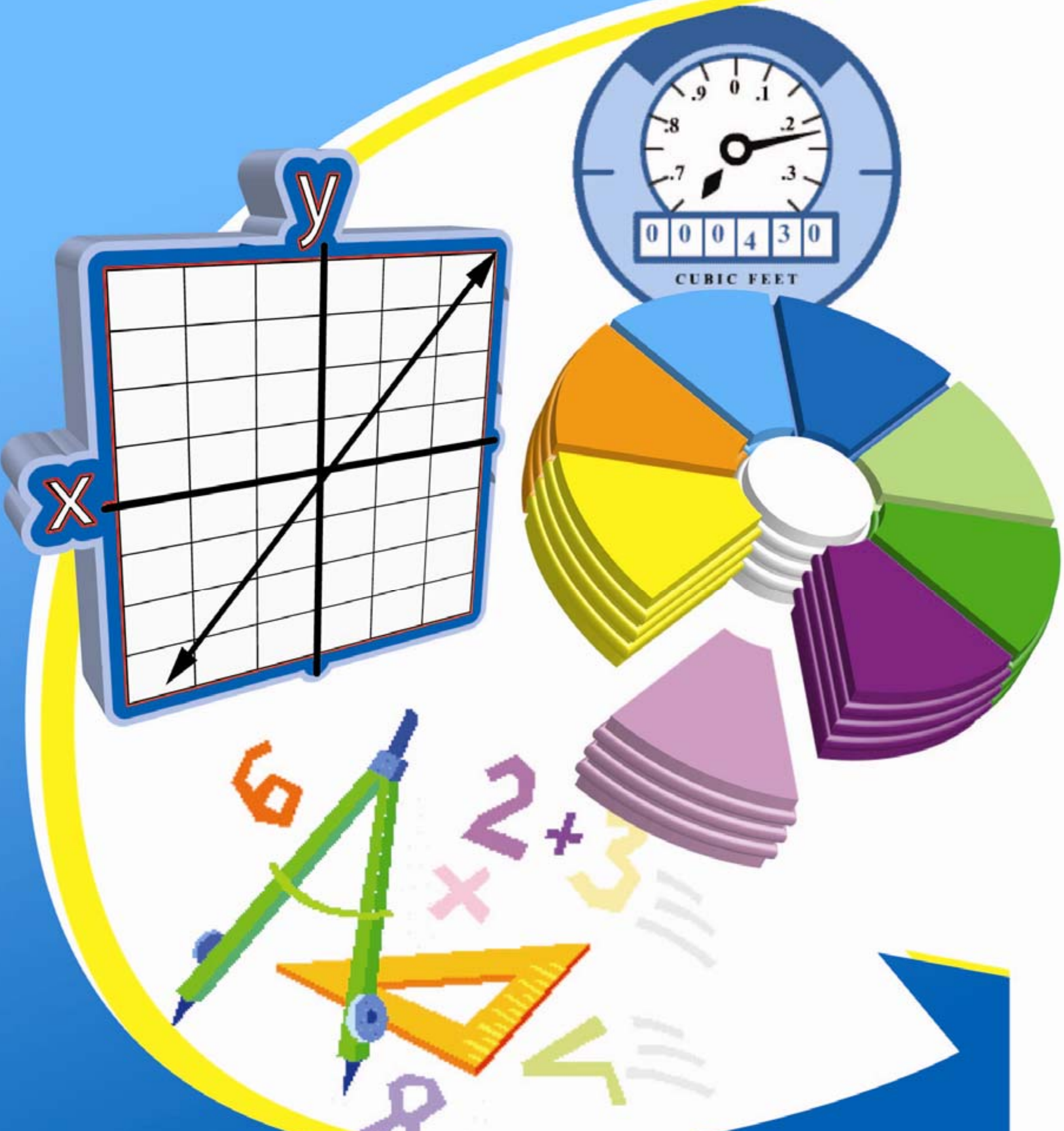
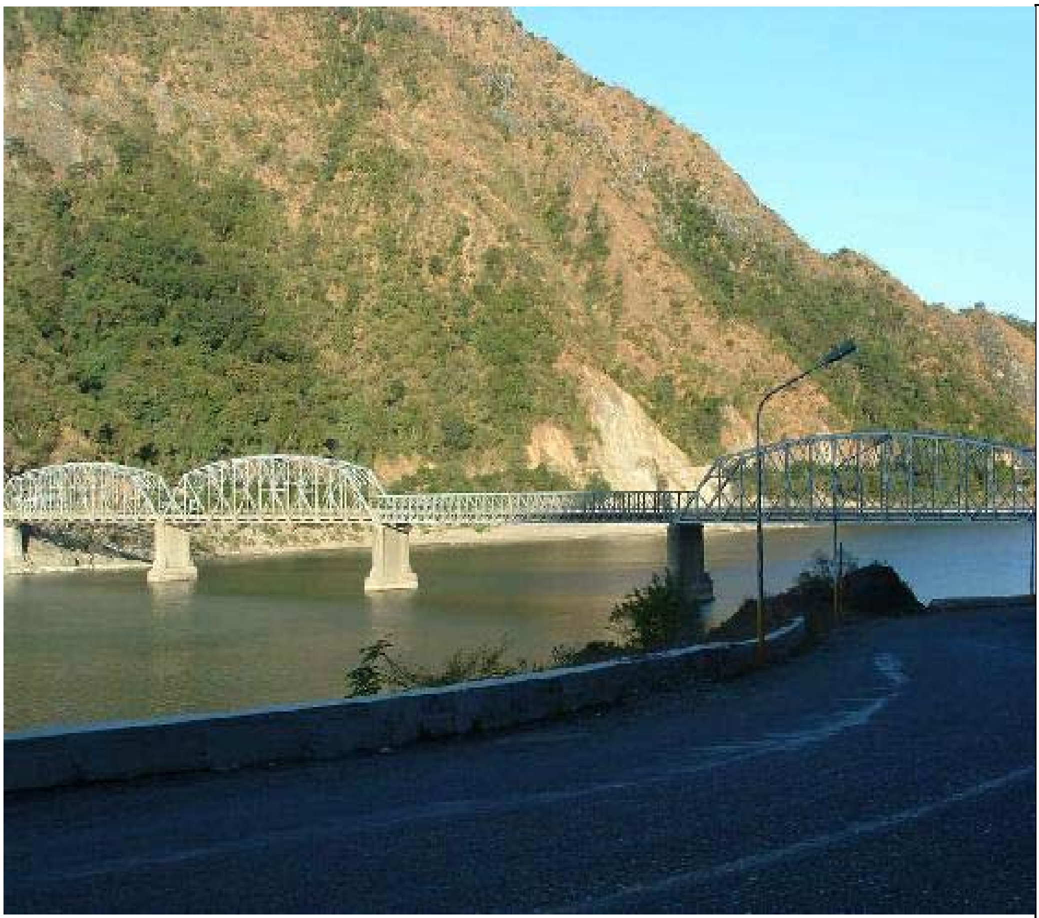
BUREAU OF SECONDARY EDUCATION
DEPARTMENT OF EDUCATION

DISTANCE LEARNING MODULE
MATHEMATICS 1



RATIONAL ALGEBRAIC EXPRESSIONS

UNIT IV

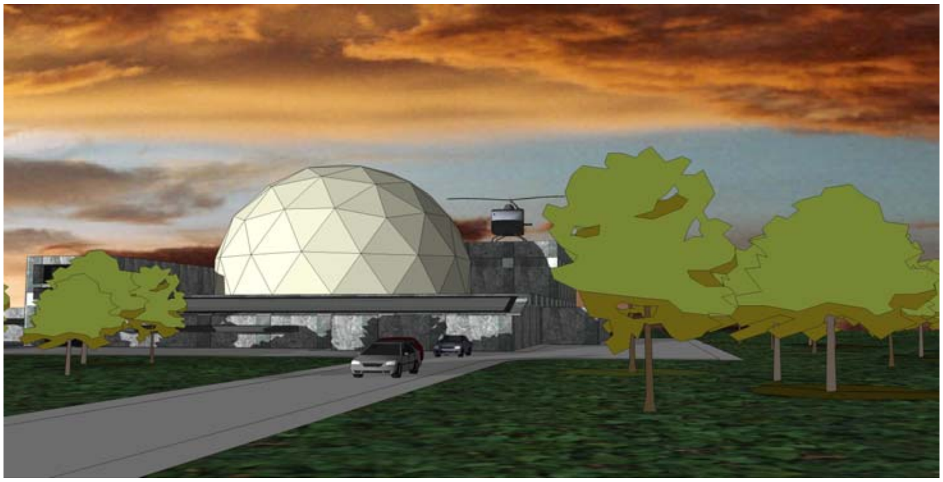


The Quirino Bridge in Santa, Ilocos Sur is composed of metal members, strategically and equally placed to hold the weight and stabilize the whole bridge.

Thus, to be able to put up this architectural design, mathematical skills in solving the exact distance and location of each structural member are necessary. Rational algebraic expressions such as $\frac{3wl}{8}$ and $\frac{wl^2}{8}$, where w is width and l is length are considered in the construction of this bridge.

In this unit, you will learn to simplify and perform operations on rational algebraic expressions. Each lesson has been provided with examples. The solution to each example is explained in detail so as to give you a deeper understanding of each lesson.

RATIONAL ALGEBRAIC EXPRESSIONS



The figure above was designed applying rational algebraic expressions. You will see that the figure shows a geometric pattern which uses repeated shapes to cover the plane without gaps or overlaps. This geometric pattern is called *tessellation*. Tessellations are made up of regular polygons often used in architecture. Not all regular polygons tessellate the plane.

In the expression $\frac{2n}{n-2}$, n represents the number of sides of a regular polygon. If the value of the expression is a whole for n, an integer ≥ 3, then the regular polygon with n sides will tessellate the plane.

Try finding three values of n that will make the expression a whole number.

Rational Algebraic Expressions

Algebraic expressions containing variables that are written the form $\frac{p}{q}$, p and q are polynomial expressions, $q \neq 0$, are called ***rational algebraic expressions.***
Examples of rational expressions are:

$\frac{a+b}{a-b}$, $\frac{8x^3-3x^2+2x+6}{4x^2+5x}$ and $\frac{1}{k^5}$.

You can simplify rational algebraic expressions by dividing both numerator and denominator by their greatest common factor (GCF). The GCF of two numbers is the product of their common prime factors.

Examples:

1. The GCF of 24 and 72.

   The factors are shown below.

   $24 = 2 \cdot 2 \cdot 2 \cdot 3$

   $72 = 2 \cdot 2 \cdot 2 \cdot 3$

   The GCF is 24

2. The GCF of $34x^2$ and $42x^5$.

   The factors are shown below.

   $34x^2 = 2 \cdot 17 \cdot x^2$

   $42x^5 = 2 \cdot 3 \cdot 7 \cdot x^{2\cdot} x^3$

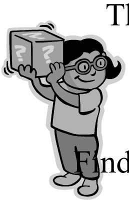
   The GCF is $2x^2$

3. The GCF of $2x^2 + 8x$ and $x^2 + x - 12$

$2x^2 + 8x = 2(x + 4)$

$x^2 + x - 12 = (x + 4)(x - 3)$

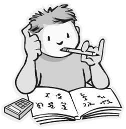
The GCF is x +4



Activity 1:

Find the GCF of the following.

1. 16 and 8
2. 48 and 12
3. 42y and 18xy
4. m + 5 and 2m + 10
5. $y^2 + 4y + 4$ and $3y^2 + 5y - 2$



Test 1:

Find the GCF of the following.

1. 21 and 35
2. 6xy and $15y^3$
3. $x^2 - 2x + 1$ and $x^2 - 1$
4. $x^2 - 25$ and $6x^2 + 29x - 5$
5. $5x - 10$ and $x^2 - 6x + 8$

Instructions:

* After answering the test, check your answers with those on the answer key.

* If your score is 3 or higher, you may proceed to the next topic; otherwise, read the lesson once more and do the Test again.

Simplifying Rational Expressions

Rational expression is in the lowest terms if the numerator and denominators have no common factor other than 1 or –1.

Examples:

1. Write $\frac{24}{72}$ in lowest term or simplest form.

Factor the numerator and denominator

$$\frac{24}{72} = \frac{(2\cancel{2\cdot 2})\cdot \overset{1}{\cancel{3}}}{(2\cdot 2\cancel{\cdot 2})\cdot \cancel{3}\cdot 3}$$ GCF is 24

$\frac{24}{72} = \frac{1}{3}$ is the lowest terms

2. Reduce $\frac{34x^2}{42x^5}$ in its lowest terms.

Factor completely the numerator and denominator.

$$34x^2 = 2\cdot 17\cdot x^2$$

$$42x^5 = 2\cdot 3\cdot 7\cdot x^{2\cdot}\ x^3$$

The GCF is $2x^2$

Divide the numerator and denominator by the GCF, $2x^2$

$\frac{34x^2}{42x^5} = \frac{17}{21x^3}$ is the lowest term

3. Write $\frac{3x-12}{5x-20}$ in lowest terms.

Factor the numerator and denominator, GCF is x - 4.

$$\frac{3x-12}{5x-20} = \frac{3\cancel{(x-4)}}{5\cancel{(x-4)}} = \frac{3}{5}$$

Divide numerator and denominator by x – 4.

$\frac{3x-12}{5x-20} = \frac{3}{5}$ is in lowest terms

4. Write $\frac{x^2+2x-8}{2x^2-x-6}$ in lowest terms.

Divide both numerator and denominator by x – 2 and cancel common factors.

$x^2 + 2x - 8 = (x + 4)(x - 2)$

$2x^2 - x - 6 = (2x + 3)(x - 2)$

$$\frac{x^2+2x-8}{2x^2-x-6} = \frac{(x+4)\cancel{(x-2)}}{(2x+3)\cancel{(x-2)}}$$

$$\frac{x^2+2x-8}{2x^2-x-6} = \frac{x+4}{2x+3}$$ is in lowest terms.

5. Express $\frac{8a^2+6a-9}{16a^2-9}$ in lowest terms

Factor the numerator and denominator, and canceling the GCF we have,

$$\frac{8a^2+6a-9}{16a^2-9} = \frac{(2a+3)\cancel{(4a-3)}}{(4a+3)\cancel{(4a-3)}}$$

$$\frac{8a^2+6a-9}{16a^2-9} = \frac{2a+3}{4a+3}$$ in lowest terms

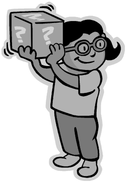
6. Write $\frac{k^3+p^3}{k^2-p^2}$ in lowest terms

Factoring, we have

$$\frac{k^3+p^3}{k^2-p^2} = \frac{(k+p)(k^2-kp+p^2)}{(k-p)(k+p)}$$

Cancelling common factors yield

$$\frac{k^2-kp+p^2}{k-p}$$



Activity 2:

A. Write in lowest terms.

1. $\frac{2n-6}{5n-15}$
2. $\frac{12x^2y^5}{-48x^2y^2}$
3. $\frac{x^2-4}{x^2+4x+4}$
4. $\frac{x^2-x-6}{x^2+x-12}$
5. $\frac{a^2+3a-4}{a^2-1}$

B. Explain why $\frac{a^2+b^2}{(a+b)^2}$ does not simplify to 1.

Test 2:

A. Simplify the following rational expressions and express your answer in lowest terms.

1. $\frac{a^2-4a}{4a-a^2}$
2. $\frac{4a^2-20a}{a^2-4a-5}$
3. $\frac{4x}{12x^2}$
4. $\frac{x+5}{x^2+3x-10}$
5. $\frac{y^2+8y-20}{y^2-4}$

Instructions:

* After answering the test, check your answers with those on the answer key.

* If your score is 3 or higher, you may proceed to the next topic; otherwise, read the lesson once more and do the Test again.

Least common denominator

The least common denominator (LCD) of two or more rational expressions is the smallest positive common multiple of the denominators of the rational expressions.

Let us recall the process of finding least common multiple (LCM) of two or more numbers. The LCM is the smallest nonzero multiple which the numbers have in common.

Examples:

1. Find the LCM of 6 and 8.

Multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48 ….

Multiples of 8: 8, 16, 24, 32, 40, 48, 56, 64…

The LCM of 6 and 8 is 24.

The least common denominator (LCD) of two or more fractions is the least number that is divisible by all denominators. It is found by multiplying the different prime factors of each denominator, taking the greatest number of times it appears in any denominator.

2. The LCD of the rational expressions $\frac{2}{6x^5y}$ and $\frac{3x}{9x^2y}$.

Find the complete factorization of all the denominators.

$6x^5y = 3 . 2 . x \cdot x \cdot x \cdot x \cdot x \cdot y$

$9x^2y^2 = 3 \;.\; x \cdot x \qquad y \cdot y \cdot 3$

$\text{LCD} = 3 \cdot 2 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot 3$

$= 18x^5y^2$

3. The LCD of $\frac{7}{6w-12}$ and $\frac{9}{9w-18}$.

$6w – 12 = 6(w – 2)$
$9w – 18 = 9(w – 2)$

$\text{LCD} = 6 \cdot 9(w – 2) = 54(w – 2)$

4. The LCD of the rational expressions

$$\frac{2}{a^2-9};\ \frac{1}{3a^2+3a-18};\ \frac{3}{a^2-4a+4}$$

$a^2 – 9 = (a + 3)(a – 3)$

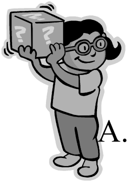
$3a^2 + 3a + 8 = 3(a^2 + a – 6)$

$= 3(a + 3)(a – 2)$

$a^2 – 4a + 4 = (a – 2)(a – 2)$

$= (a – 2)^2$

Thus, the LCD is
$3(a + 3)(a – 3)(a – 2)^2$



Activity 3:

A. The LCD of the following pairs of rational expressions.

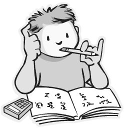
1. $\frac{2}{15k}$ and $\frac{8}{4k}$

2. $\frac{5}{5y^3}$ and $\frac{2}{15y^5}$

3. $\frac{7}{5y-30}$ and $\frac{1}{6y-36}$

4. $\frac{1}{k^2+4k-12}$ and $\frac{9}{k^2+k-90}$

5. $\frac{8}{2d^2-11d+14}$ and $\frac{10}{2d^2-d-21}$



Test 3:

A. Find the LCD of the following pairs of rational expressions.

1. $\frac{6}{18}$ and $\frac{1}{36}$
2. $\frac{3}{4ab^2}$ and $\frac{5}{6ab}$
3. $\frac{1}{4a-8b}$ and $\frac{3}{3a-6b}$
4. $\frac{b}{2b^2-b-3}$ and $\frac{3b}{3b^2+5b+2}$
5. $\frac{2x}{x^2-2x-8}$ and $\frac{1}{x^2-6x+8}$

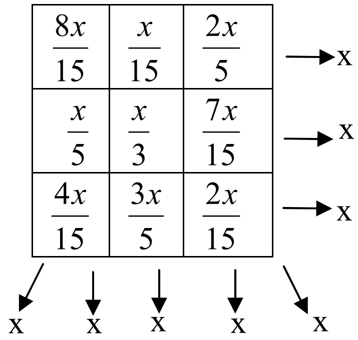
Instructions:

* After answering the test, check your answers with those on the answer key.

* If your score is 3 or higher, you may proceed to the next topic; otherwise, read the lesson once more and do the Test again.

Before you proceed to the next concept **TAKE A BREAK** and do the magic square below.

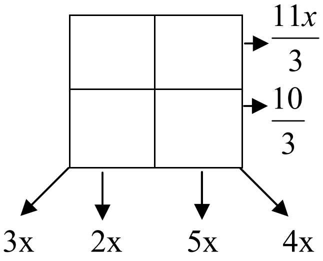
Below is an example of a 3 x 3 magic square. Notice that when you add every row, column, and diagonal the sum is x.



$\frac{8x}{15}$	$\frac{x}{15}$	$\frac{2x}{5}$
$\frac{x}{5}$	$\frac{x}{3}$	$\frac{7x}{15}$
$\frac{4x}{15}$	$\frac{3x}{5}$	$\frac{2x}{15}$

Now it's your turn:

Complete the entries to form a magic square whose entries in each row, each column and each diagonal sum up to numbers written outside the grid.



Adding Rational Expressions

To add two or more rational expressions, use a procedure similar to that of adding two numerical fractions.

Adding Similar Rational Expression

If $\frac{p}{q}$ and $\frac{r}{q}$ are rational expressions,

then $\frac{p}{q}+\frac{r}{q} = \frac{p+r}{q}$.

Addition of two similar fractions

Examples:

1. $\frac{7}{12}+\frac{3}{12}$

Since the denominators are the same, the sum is found by adding the two numerators and keeping the same (common) denominator.

$$\frac{7}{12} + \frac{3}{12} = \frac{7+3}{12} = \frac{10}{12} = \frac{5\cdot 2}{6\cdot 2}$$

by canceling common factors.

$$\frac{7}{12} + \frac{3}{12} = \frac{5}{6}$$

2. Add : $\frac{3y}{y-5}$ and $\frac{4y}{y-5}$

Since the denominators are the same, the sum is found by adding the two numerators and keeping the same (common) denominator.

$$= \frac{3y+4y}{y-5}$$

$$= \frac{7y}{y-5}$$

Addition of Dissimilar Rational Algebraic Expressions

To add dissimilar rational algebraic expressions, use the same steps that are used to add dissimilar fractions.

Examples:

1. Add: $\frac{1}{12} + \frac{2}{15}$

Find the least common denominator (LCD)

12 = 2 · 2 · 3

15 = 3 · 5

LCD = 2 · 2 · 3· 5 = 60

Rewrite the rational expressions as fractions denominator 60

$$\frac{1}{12} + \frac{2}{15} = \frac{1(5)}{12(5)} + \frac{2(4)}{15(4)}$$

$$= \frac{5}{60} + \frac{8}{60}$$

Since the fractions are now having common denominators, add the numerators copy the same denominator and write in lowest terms if necessary.

$$= \frac{5+8}{60}$$

$$= \frac{13}{60}$$

2. Find: $\frac{2}{2x} + \frac{3}{6x}$

Find the LCD of the fractions

2x = 2 · x
6x = 2 · 3 · x

LCD = 2 · x · 3 = 6x

Rewrite the rational expressions as fractions with LCD, 6x.

$$\frac{2}{2x} + \frac{3}{6x} = \frac{2(3)}{2x(3)} + \frac{3(1)}{6x(1)}$$

$$= \frac{6}{6x} + \frac{3}{6x}$$

.

$$= \frac{6+3}{6x}$$

$$= \frac{9}{6x} = \frac{3\cdot 3}{3\cdot 2\cdot x}$$

$$= \frac{3}{2x}$$

3. Find: $\frac{x}{x^2-1} + \frac{x}{x+1}$

Rewrite the fractions and find the LCD

$$\frac{x}{(x+1)(x-1)} + \frac{x}{x+1}$$

The LCD is (x + 1)(x - 1)

Rewrite the rational expressions as fractions with LCD (x + 1)(x – 1)

$$\frac{x}{(x+1)(x-1)} + \frac{x(x-1)}{(x+1)(x-1)}$$

Multiply the numerator and denominator of the second fraction by (x - 1). Add the numerators.

$$\frac{x+x(x-1)}{(x+1)(x-1)}$$

Simplify the numerator

$$= \frac{x+x^2-x}{(x+1)(x-1)}$$

$$= \frac{x^2}{(x+1)(x-1)}$$

4. Add $\frac{2x}{x^2+5x+6}$ and $\frac{x+1}{x^2+2x-3}$

Begin by factoring the denominators

$$\frac{2x}{(x+2)(x+3)} + \frac{x+1}{(x+3)(x-1)}$$

The LCD is (x + 2)(x + 3)(x - 1)

Rewrite the rational expressions as fractions with (x + 1)(x – 1)(x + 3) as denominator.

$$= \frac{2x(x-1)}{(x+2)(x+3)(x-1)} +$$

$$\frac{(x+1)(x+2)}{(x+3)(x-1)(x+2)}$$

Now that the two rational expressions have the same denominator, simplify the numerator.

$$= \frac{2x(x-1)+(x+1)(x+2)}{(x+2)(x+3)(x-1)}$$

$$= \frac{2x^2-2x+x^2+3x+2}{(x+2)(x+3)(x-1)}$$

$$= \frac{3x^2+x+2}{(x+2)(x+3)(x-1)}$$

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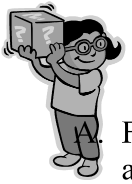
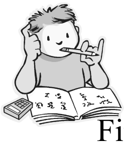


Activity 4:

A. Find the sum and express your answer in the lowest terms.

1. $\frac{3}{s} + \frac{2}{s}$
2. $\frac{x}{p-5} + \frac{2x}{p-5}$
3. $\frac{2n+3}{n^2-4n+4} + \frac{n-4}{n^2+n-6}$
4. $\frac{3}{a^2-5a+6} + \frac{-2}{a^2-a-2}$
5. $\frac{r+1}{r^2-3r-10} + \frac{r-1}{r^2+r-30}$



Test 4:

Find the sum and express your answer in lowest terms.

1. $\frac{3a}{9a-8}+\frac{5a}{9a-8}$

2. $\frac{2x}{2x^2+7x+3}+\frac{1}{2x^2+7x+3}$

3. $\frac{2x}{x+y}+\frac{3x}{2x+2y}$

4. $\frac{5}{t-6}+\frac{4}{t+6}$

5. $\frac{3a}{a^2+ab-2b^2}+\frac{4a-1}{a^2-b^2}$

Instructions:

* After answering the test, check your answers with those on the answer key.

* If your score is 3 or higher, you may proceed to the next topic; otherwise, read the lesson once more and do the Test again.

Try more:

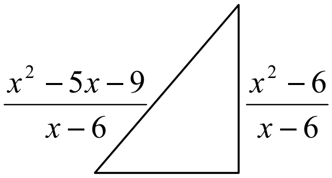
Solve the following problems:

1. Johnny, a farm caretaker, was asked to fence the farm he is working. The farm has a length of $\frac{y+4}{5}$ m and width of $\frac{y-2}{5}$ m. Find the perimeter of the farm.

   (Use formula: P = 2l + 2w).

2. The perimeter of the figure is (2x + 5)cm. Find the length of the missing side.

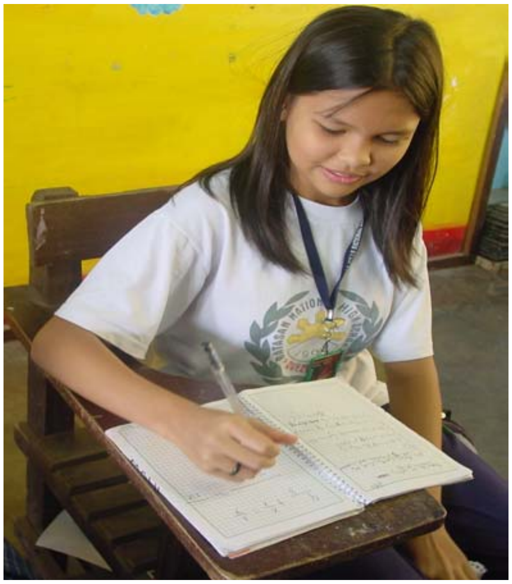
   (Use the formula, $P = s_1 + s_2 + s_3$), where s_1, s_2, and s_3 are the lengths of the sides of the triangle.



Problem solving applying addition of rational algebraic expressions

Examples:

1. When a number is subtracted from the numerator and added to the denominator of $\frac{17}{22}$, the result is $\frac{5}{8}$. What is the number?



Solution:
Let x = the number to be added and subtracted.

Original fraction	New fraction
$\frac{17}{22}$	$\frac{17-x}{22+x}$

Equation: $\frac{17-x}{22+x} = \frac{5}{8}$

LCD = 8(22 + x)

$$8(22 + x)\ \frac{17-x}{22+x} = \frac{5}{8}8(22 + x)$$

$$8(17- x) = (22 + x)\ 5$$

$$136 - 8x = 110 + 5x$$

$$136 - 110 = 8x + 5x$$

$$26 = 13x$$

$$2 = x$$

$$\text{or } x = 2$$

Check:

$$\frac{17-2}{22+2} = \frac{5}{8}$$

$$\frac{15}{24} = \frac{5}{8}$$

Therefore, the required number is 2.

2. Ed can build a fence in 2 hours. His brother can do it in 4 hours. How long would it take them working together?

Solution:

Let x = number of hours for both to work together.

$\frac{x}{2}$ = part of the job Ed can finish in 1 hour

$\frac{x}{4}$ = part of the job brother can finish in 1 hour

Equation:

$$\frac{x}{2} + \frac{x}{4} = 1$$

LCD = 4

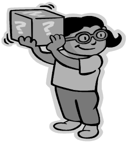
$$4(\frac{x}{2}) + 4(\frac{x}{4}) = 4(1)$$

$$2x + x = 4$$

$$3x = 4$$

$$x = 1\frac{1}{3} \text{ hrs.}$$

Thus, Ed and his brother can finish it in 1 $\frac{1}{3}$ hours.

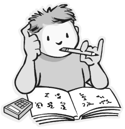


Activity 5:

Solve the following problems.

1. What number must be added to both numerator and denominator of the fraction $\frac{5}{8}$ equal to $\frac{3}{4}$?

2. Margie weeds the farm in 3 hours. Mae in 5 hours, Mia in 8 hours. If all three work together, how long will it take them to weed the farm?



Test 5:

1. What number can be subtracted from both the numerator and denominator of the fraction $\frac{7}{12}$ to make a fraction equal to $\frac{1}{2}$?

2. Ken can paint a room in 4 hours. His brother Nikki can do it in 5 hours. How long would it take them working together?

Instructions:

* After answering the test, check your answers with those on the answer key.

* If your score is 1 or higher, you may proceed to the next topic; otherwise, read the lesson once more and do the Test again.

Subtracting Rational Expressions

To subtract two rational expressions, you use the rule similar to subtracting numerical fractions.

Subtracting Rational Expressions

If $\frac{p}{q}$ *and* $\frac{r}{q}$ *are rational similar expressions then,* $\frac{p}{q} - \frac{r}{q} = \frac{p-r}{q}$.

Examples:

1. Find : $\frac{18}{30} - \frac{6}{30}$

Notice that their denominators are the same, Thus, the difference is found by subtracting the two numerators and copying the common denominator.

$\frac{18}{30} - \frac{6}{30} = \frac{12}{30}$ or $\frac{2}{5}$ in lowest terms.

2.. Find: $\frac{10x}{x^2-12} - \frac{5x}{x^2-12}$

$$= \frac{10x-5x}{x^2-12}$$

Simplifying

$$= \frac{5x}{x^2-12}$$

3. Find: $\frac{11}{12} - \frac{7}{15}$

Notice that their denominators are different. Thus, we find the LCD.

$12 = 2 \cdot 2 \cdot 3$

$15 = 3 \cdot 5$

LCD = $2 \cdot 2 \cdot 3 \cdot 5 = 60$

Rewrite the rational expressions as fractions with 60 as the denominator

$$\frac{11}{12} - \frac{7}{15} = \frac{11(5)}{12(5)} - \frac{7(4)}{15(4)}$$
$$= \frac{55}{60} - \frac{28}{60}$$

Since the fractions are now similar, you have to subtract the numerators and copy the denominator. Write the result in lowest terms.

$$= \frac{55-28}{60}$$
$$= \frac{27}{60} = \frac{\cancel{3} \cdot 9}{\cancel{3} \cdot 20}$$
$$= \frac{9}{20}$$

4. Find : $\frac{2}{3b} - \frac{2}{7b}$

Find the LCD.

$3b = 3 \cdot b$

$7b = 7 \cdot b$

LCD = $3 \cdot 7 \cdot b = 21b$

Rewrite the rational expressions as fractions with 21b as the denominator

$$\frac{2}{3b} - \frac{2}{7b} = \frac{2(7)}{3b(7)} - \frac{2(3)}{7b(3)}$$

$$= \frac{14}{21b} - \frac{6}{21b}$$

Since the fractions are now similar, subtract the numerator, copy the same denominator and write the difference in lowest terms.

$$= \frac{14-6}{21b}$$

$$= \frac{8}{21b}$$

5. Subtract $\frac{3}{x}$ from $\frac{9}{x-2}$

The LCD is (x - 2)(x)

$$\frac{9}{x-2} - \frac{3}{x} = \frac{9(x)}{(x-2)(x)} - \frac{3(x-2)}{x(x-2)}$$

$$= \frac{9(x)}{x(x-2)} - \frac{3x-6}{x(x-2)}$$

$$= \frac{9x-3x+6}{x(x-2)}$$

$$= \frac{6x+6}{x(x-2)}$$

6. Find : $\frac{1}{p-1}$ - $\frac{1}{p+1}$

The LCD is (p - 1)(p + 1)

Rewrite the rational expressions as fractions with (p – 1)(p + 1) as the denominator

$$\frac{1}{p-1} - \frac{1}{p+1} = \frac{1(p+1)}{p-1(p+1)} - \frac{1(p-1)}{p+1(p-1)}$$

$$= \frac{p+1-(p-1)}{(p-1)(p+1)}$$

$$= \frac{p+1-p+1}{(p-1)(p+1)}$$

$$= \frac{2}{(p-1)(p+1)}$$

7. Perform the indicated operation.

$$\frac{x}{x^2-4x-5} - \frac{3}{2x^2-13x+15}$$

Rewrite each fraction

$$= \frac{x}{(x-5)(x+1)} - \frac{3}{(x-5)(2x-3)}$$

The LCD is (x - 5)(x + 1)(2x – 3).

Rewrite each rational expression with (x - 5) (x + 1)(2x – 3) as denominator.

$$= \frac{x(2x-3)}{(x-5)(x+1)(2x-3)} -$$

$$\frac{3(x+1)}{(x-5)(2x-3)(x+1)}$$

$$= \frac{2x^2-3x}{(x-5)(x+1)(2x-3)} -$$

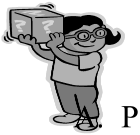
$$\frac{3x+3}{(x-5)(2x-3)(x+1)}$$

Since the fractions are now similar, subtract the numerators, copy the denominator and write the result in lowest terms.

$$= \frac{2x^2-3x-(3x+3)}{(x-5)(x+1)(2x-3)}$$

$$= \frac{2x^2 - 3x - 3x - 3}{(x-5)(x+1)(2x-3)}$$

$$= \frac{2x^2 - 6x - 3}{(x-5)(x+1)(2x-3)}$$



Activity 6:

A. Perform the indicated operation and write the answer in lowest terms.

1. $\frac{1}{b^2 - 1}$ - $\frac{1}{b^2 + 3b + 2}$

2. $\frac{8}{3 - 7y}$ - $\frac{2}{7y - 3}$

3. - $\frac{2y+1}{9 - y^2} + \frac{2}{y-3} - \frac{1}{y+3}$

4. $\frac{-1}{m^2 + mn - 2m^2}$ - $\frac{3}{m^2 - 3mn + 2n^2}$

5. $\frac{4m - 3n}{16m^2 - n^2}$ - $\frac{4m - n}{16m^2 + 8mn + n^2}$

B. Subtract the rational expressions. decode the word by writing the letter that corresponds to the correct number in the blank provided.

1. $\frac{2x}{x+y}$ - $\frac{3x}{2x+2y}$ O x

2. $\frac{4m+5}{3}$ - $\frac{m+2}{6}$ N $\frac{(x-3)(x+3)}{3x}$

3. $\frac{1}{a-b}$ - $\frac{a}{4a-4b}$ E $\frac{7m+8}{6}$

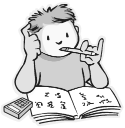
4. $\frac{6x}{3}$ - $\frac{3x}{3}$ B $\frac{x}{2(x+y)}$

5. $\frac{x}{3}$ - $\frac{3}{x}$ H 4(a- b)

6. $\frac{15}{4k^2}$ - $\frac{3}{k+2}$ T $\frac{6-x}{3(x+3)(x-3)}$

7. $\frac{1}{x^2 - 9}$ - $\frac{1}{3x+9}$ S $\frac{15 + 30 - 12k^2}{4k^2(k+2)}$

____	____	____	____	____	____	____	____
1	2	3	4	5	2	6	7



Test 6

Subtract the rational expressions and express your answer in lowest terms.

1. $\frac{3}{p}$ - $\frac{2}{p}$

2. $\frac{x}{k-5}$ - $\frac{2x}{k-5}$

3. $\frac{15}{4k^2}$ - $\frac{3}{k+2}$

4.. $\frac{m}{m^2 + 3m - 18}$ - $\frac{3}{m^2 + 3m - 18}$

5. $\frac{x}{x^2 + xy - 2y^2}$ - $\frac{3x}{x^2 - 3xy + 2y^2}$

Instructions:

* After answering the test, check your answers with those on the answer key.

* If your score is 3 or higher, you may proceed to the next topic; otherwise, read the lesson once more and do the Test again

Multiplying Rational Expressions

When you multiply two rational expressions, adopts the same procedure in multiplying two rational numbers.

Rule for Multiplying Fractions

If p, q, r and s are real numbers with $q \neq 0$ and $s \neq 0$, then

$$\frac{p}{q} \cdot \frac{r}{s} = \frac{pr}{qs}$$

The same rule applies to multiplication of rational expressions.

Examples:

Find:

1. $\frac{3}{7} \cdot \frac{4}{5} = \frac{3 \cdot 4}{7 \cdot 5} = \frac{12}{35}$

2. $\frac{2x}{3y} \cdot \frac{9}{8y} = \frac{2x \cdot 9}{3y.8y} = \frac{18x}{24y}$ or $\frac{3x}{4y}$

3. Multiply: $\frac{4m^3}{m^2 - 5m} \cdot \frac{m-5}{12}$

factor $m^2 - 5m$

$$= \frac{4m^3 \cdot (m-5)}{m(m-5)12}$$

Divide both numerator and denominator by (m- 5) and simplify.

$$\frac{4m^3}{12m} = \frac{m^2}{3}.$$

4. Multiply: $\frac{x^2 - 4}{x^2 - 5x + 6} \cdot \frac{2x+1}{6x+3}$

Factoring the numerator and the denominator

$$\frac{(x-2)(x+2)(2x+1)}{(x-2)(x-3)[3(2x+1)]}$$

Dividing both numerator and denominator by (x –2)(2x + 1).

$$\frac{x+2}{3(x-3)}$$

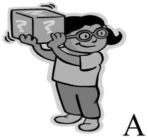
5. Find: $\frac{x^2 - 3x - 18}{x^2 - x - 2} \cdot \frac{3x+3}{x^2 - 2x - 15}$

Factoring the numerator and denominator will yield

$$\frac{(x-6)(x+3)}{(x-2)(x+1)} - \cdot \frac{3(x+1)}{(x-5(x+3)}$$

Dividing both numerator and denominator by (x+3)(x+1) will result in

$$\frac{3(x-6)}{(x-2)(x-5)}$$



Activity 7:

A. Multiply as indicated and express each product in simplest form.

1. $\frac{3}{8} \cdot \frac{12}{15}$

2. $\frac{2ab}{3c^2} \cdot \frac{c}{a^2}$

3. $\frac{9 - x^2}{x+3} \cdot \frac{x}{3-x}$

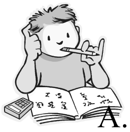
4. $\frac{a^2 + 5a}{a^2 - 16} \cdot \frac{a^2 - 4a}{a^2 - 25}$

5. $\frac{6x - 3y}{4x^2 + 4xy + y^2} \cdot \frac{2x + y}{4x^2 - 4xy + y^2}$

Take a break:
Solve this problem.

Find two different pairs of rational expressions whose product is

$$\frac{8a^2+16a-24}{a^2+13a+40}$$



Test 7:

A. Multiply and express each product in the simplest form.

1. $\frac{12}{7}\cdot\frac{21}{54}$
2. $\frac{4a^2}{3bc}\cdot\frac{8b^2c}{6a^2}$
3. $\frac{t^2+5t}{5+t}\cdot\frac{t+1}{t^2-25}$
4. $\frac{x^2+8x+16}{x^2-9}\cdot\frac{x-3}{x+4}$
5. $\frac{x^2-6x+5}{x-1}\cdot\frac{x-1}{x-5}$

Instructions:

* After answering the test, check your answers with those on the answer key.

* If your score is 3 or higher, you may proceed to the next topic; otherwise, read the lesson once more and do the Test again

Dividing Rational Expressions

Dividing rational expressions is the same as multiplying the dividend by the multiplicative inverse or reciprocal of the divisor.

Rule for Dividing Fractions

If p, q, r and s are real numbers, with

$q \neq 0$ and $r \neq 0$ and $s \neq 0$, then

$$\frac{p}{q}\div\frac{r}{s}=\frac{p}{q}\bullet\frac{s}{r}=\frac{ps}{qr}$$

Examples:

1. Divide $\frac{5}{9}$ by $\frac{4}{15}$

Multiply $\frac{5}{9}$ by the multiplicative inverse or reciprocal of $\frac{4}{15}$ which is $\frac{15}{4}$.

$$=\frac{5}{9}\cdot\frac{15}{4}$$

Factor the numerator and the denominator and cancel common factors

$$\frac{5}{9}\cdot\frac{15}{4}=\frac{5}{3\cdot 3}\cdot\frac{5\cdot 3}{2\cdot 2}=\frac{25}{12}$$

2. Find: $\frac{5y}{9xz^2}\div\frac{15y^3}{18x^2z^2}$

Multiplying $\frac{5y}{9xz^2}$ by the reciprocal of

$\frac{15y^3}{18x^2z^2}$ which is $\frac{18x^2z^2}{15y^3}$, we get

$$\frac{5y}{9xz^2}\cdot\frac{18x^2z^2}{15y^3}$$

Factor the numerators and denominators and cancel common factors.

$$=\frac{5y}{9xz^2}\cdot\frac{9\cdot 2x^2z^2}{5\cdot 3y^3}$$

$$= \frac{2x}{3y^2}$$

3. Find: $\frac{x^2}{x^2 - 25x^2} \div \frac{x}{x+5}$

Multiply $\frac{x^2}{x^2 - 25x^2}$ by the reciprocal of

$\frac{x}{x+5}$ which is $\frac{x+5}{x}$

$$= \frac{x^2}{x^2 - 25x^2} \cdot \frac{x+5}{x}$$

Factor the numerators and denominators.

$$= \frac{x \cdot x}{(x+5x)(x-5x)} \cdot \frac{x+5}{x}$$
$$= \frac{x}{x-5}$$

Complex Rational Expressions

A complex rational expression is a rational expression whose numerator or denominator contains one or more rational expressions.

Examples:

$$\frac{\frac{3}{x}}{\frac{5}{x^2}} \; ; \; \frac{\frac{1}{m} + \frac{1}{n}}{\frac{2}{3m} + \frac{2}{3n}} \; : \; \frac{a - \frac{2}{2a-3}}{2a - 1 - \frac{8}{2a-3}}$$

To simplify a complex rational expression, consider the example below:

Simplify complex fraction

$$\frac{\frac{4-x^2}{2}}{\frac{2-x}{5}}$$

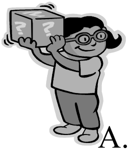
Rewrite $\frac{\frac{4-x^2}{2}}{\frac{2-x}{5}}$ as $\frac{4-x^2}{2} \div \frac{2-x}{5}$

By definition of division of fractions, we proceed as follows:

$$= \frac{4-x^2}{2} \cdot \frac{5}{2-x}$$

Factor the numerator and denominator. Cancel the common factor.

$$= \frac{(2-x)(2+x)}{2} \cdot \frac{5}{2-x}$$
$$= \frac{5(2+x)}{2}$$



Activity 8:

A. State the multiplicative inverse or reciprocal of each expression below..

1. $\frac{3}{8}$
2. $\frac{1}{x-y}$
3. $\frac{7x}{9y}$
4. $\frac{x+y}{2}$
5. $\frac{18x}{7}$

B. Perform each indicated division and write the answer in simplest form.

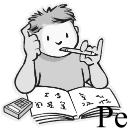
1. $\frac{21}{7} \div \frac{3}{14}$

2. $\frac{7}{a} \div \frac{14}{a^2}$

3. $\frac{5}{m-3} \div \frac{10}{m-3}$

4. $\frac{3x-21}{x^2-49} \div \frac{3x}{x^2+7x}$

5. $\dfrac{\frac{x^2-y^2}{2}}{\frac{x-y}{4}}$



Test: 8

Perform each indicated division and write the answer in simplest form.

1. $\frac{3d^2c}{a^4} \div \text{-} \frac{6dc}{a^5}$

2. $\frac{a^2-b^2}{x^2-y^2} \div \frac{a+b}{x+y}$

3. $\frac{x-y}{x+y} \div \frac{5x^2-5y^2}{3x-3y}$

4. $\frac{5a^2-5ab}{ab+b^2} \div \frac{5a^2+5b^2}{b}$

5 $\dfrac{\frac{3x}{y}}{\frac{5y^2}{2x^3}}$

Instructions:

* After answering the test, check your answers with those on the answer key.

* If your score is 3 or higher, you may proceed to the next topic; otherwise, read the lesson once more and do the Test again

Summary:

- To simplify a rational algebraic expression, divide both numerator and denominator by their greatest common factor (GCF).
- The GCF is the greatest factor that a set of terms has in common.
- The LCM of two or more numbers is the smallest nonzero multiple which numbers have in common.
- Addition and Subtraction of Rational Expressions
  1. Similar rational expressions

     If $\frac{p}{q}$ and $\frac{p}{q}$ are rational expressions then, $\frac{p}{q} + \frac{r}{q} = \frac{p+r}{q}$ and

     $$\frac{p}{q} - \frac{r}{q} = \frac{p-r}{q}$$

  2. Dissimilar rational expressions

     To add or subtract rational expressions with dissimilar denominators, write them as equivalent expressions with a least common denominator.
- Multiplication of Rational Expressions

If $\frac{p}{q}$ and $\frac{r}{s}$ are rational expressions,

then, $\frac{p}{q} \cdot \frac{r}{s} = \frac{p \cdot r}{q \cdot s}$

- Division of Rational Expressions

If $\frac{p}{q}$ is divided by $\frac{r}{s}$, multiply $\frac{p}{q}$ by the multiplicative inverse of $\frac{r}{s}$, for $r \neq 0$

$$\frac{p}{q} \div \frac{r}{s} = \frac{p}{q} \cdot \frac{s}{r}$$

CHAPTER TEST:

1. The simplified form of $\frac{m^2n - mn^2}{m^2 - mn}$ is

a. m c. mn

b. n d. $\frac{m}{n}$

2. Which is the sum of $\frac{5y}{10}$ and $\frac{15}{10}$?

a. $\frac{y+3}{2}$ c. $\frac{5y}{10}$

b. $\frac{y+3}{10}$ d. $\frac{y+2}{5}$

3. When simplified, $\frac{7x-21}{x^2-x-6}$ is equal to

a. $\frac{7}{x+2}$ c. $\frac{7}{x-2}$

b. $\frac{-7}{x+2}$ d. $\frac{-7}{x-2}$

4. The difference of $\frac{7}{x^2-x}$ - $\frac{3}{x^2-1}$ is

a. $\frac{4x-7}{x(x-1)(x+1)}$ c. $\frac{4x+7}{x(x-1)(x+1)}$

c. $\frac{4x-7}{(x-1)(x+1)}$ d. $\frac{4x-7}{x(x-1)}$

5. What is the reciprocal of $\frac{6x-9}{25}$?

a. $\frac{6x+9}{25}$ c. $\frac{25}{6x+9}$

b. $\frac{25}{6x-9}$ d. none of these

6. The least common denominator $\frac{2}{8x^2y^2}$ and $\frac{5}{12xy^3}$ is

a. 24xy c. $24x^2y^3$

b. $24x^3y^2$ d. $24xy^3$

7. The greatest common factor of 24x and $14x^2$ is

a.2x c. 4x

b. $2x^2$ d. $24x^2$

8. When simplified, $\frac{3}{x^2} + \frac{5}{2xy}$ - $\frac{4}{3y^2}$ is equal to

a. $\frac{18y^2 + 15xy - 8x^2}{6x^2y^2}$

b. $\frac{18y \ + 15xy - 8x^2}{6x^2y^2}$

c. $\dfrac{18y^2+15xy-8x}{6x^2y^2}$

d. $\dfrac{18y^2+15xy+8x^2}{6x^2y^2}$

9. The simplest form of $\dfrac{a+b}{ab} - \dfrac{3}{2a}$ is

a. $\dfrac{2a+b}{2ab}$ b. $\dfrac{2a-b}{2ab}$

c. $\dfrac{a-b}{2ab}$ d. $\dfrac{2a-b}{ab}$

10. The quotient of $\dfrac{6a^2b}{5c} \div \dfrac{3ac}{5b}$ is equal to

a. $\dfrac{2ab}{c^2}$ c. $\dfrac{2a^2b}{c^2}$

b. $\dfrac{2ab^2}{c^2}$ d. $\dfrac{ab}{c^2}$

Perform the indicated operation and write each answer in simplest form.

11. $\dfrac{7ab}{9c} \cdot \dfrac{81c^2}{91a^2b}$

12. $\dfrac{x^2-y^2}{a^2-b^2} \cdot \dfrac{a+b}{x-y}$

13. $\dfrac{y+6}{2y} \cdot \dfrac{4y^2}{y+6}$

14. $\dfrac{x^2-2x+1}{y-5} \cdot \dfrac{a^2-3a}{a^2+a-12}$

15. $\dfrac{a-1}{a+2} \cdot \dfrac{a+1}{a-1}$

16. $\dfrac{a^2-ab}{3a} - \dfrac{a-b}{15b^2}$

17. $\dfrac{x^2-2x+1}{y-5} \div \dfrac{x-1}{y^2-25}$

18. $\dfrac{c^2+3c}{c^2+2c-3} + \dfrac{c}{c+1}$

19. $\dfrac{a^2-49}{(a+7)^2} + \dfrac{3a-21}{2a+14}$

20. $\dfrac{a^2-b^2}{x^2-y^2} - \dfrac{a-b}{x+y}$

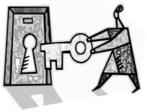
21. $\dfrac{8}{3-7y} - \dfrac{2}{7y-3}$

22. $\dfrac{p+q}{3p^2+2pq-q^2} - \dfrac{q-p}{6p^2-5pq+q^2}$

23. $\dfrac{m-n}{m^2+2mn+n^2} - \dfrac{m+n}{m^2-mn-2n^2}$

24. $\dfrac{1}{b^2-1} - \dfrac{1}{b^2+3b+2}$

25. $\dfrac{2t-1}{t+1} - \dfrac{2t+1}{t-1}$



Answer Key

Activity 1

Find the (GCF) of the following numbers.

1. 16 and 8 GCF = 8
2. 48 and 12 GCF = 12
3. 42y and 18xy GCF = 6y
4. n+5 and 2m + 10 GCF = m +5
5. y^2 +4y + 4 and 3^2 + 5y -2
   GCF = (y + 2)

Test: 1

Find the greatest (GCF) of the following numbers.

1. 21 and 35 GCF = 7
2. 6xy and $15y^3$ GCF = 3y
3. x^2 - 2x +1 and x^2 -1 GCF = (x - 1)
4. x^2 – 25 and $6x^2$ + 29 -5 GCF = (x + 5)
5. 5x – 10 and x^2 -6x +8 GCF = (x - 2)

Activity 2:

A. Simplify the following rational expressions and express your answer in lowest terms.

1. $\frac{2n-6}{5n-15} = \frac{2(n-3)}{5(n-3)} = \frac{2}{5}$

2. $\frac{12x^2y^5}{-48x^2y^2} = \frac{3\cdot 4\cdot x\cdot x\cdot y\cdot y\cdot y\cdot y\cdot y}{-3\cdot 4\cdot 4\cdot x\cdot x\cdot y\cdot y}$

   $= \frac{-y^3}{4}$

3. $\frac{x^2-4}{x^2+4x+4} = \frac{(x-2)(x+2)}{(x+2)(x+2)}$

   $= \frac{(x-2)}{(x+2)}$

4. $\frac{x^2-x-6}{x^2+x-12} = \frac{(x-3)(x+2)}{(x-3)(x+4)}$

   $= \frac{(x+2)}{(x+4)}$

5. $\frac{a^2+3a-4}{a^2-1} = \frac{(a+4)(a-1)}{(a+1)(a-1)}$

   $= \frac{(a+4)}{(a+1)}$

B. Explain why $\frac{a^2+b^2}{(a+b)^2}$ does not simplify to 1.

This expression will not be equal to 1.
Because see the solution:

$\frac{a^2+b^2}{(a+b)(a+b)}$ since $(a + b)^2$ is same as (a + b)(a + b)

$\frac{a^2+b^2}{a^2+2ab+b^2}$ the result is not equal to 1

Test 2:

Simplify the following rational expressions and express your answer in lowest terms.

1. $\frac{a^2-4a}{4a-a^2} = \frac{a(a-4)}{a(4-a)} = \frac{a-4}{4-a}$

   $= -\frac{(4-a)}{(4-a)} = -1$

2. $\frac{4a^2-20a}{a^2-4a-5} = \frac{4a(a-5)}{(a-5)(a+1)}$

   $= \frac{4a}{a+1}$

3. $\frac{4x}{12x^2} = \frac{1}{3x}$

4. $\frac{x+5}{x^2+3x-10} = \frac{x+5}{(x+5)(x-2)}$

   $= \frac{1}{x-2}$

5. $\frac{y^2+8y-20}{y^2-4} = \frac{(y+10)(y-2)}{(y+2)(y-2)}$

$$= \frac{y+10}{y+2}$$

Activity 3

Find the LCD of the following rational expressions.

1. $\frac{2}{15k}$; $\frac{8}{4k}$ LCD =60k
2. $\frac{5}{5y^3}$; $\frac{2}{15y^5}$ LCD = $15y^5$
3. $\frac{7}{5y-30}$; $\frac{1}{6y-36}$ LCD= 30(y - 6)
4. $\frac{1}{k^2+4k-12}$; $\frac{9}{k^2+k-90}$

$k^2 + 4k – 12 = (k + 6)(k - 2)$
$k^2 + k – 90 = (k + 10)(k - 9)$

LCD = (k + 6)(k - 2)(k + 10)(k - 9)

5. $\frac{8}{2d^2-11d-14}$; $\frac{10}{2d^2-d-21}$

$2d^2 – 11d + 14 = (2d - 7)(d - 2)$
$2d^2 – d – 21 = (2d - 7)(d + 3)$

LCD = (2d - 7)(d - 2)(d + 3)

Test 3

Find the LCD of the following rational expressions.

1. $\frac{6}{18}$; $\frac{1}{36}$

18 = 2 · 3 · 3
36 = 2 · 3 · 3 · 2

LCD = 2 · 3 · 3 · 2 = 36

2. $\frac{3}{4ab^2}$; $\frac{5}{6ab}$

$4ab^2 = 2 \cdot 2 \cdot a \cdot b \cdot b$
$6ab = 2 \cdot 3 \cdot a \cdot b$

LCD = $2 \cdot 3 \cdot 2 \cdot a \cdot b \cdot b = 12ab^2$

3. $\frac{1}{(4a-8b)}$; $\frac{3}{(3a-6b)}$

4a – 8b = 4(a – 2b)
3a – 6b = 3(a – 2b)

LCD = 12(a – 2b)

4. $\frac{b}{2b^2-b-3}$; $\frac{3b}{3b^2+5b+2}$

$2b^2 – b – 3 = (2b - 3)(b + 1)$
$3b^2 + 5b + 2 = (3b + 2)(b + 1)$

LCD = (2b - 3)(3b + 2)(b + 1)

5. $\frac{2x}{x^2-2x-8}$; $\frac{1}{x^2-6x+8}$

$x^2 – 2x – 8 = (x - 4)(x + 2)$
$x^2 – 6x + 8 = (x - 4)(x - 2)$

LCD = (x - 4)(x + 2)(x - 2)

Activity 4

A. Find the sum and express your answer in lowest terms.

1. $\frac{3}{s} + \frac{2}{s} = \frac{5}{s}$

2. $\frac{x}{p-5} + \frac{2x}{p-5} = \frac{3x}{p-5}$

3. $\frac{2n+3}{n^2-4n+4} + \frac{n-4}{n^2+n-6}$

$n^2 – 4n + 4 = (n - 2)(n - 2)$

$n^2 + n – 6 = (n + 3)(n - 2)$

LCD = (n - 2)(n - 2)(n + 3)

$$= \frac{(2n+3)(n+3)+(n-4)(n-2)}{(n-2)(n+3)(n-2)}$$

$$= \frac{2n^2 + 6n + 3n + 9 + n^2 - 2n - 4n + 8}{(n - 2)(n + 3)(n - 2)}$$

$$= \frac{3n^2 + 9n - 6n + 17}{(n - 2)(n + 3)(n - 2)}$$

$$= \frac{3n^2+3n+17}{(n-2)(n+3)(n-2)}$$

4. $\frac{3}{a^2-5a+6} + \frac{-2}{a^2-a-2}$

$a^2 - 5a + 6 = (a - 3)(a - 2)$
$a^2 - a - 2 = (a + 1)(a - 2)$

LCD = $(a - 2)(a - 3)(a + 1)$

$$= \frac{3(a+1)+(-2)(a-3)}{(a-2)(a-3)(a+1)}$$

$$= \frac{3a+3-2a+6}{(a-2)(a-3)(a+1)}$$

$$= \frac{a+9}{(a-2)(a-3)(a+1)}$$

5. $\frac{r+1}{r^2-3r-10} + \frac{r-1}{r^2+r-30}$

$r^2 - 3r - 10 = (r - 5)(r + 2)$
$r^2 + r - 30 = (r - 5)(r + 6)$

LCD = $(r - 5)(r + 2)(r + 6)$

$$= \frac{(r+1)(r+6)+(r-1)(r+2)}{(r-5)(r+2)(r+6)}$$

$$= \frac{r^2+6r+r+6+r^2+2r-r-2}{(r-5)(r+2)(r+6)}$$

$$= \frac{2r^2+8r+4}{(r-5)(r+2)(r+6)}$$

Test 4

Add and express your answer in lowest terms if necessary.

1. $\frac{3a}{9a-8} + \frac{5a}{9a-8} = \frac{8a}{9a-8}$

2. $\frac{2x}{2x^2+7x+3} + \frac{1}{2x^2+7x+3}$

$$= \frac{2x+1}{2x^2+7x+3} \text{ or } \frac{2x+1}{(2x+1)(x+3)}$$

3. $\frac{2x}{x+y} + \frac{3x}{2x+2y}$

$x + y = x + y$
$2x + 2y = 2(x + y)$

LCD = $2(x + y)$

$$= \frac{2x(2)}{2(x+y)} + \frac{3x}{2(x+y)} = \frac{4x+3x}{2(x+y)}$$

4. $\frac{5}{t-6} + \frac{4}{t+6}$

LCD = $(t - 6)(t + 6)$

$$= \frac{5(t+6)}{(t-6)(t+6)} + \frac{4(t-6)}{(t-6)(t+6)}$$

$$= \frac{5t+30+4t-24}{(t-6)(t+6)}$$

$$= \frac{9t+6}{(t-6)(t+6)}$$

5. $\frac{3a}{a^2+ab-2b^2} + \frac{4a-1}{a^2-b^2}$

$$= \frac{3a}{(a\ \ +2b)(a-b)} + \frac{4a-1}{(a-b)(a+b)}$$

$$= \frac{3a(a+b)}{(a+2b)(a+b)(a-b)} + \frac{(4a-1)(a+2b)}{(a+2b)(a+b)(a-b)}$$

$$= \frac{3a^2+3ab+4a^2+8ab-a-2b}{(a+2b)(a+b)(a-b)}$$

$$= \frac{7a^2+11ab-2b-a}{(a+2b)(a+b)(a-b)}$$

Activity 5:

1. Let x = the number to be added to both numerator and denominator .

Original fraction	New fraction
$\frac{5}{8}$	$\frac{5}{8}+\frac{x}{x}$

Equation:

$$\frac{5}{8}+\frac{x}{x} = \frac{3}{4}$$

LCD = 4(8 + x)

$$4(8 + x)\left(\frac{5}{8}+\frac{x}{x}\right) = 4(8 + x)\ (\frac{3}{4}$$

$$20 + 4x = 24 + 3x$$

$$4x - 3x = 24 - 20$$

$$x = 4$$

Check:

$$\frac{5}{8}+\frac{4}{4} = \frac{3}{4}$$

$$\frac{9}{12} = \frac{3}{4}$$

Therefore, the required number is 4.

2. Let x = number of hours for the three to work together.

$\frac{x}{3}$ = part of the job Margie can finish in 1 hour.

$\frac{x}{5}$ = part of the job Mae can finish in 1 hour.

$\frac{x}{8}$ = part of the job Mia can finish in 1 hour

Equation:

$$\frac{x}{3} + \frac{x}{5} + \frac{x}{8} = 1$$

LCD = 120

$$120(\frac{x}{3}) + 120(\frac{x}{5}) + 120(\frac{x}{8}) = 1(120)$$

$$40x + 24x + 15x = 120$$

$$79x = 120$$

$$x = 1\frac{41}{79}$$

Thus, Margie, Mae, Mia can finish it in $1\frac{41}{79}$ hours.

Test 5:

1. Let x = the number to be subtracted from both numerator and denominator.

Original fraction	New fraction
$\frac{7}{12}$	$\frac{7}{12}-\frac{x}{x}$

Equation:

$$\frac{7}{12}-\frac{x}{x} = \frac{1}{2}$$

LCD = 2(12 –x)

$$2(\ 12\text{-x})\left(\frac{7-x}{12-x}\right)= 2(\ 12\text{-x})\ \frac{1}{2}$$

$$2(7 - x) = 12 - x$$

$$14 - 2x = 12 - x$$

$$14 - 12 = 2x - x$$

$$2 = x \quad \text{or} \quad x = 2$$

Therefore, the required number is 2.

2. Let x = number of hours for both to work together.

$\frac{x}{4}$ = part of the job Ken can finish in 1 hour.

$\frac{x}{5}$ = part of the job Nikki can finish in 1 hour.

Equation:

$$\frac{x}{4} + \frac{x}{5} = 1$$

LCD = 20

$$20(\frac{x}{4}) + 20(\frac{x}{5}) = 1(20)$$

$$5x + 4x = 20$$

$$9x = 20$$

$$x = 2\frac{2}{9}$$

Thus, Ken and Nikki can finish it in $2\frac{2}{9}$ hrs

Activity 6::

Perform the indicated operation and express the answer in the lowest terms.

1. $\frac{1}{b^2-1}$ - $\frac{1}{b^2+3b+2}$

$$= \frac{1(b+2)}{(b+1)(b-1)(b+2)} -$$

$$\frac{1(b-1)}{(b+2)(b+1)(b-1)}$$

$$= \frac{(b+2)-(b-1)}{(b+1)(b-1)(b+2)}$$

$$= \frac{3}{(b+1)(b-1)(b+2)}$$

2. $\frac{8}{3-7y}$ - $\frac{2}{7y-3}$

$$= \frac{8}{-(3-7y)} - \frac{2}{7y-3}$$

$$= \frac{8-(-2)}{-(7y-3)}$$

$$= \frac{10}{-(7y-3)}$$

$$= \frac{10}{3-7y}$$

3. $\frac{2y+1}{9-y^2} + \frac{2}{y-3}$ - $\frac{1}{y+3}$

$$\frac{2y+1}{9-y^2} + \frac{2}{y-3} - \frac{1}{y+3}$$

$$= \frac{2y+1}{(3-y)(3+y)} + \frac{2}{y-3} - \frac{1}{y+3}$$

$$= \frac{2y+1}{-(y+3)(y-3)} + \frac{2}{y-3} - \frac{1}{y+3}$$

$$= \frac{2y+1}{-(y+3)(y-3)} + \frac{2}{y-3} \cdot \frac{y+3}{y+3}$$

$$- \frac{1}{y+3} \cdot \frac{y-3}{y-3}$$

$$= \frac{-(2y+1)+2(y+3)-1(y-3)}{(y+3)(y-3)}$$

$$= \frac{-2y-1+2y+6-y+3}{(y+3)(y-3)}$$

$$= \frac{8-y}{(y+3)(y-3)}$$

4. $\frac{-1}{m^2+mn-2n^2}$ - $\frac{3}{m^2-3mn+2n^2}$

$$= \frac{-1}{(m+2n)(m-n)} - \frac{3}{(m-2n)(m-n)}$$

$$= \frac{-1(m-2n)-3(m+2n)}{(m+2n)(m-n)(m-2n)}$$

$$= \frac{-m+2n-3m-6n}{(m+2n)(m-n)(m-2n)}$$

$$= \frac{-4m-4n}{(m+2n)(m-n)}$$

5. $\frac{4m-3n}{16m^2-n^2}$ - $\frac{4m-n}{16m^2+8mn+n^2}$

$$= \frac{4m-3n}{(4m+n)(4m-n)} - \frac{4m-n}{(4m+n)(4m+n)}$$

$$= \frac{(4m-3n)(4m+n)-(4m-n)(4m-n)}{(4m+n)(4m-n)(4m+n)}$$

$$= \frac{16m^2-8mn-3m^2-16m^2+8mn-n^2}{(4m+n)(4m-n)(4m+n)}$$

$$= \frac{-4n^2}{(4m+n)(4m-n)(4m+n)}$$

Activity B:

<u>B</u> <u>E</u> <u>H</u> <u>O</u> <u>N</u> <u>E</u> <u>S</u> <u>T</u>

Test 6:

1. $\frac{3}{p}$ - $\frac{2}{p}$ $= \frac{1}{p}$

2. $\frac{x}{x-5}$ - $\frac{2x}{x-5} = \frac{x-2x}{x-5} = \frac{-x}{x-5}$

3. $\frac{15}{4k^2}$ - $\frac{3}{k+2} = \frac{15(k+2)-3(4k^2)}{4k^2(k+2)}$

$$= \frac{15k+30-12k^2}{4k^2(k+2)}$$

4. $\frac{m}{m^2+3m-18}$ - $\frac{3}{m^2+3m-18}$

$$= \frac{m-3}{m^2+3m-18}$$

5. $\frac{x}{x^2+xy-2y^2}$ - $\frac{3x}{x^2-3xy+2y^2}$

$$= \frac{x}{(x+2y)(x-y)} - \frac{3x}{(x-2y)(x-y)}$$

$$= \frac{x(x-2y)-3x(x+2y)}{(x+2y)(x-y)(x-2y)}$$

$$= \frac{x^2-2xy-3x^2-6xy}{(x+2y)(x-y)(x-2y)}$$

$$= \frac{-2x^2-8xy}{(x+2y)(x-y)(x-2y)}$$

Activity 7

Multiply as indicated and express each product in simplest form.

1. $\frac{3}{8} \cdot \frac{12}{15} = \frac{36}{120} = \frac{3}{10}$

2. $\frac{2ab}{3c^2} \cdot \frac{c}{a^2} = \frac{2abc}{3c^2a^2} = \frac{2b}{3ca}$

3. $\frac{9-x^2}{x+3} \cdot \frac{x}{3-x}$

$= \frac{(3-x)(3+x) \cdot x}{(x+3)(3-x)}$

= x

4. $\frac{a^2+5a}{a^2-16} \cdot \frac{a^2-4a}{a^2-25}$

$= \frac{a(a+5) \cdot a(a-4)}{(a-4)(a+4)(a-5)(a+5)}$

$= \frac{a^2}{(a+4)(a-5)}$

$= \frac{a^2}{a^2-a-20}$

5. $\frac{6x-3y}{4x^2+4xy+y^2} \cdot \frac{2x+y}{4x^2-4xy+y^2}$

$= \frac{3(2x-y) \cdot 2x+y}{(2x+y)(2x+y)(2x-y)(2x-y)}$

$= \frac{3}{(2x+y)(2x-y)}$

$= \frac{3}{4x^2-y^2}$

Test 7
Multiply and express each product in simplest form.

1. $\frac{12}{7} \cdot \frac{21}{54} = \frac{2}{3}$

2. $\frac{4a^2}{3bc} \cdot \frac{8b^2c}{6a^2}$

$= \frac{2 \cdot 2 \cdot a \cdot a \cdot 2 \cdot 2 \cdot 2 \cdot b \cdot b \cdot c}{3 \cdot b \cdot c \cdot 3 \cdot 2 \cdot a \cdot a}$

$= \frac{16b}{9}$

3. $\frac{t^2+5t}{5+t} \cdot \frac{t+1}{t^2-25}$

$= \frac{t(t+5) \cdot (t+1)}{(5+t)(t-5)(t+5)}$

$= \frac{t(t+1)}{(t+5)(t-5)}$

$= \frac{t^2+1}{t^2-25}$

4. $\frac{x^2+8x+16}{x^2-9} \cdot \frac{x-3}{x+4}$

$= \frac{(x+4)(x+4)}{(x-3)(x+3)} \cdot \frac{(x-3)}{(x+4)}$

$= \frac{x+4}{x+3}$

5. $\frac{x^2-6x+5}{x-1} \cdot \frac{x-1}{x-5}$

$= \frac{(x-5)(x-1)}{x-5}$

= x – 1

Activity 8
A. State the multiplicative inverse or reciprocal of each expression below.

1. $\frac{3}{8}$ reciprocal is $\frac{8}{3}$

2. $\frac{1}{x-y}$ reciprocal is $x-y$

3. $\frac{7x}{9y}$ reciprocal is $\frac{9y}{7x}$

4. $\frac{x+y}{2}$ reciprocal is $\frac{2}{x+y}$

5. $\frac{18x}{7}$ reciprocal is $\frac{7}{18x}$

B. Perform the indicated division and write the answer in simplified form.

1. $\frac{21}{7} \div \frac{3}{14} = \frac{21}{7} \cdot \frac{14}{3} = 14$

2. $\frac{7}{a} \div \frac{14}{a^2} = \frac{7}{a} \cdot \frac{a^2}{14} = \frac{a}{2}$

3. $\frac{5}{m-3} \div \frac{10}{m-3} = \frac{5}{m-3} \cdot \frac{m-3}{10} = \frac{1}{2}$

4. $\frac{3x-21}{x^2-49} \div \frac{3x}{x^2+7x}$

$$= \frac{3x-21}{x^2-49} \cdot \frac{x^2+7x}{3x}$$

$$= \frac{3(x-7)}{(x+7)(x-7)} \cdot \frac{x(x+7)}{3x}$$

$$= \frac{3x}{3x} = 1$$

5. $\frac{\frac{x^2-y^2}{2}}{\frac{x-y}{4}} = \frac{x^2-y^2}{2} \cdot \frac{4}{x-y}$

$$= \frac{(x-y)(x+y)}{2} \cdot \frac{2 \cdot 2}{x-y}$$

$$= 2(x + y)$$

Test 8

Perform the indicated division and write the answer in simplest form.

1. $\frac{3d^2c}{a^4} \div \frac{-6dc}{a^5}$

$$= \frac{3d^2c}{a^4} \cdot \frac{a^5}{-6dc}$$

$$= \frac{3 \cdot d \cdot d \cdot c \cdot a^4 \cdot a}{3 \cdot 2 \cdot d \cdot c \cdot a^4}$$

$$= \frac{-dca}{2c}$$

2. $\frac{a^2-b^2}{x^2-y^2} \div \frac{a+b}{x+y}$

$$= \frac{a^2-b^2}{x^2-y^2} \cdot \frac{x+y}{a+b}$$

$$= \frac{(a-b)(a+b)}{(x-y)(x+y)} \cdot \frac{x+y}{a+b}$$

$$= \frac{a-b}{x-y}$$

3. $\frac{x-y}{x+y} \div \frac{5x^2-5y^2}{3x-3y}$

$$= \frac{x-y}{x+y} \cdot \frac{3x-3y}{5x^2-5y^2}$$

$$= \frac{x-y}{x+y} \cdot \frac{3(x-y)}{5(x^2-y^2)}$$

$$= \frac{3(x-y)(x-y)}{5(x^2-y^2)(x+y)}$$

$$= \frac{3(x-y)}{5(x+y)^2}$$

4. $\dfrac{5a^2 - 5ab}{ab + b^2} \div \dfrac{5a^2 - 5b^2}{b}$

$= \dfrac{5a^2 - 5ab}{ab + b^2} \cdot \dfrac{b}{5a^2 - 5b^2}$

$= \dfrac{5a(a - b)}{b(a + b)} \cdot \dfrac{b}{5(a^2 - b^2)}$

$= \dfrac{5a(a - b)(b)}{5b(a + b)(a - b)(a + b)}$

$= \dfrac{a}{(a + b)^2}$

5. $\dfrac{\dfrac{3x}{y}}{\dfrac{5y^2}{2x^2}} = \dfrac{3x}{y} \cdot \dfrac{2x^2}{5y^2} = \dfrac{6x^3}{5y^3}$

Chapter Test

1. b

2. a

3. a

4. c

5. .b

6. c

7. a

8. a

9. b

10. c

11. $\dfrac{7ab}{9c} \cdot \dfrac{81c^2}{91a^2b} = \dfrac{9c}{13a}$

12. $\dfrac{x^2 - y^2}{a^2 - b^2} \cdot \dfrac{a + b}{x - y}$

$= \dfrac{(x + y)(x - y) \cdot (a + b)}{(a + b)(a - b)(x - y)}$

$= \dfrac{x + y}{a - b}$

13. $\dfrac{y + 6}{2y} \cdot \dfrac{4y^2}{y + 6} =$ 2y

14. $\dfrac{a^2 - 9}{a^2} \cdot \dfrac{a^2 - 3a}{a^2 + a - 12}$

$= \dfrac{(a - 3)(a + 3)}{a} \cdot \dfrac{a(a - 3)}{(a + 4)(a - 3)}$

$= \dfrac{(a - 3)(a + 3)}{a(a + 4)}$

15. . $\dfrac{a - 1}{a + 2} \cdot \dfrac{a + 1}{a - 1}$

$= \dfrac{a + 1}{a + 2}$

16. $\dfrac{a^2 - ab}{3a} \div \dfrac{a - b}{15b^2}$

$= \dfrac{a^2 - ab}{3a} \cdot \dfrac{15b^2}{a - b}$

$= \dfrac{a(a - b)}{3a} \cdot \dfrac{15b^2}{a - b}$

= 5b^2

17. $\dfrac{x^2 - 2x + 1}{y - 5} \div \dfrac{x - 1}{y^2 - 25}$

$$= \frac{x^2 - 2x + 1}{y - 5} \cdot \frac{y^2 - 25}{x - 1}$$

$$= \frac{(x - 1)(x - 1)}{y - 5} \cdot \frac{(y - 5)(y + 5)}{x - 1}$$

= (x - 1)(y + 5)

18. $\frac{c^2 + 3c}{c^2 + 2c - 3} \div \frac{c}{c + 1}$

$$= \frac{c^2 + 3c}{c^2 + 2c - 3} \cdot \frac{c + 1}{c}$$

$$= \frac{c(c + 3)}{(c - 1)(c + 3)} \cdot \frac{c + 1}{c}$$

$$= \frac{c + 1}{c - 1}$$

19. . $\frac{a^2 - 49}{(a + 7)^2} \div \frac{3a - 21}{2a + 14}$

$$= \frac{a^2 - 49}{(a + 7)^2} \cdot \frac{2a + 14}{3a - 21}$$

$$= \frac{(a + 7)(a - 7)}{(a + 7)(a + 7)} \cdot \frac{2(a + 7)}{3(a - 7)}$$

$$= \frac{2(a + 7)}{3(a + 7)} = \frac{2}{3}$$

20. $\frac{a^2 - b^2}{x^2 - y^2} \div \frac{a - b}{x - y}$

$$= \frac{a^2 - b^2}{x^2 - y^2} \cdot \frac{x - y}{a - b}$$

$$= \frac{(a - b)(a + b)}{(x - y)(x + y)} \cdot \frac{x - y}{a - b}$$

$$= \frac{a + b}{x + y}$$

21. $\frac{8}{3 - 7y} - \frac{2}{7y - 3}$

$$= \frac{8}{-(7y - 3)} - \frac{2}{7y - 3}$$

$$= \frac{8 - (-2)}{-(7y - 3)}$$

$$= \frac{10}{-(7y - 3)}$$

22. $\frac{p + q}{3p^2 + 2pq - q^2} - \frac{q - p}{6p^2 - 5qp + q^2}$

$$= \frac{p + q}{(3p - q)(p + q)} - \frac{q - p}{(3p - q)(2p - q)}$$

$$= \frac{(p + q)(2p - q) - (q - p)(p + q)}{(3p - q)(p + q)(2p - q)}$$

$$= \frac{2p^2 + pq - q^2 - (q^2 - p^2)}{(3p - q)(p + q)(2p - q)}$$

$$= \frac{2p^2 + pq - q^2 - q^2 + p^2}{(3p - q)(p + q)(2p - q)}$$

$$= \frac{3p^2 + pq - 2q^2}{(3p - q)(p + q)(2p - q)}$$

23. $\frac{m - n}{m^2 + 2mn + n^2} - \frac{m + n}{m^2 - mn - 2n^2}$

$$= \frac{m - n}{(m + n)(m + n)} - \frac{m + n}{(m - 2n)(m + n)}$$

$$= \frac{(m - n)(m - 2n) - (m + n)(m + n)}{(m + n)(m + n)(m - 2n)}$$

$$= \frac{m^2 - 3mn + 2n^2 - (m^2 + 2mn + n^2)}{(m+n)(m+n)(m-2n)}$$

$$= \frac{m^2 - 3mn + 2n^2 - m^2 - 2mn - n^2}{(m+n)(m+n)(m-2n)}$$

$$= \frac{-5mn + n^2}{(m+n)(m+n)(m-2n)}$$

24. $\frac{1}{b^2 - 1}$ - $\frac{1}{b^2 + 3b + 2}$

$$= \frac{1}{(b+1)(b-1)} - \frac{1}{(b+2)(b-1)}$$

$$= \frac{(b+2) - (b-1)}{(b+1)(b-1)(b+2)}$$

$$= \frac{b + 2 - b + 1}{(b+1)(b-1)(b+2)}$$

$$= \frac{3}{(b+1)(b-1)(b+2)}$$

25. $\frac{2t-1}{t+1}$ - $\frac{2t+1}{t-1}$

$$= \frac{(2t-1)(t-1) - (2t+1)(t+1)}{(t+1)(t-1)}$$

$$= \frac{(2t^2 - 3t + 1) - (2t^2 + 3t + 1)}{(t+1)(t-1)}$$

$$= \frac{2t^2 - 3t + 1 - 2t^2 - 3t - 1}{(t+1)(t-1)}$$

$$= \frac{-6t}{(t+1)(t-1)}$$

Common Errors / Misconceptions in Unit IV

1. What is the sum of $\frac{5y}{10}$ and $\frac{15}{10}$?

Student's answer: $\frac{20y}{10}$ or 2y

This answer is wrong because 5y and 15 cannot be added since they are dissimilar fractions.

2. Find $\frac{11x}{15}$ - $\frac{7x}{12}$.

Student's answer: $\frac{4x}{3}$

This answer is wrong. Notice that their denominators are different. Thus, we find the LCD and the LCD is 60. Rewrite the rational expression as fraction with 60 as the denominator.

$$\frac{11}{15} - \frac{7}{12} = \frac{11(4)}{15(4)} - \frac{7(5)}{12(5)}$$

$$= \frac{44}{60} - \frac{35}{60}$$

$$= \frac{9}{60} \text{ or } \frac{3}{20}$$

3. Simplify the expression $\frac{(x+3)}{(x+3)+2}$.

Student's answer: $\frac{1}{3}$

This answer is wrong. You cannot cancel (x + 3) in the numerator and denominator, since the denominator is taken as a single expression.