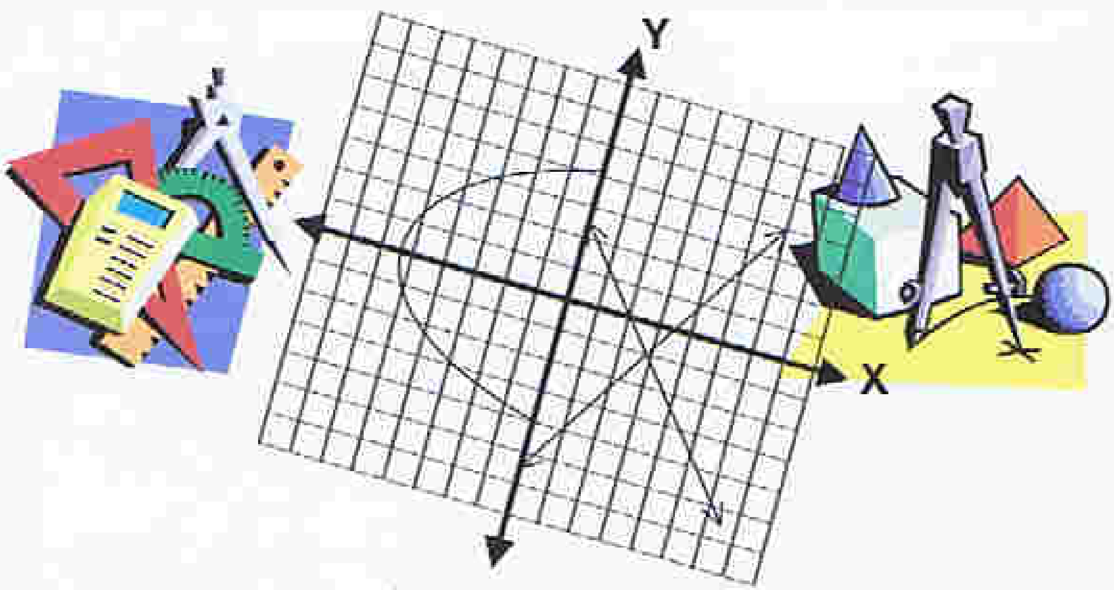
Project EASE

(Effective and Alternative Secondary Education)

MATHEMATICS I



MODULE 8

Powerful of "O"

BUREAU OF SECONDARY EDUCATION
Department of Education
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Module 8
Powerful "O"

What this module is all about

This module will focus on a special kind of algebraic expressions called **polynomials**. You will perform operations on polynomials and solve problems involving polynomials.

This module contains the following lessons:

Lesson 1 Polynomials
Lesson 2 Addition and Subtraction of Polynomials
Lesson 3 Multiplication of Polynomials
Lesson 4 Division of Polynomials
Lesson 5 Application of the Operations on Polynomials

What you are expected to learn

After going through this module, you are expected to:

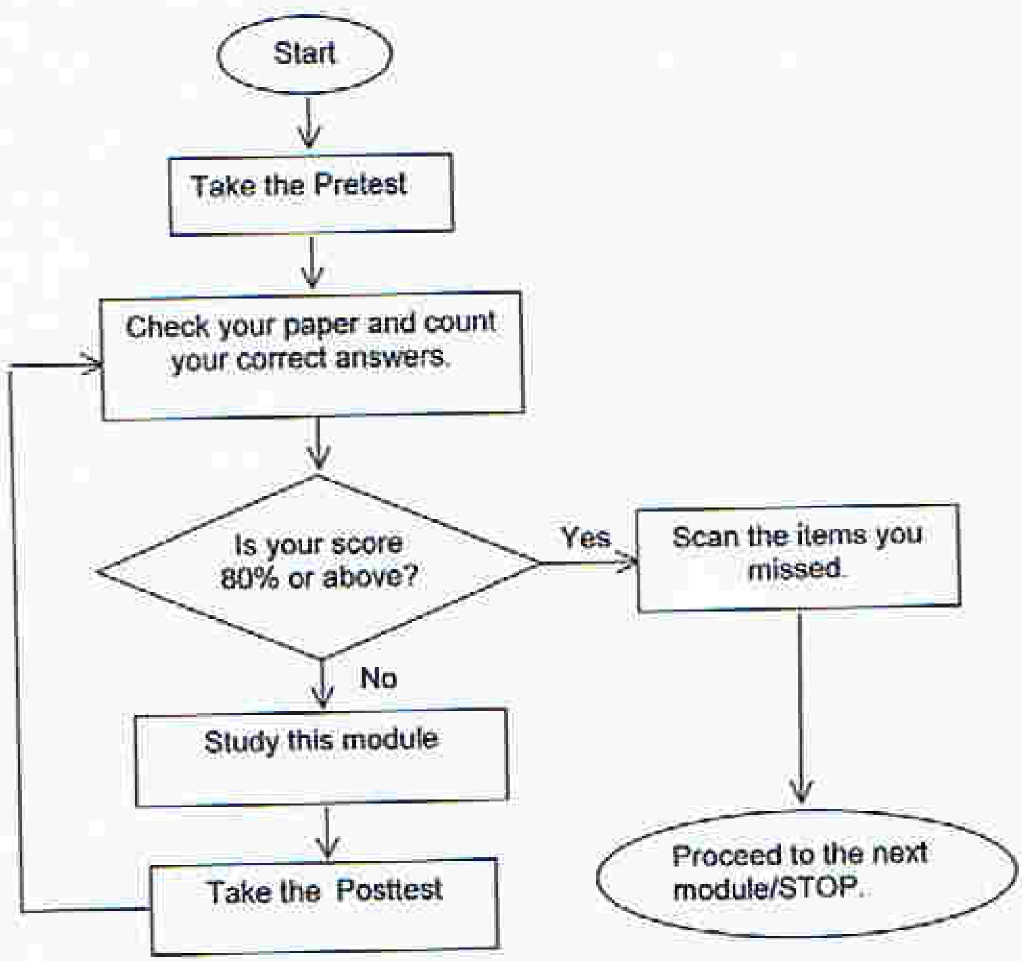
- define polynomials;
- classify algebraic expressions as polynomials and non-polynomials;
- perform operations on polynomials
  - addition and subtraction
  - multiplication: polynomial by a monomial
  - multiplication of polynomial by another polynomial
  - division of polynomials by monomial and by another polynomial; and
- solve some problem involving polynomials.

How to learn from this module

This is your guide for the proper use of the module:

1. Read the items in the module carefully.
2. Follow the directions as you read the materials.
3. Answer all the questions that you encounter. As you go through the module, you will find help to answer these questions. Sometimes, the answers are found at the end of the module for immediate feedback.
4. To be successful in undertaking this module, you must be patient and industrious in doing the suggested tasks.
5. Take your time to study and learn. **Happy learning!**

The following flowchart serves as your quick guide in using this module.



What to do before (Pretest)

Direction: Choose the letter of the correct answer.

1. Which of the following algebraic expressions is a polynomial?

   a. $\frac{3}{x} - 2$ c. $3a^{-2}+1$

   b. $\sqrt{y+1}$ d. x^2+3

2. Which of the following is not a monomial?

   a. $3(2x+y)$ c. $2x^2y$

   b. $(xy)^3$ d. $3x+y$

3. What is the degree of the monomial $3xy^2z^3$?

   a. 1   c. 5

   b. 3   d. 6

4. What must you add to $3x^2-2x+1$ to get $-x^2+x-2$?

   a. $-4x^2+3x-3$ c. x^2-x-1

   b. $-4x^2+3x-3$ d. $2x^2-x+1$

5. What must be subtracted from a^3-2a^2+3a+4 to get $2a^3+4a^2+6a+8$

   a. $-a^3-6a^2-3a-4$ c. a^3-2a^2+3a-4

   b. $a^3+2a^2+3a-14$ d. a^3-2a^2+3a+4

6. Find the product of this expression $2x(2x + 3y - 4)$.

   a. $4x^2 + 6xy - 8x$ c. $2x^2 - 6xy - 8$

   b. $4x + 6y - 8$ d. $4x^2 - 6xy - 8x$

7. Divide $32ay^2 - 16a^2y^3 + 8ay$ by $4ay$.

   a. $8y - 4ay^2 + 2$ c. $8y^3 + 4ay + 2$

   b. $7y - 3ay + 2y$ d. $8y^2 - 4a^2y^2 + 2a^2y^2$

8. Find the area of the rectangle if its length is $3x + 2$ and the width is $x+2$.

a. $4x^2 + 6x + 4$
b. $3x^2 + 8x + 4$
c. $3x^2 - 8x + 4$
d. $4x^2 + 8x + 4$

9. The area of a rectangular garden is $4x^2+2x$. If its width is $2x$, what is its length?

a. $2x+2$
b. $2x+1$
c. $x+1$
d. $x+2$

10. Which of the following statements is true about algebraic expressions and polynomials?

a. Some polynomials are algebraic expressions.
b. All polynomials are algebraic expressions.
c. All algebraic expressions are polynomials.
d. Algebraic expressions are special types of polynomials.

Answer Key on page 33

What you will do

Read the following lessons carefully and do the suggested activities patiently.

Lesson 1 *Polynomials*

Exploration

Consider the following examples and non-examples of polynomials.

All of these are polynomials.

$3x^2y$ 6 $\sqrt{2}x^3 - 6$ $4a^2+3a+1$

All of these are NOT polynomials.

$$\frac{3}{x} \qquad 2x^{-2} \qquad \sqrt{a}+6 \qquad \frac{x}{y} \qquad \frac{1}{3}x^{\frac{1}{3}}$$

What is a polynomial?

Did you know?

Remember

A **polynomial** is a special kind of algebraic expression where each term is a constant, a variable, or a product of constants and variables raised to whole number exponents. An algebraic expression is not a polynomial when

- the variable is in the denominator, such as $\frac{3}{x}, \frac{x}{y}$
- the exponent of the variable is not a whole number, such as $2x^{-2}$, $\frac{1}{3}x^{\frac{1}{3}}$
- the variable is under a radical sign, such as $\sqrt{a}+6$

Can you identify which of the following algebraic expressions are polynomials by placing a check mark before the number?

_____1. 4x-y

_____2. $\frac{3a}{b}$

_____3. $4a^2b^2$

_____4. $\sqrt{3x+1}$

_____5. $4x^2+2x+1$

Did you check 1, 3 and 5? You are right!

A better way to describe these polynomials is by identifying them according to the number of terms they contain.

- $4x - y$ is a polynomial of two terms. What is the special name for this polynomial?_________
- $4a^2b^2$ is a polynomial of one term. What is the special name for this polynomial?_________
- $4x^2 + 2x + 1n$ is a polynomial of three terms. What is the special name for this polynomial?_________

If you answered **binomial, monomial and trinomial** respectively, then you're correct!

Remember

There are special names for polynomials according to the number of terms.

A **monomial** has one term.
A **binomial** has two terms.
A **trinomial** has three terms.
A **multinomial** has four or more terms.

The **degree of a monomial or each term** of a polynomial is the exponent of its variables or the sum of the exponents of its variables. The **degree of the polynomial** is the highest degree of its term or the highest degree of a monomial.

Study the following examples in determining the degree of polynomials.

- The degree of -8 is zero, since $-8 = -8x^0$ which is a constant term.
- The degree of y^2 is 2 in variable y.
- The degree of 3xy is 2 in variables x and y.
- The degree of $4x^4-3x^3+2x^2+1$ is 4 in variable x.
- The degree of $x^3y-x^2y+xy^2$ is 4 in variables x and y.

Can you tell the degree of each polynomial?

- 6x, the degree is ______ in variable_____
- $7x^2y$, the degree is_____ in variables_____
- $3a+4a^2b^2$, the degree is _____ in variables____
- $3x^2+4x^3-5x^5$, the degree is____ in variable____

Are your answers correct? Compare them with my answers.

My answers:

✓ The degree of 6x is **1** in **variable x.**
✓ The degree of $7x^2y$ is **3** in **variables x and y.**
✓ The degree of $3a+4a^2b^2$ is **4** in **variables a and b.**
✓ The degree of $3x^2+4x^3-5x^5$ is **5** in **variable x.**

Study the example below.

The polynomial $3x^2 + 4x^3 - 5x^5$ is a polynomial in one variable x. Arrange the terms in descending powers of the variable x.

_____ 1st _____ 2nd _____ 3rd

Did you write $-5x^5 + 4x^3+3x^2$? See, you can do it!

When you arrange polynomials in one variable in descending powers of its variables, you can easily determine the degree of the polynomials.

> **Remember**
>
> The degree of a polynomial in one variable is the value of the largest exponent of the variable that appears in any term.

Could you arrange the polynomials in descending powers of its variable? _____
Try. You can do it!

Polynomials	Descending Order	Degree
• $-3x^2+6x^3$	__________	____
• $b-3b^2$	__________	____
• $m - 3m^2+ 5m^3$	__________	____
• $-8 + 6x^8$	__________	____
• $14a^6 +14+ 14a$	__________	____

Compare your answers with mine. Check if you got the correct answers.

My answers:

Polynomials	Descending Order	Degree
✓ $-3x^2+6x^3$	$6x^3-3x^2$	3
✓ $b-3b^2$	$-3b^2+b$	2
✓ $m-3m^2+5m^3$	$5m^3-3m^2+m$	3
✓ $-8+6x^8$	$6x^8-8$	8
✓ $14+14a+14a^6$	$14a^6+14a+14$	6

Self-check 1

Directions: Answer as indicated.

1. The expression $\frac{3}{x}+3y^2$ is not a polynomial. (True, False)

2. What is the degree of –10?

   a. –1 b. 0 c. 1 d. 10

3. Arrange the following terms in descending order by numbering 1, 2, 3, 4...

$2m$	-4	$3m^2$	m
____	____	____	____

4. Decide whether the polynomial in number 3 is a monomial, binomial, trinomial or multinomial. ________

5. What is the degree of the polynomial in number three? __________

Compare your answers with those in the Answer key. If you got all the answers correct, CONGRATULATIONS! You did well in the lesson and can proceed to the next lesson. If not, you need to go over the same lesson and take note of the mistakes committed. It pays to have a second look.

Answer Key on page 33

Lesson 2 *Addition and Subtraction of Polynomials*

Exploration

To add and subtract polynomials, you must recall the difference between like and unlike terms.

Study the examples of like terms and unlike terms.

Like terms	**Unlike terms**
2, -5, 7.5	3, y
x, 8x, $\frac{-2}{3}x$	$3x^2$, $\frac{3}{4}x^3$
$3xy^2$, $-4xy^2$	$3xy^2$, $3x^2y$

Determine which of the following are pairs of like terms by placing a check before the number.

_____1. $25b$, $25b^2$ _____4. a^4b^2c, $-11a^4b^2c$

_____2. $-3x^3y^2$, $2x^3y^2$ _____5. -0.4, $\frac{2}{3}$

_____3. $-0.5c$, $\frac{1}{3}c$ _____6. $3x^5y$, $3xy^5$

Did you check 2, 3, 4 and 5? You're correct!

> **Remember**
> **Like terms** *or* **similar terms** are monomials that contain the same literal coefficients, that is, the terms have exactly the same variables and exponents.

A polynomial is in the **simplest form** when all like terms are combined.

Study and learn from the illustration below.

Simplify

1. $11t + 7t = (11+7)t$
   $= 18t$

2. $2xy - 5xy + 4xy = (2-5+4)xy$
   $= xy$

3. $2x^2 - 5x + 6x - x^2 + 11 = (2x^2 - x^2) + (-5x + 6x) + 11$ → Group like terms
   $= (2-1)x^2 + (-5+6)x + 11$
   $= x^2 + x + 11$

What property is applied in simplifying polynomials?__________

Did you answer **distributive property**? You're right!

> **Remember**
> - Distributive Property: **a(b+c) = ab + bc**
> - In simplifying polynomials, combine like terms by adding or subtracting their numerical coefficients and multiplying the result by the common literal coefficient.

Your skills in combining like terms can be used in adding and subtracting polynomials.

Consider these examples:

1. Add $x^2 - 2x + 3$ and $4x^2 + x - 2$

   $(x^2 - 2x + 3) + (4x^2 + x - 2) = (x^2 + 4x^2) + (-2x + x) + (3-2)$ → Grouping like terms
   $= (1+4)x^2 + (-2+1)x + (3-2)$ → Distributive property
   $= 5x^2 - x + 1$

2. Add: $(3 - 2x + x^2) + (6x^2 + 5x - 4)$

A vertical arrangement may be used to add polynomials. Like terms are written in the same column.

$$\begin{array}{r} x^2 - 2x + 3 \\ 6x^2 + 5x - 4 \\ \hline 7x^2 + 3x - 1 \end{array}$$

Since subtraction is defined as addition of the opposite or additive inverse, subtraction is very similar to addition of polynomials.

3. Subtract:: $(4x^2\text{-}4x) - (7x^2 - 3x)$

$(4x^2\text{-}4x) - (7x^2 - 3x) = (4x^2 - 4x) + (-7x^2 + 3x)$ → Add the additive inverse

$= (4x^2 - 7x^2) + (-4x + 3x)$ → Grouping like terms

$= (4\text{-}7)x^2 + (-4+3)x$ → Distributive property

$= -3x^2 - x$

4. Subtract $4a^3 - 2a^2 + 6a - 5$ from $6a^3 - 5a^2 + 11a + 8$

What is the minuend? ____________________

What is the subtrahend? ____________________

Check your answers with mine.

My answer:

The minuend is $\mathbf{6a^3 - 5a^2 + 11a + 8}$

The subtrahend is $\mathbf{4a^3 - 2a^2 + 6a - 5}$

The mere identification of the minuend and the subtrahend facilitate accuracy in subtracting polynomials.

Subtracting polynomials vertically,

$$\begin{array}{r} 6a^3 - 5a^2 + 11a + 8 \\ -\ \ 4a^3 - 2a^2 + 6a - 5 \\ \hline \end{array} \quad \longrightarrow \quad \begin{array}{r} 6a^3 - 5a^2 + 11a + 8 \\ +\ \ -4a^3 + 2a^2 - 6a + 5 \\ \hline 2a^3 - 3a^2 + 5a + 13 \end{array}$$

Self-check 2

Do the indicated operation:

1. Find the sum :

$$\begin{array}{r} 8x^4 - 5x^3 + 15x^2 - 7 \\ -3x^4 + 6x^3 - 8x^2 \; - 2 \\ \underline{14x^4 + x^3 \; - \; x^2 \; + 3} \end{array}$$

2. Find the difference: $(25m^3 + 6m^2 - 7m) - (9m^3 + 12m^2 + 4m)$

3. Add: (-4a + ab + 3b) + (2ab – 8b – 6a) + (3 + a – ab)

4. Subtract $30x^2y - 15xy + 6y$ from $(25x^2y + 9xy - 7y)$.

Compare your answers with those in the Answer key. If you got all the answers correct, CONGRATULATIONS! You did well in the lesson and can proceed to the next lesson. If not, you need to go over the same lesson and take note of the mistakes committed. It pays to have a second look.

Answer Key on page 33

Lesson 3 *Multiplication of Polynomials*

Exploration

In multiplying polynomials, the distributive property and the laws of exponents are used extensively. Observe how these two concepts are used in the following illustrations.

1. $2x(x + y - 2) = 2x(x) + 2x(y) + 2x(-2)$ ⟶ Distributive property

$= 2x^2 + 2xy - 4x$ ⟶ $a^m a^n = a^{m+n}$ that is, $2x \cdot x = 2x^2$

2. $3a^2(a^2 + 2ab + 3b^2 - 3) = 3a^2(a^2) + 3a^2(2ab) + 3a^2(3b^2) + 3a^2(-3)$

$$= 3a^4 + 6a^3b + 9a^2b^2 - 9a^2$$

Try this one.

Find the product of $3n(2m^2 - 3mn + 4)$

- How many terms are there in the product?______
- The first term is the product of $3n(2m^2)$ = _______
- The second term is the product of $3n(-3mn)$ = _____
- The third term is the product of $3n(4)$ = ______

If you answered…

- There are three terms in the product.
- The product is $6m^2n - 9mn^2 + 12n$

Then you are correct!

Remember

The **distributive property** is used to multiply a polynomial by a monomial. The monomial is distributed over each of the terms of the polynomial.

The laws of exponents, $a^m a^n = a^{m+n}$ is used to find the product of each monomial.

To multiply binomial by another binomial, use the distributive property twice. Observe how it is done in the illustration below.

1. Multiply $(4a+b)(x - 2y)$

$(4a+b)(x - 2y) = 4a(x - 2y) + b(x - 2y)$ ⟶ Distributive property

$= 4a(x) + 4a(-2y) + b(x) + b(-2y)$ ⟶ Distributive property

$= 4ax - 8ay + bx - 2by$

To help you remember which terms to multiply in multiplying two binomials a simple memory aid called FOIL is used . FOIL stands for F(First), O(Outer), I(Inner) and L(Last). See below how this FOIL method is used.

2. Multiply (2x –3y) (x-2y)

F:	(2x –3y) (x-2y)	Multiply first terms: $2x^2$
O:	(2x –3y) (x-2y)	Multiply outer terms : -4xy
I:	(2x –3y) (x-2y)	Multiply inner terms : -3xy
L:	(2x –3y) (x-2y)	Multiply last terms : $6y^2$

The result is $2x^2$-4xy –3xy + $6y^2$. Combining like terms, the answer is $2x^2 - 7xy + 6y^2$.

Try this one.

3. Multiply (3x-4) (4x + 5) using the FOIL method.

F: ____________________

O: ____________________

I: ____________________

L: ____________________

The product is ____________

Compare your answer with my answer.

My answers:

F: $12x^2$ O: 15x I: -16x L: -20

The product is **$12x^2$-x –20** by combining like terms.

Remember

The FOIL method is a simple memory aid for multiplying binomials using the distributive property.

The distributive property can also be used to multiply polynomials of any number of terms. Observe how this is done below.

3. Multiply $(x - 4)(x^2 - 5x + 4)$

$(x - 4)(x^2 - 5x + 4) = x(x^2 - 5x + 4) - 4(x^2 - 5x + 4)$ ⟶ Distributive property

$= x^3 - 5x^2 + 4x - 4x^2 + 20x - 16$ ⟶ Distributive property

$= x^3 - 9x^2 + 24x - 16$ ⟶ Combining like terms

Try this one.

4. Multiply $(2x + 5)(x^2 - x - 1)$

$(2x + 5)(x^2 - x - 1) =$ ______________

$=$ ______________

$=$ ______________

Compare your answer with my answer.

My answer:

$(2x + 5)(x^2 - x - 1) = 2x(x^2 - x - 1) + 5(x^2 - x - 1)$

$= 2x^3 - 2x^2 - 2x + 5x^2 - 5x - 5$

$= 2x^3 + 3x^2 - 7x - 5$

Another way to do it is by the vertical method. Study and learn from the following illustration.

5. Multiply $(2x + 7)(3x - 2)$ by vertical method.

$$
\begin{array}{r}
2x + 7 \\
3x - 2 \\
\hline
-4x - 14 \\
6x^2 + 21x \quad \\
\hline
6x^2 + 17x - 14
\end{array}
$$

- $-4x - 14$ → Multiply 2x +7 by -2
- $6x^2 + 21x$ → Multiply 2x + 7 by 3x
- $6x^2 + 17x - 14$ → Add

Try this.

6. Multiply (2x + 5) (x-2) by vertical method.

$$
\begin{array}{r}
2x + 5 \\
x - 2 \\
\hline
\end{array}
$$

Compare your answer with mine.

My answers:

$$
\begin{array}{r}
2x + 5 \\
x - 2 \\
\hline
-4x - 10 \\
2x^2 + 5x \quad \\
\hline
2x^2 + x - 10
\end{array}
$$

Remember

In using the vertical method, make sure to include the sign of the term in multiplying that term to each of the terms in the first factor.

Self-check 3

Multiply. You may use any style of multiplying the polynomials.

1. $3abc(-4abc + 2a^2b^2c^2 - 3a^3b + c)$
2. $(3x+2)(5x-4)$
3. $(3x-2)(x+4)$
4. $(5a^2 - 6a + 3)(2a + 5)$
5. $(x^2 - 4x + 2)(x - 4)$

Compare your answers with those in the Answer key. If you got all the answers correct, CONGRATULATIONS! You did well in the lesson and can proceed to the next lesson. If not, you need to go over the same lesson and take note of the mistakes committed. It pays to have a second look.

Answer Key on page 33

Lesson 4 *Division of Polynomials*

Exploration

The laws of exponents are used when one monomial is divided by another. Below is an illustration on how these laws are used.

Divide the following.

1. $2^5 \div 2^3 = 2^{5-3} = 2^2$ ⟶ $a^m \div a^n = a^{m-n}......if...m > n$

2. $x^4 \div x^7 = \frac{1}{x^{7-4}} = \frac{1}{x^3}$ ⟶ $a^m \div a^n = \frac{1}{a^{n-m}}......if...m < n$

3. $y^6 \div y^6 = y^0 = 1$ ⟶ $a^m \div a^n = a^0 = 1......if...m = n$

Another way to answer 2 is shown below.

$$x^4 \div x^7 = x^{4-7} = x^{-3}$$

Thus, $x^{-3} = \frac{1}{x^3}$.

Remember

- ***Definition of a Negative Exponent***

  For $a \neq 0,..a^{-n} = \frac{1}{a^n}$

- ***Laws of Exponent***

  $a^m \div a^n = a^{m-n}......if...m > n$

  $a^m \div a^n = \frac{1}{a^{n-m}}......if...m < n$

  $a^m \div a^n = a^0 = 1......if...m = n$

You can now use these laws and the definition together with the operations on real numbers in dividing monomials.

Study and learn from the following illustration.

Divide the following monomials.

1. $48x^3 \div 16x^2 = \frac{48}{16}x^{3-2} = 3x$ or $\frac{3(16)x \cdot x \cdot x}{16 \cdot x \cdot x} = 3x$

2. $-24xy^3 \div 4x^2y^5 = \frac{-24}{4} \cdot \frac{1}{x^{2-1}} \cdot \frac{1}{y^{5-3}} = \frac{-6}{xy^2}$

3. $\frac{18ab^2c^3d^4}{-4a^2bc^4d^3} = \frac{-9}{2}a^{1-2}b^{2-1}c^{3-4}d^{4-3} = \frac{-9}{2}a^{-1}bc^{-1}d = \frac{-9bd}{2ac}$

Try this

Divide the following.

1. $-27xz^2 \div -9xz$ = ____________________

2. $\frac{81c^4d^3}{-3c^3d^4}$ = ____________________

If you answered. . .

1. $-27xz^2 \div -9xz = 3x^{1-1}z^{2-1} = 3x^0z = 3z$

2. $\frac{81c^4d^3}{-3c^3d^4} = -27c^{4-3}d^{3-4} = -27cd^{-1} = \frac{-27c}{d}$

Then you're correct!

> **Remember**
>
> To divide a monomial by a monomial:
> 1. Divide the numerical coefficients. Express the quotient as a rational number in simplest form.
> 2. Apply the laws of exponent and make all exponents positive.

To divide a polynomial by a monomial, you may use the distributive property. See the illustration to understand this task.

Divide.

1. $\frac{x^3 - 3x^2 + 4x - 6}{x} = \frac{x^3}{x} - \frac{3x^2}{x} + \frac{4x}{x} - \frac{6}{x}$ ⟶ Distributive Property

$$= x^2 - 3x + 4 - \frac{6}{x} \longrightarrow$$ Dividing each monomial

2. $$\frac{28a^3b^5 - 21a^4b^3 + 49a^3b^6}{7a^2b^3} = \frac{28a^3b^5}{7a^2b^3} - \frac{21a^4b^3}{7a^2b^3} + \frac{49a^3b^6}{7a^2b^3}$$ → Distributive Property

$$= 4ab^2 - 3a^2 + 7ab^3$$

Try this.

Divide $\frac{-12x^2y^2 - 20x^2y + 4x}{-4x}$.

Compare your answer with mine.

My answer is **3xy²+5xy – 1.**

> **Remember**
>
> To divide a polynomial by a monomial
> 1. Divide each term of the polynomial by the monomial.
> 2. Add the resulting quotient. Be sure to indicate the sign before each term using the rule of sign.

The process of dividing a polynomial by a polynomial is similar to the long division process in arithmetic.

Look at the illustration below for this comparison.

- Divide 107 by 3.

	35	← Quotient	
Divisor →	$3\overline{)107}$	← Dividend	Divide: 10 ÷ 3
	9		Multiply: 3X3
	17		Subtract. Bring down the next digit.
	15		Repeat the steps.
Remainder →	2		Stop when the remainder is 0 or it is less than the divisor.

Thus, $\frac{107}{3} = 35 + \frac{2}{3}$.

Generally, you write $\frac{dividend}{divisor} = quotient + \frac{remainder}{divisor}$

- Divide: $(8a^2+10a-7)\div(2a-1)$

$$\begin{array}{r|l} & 4a \\ 2a-1 \,\big)\, & 8a^2+10a-7 \\ & 8a^2-4a \\ \hline & 14a-7 \end{array}$$

Divide $8a^2$ by $2a$

Multiply $2a-1$ by $4a$

Subtract and bring down the next term.

$$\begin{array}{r|l} & 4a+7 \\ 2a-1 \,\big)\, & 8a^2+10a-7 \\ & 8a^2-4a \\ \hline & 14a-7 \\ & 14a-7 \\ \hline & 0 \end{array}$$

Divide $14a$ by $2a$

Multiply $2a-1$ by 7

Stop when the remainder is 0.

Thus, the quotient is $4a+7$.

> **Remember**
>
> 1. Before dividing, arrange the terms in descending power of the variables.
> 2. Insert 0 for the missing terms of the dividend or the divisor.

Try this.

Divide $(x^3-13x-12)$ by $(x-4)$.

Did you answer this way?

$$\begin{array}{r|l} & x^2+4x+3 \\ x-4 \,\big)\, & x^3+0x^2-13x-12 \\ & x^3-4x^2 \\ \hline & 4x^2-13x \\ & 4x^2-16x \\ \hline & 3x-12 \\ & 3x-12 \\ \hline & 0 \end{array}$$

Insert $0x^2$

The remainder is 0.

Thus, $\frac{x^3 - 13x - 12}{x - 4} = x^2 + 4x + 3$

Then you are right!

Self-check 4

Divide.

1. $\frac{2a^3b^2 - 10a^4b^3}{5a^2b^2}$

2. $\frac{9m^4n^5 + 15m^3n^4 - 21m^2n^2}{3m^2n}$

3. $x + 5\overline{)x^2 + 7x + 3}$

4. $x - 6\overline{)3x^2 - 14x - 24}$

Compare your answers with those in the Answer key. If you got all the answers correct, CONGRATULATIONS! You did well in the lesson and can proceed to the next lesson. If not, you need to go over the same lesson and take note of the mistakes committed. It pays to have a second look.

Answer Key on page 33

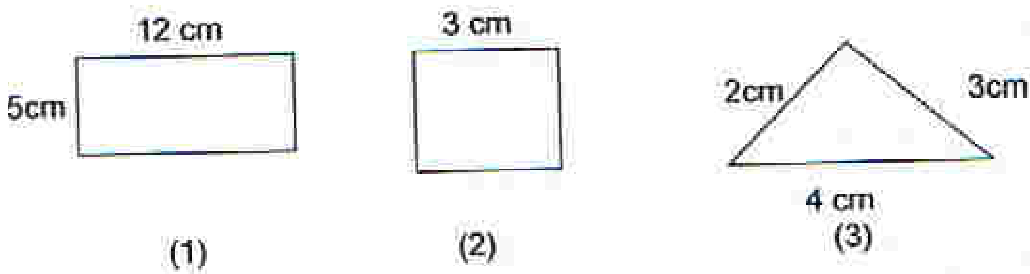
Lesson 5 Application of the Operations on Polynomials

Exploration

The skills developed in performing operations on polynomials would be more meaningful if you can solve problems. Furthermore, solving word problems will develop your reasoning and thinking power.

Do the activity below.

Can you identify the figures below?



- Figure 1 is ________
- Figure 2 is ________
- Figure 3 is ________

Did you answer **rectangle, square,** and **triangle**? You're right!

Can you find the perimeter of each figure?

- The perimeter of figure 1 is _____ cm
- The perimeter of figure 2 is _____ cm
- The perimeter of figure 3 is _____cm
- How do you compute for the perimeter of each of these figures?

____

Compare your answers with mine.

My answers:

- The perimeter of figure 1 is 5+5+12+12= 2(5)+2(12)= **34 cm**
- The perimeter of figure 2 is 3+3+3+3= 4(3)= **12 cm**
- The perimeter of figure 3 is **2+3+4= 9cm**
- How do you compute for the perimeter of each of these figures?

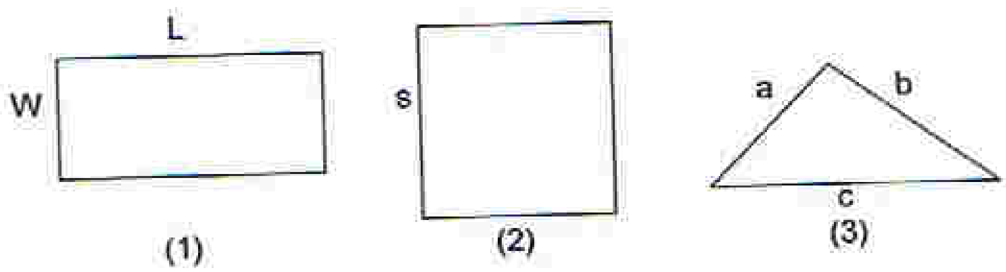
  I find the perimeter of each figure by adding the measure of each side.

Remember

The **perimeter** is the sum of the measure of the sides of any polygon.

- A **rectangle** is a four-sided polygon with two pairs of opposite sides are congruent and having four right angles.
- A **square** is a polygon with four sides congruent and having four right angles.
- A **triangle** is a three-sided polygon. It can be a **scalene triangle** (no sides congruent), an **isosceles triangle** (2 sides congruent), or an **equilateral triangle** (3 sides congruent).

Your knowledge in combining similar terms can be used to find formulas for the perimeter of these polygons.



Find the perimeter of each figure above.

- The perimeter of (1) is ______ units
- The perimeter of (2) is ______ units

- The perimeter of (3) is ______ units

If you answered. . .

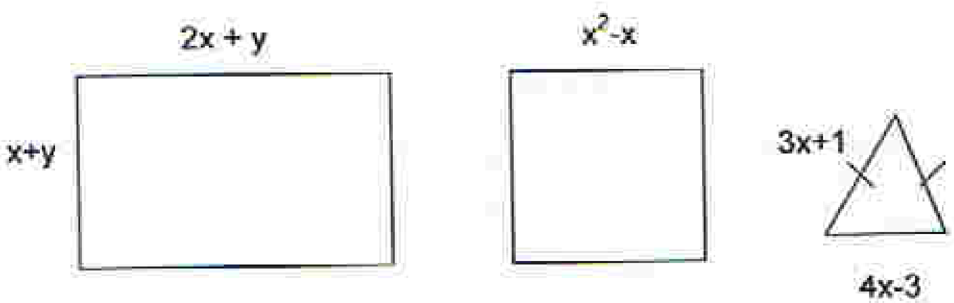
- The perimeter of (1) is L+L+W+W= **2L+ 2W units**
- The perimeter of (2) is s+s+s+s = **4s units**
- The perimeter of (3) is **a+b+c units.**

Then you're correct!

Notice that these are the formulas for the perimeter of these polygons.

> **Remember**
>
> The perimeter of a rectangle = 2L + 2W, where L is the length and W is the width.
> The perimeter of a square = 4s, where s is the length of sides.
> The perimeter of a triangle= a+b+c, where a, b and c are the sides.

You can also apply your skills in addition of polynomials in finding the perimeter of these polygons if the sides are polynomials.



(1) (2) (3)

P= ________ P= ________ P= ________

Compare your answers with mine.

My answers:

(1) $P = 2(x+y) + 2(2x+y) = 2x+2y + 4x + 2y =$ $\mathbf{6x + 4y}$

(2) $P = 4(x^2=x) =$ $\mathbf{4x^2 - 4x}$

(3) $P= (3x+1) +(3x+1) +(4x-3) =$ $\mathbf{10x -2}$

You are now ready to solve problems about perimeter using your skills in performing operations on polynomials.

Read and analyze this problem below.

Problem: The length of a rectangle is 3 times greater than its width. Write an expression to represent its perimeter.

If you represent **x = width,** how will you represent the length? _____

Did you answer **3x**? You're right.

Since the length is 3 times greater than the width.

What formula should you use to answer this problem? __________

Yes, the perimeter of the rectangle P= 2L +2w, where L is the length and W is the width.

How will you solve this problem?__________

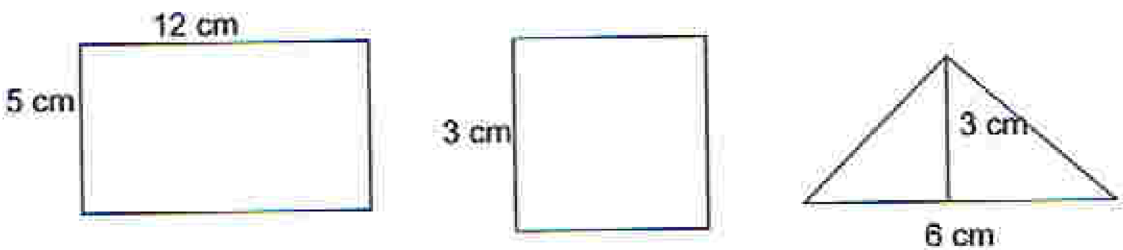
Compare your solution with mine.

My solution:

$P= 2L + 2W$
$= 2(3x) + 2(x)$
$= 6x + 2x$
$=$ **8x units**

Do the activity that follows.

Find the area of the following figures.



(1) (2) (3)

- The area of figure 1 is ________ sq cm
- The area of figure 2 is ________ sq cm
- The area of figure 3 is ________ sq cm

Compare your answer with my answer.

My answers:

- The area of figure 1 is (5cm) (12cm)= **60 sq cm**
- The area of figure 2 is (3cm)(3cm)=$(3cm)^2$= **9 sq cm**
- The area of figure 3 is sq cm $\frac{1}{2}(6cm)(3cm)$ = **9 sq cm**

How do you find the area of these figures? Did you use these formulas?

Remember

- Area of a rectangle is length times the width: **A=LW**
- Area of a square is the square of its side: **A= s^2**
- Area of a triangle is one-half times its base and height: **A =** $\frac{1}{2}bh$

You can also apply your skills in multiplying of polynomials in finding the areas of these polygons if the sides are polynomials.

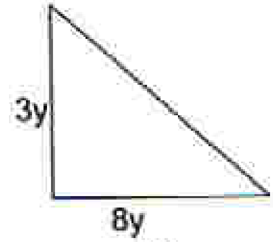
Find the area of the following figures.

(1)

A =________ sq units

(2)

A = ______ sq units



(3)

A = _____sq units

Compare your answers with mine.

My answers:

(1) A= 3x(6x)= $\mathbf{18x^2}$ **sq units**

(2) A = $(6xy)^2$= $\mathbf{36x^2y^2}$ **sq units**

(3) A = $\frac{1}{2}(3y)(8y)$= $\mathbf{12y^2}$ **sq units**

Read and analyze the problem and answer the questions that follow.

Problem: One side of a square is 2x+3, find the area of the square.

- What is 2x + 3 in the problem? _______
- What is asked in the problem? _______
- What formula can you use? ______
- What is your answer? ________

Compare your answer with my answers.

My answers:

- $2x + 3$ is the **side of the square**
- the area of the square
- $A = s^2$
- $A = (2x+3)^2 =$ $\mathbf{4x^2+12x+9}$ **sq units**

Self-check 5

1. A triangle has 3 sides. The first side is $x + 5m$, second side is $x + 7m$ and the third side is $2x + 3m$. Find the sum of the lengths of the sides of the triangle.
2. One side of a square is $2x + 2$. Find the area of the square. Follow this formula: $\mathbf{A = (s)^2}$.
3. A garden lot has an area of $8a^2 + 10a - 63$. If the width is $2a + 7$, find the length.

Answer Key on page 33

Let's summarize

- A **polynomial** is a special kind of algebraic expression where each term is a constant, a variable, or a product of constants and variables raised to whole number exponents.
- There are special names for polynomials according to the number of terms.
  A **monomial** has one term.
  A **binomial** has two terms.
  A **trinomial** has three terms.
  A **multinomial** has four or more terms.
- The **degree of a polynomial in one variable** is the value of the largest exponent of the variable that appears in any term.
- **Like terms** *or* **similar terms** are monomials that contain the same literal coefficients, that is, the terms have exactly the same variables and exponents.
- In simplifying polynomials, combine like terms by adding or subtracting their numerical coefficients and multiplying the result by the common literal coefficient.
- The **distributive property** is used to multiply a polynomial by a monomial. The monomial is distributed over each of the terms of the polynomial.
- The laws of exponents, $\mathbf{a^m\ a^n = a^{m+n}}$ is used to find the product of each monomial.
- To divide a monomial by a monomial,
  i). Divide the numerical coefficients. Express the quotient as a rational number in simplest form.
  ii) Apply the laws of exponent and make all exponents positive.
- To divide a polynomial by a monomial
  i). Divide each term of the polynomial by the monomial.
  ii). Add the resulting quotient. Be sure to indicate the sign before each term using the rule of sign.

What to do after (Posttest)

Choose the letter of the correct answer.

1. In an algebraic expression, the degree of a monomial is the sum of the _____ of its variables.
   a. base c. terms
   b. exponents d. constant

2. Of the given expressions, which is a polynomial?
   a. $6x + 3$ c. $\frac{10}{x^2}$
   b. $\frac{3m^2 - 3}{m}$ d. m^{-2}

3. Look for the similar terms in the given expressions.
   a. $2b, 2a, -3ab$ c. $6ac, 4a^2c, 7ac^2$
   b. $8x, -3x, 4x$ d. $4x^2, 3x, 2ax$

4. What must you add to $x^2 + 3x + 4$ to get $2x^2 - 5x + 6$?
   a. $x^2 - 8x + 2$ c. $2x^2 + 2x + 2$
   b. $-x^2 + 8x + 2$ d. $-2x^2 - 2x - 10$

5. What must be subtracted from $2a^3 + 4a^2 + 6a + 8$ to get $a^3 - 2a^2 + 3a + 4$?
   a. $-a^3 + 6a^2 + 3a + 4$ c. $a^3 - 2a^2 + 3a - 4$
   b. $a^3 + 2a^2 + 3a - 14$ d. $a^3 - 2a^2 + 3a + 4$

6. Multiply the polynomials: $2x^2(2x + 3y - 4)$
   a. $4x^2 + 6xy - 8x$ c. $4x^3 + 6x^2y - 8x^2$
   b. $4x^3 + 6xy - 8$ d. $4x^3 - 6x^2y + 8x^2$

7. Divide $4y^2 + 29y + 7$ by $y + 7$.
   a. $y - 7$ c. $y + 7$
   b. $2y - 7$ d. $4y + 1$

8. Find the perimeter of a square whose side is $3x-1$.
   a. $12x - 4$ c. $9x^2 - 1$
   b. $12x - 1$ d. $9x^2 - 6x + 1$

9. Find the area of the rectangle if its width is $x + 2$ and its length is $2x + 6$.
   a. $2x^2 + 10x - 12$ c. $2x^2 + 12x + 10$
   b. $2x^2 + 10x + 12$ d. $x^2 + 10x + 12$

10. Which of the following statements is NOT true about algebraic expressions and polynomials?
   a. Some algebraic expressions are polynomials.
   b. All polynomials are algebraic expressions.
   c. All algebraic expressions are polynomials.
   d. Polynomials are special type of algebraic expressions.

Answer Key on page 33

Answer Key

Pretest page 3

1. c
2. d
3. d
4. a
5. a
6. a
7. a
8. b
9. b
10. b

Lesson 1 *Self-Check 1* page 8

1. True
2. 0
3.

$2m$	-4	$3m$	m
3	1	4	2

4. multinomial
5. 2

Lesson 2 *Self-Check 2* page 12

1. $19x^4 + 2x^3 + 6x^2 - 6$
2. $16m^3 - 6m^2 - 11m$
3. $-91 + 2ab - 5b + 3$
4. $-5x^2y + 24xy - 13y$

Lesson 3 *Self-Check 3* page 17

1. $-12a^2b^2c^2 + 6a^3b^3c^3 - 9a^4b^2c + 3abc^2$
2. $15x^2 - 2x - 8$
3. $3x^2 + 10x - 22$
4. $10a^3 + 13a^2 - 24a + 15$
5. $x^3 - 8x^2 + 18x - 8$

Lesson 4 *Self-Check 4* page 22

1. $\frac{2}{5}a - 2a^3b$
2. $3m^2n^2 + 5mn^3 - 7n$
3. $x + 2 - \frac{7}{x+5}$
4. $3x + 4$

Lesson 5 *Self-Check 1* page 29

1. $4x + 15m$ units
2. $4x^2 + 8x + 4$ sq units
3. $4a - 9$ unit

Posttest page 31

1. b
2. a
3. b
4. a
5. a
6. 6
7. d
8. a
9. b
10. c