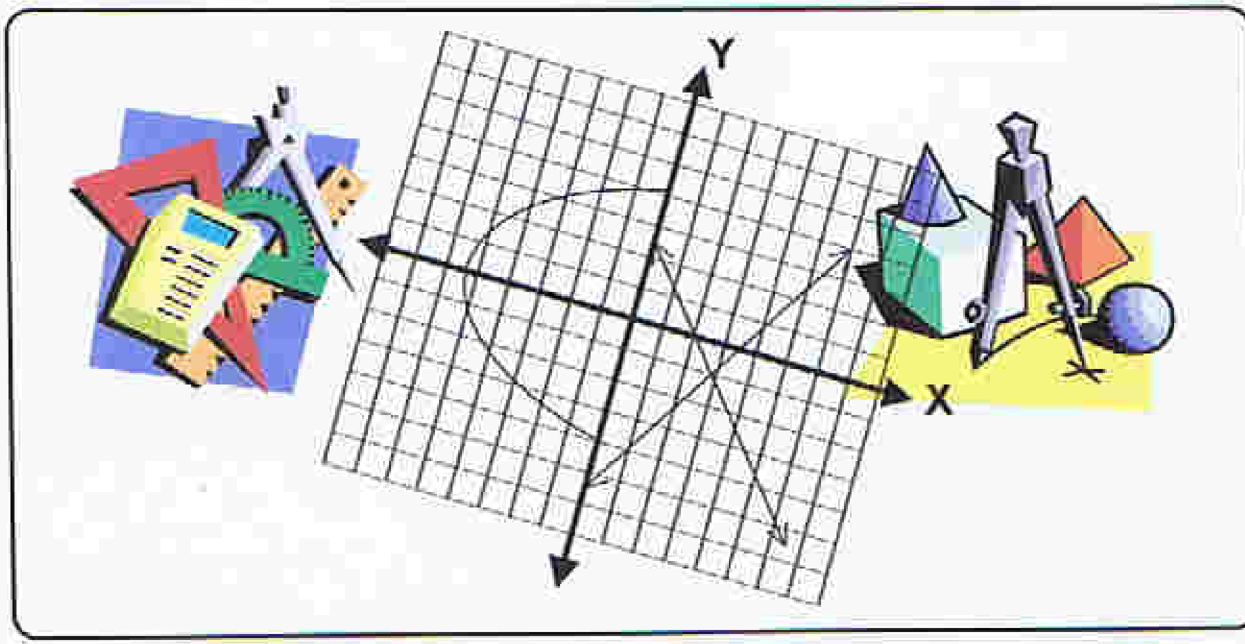
Project EASE

(Effective and Alternative Secondary Education)

MATHEMATICS I



MODULE 7

Terms and Powers

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Module 7
Terms and Powers

What this module is all about

This module is basically a continuation of the lessons in Module 6. Specifically, it deals with the terms of an algebraic expression, coefficient/s, base, and exponents in a term, simplification of monomials with the application of the laws of exponents, operations on monomials and scientific notation.

The lessons in this module will help you identify the parts of a term such as the numerical and literal coefficients, and the exponents of the bases in a term. It will also enable you to classify similar or dissimilar terms. Likewise, the module will discuss how the laws of exponents work in the simplification of monomials, and how to write numbers in scientific notation.

This consists of the following lessons:
Lesson 1 Terms
Lesson 2 The Base and Exponent in a Term
Lesson 3 Simplifying Terms Using the Laws of Exponents
Lesson 4 Operations on Terms
Lesson 5 Scientific Notation

What you are expected to learn

After going through this module, you are expected to:

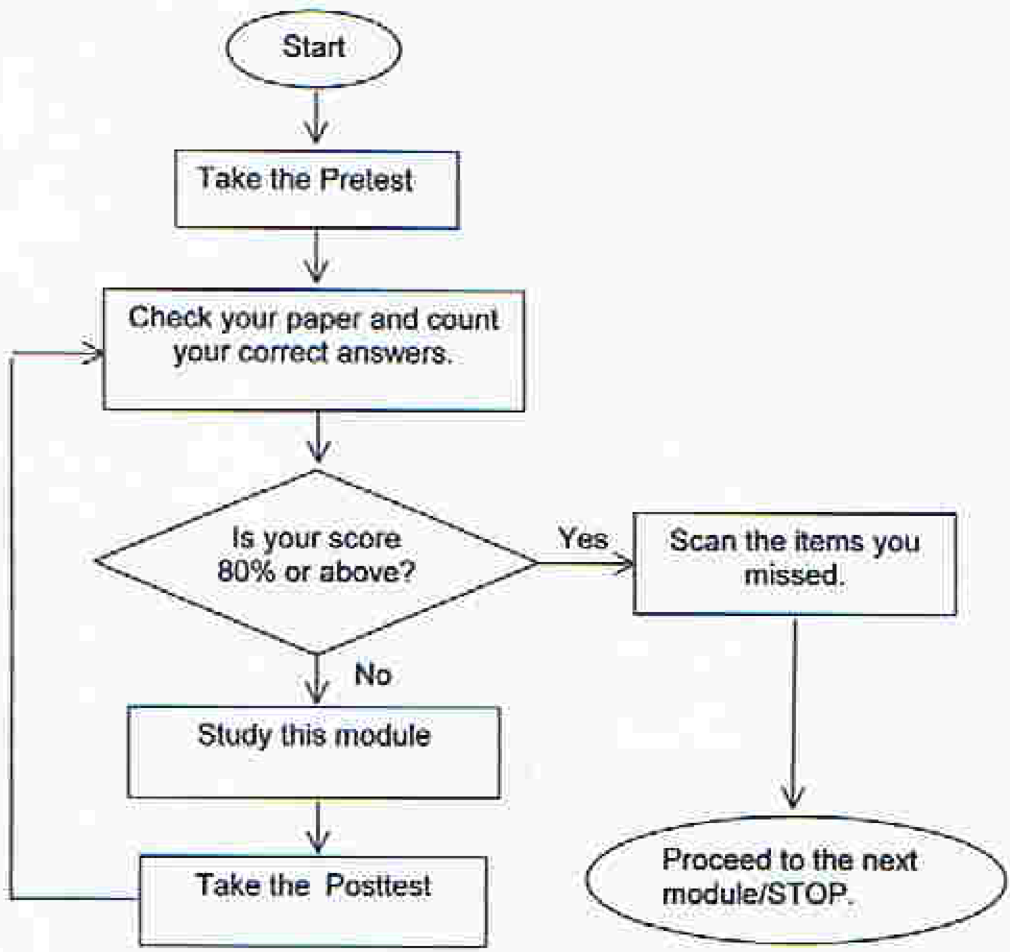
- determine if terms are similar or dissimilar;
- identify the base, coefficient and exponent in a term;
- simplify a term using the laws on exponents;
- perform operations on terms; and
- express numbers in scientific notation.

How to learn from this module

This is your guide for the proper use of the module:

1. Read the items in the module carefully.
2. Follow the directions as you read the materials.
3. Answer all the questions that you encounter. As you go through the module, you will find help to answer these questions. Sometimes, the answers are found at the end of the module for immediate feedback.
4. To be successful in undertaking this module, you must be patient and industrious in doing the suggested tasks.
5. Take your time to study and learn. **Happy learning!**

The following flowchart serves as your quick guide in using this module.



What to do before (Pretest)

Slow Down!

Answer the pretest first before you proceed with the module.

Pretest

Directions: Read each item carefully and choose the letter of the correct answer.

1. If two terms have the same literal coefficients, then they are called __________.
   a. Like terms b. Dissimilar terms c. Similar Terms d. Both a and c

2. What is ***m*** in the term 12m?
   a. It is the exponent of 12.
   b. It is the literal coefficient of 12.
   c. It is the numerical coefficient of 12.
   d. It is the literal coefficient of the term.

3. How many terms are there in the expression **3*x* – 2*y* + 5**?
   a. 1 b. 2 c. 3 d. 4

4. In the expression $\frac{-ab}{3}$, which of the following statements is **NOT** correct?
   a. $\frac{-1}{3}$ is the numerical coefficient of ab.
   b. - ab is the coefficient of 3.
   c. a is a coefficient of $\frac{-b}{3}$.
   d. –b is a coefficient of $\frac{a}{3}$.

5. What is the sum of 6x, - 4x, and – 7x?
   a. -5x b. 5x c. 9x d. -9x

6. What is/are the base/s in the term $5x^2$?
   a. 5 b. x c. x^2 d. 5 and x

7. What is the product of $4x^2$ and $-2x^3$?
   a. $-8x^6$ b. $-8x^5$ c. $8x^5$ d. $8x^6$

8. What is the quotient of $12x^4y^5$ and $-3x^3y^3$?
   a. $4x^7y^8$ b. $-4x^7y^8$ c. $-4xy^2$ d. $4xy^2$

9. When a number is written in scientific notation, where is the decimal point located?
   a. Decimal point is located between the last two digits of the given number.
   b. Decimal point is located after the last non-zero digit of the given number.
   c. Decimal point is located after the first non-zero digit of the given number.
   d. Decimal point is located between any two non-zero digits of the number.

10. What is the difference when $5m^2$ is subtracted from $-7m^2$?
    a. $-12m^2$ b. $-12m^4$ c. $12m^2$ d. -12

11. What is 25 643 in scientific notation?
    a. 2.5643×10^4 b. 2.5643×10^{-4} c. 25.643×10^3 d. 25.643×10^{-3}

12. What is the product of 2^3x^5 and $2x^3$?
    a. 4^4x^8 b. 2^3x^8 c. 2^4x^8 d. $12x^8$

13. What is the simplest form of the expression $\frac{x^4y^2}{x^2y^3}$?
    a. xy b. $\frac{x^2}{y}$ c. x^2y d. $\frac{x^6}{y^5}$

14. What is 2.5×10^4 in standard notation?
    a. 25.00 b. 250.00 c. 2500.0 d. 25000

15. Which of the following sets contains similar terms?
    a. $\left(-2x^6y^4, -2x^4y^6, -2x^{10}y^{10}\right)$
    b. $\left(\frac{3x^6}{y^4}, \frac{-5}{x^6y^4}, \frac{-x^6y^4}{8}\right)$
    c. $\left(4x^6y^4, -x^6y^4, 3y^4x^6\right)$
    d. $\left(3x^6y^4, -4x^3y^2, -12x^9y^8\right)$

Check your answers in the pretest using the correction key at the end of this module. If your score is 13 or 14, scan the material as you review the missed item/s. You may skip the activities following the pretest and proceed to the posttest. If your score is 15, you may just scan the material then proceed to the next module. If your score is below 13, study the whole module patiently then proceed to the posttest.

Answer Key on page 32

What you will do

Lesson 1: Terms

In the previous lesson, you learned that a term is a part of an algebraic expression indicated as a symbol, product or quotient of coefficients. Terms are parts of an algebraic expression separated by plus sign (+) or minus sign (-). In this lesson, you will learn to classify similar and dissimilar terms.

Exploration

In the expression 2xy, 2 is the numerical coefficient of xy, and x and y are the literal coefficients of 2. What is the operation between the numerical coefficient and literal coefficients? The operation used between the numerical and literal coefficients is __________.

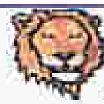
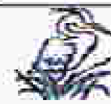
In the expression $\frac{-xy}{5}$, $\frac{-1}{5}$ is the numerical coefficient of x and y, and x and y are the literal coefficients of $\frac{-1}{5}$. What is the operation involved between the numerical and literal coefficients? The operation used between the numerical and literal coefficients is __________.

But, what is the operation used between –xy and 5? __________

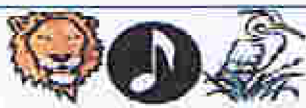
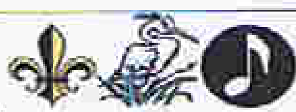
In the expression 2xy, the operation used between the numerical and literal coefficients is multiplication. The expression can be read as the product of 2, x, and y. In the expression $\frac{-xy}{5}$, the operation used between the numerical and literal coefficients is also multiplication. It can also be read as the product of $\frac{-1}{5}$, x, and y. The expression can also be treated as the quotient of –xy and 5 where the operation division is used between –xy and 5. Each of the given expressions is a single term.

Activity 1 Classifying Terms

Let the following icons stand for the given variables.

 $= x^2$	 $= y^2$	 $= z^2$
 $= x$	 $= y$	 $= z$

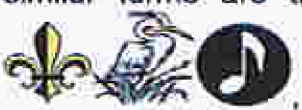
Every set of pictures in the table on page 6 is represented mathematically using the variables shown above. Let the operation between the pictures be multiplication.

Set of Pictures	Algebraic Representation
1. 	z^2yx^2
2. 	xy^2z
3. 	x^2y^2z
4. 	x^2zy
5. 	xz^2y
6. 	x^2zy^2
7. 	zxy^2
8. 	z^2yx

Are there sets of pictures that contain the same pictures? ___________

If there are, how are they represented? __________
Are the variables in the representations the same? __________

Exploration

The sets of pictures containing the same pictures represent ***similar terms*** while the sets of pictures containing different pictures represent ***dissimilar terms***. Examples of similar terms are the sets of pictures in item #3  and item #6 , which are mathematically represented by x^2y^2z and x^2zy^2. Can you give another two sets of pictures that represent similar terms? __________

The commutative property of real numbers tells us that the order of the addends or factors does not affect the result. Do you know why xz^2y and z^2yx are called similar terms? ____________________
What can you say about their literal coefficients and their corresponding exponents? ____________________

Do the sets of pictures in item #4  and item #5  represent similar terms? __________
Why or why not? ____________________
How are these terms called? ____________________

Look at these terms **2*mn*, -4*mn*, 6*mn***, do the terms have the same literal coefficients? ____

Do they have the same numerical coefficients? __________.

Did you know?

The numerical coefficients of terms do not affect the similarity of those terms. Only the literal coefficients can determine the similarity or dissimilarity of terms. Hence, the terms **2*mn*, -4*mn*,** and **6*mn*** are **similar terms**. While the terms $-5x$, $2xy$, and $-3y^2$ are not similar terms. They are called **dissimilar terms**.

Furthermore, in classifying algebraic expressions, only the distinct terms should be counted. If there are similar terms, they should be combined.

Try This

Consider the following terms. Write the terms that are similar in column.

$-2x^2y$	y^2x	$4yx$	$-6x^2$	$3yx^2$
$6xy$	$-2x^2y^2$	$10x^2$	$4x^2y^2$	$3x^2$
$3xy^2$	$4yx^2$	$-2xy$	$-10y^2x$	x^2y^2

Remember that similar terms have the same literal coefficient. The phrase 'the same literal coefficient/s' implies the sameness of exponents of the literal coefficients or variables. So, your groupings must be like these:

$-2x^2y$	$6xy$	$3xy^2$	$-2x^2y^2$	$10x^2$
$4yx^2$	$4yx$	Y^2x	$4x^2y^2$	$-6x^2$
$3yx^2$	$-2xy$	$-10y^2x$	x^2y^2	$3x^2$

Have you done it correctly? Look at the terms in the same column. Take note of the literal coefficients. You cannot combine the terms in the first column with the terms in the second column. They are dissimilar terms. Also the terms in the third, fourth, and fifth columns have different literal coefficients.

Self-check 1

A. From the given set of terms, put similar terms in the same column.

$4ab^2$	$6ab$	$-4a^2$	$-7b^2$	$2a^2b$
$-2a^2$	$-2b^2$	a^2b	$-10ba$	$11b^2$
$-4a^2b$	$-5ab^2$	$10a^2$	$2b^2a$	$-15ab$

Answer Key on page 32

Lesson 2: The Base and Exponent in a Term

'Math In Action'

In a computer, information is read in units called "bits" and "bytes". A bit is like an on-off switch and is read by the computer as 1 (on) or 0 (off). A byte is a group of 8 bits, put together to represent one unit of data such as a letter, digit, or a special character. Each byte, therefore, can represent 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 or 256 different characters.

Did you know?

A product in which the factors are the same is called a **power**. We can write $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ as 2^8. The number 8 is called the **exponent**, and 2 is called the **base**. The exponent tells how many times the base is used as a factor. Similarly, we can write $a \cdot a \cdot a = a^3$. Here the exponent is 3 and the base is 2. When the base in an expression is written with exponents higher than 1, we say that the expression is written in **exponential notation**. For example, $\mathbf{b^n}$, can be read as the 'nth power of b', or simply 'b to the nth', or 'b to the n', or 'b raised to the n'. We may also read $\mathbf{b^2}$ as 'b squared' or 'the second power of b'.

If the exponent of the base is 1, it may be omitted. For example, in the expression ***2b,*** 2 and ***b*** are the bases, where 2 is the numerical coefficient of b and **b** is the literal coefficient of 2. Each base has an exponent of 1. Usually, the base being referred to in algebra is the literal coefficient. Also, if the numerical coefficient of a term is 1, it may be omitted. This algebraic term y^2 means $\mathbf{1y^2}$.

Example 1: What is the meaning of each expression?

1. 2^3	$\mathbf{2^3}$ means $2 \cdot 2 \cdot 2$
2. n^4	$\mathbf{n^4}$ means $n \cdot n \cdot n \cdot n$
3. $9b^3$	$\mathbf{9b^3}$ means $3 \cdot 3 \cdot b \cdot b \cdot b$
4. $(x+2)^3$	$\mathbf{(x+2)^3}$ means $(x+2)\,(x+2)\,(x+2)$
5. $[2-(x+y)]^2$	$\mathbf{[2-(x+y)]^2}$ means $[2-(x+y)]\,[2-(x+y)]$

Example 2: Write each in exponential notation.

1. $7 \cdot 7 \cdot 7 \cdot 7 = 7^4$
2. $2 \cdot 2 \cdot 2 \cdot n \cdot n \cdot n = 2^3n^3$ or $8n^3$
3. $10 \cdot 10 \cdot b \cdot b \cdot b \cdot b = 10^2b^4$ or $2^25^2b^4$
4. $(a - 1)\,(a - 1)\,(a - 1)\,(a - 1)\,(a - 1) = (a - 1)^5$
5. $\{(a-1) - 2b\}\,\{(a-1) - 2b\}\,\{(a-1) - 2b\} = \{(a-1) - 2b\}^3$

Self-check 2

Write each in exponential notation and indicate the base and exponent of the result.

Factor Form	Exponential Notation	Base	Exponent
1. $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$			
2. $b \cdot b \cdot b \cdot b \cdot b \cdot b$			
3. (2y)(2y)(2y)(2y)			
4. (z/2) (z/2) (z/2) (z/2)			
5. (b+c)(b+c)(b+c)			

Answer Key on page 32

Lesson 3 Operations on Terms

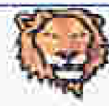
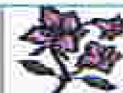
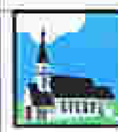
Did you know?

Classifying terms as similar and dissimilar is very useful in doing operations because only similar terms can be combined through addition and subtraction. We cannot combine dissimilar terms. To add or subtract similar terms, add or subtract their numerical coefficients following the laws of signed numbers, then copy the common literal coefficients of the given terms. The result is expressed as the sum or difference of the numerical coefficients multiplied by the common literal coefficients.

Activity 1: Addition of Terms

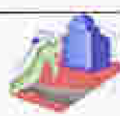
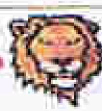
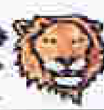
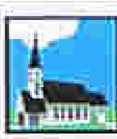
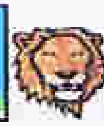
How do we add terms? Let us consider the table of equivalence and examples below.

Let the following icons stand for the given variables and constants.

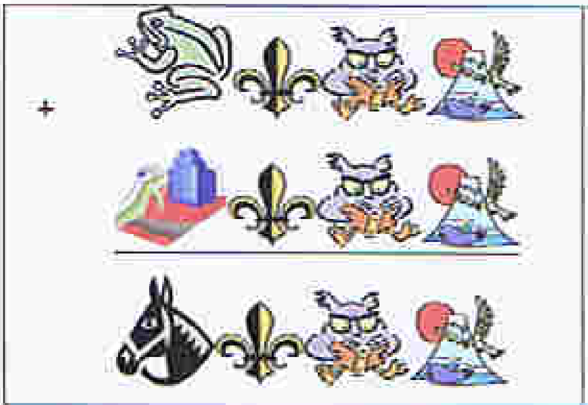
Variables		Constants	
 $= x^2$	 $= x$	 $= 6$	 $= -2$
 $= y^2$	 $= y$	 $= 4$	 $= -4$
 $= z^2$	 $= z$	 $= 2$	 $= -6$

Use these representations; study how addition of terms is performed.

Illustrative Example1: Add the following:

+   	$- 4xy^2$
  	+ $\underline{- 2xy^2}$
  	$- 6xy^2$

Illustrative Example #2

	$$\begin{array}{r} 6x^2yz^2 \\ +\ \underline{-4x^2yz^2} \\ 2x^2yz^2 \end{array}$$

In the first column, the pictures representing the variables in the addends are the same as the pictures in the sum. Does it mean that the sum of similar terms is also similar to the result? __________

In item #1, how is the sum $-6xy^2$ obtained? ________________________________

Why is the numerical coefficient in the sum -6 and not 6? ____________________

Addition of terms requires the application of the rules on how to add integers. If the integers to be added have like signs, add their absolute values then affix their common sign like in this example:

$$\begin{array}{r} -4xy^2 \\ +\ \underline{-2xy^2} \\ -6xy^2 \end{array}$$

If the integers to be added have unlike signs, find the difference of their absolute values, then affix the sign of the integer having the greater absolute value as in this example:

$$\begin{array}{r} 6x^2yz^2 \\ +\ \underline{-4x^2yz^2} \\ 2x^2yz^2 \end{array}$$

Are the literal coefficients of the addends the same as the literal coefficients of the sum? ____

Finally, how do we add similar terms? ____________________________________

Try This

Find the sum of the following terms.

1. $-6x^2yz$, 4x2yz, 3x2yz
2. $2ab$, $-8ab$, $5ab$, $-6ab$, $4ab$
3. $3x^2y$, $-9x^2y$, $6yx^2$, $-x^2y$, $4yx^2$

It is easier to determine whether terms are similar if these are arranged in column. Did you get all answers correctly? ______________________________

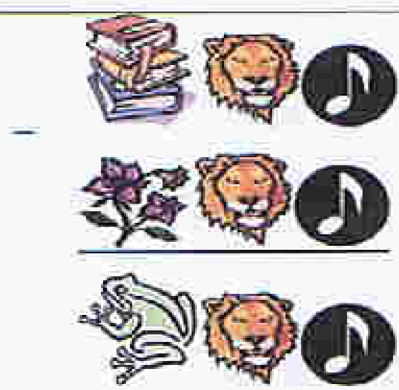
Answers: 1) x^2yz 2) $-3ab$ 3) $3x^2y$

Activity 2: Subtraction of Terms

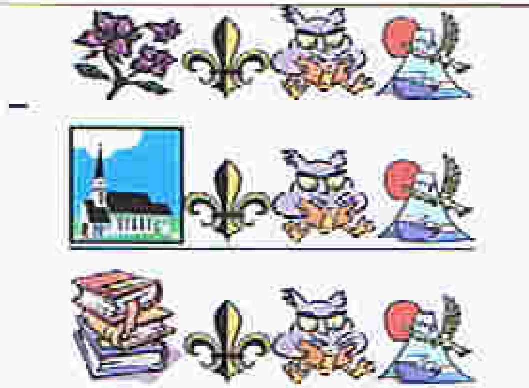
How do we subtract terms?

Let us also consider the table of equivalence used in adding terms in analyzing the examples that follow:

Illustrative Example #1

	$4xy^2$ $-$ $\underline{-2xy^2}$ $6xy^2$

Illustrative Example #2



$$\begin{array}{r} -2x^2yz^2 \\ -\ \underline{-6x^2yz^2} \\ 4x^2yz^2 \end{array}$$

In the first column, the pictures representing the variables are the same in the minuend, in the subtrahend and in the difference. Does it mean that the difference of similar terms also contains exactly the same literal coefficient? ______________________________

In item #1, how is the difference between $4xy^2$ and $-2xy^2$ obtained? ______________________________

Why is the numerical coefficient in the sum 6 and not - 6? ______________________

Subtraction of monomials requires the application of the rules on how to subtract integers. In subtracting integers, change the sign of the subtrahend and proceed as in addition of integers like in these examples

$$\begin{array}{r} 4xy^2 \\ -\ \underline{-2xy^2} \end{array} \longrightarrow \begin{array}{r} 4xy^2 \\ ++\ \underline{(-)\ 2xy^2} \\ 6xy^2 \end{array}$$

and

$$\begin{array}{r} -2x^2yz^2 \\ -\ \underline{-6x^2yz^2} \end{array} \longrightarrow \begin{array}{r} -2x^2yz^2 \\ ++\ \underline{(-)6x^2yz^2} \\ 4x^2yz^2 \end{array}$$

Are the literal coefficients of the terms being subtracted the same as the literal coefficients of their difference? ______________________________

Finally, how do we subtract similar monomials?

____

Analyze further the examples below.

1. $4a^2 - (+6a)$	Change the sign of the subtrahend and
$= 4a^2 + (-6a^2)$	proceed to addition of signed numbers.
$= -2a^2$	Bring down the literal coefficient.

2. $4a^2b - (-6a^2b)$	Change the sign of the subtrahend, then
$= 4a^2b + (+6a^2b)$	add.
$= 10a^2b$	Bring down the literal coefficient.

Try This

Find the difference between the given terms in each item.

1. 6mn	2. -36y	3. $-3a^2b$	4. $7x^2yz^3$
<u>8mn</u>	<u>10y</u>	<u>$- 6a^2b$</u>	<u>$-3x^2yz^3$</u>

Have you answered the items correctly? ___________

Compare your answers with the following: 1) –2mn; 2) – 46y; 3) $3a^2b$; and 4) $10x^2yz^3$.

Self-check 3

A. Find the sum of the terms in each of the items.	
Terms	**Answer**
1. 8ab, -11ab, 6ab	1.
2. 6xy, 8xy, -16xy, 5xy	2.
3. -5mn, -3mn, 9mn, -5mn	3.
4. bc, -8bc, 3bc, 5bc, -2bc	4.
5. -35ax, -16ax, 45ax, -12ax, 12ax	5.

B. Find the difference between the terms in each item. Subtract the second term from the first term.	
Terms	**Answer**
1. -19y, -30y	1.
2. 36xy, 45xy	2.
3. -4ac, 3ac	3.
4. $-48a^2b$, $-32a^2b$	4.
5. $-37x^2$, $-48x^2$	5.

Answer Key on page 32

Lesson 4 Simplifying Terms Using the Laws of Exponents

Exploration

If the same number is multiplied to itself for a number of times, we can write it in a shorter way. The number is used as a **base** and the number of times the number or base is used as a factor becomes the **exponent**. If you use 3 five times as a factor as in 3 · 3 · 3 · 3 · 3, it could be written as 3^5. In 3^5, 5 is the exponent indicating the number of times 3 is used as a factor. Similarly, $x \cdot x \cdot x \cdot x$ can be written as x^4. This manner of writing numbers is called the **exponential notation**.

Activity 1: Multiplying Powers with Like Bases

Multiplying Powers
For any rational number ***n***, and for all whole numbers ***a*** and ***b***, $(n^a)(n^b) = n^{a+b}$.

Study these examples.

1. $2 \cdot 2 \cdot 2 \cdot 2 = 2^4$
2. $a \cdot a \cdot a \cdot b \cdot b = a^3b^2$

Why is $\mathbf{2 \cdot 2 \cdot 2 \cdot 2}$ equal to $\mathbf{2^4}$? ____________________
Why is 4 used as the exponent of 2? ____________________
How many times is the base 2 used as a factor? __________ Is there any exponent of 2 when used as a factor? __________ If ever there were, what is the exponent and what did you do to get 4 as the exponent of the base 2 in the product?
____

Why is $\mathbf{a \cdot a \cdot a \cdot b \cdot b}$ equal to $\mathbf{a^3b^2}$ and not equal to $(ab)^5$? ____________________
Why are 3 and 2 used as exponents of **a** and **b**, respectively? ____________________
Why can't we add the exponents of a and b to get $(ab)^5$? ____________________

The law on multiplying powers is used in these examples.

Illustrative example #1

$$\begin{aligned} a^2 \cdot a^4 &= (a \cdot a)(a \cdot a \cdot a \cdot a) \\ &= a^{2+4} \\ &= a^6 \end{aligned}$$

Illustrative example #2

$$\begin{aligned} 3x^3y^2 \cdot 5xy^3 &= 3 \cdot 5 \cdot x^3 \cdot x \cdot y^2 \cdot y^3 \\ &= 15x^{3+1}y^{2+3} \\ &= 15x^4y^5 \end{aligned}$$

In example #1, the base ***a*** with 2 as the exponent is multiplied with the same base a with 4 as exponent.

What do you notice with the exponents of base *a*? ____________________

Are the exponents 2 and 4 added to get a^6? ____________________

In example #2, there is a numerical coefficient in each factor. As you can see, 3 and 5 are multiplied to get 15. Are the exponents of *x* in the two factors added to get x^4?
____

How did you get y^5 in the product? ____________________

Error Analysis: Find and correct each error in the following exercises.

a. $(3x^2)(2x^5) = 6x^{(2)(5)} = 6x^{10}$
b. $(x^5)(x)(x^2) = x^{5+2} = x^7$

Challenge: Write each of the following as a power of 2.

a. 16 b. 4^3 c. 8^2 d. $(4^3)(8)(16)$

Activity 2 Raising a Power to a Power

We can use the meaning of an exponent to simplify an expression like $(3^2)^4$.

$$\begin{aligned}(3^2)^4 &= (3^2)\,(3^2)\,(3^2)\,(3^2)\\ &= 3^{2+2+2+2} \quad \text{Using the rule for multiplying powers with like bases}\\ &= 3^8\end{aligned}$$

Notice that we get the same result if we multiply the exponents.

$$\begin{aligned}(3^2)^4 &= 3^{(2)(4)}\\ &= 3^8\end{aligned}$$

In general, we can state the following rule for raising a power to a power.

For any rational number ***n***, and any whole numbers ***a*** and ***b***,

$$(a^m)^n = a^{mn}$$

Study the following examples:

Example 1:

$$\begin{aligned}(xy)^3 &= (xy)(xy)(xy)\\ &= (x \cdot x \cdot x)(y \cdot y \cdot y)\\ &= x^3y^3\end{aligned}$$

Example 2:

$$\begin{aligned}(4x^3y^2)^2 &= (4x^3y^2)(4x^3y^2)\\ &= (4\cdot4)(x^3 \cdot x^3)(y^2 \cdot y^2) \qquad \text{or} \qquad 4^{(1)(2)}x^{(3)(2)}y^{(2)(2)}\\ &= 4^2x^6y^4 \qquad \text{or} \qquad 4^2x^6y^4\\ &= 16x^6y^4 \qquad \text{or} \qquad 16x^6y^4\end{aligned}$$

Look at illustrative example #1. The exponent 3 in expression (xy) tells how many times each base is used as a factor. In Illustrative example #2, the numerical coefficient 4 is also squared because it is also a base within the grouping symbol.

Thus,

$(4\cdot4)(x^3 \cdot x^3)(y^2 \cdot y^2)$ or $4^{(1)(2)}x^{(3)(2)}y^{(2)(2)}$ may be used to get $16x^2y^2$.

Try This

Simplify each of the following using the laws of exponents discussed above.

Given	Answer
1. $3x \cdot 4x^2$	
2. $(3a^2b)^3$	
3. $(-2a^3b^2)(3a^3b^5)$	
4. $(-2a^2b^3c)^3$	
5. $(2m^2n)(-4mn)(3m^3n^2)$	

Check your answers using this answer key.
1. $12x^3$ 2. $27a^6b^3$ 3. $-6a^6b^7$ 4. $-8a^6b^9c^3$ 5. $-24m^6n^4$

Notice that in items 1, 3, and 5, numerical coefficients and literal coefficients are just multiplied to get the numerical coefficients and literal coefficients of the results. However, in items 2 and 4, powers of the numerical coefficients and literal coefficients are obtained to get the numerical coefficients and the literal coefficients of the results. If you did not get the answers correctly, go back to the examples given in this lesson

Self-check 4

Multiplication

Simplify each of the following	
Factors	**Product**
1. $3xy^2 \cdot 6x^3y^3$	1.
2. $(3x^2y^2)^3$	2.
3. $-3x \cdot 6x^2 \cdot 2x^3$	3.
4. $(-4x^2)^2$	4.
5. $7a^2b^3c \cdot 3abc$	5.

Answer Key on page 32

Critical Thinking: Is $(a + b)^m = a^m + b^m$ true for all numbers? If yes, justify your answer. If no, give a counterexample.

Activity 3: Dividing Powers with Like Bases

The following suggests a rule for simplifying expressions in the form $\frac{a^m}{a^n}$.

$$\frac{3^5}{3^3} = \frac{\cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot 3 \cdot 3}{\cancel{3} \cdot \cancel{3} \cdot \cancel{3}} = 3 \cdot 3 = 3^2$$

Notice that we can subtract the exponents to find the exponent of the quotient.

Dividing powers For any rational number ***a*** except 0, and for all whole numbers ***m*** and ***n***, $\frac{a^m}{a^n} = a^{m-n}$

Definition of a Negative Exponent

For any rational number ***a*** except 0, and for all whole numbers m, $a^{-m} = \frac{1}{a^m}$.

Definition

For any rational number ***a*** except 0, $a^0 = 1$.

Study the following examples as to show how the laws of exponents work in division.

Example #1

1. $\frac{x^6}{x^2} = \frac{\cancel{x} \cdot \cancel{x} \cdot x \cdot x \cdot x \cdot x}{\cancel{x} \cdot \cancel{x}} = x^{6-2} = x^4$

2. $\frac{12a^5b^6c^3}{3a^3b^4c} = \frac{2 \cdot 2 \cdot 3a^5b^6c^3}{3a^3b^4c} = 2^2 3^{1-1} a^{5-3} b^{6-4} c^{3-1} = 4a^2b^2c^2$

In $\frac{x^6}{x^2}$, x^6 is the dividend and x^2 is the divisor.

In division, we cancel the same factors in both dividend and divisor. If the dividend is y^7 and the divisor is y^5, what do you think is the answer?

$$\frac{y^7}{y^5} = \frac{y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y}{y \cdot y \cdot y \cdot y \cdot y} = y^2$$

Look at example 2. Both the dividend and divisor have numerical coefficients other than 1. So, twelve is written in factor form so that it will be divided by 3 following the rule or law of exponent to get the quotient 4. Look at how the same factors are cancelled applying the law of dividing powers with the same bases. The exponents are subtracted, aren't they? Could you give the quotient to this expression $\frac{-16m^6n^5}{8m^3n^3}$? It should be done like this:

$$= -(2^{4-3}m^{6-3}n^{5-3})$$
$$= -2m^3n^2$$

Analyze further the following examples:

1. $\frac{x^3}{x^6} = \frac{x \cdot x \cdot x}{x \cdot x \cdot x \cdot x \cdot x \cdot x} = \frac{1}{x^3}$

Also, $\frac{x^3}{x^6} = x^{3-6} = x^{-3} = \frac{1}{x^3}$

2. $\frac{-10m^8n^4}{5m^7n^6} = \frac{-2 \cdot 5 \cdot m \cdot m \cdot m \cdot m \cdot m \cdot m \cdot m \cdot m \cdot n \cdot n \cdot n \cdot n}{5 \cdot m \cdot m \cdot m \cdot m \cdot m \cdot m \cdot m \cdot n \cdot n \cdot n \cdot n \cdot n \cdot n}$

$= \frac{-2}{m \cdot n \cdot n} = \frac{-2}{mn^2}$

Therefore, $\frac{-10m^8n^4}{5m^7n^6} = -2m^{8-7}n^{4-6} = -2m^{-1}n^{-2}$

$= \frac{-2}{mn^2}$

3. $\frac{8a^6b^4}{4a^5b^4} = \frac{2 \cdot 2 \cdot 2 \cdot a \cdot a \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b}{2 \cdot 2 \cdot a \cdot a \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b} = 2(1)(1) = 2$

or $= 2^{3-2}a^{5-5}b^{4-4}$

$= 2^1a^0b^0$ *(Definition 2)*

$= 2 \cdot 1 \cdot 1$

$= 2$

Try This

Simplify each of the following using the laws of exponents.

Given	Answer
1. $\frac{8a^8}{2a^5}$	
2. $\frac{-12x^5y^3}{6x^3y^2}$	
3. $\frac{6m^5n^4}{2m^6n7}$	
4. $\frac{x^4y^3}{x^4y^3}$	
5. $\frac{2a^3 \cdot 4b^2}{8a^3b^2}$	

If you have followed the given examples, you could have done those items above correctly. Check your answer and review if you made a mistake.

1. $4a^3$ 2. $-2x^2y$ 3. $\frac{3}{mn^3}$ 4. 1 5. 1

In Items 4 and 5, the exponents of the same bases in both divisor and dividend are equal, so, if you subtract the exponents of the same bases, you get a zero exponent.

For any base (except 0) raised to a zero exponent is always 1.

Critical Thinking:

How are the following items below simplified to get the indicated answers?

1. $\left(\frac{x}{y}\right)^2 = \frac{x^2}{y^2}$	2. $\left(\frac{a^2}{b^3}\right)^4 = \frac{a^8}{b^{12}}$	3. $\left(\frac{2a^2}{3b^3}\right) = \frac{4a^4}{9b^6}$

Error Analysis: *Elaine wrote in her Math journal "The square of any number is always greater than the number". Find a counterexample to show that Elaine's statement is incorrect.*

Mathematical Reasoning: Square any number, and then double the result. Is your answer always, sometimes, or never greater than the result of doubling the number, then squaring it? Justify your answer.

Self-check 4

Do the indicated operation.

Division

Simplify the following monomials using the laws of exponents in division.

	1.	2.	3.	4.	5.
Expression	$\frac{3x^5}{x^2}$	$\frac{b^6c^5d^2}{b^4c^7d^3}$	$\frac{-16x^3y^6}{8x^5y^3}$	$\frac{15a^6y^7}{5a^6y^7}$	$(3x^3/4y^4)^3$
Answer					

Answer Key on page 32

Lesson 5 Scientific Notation

Read the following article carefully. It is about **'Math in Action'.**

<table><tr><td>
The distance from Earth to the North Star is about 10 000 000 000 000 000 000 meters. The thickness of a soap bubble is about 0.0000001 meter. It is easy to make errors when working with numbers involving many zeros. If an extra zero is included, the resulting number is ten times larger or ten times smaller.
To prevent this type of error and to make it easier to work with very large numbers and very small numbers, we can write these numbers in a form called scientific notation. Using scientific notation, we can write a number as the product of a power of 10 and a number greater or equal to 1, but less than 10. In scientific notation, the distance to the North Star is 1.0×10^{19} meters and the thickness of a soap bubble is about 1.0×10^{-7} meter. The numbers 10 000 000 000 000 000000 and 0.0000001 are expressed using the standard notation.
</td><td></td></tr></table>

Can you still remember how to express multiplication phrase in exponential form? The expression 4^2 means $4 \cdot 4 = 16$, 5^3 means $5 \cdot 5 \cdot 5 = 125$. How about 10^3? Its base is 10 so you multiply 10 by itself three times → $10 \cdot 10 \cdot 10 = 1000$.

1. How do you express 10 000 using 10 as the base? __________; 100 000? __________

2. How about $\frac{1}{100}$? __________, $\frac{1}{1000}$? __________, $\frac{1}{10000}$? __________

In item #1, using 10 as a base, 10 000 can be written as 10^4, while 100 000 can also be written as 10^5.

In item #2, $\frac{1}{100}$ is written as $\frac{1}{10^2}$ or 10^{-2}

$$\frac{1}{1000} = \frac{1}{10^3} \text{ or } 10^{-3}$$

$$\frac{1}{10000} = \frac{1}{10^4} \text{ or } 10^{-4}$$

In the examples, the base is 10. Unlike in a^6, the base is *a*. The expression a^{-6} also means $\frac{1}{a^6}$.

Study This

Activity 1: Computing the Product of a Number and a Power of 10

Try to find the products of the following.

a) 24×10 c) 24×10^3 e) 24.567×10^2
b) 24×10^2 d) 24.567×10 f) 24.567×10^3

Let us look at the answers. Is there any pattern? __________ What pattern can you derive from the products? ____

a) $24 \times 10 = 240$ d) $24.567 \times 10 = 245.67$
b) $24 \times 10^2 = 2{,}400$ e) $24.567 \times 10^2 = 2456.7$
c) $24 \times 10^3 = 24{,}000$ f) $24.567 \times 10^3 = 24567$

Do you know how each product is obtained? To multiply a number by any positive power of 10, you simply move the decimal point to the right by as many places as the exponent of 10.

Activity 2: Computing the Quotient of a Number and a Power of 10

Try to get the quotients of the following. Each number is divided by a positive power of 10.

a) $165 \div 10$ d) $25.8 \div 10$
b) $165 \div 10^2$ e) $25.8 \div 10^2$
c) $165 \div 10^3$ f) $25.8 \div 10^3$

How did you do it? ____ Do you see a pattern? __________ If there is any, what is it? ____ Is the pattern you derived from dividing the numbers the same as the pattern you derived from the multiplication of the numbers? ______________________________

The quotient can be obtained by moving the decimal point to the left as many places as the exponent of 10.

So, the answers are the following:

a) $165 \div 10 = 16.5$	Move 1 place to the left.
b) $165 \div 10^2 = 1.65$	Move 2 places to the left.
c) $165 \div 10^3 = .165$	How many places to the left?
d) $25.8 \div 10 = 2.58$	How many places to the left?
e) $25.8 \div 10^2 = .258$	How many places to the left?
f) $25.8 \div 10^3 = .0258$	You move the decimal point 3 places to the left; there's no other digit, so you add a cipher before the last non-zero digit from the right then put the decimal point.

The procedures learned from the multiplication and division of numbers by the powers of ten help you understand how to write numbers in scientific notation. This technique of writing numbers is based on the powers of 10. It is very useful in expressing very large or very small numbers in a way that is easier to read.

Did you know?

Consider the following information:

1. The earth's distance from the sun is about 149 590 000 km. This number can be rewritten as 1.4959 x 100 000 000 or 1.4959×10^8 km in scientific notation.
2. A light year is the distance that light travels in a year. It is approximately 9 460 800 000 000 km. It can be rewritten as 9.408 x 100 000 000 or 9.408×10^8 in scientific notation.
3. The diameter of a red blood cell is about 0.00075 cm. It can be rewritten as

$7.5 \times \frac{1}{10000}$ or 7.5×10^{-4} cm in scientific notation.

Can you see the equivalence of these numbers? __________

1) $149\,590\,000 = 1.4959 \times 100\,000\,000 = 1.4959 \times 10^8$
2) $940\,800\,000\,000 = 9.408 \times 100\,000\,000 = 9.408 \times 10^8$
3) $0.00075 = 7.5 \times \frac{1}{10000} = 7.5 \times 10^{-4}$

Study the table.

Standard Notation	*Scientific Notation*
1. 149 590 000	1. 1.4959×10^8
2. 940 800 000 000	2. 9.408×10^8
3. 0.00075	3. 7.5×10^{-4}

Look at the location of the decimal point in the scientific notations of the numbers given above. Can you describe where the decimal point is located? ____

How is the exponent of the factor 10 obtained? ____________________

You should have discovered that the decimal point in the scientific notation of a number is located just after the first non-zero digit from the left, which is known as its standard location in scientific notation. Also, the exponent of 10 depends on how many times you move the decimal from its given location to its standard location.

Try This

A. Write each number in standard notation.

Given Number	Standard Notation
1. 35.345×10^3	
2. $35.345 \div 10^2$	
3. $5.35 \div 10^3$	

B. Express the following information in scientific notation.

Information	Scientific Notation
1. The earth's diameter is 12 760 km.	
2. The speed of light is 279 600 km/s.	

Check your answers.

A. Standard Notation

1. 35 345 — Just move three places to the right.
2. .35345 — Just move two places to the left.
3. .00535 — Just move two places to the left.

Why move the decimal point to the left for item numbers 2 and 3?

B. Scientific Notation

1. 1.276×10^4
2. 2.796×10^5

Did you get all the answers? That's very good! You are now ready to express those big numbers and small numbers in scientific notation.

Self-check 5

Express each number in scientific notation.

Standard Notation	Scientific Notation
1) 56,700,000	
2) 876,000	
3) 0.00134	
4) 0.03720	

Answer Key on page 32

Let's summarize

Kinds of Terms

Similar terms are terms with the same literal coefficients.
Dissimilar terms are terms with different literal coefficients.

Similar terms in an algebraic expression may be combined in to a single term by adding or subtracting their numerical coefficients, as indicated by the signs, keeping the identical literal factors.

Exponential Notation

Any number expressed in the form b^n is in **exponential notation** where ***b*** is the base and ***n*** is the exponent.

Exponent is a symbol or a number at the upper right hand corner of a variable or constant. It tells how many times a base is used as a factor.

Base is the repeated factor in a power.

Addition and Subtraction of Similar Monomials

To add or subtract similar terms, add or subtract their numerical coefficients following the laws of integers and copy their common literal coefficients.

Simplification of Terms Expressed as Product or Quotient

To simplify terms expressed as product, multiply the numerical coefficients of the factors following laws of integers and multiply the literal coefficients of the factors following the rules of exponents $(n^a)(n^b) = n^{a+b}$ and $(a^m)^n = a^{mn}$.

To simplify terms expressed as quotient, divide the numerical coefficients of the numerator and denominator following the laws of integers and divide the literal coefficients of the numerator and denominator following the rules of exponents in division $\frac{a^m}{a^n} = a^{m-n}$ and $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$.

Scientific Notation

A number is expressed in **scientific notation** when it is in the form $a \times 10^n$, where $1 \le a < 10$ and n is an integer.

What to do after (Posttest)

Direction: Choose the letter of the correct answer.

1. Which of the following sets contains similar terms?

   a. $-3x^2y$, $6xy^2$, x^2y^2, $-9x^2y$ c. $-a^2b$, $8ba^2$, $6a^2b$, $-5ba^2$

   b. xy^2, $-7xy$, $2x^2y^2$, $-5xy^2$ d. abc, 3bca, $6a^2bc$, $-8a^2c$

2. In the expression $12m^2$, m is the coefficient of __________.

   a. 12 b. $12m^2$ c. 12m d. m^2

3. What value of the variable ***x*** makes the statement ($x^0 = 1$) false?

   a. negative integer b. fraction c. positive integer d. zero

4. What is the sum of the terms 8x, -4x, and -6x?

   a. $-4x$ b. $-2x$ c. $2x$ d. $4x$

5. What is the product of $-2x^3$ and $5x$?

   a. $10x^4$ b. $10x^3$ c. $-10x^4$ d. $-10x^3$

6. What is the difference between $-8mn$ and $-5mn$?

   a. $-13mn$ b. $-3mn$ c. $13mn$ d. $3mn$

7. What is the quotient if $24a^6b^5$ is divided by $-8a^3b^2$?

   a. $-3a^2b^2$ b. $-3a^3b^3$ c. $3a^3b^3$ d. $3a^9b^7$

8. What is 345 600 in scientific notation?

   a. 3.456×10^2 b. 3.456×10^3 c. 3.456×10^4 d. 3.456×10^5

9. What is $\frac{1}{10000}$ if it is written in the power of 10?

   a. 10^{-2} b. 10^{-3} c. 10^{-4} d. $1/10^4$

10. Which of the following is **true** about the expression $-5x^3$?

    a. x is the literal coefficient of $5x^3$.
    b. -5 is the numerical coefficient of x^3.
    c. 5 is the numerical coefficient of $-5x^3$.
    d. 3 is the common exponent of -5 and x.

11. What is the standard notation of 2.35×10^2?

a. 2.35 b. 23.5 c. 235 d. 2 350

12. When the exponent of 10 in the scientific notation of a number is negative, it means that the number

a. is less than 1.
b. is greater than 1.
c. is equal to 1.
d. could not be determined.

13. If 4.506 is written in scientific notation, what is the exponent of 10?

a. –1 b. 0 c. 1 d. 2

14. What do you call the repeated factor in a power?

a. term b. product c. exponent d. base

15. When $(2^2x^3)^3$ is simplified, what will be the numerical coefficient of the result?

a. 64 b. 32 c. 8 d. 4

Answer Key on page 32

Answer Key

Pretest page 3

1. d	4. b	7. b	10. a	13. b
2. b	5. a	8. c	11. a	14. d
3. c	6. d	9. c	12. c	15. c

Lesson 1 Self-Check 1 page 8

$4ab^2$	$-2a^2$	$-4a^2b$	$6ab$	$-2b^2$
$-5ab^2$	$-4a^2$	a^2b	$10ab$	$-7b^2$
$2ab^2$	$10a^2$	$2a^2b$	$-15ab$	$11b^2$

Lesson 2 Self-Check 2 page 10

Factor Form	Exponential Notation	Base	Exponent
1. 3 x 3 x 3 x 3 x 3	3^5	3	5
2. b x b x b x b x b x b	b^6	b	6
3. (2y)(2y)(2y)(2y)	$(2y)^4$	(2y)	4
4. (z/2) (z/2) (z/2) (z/2)	$(z/2)^4$	(z/2)	4
5. (b+c)(b+c)(b+c)	$(b+c)^3$	(b+c)	3

Lesson 3 Self-Check 3

A. Addition page 15				
1. 3ab	2. 3xy	3. –4mn	4. -bc	5. –6ax
B. Subtraction page 16				
1. 11y	2. –9xy	3. –3ac	4. $-16a^2b$	5. $11x^2$

Lesson 4 Self-Check 4

Multiplication page 19				
1. $18x^4y^5$	2. $27x^6y^6$	3. $-36x^6$	4. $16x^4$	5. $21a^3b^4c^2$
Division page 23				
1. $3x^3$	2. b^2/c^2d	3. $-2y^3/x^2$	4. 3	5. $27x^9/64y^{12}$

Lesson 5 Self-Check 5 page 28

1. 5.67×10^7
2. 8.76×10^5
3. 1.34×10^{-3}
4. 3.72×10^{-2}

Posttest page 30

1. c	6. b	11. c
2. c	7. b	12. a
3. d	8. d	13. b
4. b	9. c	14. d
5. c	10. b	15. a

END OF MODULE

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