inequality **taken from a given replacement set** is **not necessarily** the solution set of the given equation or inequality when the given equation or inequality when the **replacement set is the set of all real numbers**.

Self-check 4

Find the solution set of each of the given sentence when the replacement set is {-2, -1, 0, 1, 2}.

1. $x + 1 = 2$
2. $2y + 1 = 9$
3. $4z = 4$
4. $y + 2 > 2$
5. $h - 1 > -2$
6. $2r + 4 \geq 8$
7. $a - 2 < 6$
8. $2c + 1 < 8$
9. $b - 1 \leq -3$
10. $2f + 1 \leq -2$

Answer Key on page 29

Lesson 5 *Methods of Solving Linear Equations and Inequalities in One Variable*

In the previous lessons, we learned how to find the solution set of first degree equations or first degree inequalities. When you are asked to solve an equation or inequality, it means that you are to find its solution set.

The method of solving equations or inequalities that was illustrated in the previous explorations is by inspection. There are several ways of doing inspection, namely: *guess-and-check, cover-up* and *working backwards.* Such methods will be discussed in this exploration.

Example 1. Consider the equation $2x + 6 = 14$.

Let us illustrate how this equation is solved using three methods.

Method 1. *Guess-and-Check*

This is the method that we used in the previous explorations. In this method, one guesses and substitutes the value to see if a true equation results.

$2x \ + 6 = 14$ $\qquad\qquad$ $2x \ + 6 = 14$

If x = 1 → 2 (1) + 6 = 14
This is false since 8 ≠ 14

If x = 2 → 2 (_) + 6 = 14
This is false since 10 ≠ 14

2x + 6 = 14
If x = 3 → 2 (_) + 6 = 14
This is false since __ ≠ 14

2x + 6 = 14
If x = __ → 2 (_) + 6 = 14
This is now ____ since __ = 14

Thus, what is the solution of the given equation? ______ We see that 4 is the solution and {4} is the solution set of the given equation.

Method 2. *Cover-up*

In this method, we *cover-up* the term with the variable.

$$2x + 6 = 14$$

$$\square + 6 = 14$$

What number should be added to 6 to get 14? _______
If your answer is 8, then you are correct.

$$2x + 6 = 14 \rightarrow 8 + 6 = 14$$

$$2\square + 6 = 14$$

What number should be multiplied by 2 to give 8? ________
Your answer must be 4.

This means that $x = 4$ and {4} is the solution set.

Method 3. Working backwards

In this method, the reverse procedure is used.

The equation $2x + 6 = 14$ shows the following.
x is multiplied by 2.
The product is added to 6 to get 14.

Thus, if the process is reversed, we have
6 is subtracted from 14. (The answer is 8)
The result is divided by 2. (The answer is 4)

This means that $x = 4$ and the solution set is {4}.

Example 2. Solve the inequality $y + 3 < 5$

Method 1. Guess-and-Check

	$y + 3 < 5$		$y + 3 < 5$
If $y = 5$ →	$(5) + 3 < 5$	If $y = 3$ →	$(3) + 3 < 5$
This is false since	$8 > 5$	This is false since	$6 > 5$

	$y + 3 < 5$
If $y = 2$ →	$(2) + 3 < 5$
This is false since	$5 = 5$

At this point, you should see that we can now substitute a value that is less than 2 to make the inequality true. Some examples are shown below.

	$y + 3 < 5$		$y + 3 < 5$
If $y = 1$ →	$(1) + 3 < 5$	If $y = \frac{1}{2}$ →	$(\frac{1}{2}) + 3 < 5$
This is true since	$4 < 5$	This is true since	$3\frac{1}{2} < 5$

Thus, what is the solution of the given inequality? ____________________ You are correct. The solution is the set of all real numbers which are less than 2.

What is the solution set? ____________________________________
The solution set is $\{ y \mid y < 2 \}$.

Method 2. Cover-up

$$y + 3 < 5$$

$$\square + 3 < 5$$

What number should be added to 3 to get a value less than 5? ______
If your answer is less than 2, then you are correct.

This means that $y < 2$ and $\{y \mid y < 2\}$ is the solution set.

Method 3. Working Backwards

The inequality $y + 3 < 5$ shows the following.
y is added to 3.
The sum is less than 5.

Thus, if the process is reversed, we have
3 is subtracted from 5. (The answer is 2)
The result is less than 2.

This means that $y < 2$ and the solution set is $\{y \mid y < 2\}$.

Remember that

> To solve an equation/inequality using
>
> 1. **Guess-and-Check**
>
>   We **guess** and **substitute** some values for the variable to see if a true equation/inequality results. The value/s which make/s the equation/inequality true is the **solution** of the equation/inequality.
>
> 2. **Cover-up**
>
>   We **cover-up** the term with the variable and solve for value/s which will make the resulting open sentence true.
>
> 3. **Working Backwards**
>
>   We **reverse** the procedure in the given equation/inequality.

Self-check 5

Do the following and write your answers on a separate sheet of paper.

I. Solve the following equations by inspection.

1. $23 = 7 + F$
2. $144 = 12a$
3. $\frac{1}{2} h = 9$
4. $4 + 2p = 10$
5. $86 = b - 2$

II. Represent the solution sets of the equations given in I on a number line.

III. Find the solution set of the following inequalities by inspection and graph the solution set on a number line.

1. $t + 7 < 8$
2. $2a \leq 6$
3. $\frac{1}{2} b > 4$
4. $2x - 2 > 6$
5. $3y \geq -9$

Answer Key on page 30

What to do after (Posttest)

Multiple Choice. Choose the letter of the correct answer.

1. The following are true EXCEPT

   a. $19 - x = 25$, if $x = 6$
   b. $7a + 2 = 16$, if $a = 2$
   c. $21 - 4m = 13$, if $m = 2$
   d. $-5 = -b + 4$, if $b = 9$

2. Which of the following is true if $a = -1$?

   a. $a - 7 > 0$
   b. $2a < 0$
   c. $3a \geq -1$
   d. $2 + a < -3$

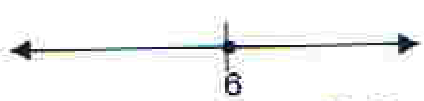
3. What equation has {-1} as solution set?

   a. $y + 1 = 0$
   b. $y - 1 = -1$
   c. $y > -1$
   d. $y < -1$

4. Determine the open sentence with -2 as one of its solutions.

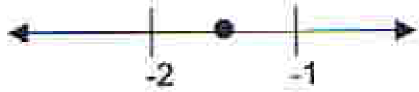
   a. $x > 1$
   b. $x < 1$
   c. $x < -2$
   d. $x > -2$

5. Which of the following equations has a solution represented by the graph below?



   c. $M + 5 = -11$
   d. $11 = A - 5$
   c. $T + 5 = 11$
   d. $H - 5 = -11$

6. Illustrate the graph of a = -½.

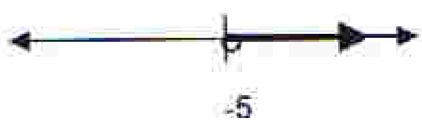
a. 

b. 

c. 

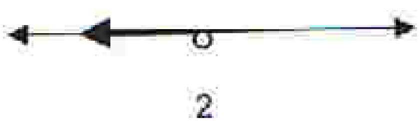
d. 

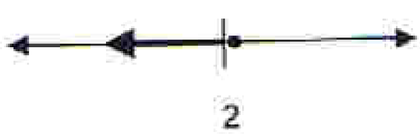
7. Represent the graph below by an open sentence.

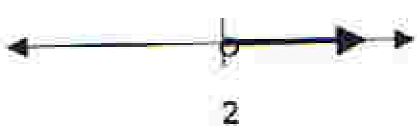


a. $x = -5$ b. $x > -5$ c. $x \leq -5$ d. $x \geq -5$

8. Which of the following graphs illustrates the solution set of $a \geq 2$?

a. 

b. 

c. 

d. 

9. Given the replacement set {1, 2, 3}, find the solution set of

$$2(n + 1) = 10.$$

a. {1} b. {2} c. {3} d. {4}

10. What is the solution set of $24 - x = 30$?

a. {6} b. {-6}c. {54} d. {-54}

11. The following equations has –3 as solution EXCEPT

a. $23 - x = 26$
b. $6x = -18$
c. $-y = 3$
d. $y - 3 = 6$

12. Determine which sentence has 7 as the **only** solution.

a. $b + 2 = 9$
b. $h/2 = 14$
c. $r \geq 7$
d. $w \leq 7$

13. Taken from the replacement set {-2, -1, 0, 1}, the solution set of the inequality $c - 5 < 5$ is

a. {0} b. {-2, -1} c. {-2, -1, 0, 1} d. { }

14. What open sentence is satisfied by all elements of the replacement set {½, 1}?

a. $4x = 2$
b. $j - 0 = 1$
c. $2n > 1$
d. $3t < 4$

15. Find the solution set of the inequality $3n > 21$.

a. $\{n/n > 0\}$
b. $\{n/n > 7\}$
c. $\{n/n < 0\}$
d. $\{n/n < 7\}$

Answer Key on page 24

Answer Key

Pretest page 3

1. c	4. b	7. c	10. b	13. c
2. a	5. b	8. c	11. c	14. a
3. b	6. b	9. a	12. d	15. d

Lesson 1 *Self-Check 1* page 9

I. 1. $14 = 3(2) + 2$ False, thus not a solution.
   2. $14 = 3(4) + 2$ True, thus a solution.
   3. $14 = 3(5) + 2$ False, thus, not a solution

II. 1. The solution set is {7} because $7 + 1 = 8$.
   2. {1/3} is the solution set because $9\,(1/3) + 2 = 5$
   3. The solution set is {5} because $4(5) = 20$.
   4. {4} because $2(4) + 1 = 9$
   5. {2} because $2(2) - 1 = 3$

Lesson 2 *Self-Check 2* page 13

I. 1. $3\,(-2) < -6$ false 2. $3\,(-3) < -6$ true 3. $3\,(-4) < -6$ true

II.

1. $\{p / p < -3\}$ If $p = -4$, $-4 + 3 < 0$, $-1 < 0$

   If $p = -5$, $-5 + 3 < 0$, $-2 < 0$

   If $p = -6$, $-6 + 3 < 0$, $-3 < 0$

2.. $\{s / s < 0\}$ If $s = -1$, $4(-1) < 0$, $-4 < 0$

   If $s = -2$, $4(-2) < 0$, $-8 < 0$

   If $s = -3$, $4(-3) < 0$, $-12 < 0$

3. $\{m / m > -1\}$ If $m = 0$, $0 + 1 > 0$, $1 > 0$

   If $m = 1$, $1 + 1 > 0$, $2 > 0$

   If $m = 2$, $2 + 1 > 0$, $3 > 0$

4. $\{y / y > -1\}$ If $y = 0$, $2\,(0) > -2$

$0 > -2$

If $y = 1$, $2(1) > -2$
$0 > -2$

If $y = 2$, $2(2) > -2$
$4 > -2$

5. $\{z/z < -3\}$ If $z = -4$, $-6(-4) > 18$
$24 > 18$

If $z = -5$, $-6(-5) > 18$
$30 > 18$

If $z = -6$, $-6(-6) > 18$
$36 > 18$

6. $\{w/w \le 10\}$ If $w = 10$, $10 + 2 \le 12$
$12 \le 12$

If $w = 11$, $11 + 2 \le 12$
$13 \le 12$

If $w = 12$, $12 + 2 \le 12$
$14 \le 12$

7. $\{c/c \le 1\}$ If $c = 1$, $2(1) - 1 \le 1$
$1 \le 1$

If $c = 0$, $2(0) - 1 \le 1$
$-1 \le 1$

If $c = -1$, $2(-1) - 1 \le 1$
$-3 \le 1$

8. $\{q/q \ge 16\}$ If $q = 16$, $16 - 5 \ge 11$
$11 \ge 11$

If $q = 17$, $17 - 5 \ge 11$
$12 \ge 11$

If $q = 18$, $18 - 5 \ge 11$
$13 \ge 11$

9. $\{r/r \ge 2\}$ If $r = 2$, $2(2) + 1 \ge 5$
$5 \ge 5$

If $r = 3$, $2(3) + 1 \ge 5$
$7 \ge 5$

If $r = 4$, $2(4) + 1 \ge 5$
$9 \ge 5$

10. $\{b/b \ge -1\}$ If $b = -1$, $2(-1) - 1 \ge -3$
$-3 \ge -3$

If $b = 0$, $2(0) - 1 \ge -3$
$-1 \ge -3$

If $b = 1$, $2(1) - 1 \ge -3$
$1 \ge -3$

Lesson 3 *Self-Check* 3 page 17

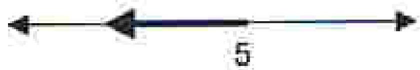
I. 1.

0 3.5

2.

3.

4.



5.

6.

7.

8.

9.

10.

II. 1. $x = 4$ 2. $x = -½$ 3. $x = 2/3$ 4. $x = -1/4$ 5. $x = -1\ 1/3$

Lesson 4 *Self-Check 4* page 20

1. If $x = 1$, $x + 1 = 2$, $1 + 1 = 2$ True Solution set: {1}

2. If all elements of the replacement set are substituted for *y*, the resulting statements are all false. Thus, the solution set is { }.

3. If $z = 1$, $4z = 4$, $4(1) = 4$ True Solution set: {1}

4. If $y = 1$, $y + 2 > 2$, $1 + 2 > 2$ True

   If $y = 2$, $y + 2 > 2$, $2 + 2 > 2$ True Solution set: {1, 2}

5. If $h = 0$, $h - 1 > -2$, $0 - 1 > -2$ True

   If $h = 1$, $h - 1 > -2$, $1 - 1 > -2$ True

   If $h = 2$, $h - 1 > -2$, $2 - 1 > -2$ True Solution set: {0, 1, 2}

6. If $r = 2$, $2r + 4 \geq 8$, $2(2) + 4 \geq 8$ True Solution set: {2}

7. If $a = -2$, $a - 2 < 6$, $-2 - 2 < 6$ True

   If $a = -1$, $a - 2 < 6$, $-1 - 2 < 6$ True

   If $a = 0$, $a - 2 < 6$, $0 - 2 < 6$ True

   If $a = 1$, $a - 2 < 6$, $1 - 2 < 6$ True

   If $a = 2$, $a - 2 < 6$, $2 - 2 < 6$ True Solution set: {-2, -1, 0, 1, 2}

8. If $c = -2$, $2c + 1 < 8$, $2(-2) + 1 < 8$ True

   If $c = -1$, $2c + 1 < 8$, $2(-1) + 1 < 8$ True

   If $c = 0$, $2c + 1 < 8$, $2(0) + 1 < 8$ True

   If $c = 1$, $2c + 1 < 8$, $2(1) + 1 < 8$ True

   If $c = 2$, $2c + 1 < 8$, $2(2) + 1 < 8$ True
   Solution set: {-2, -1, 0, 1, 2}

9. If all elements of the replacement set are substituted for b, the results are all false. Thus, the solution set is { }.

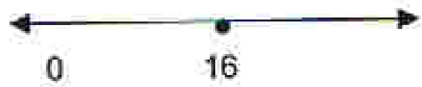
10. If $f = -2$, $2f + 1 \leq -2$, $2(-2) + 1 \leq -2$ True Solution set: {-2}

Lesson 5 *Self-Check 5* page 23

I.
1. $23 = 7 + F \rightarrow 23 = 7 + 16 \rightarrow F = 16$
2. $144 = 12a \rightarrow 144 = 12\,(12) \rightarrow a = 12$
3. $½\,h = 9 \rightarrow ½\,(18) = 9 \rightarrow h = 18$

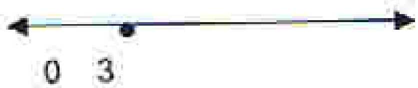
4. $4 + 2p = 10$ → $4 + 2\,(3) = 10$ → $p = 3$
5. $86 = b - 2$ → $86 = 88 - 2$ → $b = 88$

II.

1. 

2. 

3. 

4. 

5. 

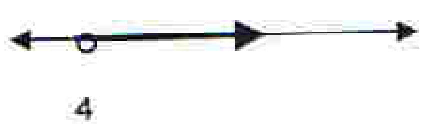
III.

1. $t + 7 < 8$ solution set: {t/t < 1}

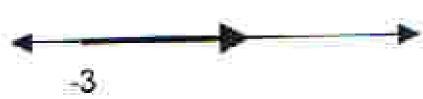
2. $2a \leq 6$ solution set: {a/a ≤ 3}

3. $\frac{1}{2}b > 4$ solution set: {b/b > 8}

4. $2x - 2 > 6$ solution set: {x/x > 4}



5. $3y \geq -9$ solution set: {y/y ≥ -3}



Posttest page 24

1. a	6. b	11. d
2. b	7. b	12. a
3. a	8. c	13. c
4. b	9. d	14. d
5. c	10. c	15. b