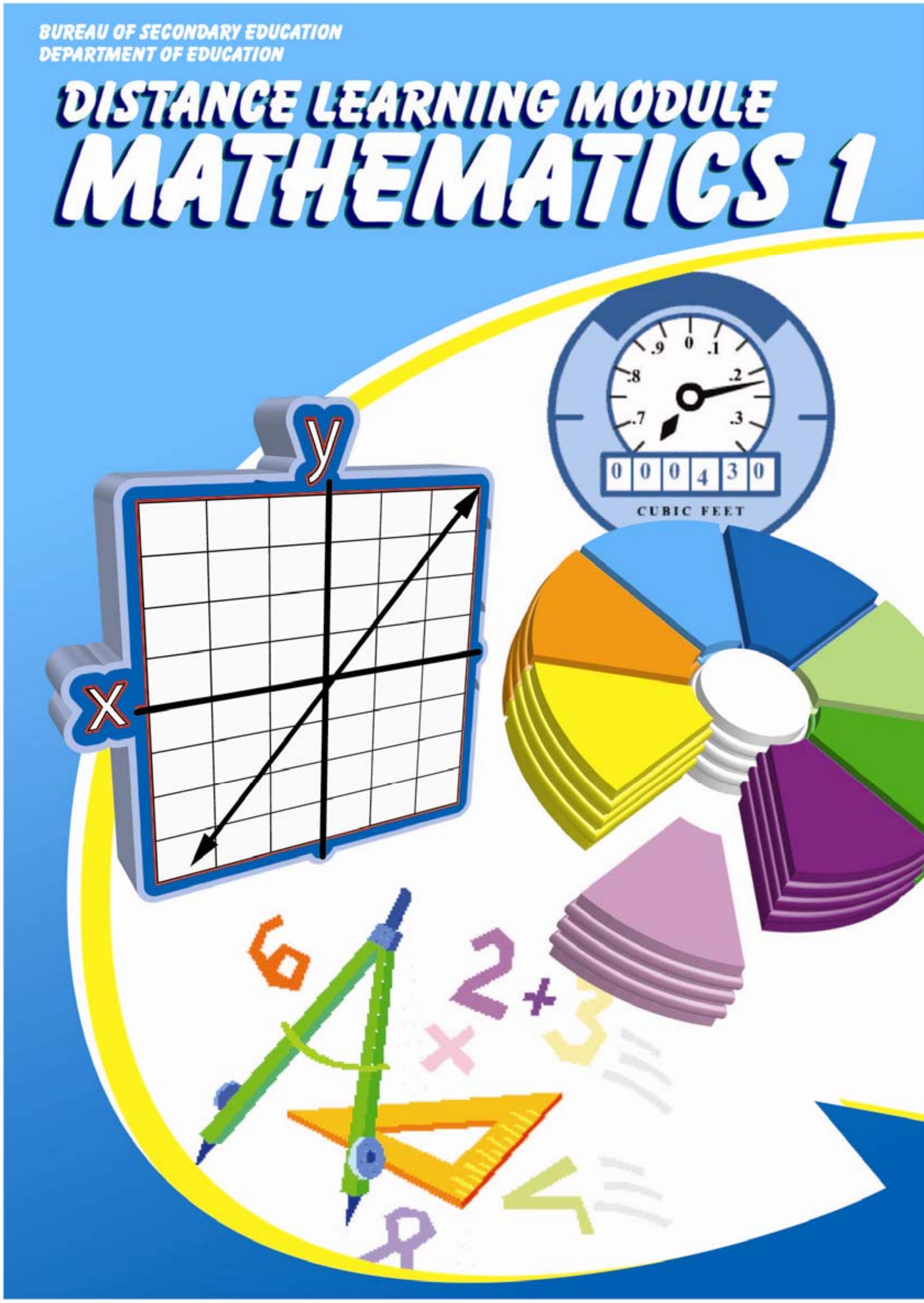

BUREAU OF SECONDARY EDUCATION
DEPARTMENT OF EDUCATION
DISTANCE LEARNING MODULE
MATHEMATICS 1
y
x
CUBIC FEET

ALGEBRAIC EXPRESSIONS

U N I T III

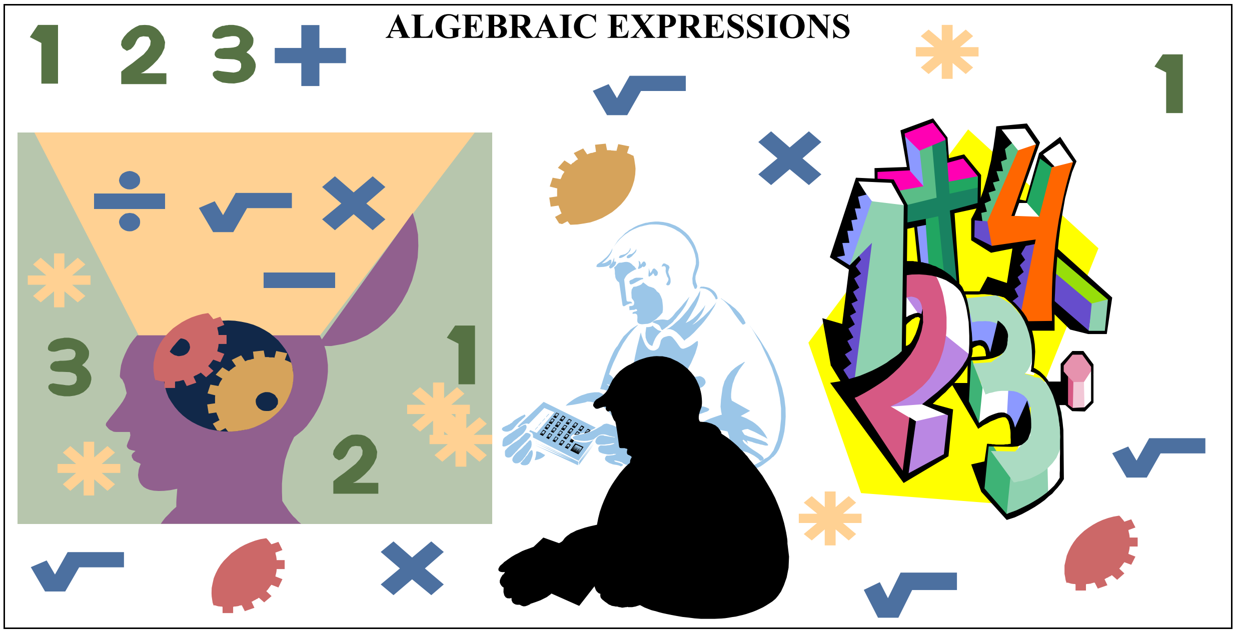
An aid for navigation and pilotage at sea is the lighthouse , a tower building or framework very familiar among the navigators, especially to those who sail by boat. This lighthouse serves as the compass for the navigators as it provides travelers and mariners information on the wind direction by displaying a light for their guidance. Lighthouses also provide coordinate location for small aircraft traveling at night.

Because of modern navigational aids, the number of operational lighthouses has declined to less than 1,500 worldwide. Lighthouses are used to mark dangerous coastlines, hazardous shoals away from the coast, and safe entries to harbors.

Just like the lighthouse, in this unit, you will be introduced to the world of symbols that are used to convey ideas in Algebra. The concept of variables and algebraic expressions are basic in the study of Algebra. The use of variables and symbols become more meaningful when word phrases or verbal phrases are translated to mathematical phrases . Identifying the key phrases in verbal phrases facilitates translating them to mathematical phrases . This process is a prerequisite in solving word problems.

Aside from translating verbal phrases to mathematical phrases, the constant use of mathematical symbols will also help you discover some patterns in the applications of algebraic concepts. Performing the operations on algebraic expressions , applying the laws of exponents , special products and factoring become more interesting when applied to real life experiences.

ALGEBRAIC EXPRESSIONS



Translation of Verbal Phrases to Mathematical Phrases and vice-versa

Julius helps his parents increase their family income by selling banana cue. To facilitate the computation of his sales, he prepared this table.

Number of Banana Cues Sold	Sales (in pesos)
1	10
2	20
3	30
4	40
6	____
7	____
.	____
.	
.	
y	____

How much will you pay for 5 pieces of banana cue? 6? 7? What expression will complete the table? Can you describe the pattern?

With y as the variable representing the number of banana cues sold at P10.00, 10y is the expression that will complete the table,

If y is the variable , 10y is called an algebraic expression.

An *algebraic expression* consists of one or more numbers and variables with one or more arithmetic operations. Following are examples of algebraic expressions

$y+2$, $\frac{b}{c}+1$, $8xy+2x$ and $a+3n$

In multiplication, the quantities being multiplied are called factors and the result is called product.

$3 \bullet n = 3n$ $\quad\quad$ $(3)(8) = 24$

Factors Product $\quad\quad$ Factors Product

A center dot or parenthesis is often used to indicate multiplication. When variables are used to represent the factors the multiplication sign is omitted as shown in the following examples:

$5x$, $2ab$, $4xyz$ and $6an$.

Fraction bars indicate division as in

$$\frac{18abc}{9ac}, \frac{36xyz}{4yz} \text{ and } \frac{12ab}{4ab}$$

To solve word problems in mathematics, you can create algebraic expressions by translating verbal phrases to mathematical phrases using symbols.

The chart below shows some of the words used to indicate the mathematical operations:

Operation	Mathematical phrases
1. Addition	-the sum or total of -plus -added to - more than -increased by
2. Subtraction	-decreased by - subtracted from -less than -minus -the difference of
3. Multiplication	-the product of - multiplied by -times
4. Division	-the quotient of -divided by - the ratio of

The following are examples of word phrases translated to mathematical phrases.

Word Phrases	Mathematical Phrases
1. x plus 2 2 is added to x x increased by 2	x+ 2
2. ten times y the product of 10 and y	10y
3. y minus 3 3 subtracted from y the difference of y and 3 y decreased by 3	y-3
4. n divided by 8 the quotient of n and 8	$\frac{n}{8}$
5. Eight divided by n	$\frac{8}{n}$
6. The product of x and y increased by 2	$xy+2$
7. The square of x less 8	x^2-8
8. Three times the quantity x squared plus 4	$3(x^2+4)$
9. Three times the quantity x minus 5	$3(x-5)$
10. Three-fourths of the quantity x plus 2	$\frac{3}{4}(x+2)$

Do the following exercises:

Translate the following verbal phrases into Mathematical phrases/Algebraic expression

Verbal Phrases	Algebraic Expression
1. 3 times y minus 1	
2. Three times x squared plus 4	
3. The product of the square of y and five	
4. The quotient of 2 and the product of x and y	
5. One-half times the quantity twice x minus y	
6. The sum of the squares of a and b	

Now, let us do the reverse operation, that is, changing mathematical phrases to verbal phrases or word phrases:

Translate the following mathematical phrases to verbal phrases

Mathematical Phrases	Word or Verbal Phrases
1. $5(z^3-9)$	Five times the quantity cubed of z minus 9
2. $2x^2+2(x-y)$	The sum of twice the square of x and twice the difference of x and y
3. k^3+12	k cubed plus twelve
4. $\frac{2(x+y)}{6(x-y)}$	The quotient of twice the sum of x and y and six times the difference of x and y
5. $\frac{a^3+b^3}{a^2-b^2}$	The sum of a cubed and b cubed divided by the difference of a squared and b squared
6. $5(x^2y-z^3)$	Five times the quantity x squared y minus z cubed.
7. $\frac{3}{4}x^5+2xy$	Three – fourths of x to the fifth power plus twice the product of x and y
8. $V=lw$	V is equal to the product of l and w
9. $I=prt$	I is equal to product of p, r and t
10. $P=2l+2w$	P is equal to the sum of twice *l* and twice w

Evaluating Algebraic Expressions

Is 3y +5 equal to 3 (y + 5)? Let us find out.

If you replace <u>y</u> by2, then:

$$3y+5=3(2)+5=6+5=11$$
$$3(y+5)=3(2+5)=3(7)=21$$

Hence, $3y+5\neq 3(y+5)$

In evaluating algebraic expressions, you simply substitute or replace variables by numbers and carry out the operations following the order of operations:

<u>Order of Operations</u>

1. Simplify the expressions inside the grouping symbols, such as braces, parenthesis or brackets and as indicated by fraction bars.
2. Evaluate all powers and extract roots
3. Do all the multiplications and divisions from left to right
4. Do all addition and subtraction from left to right.

Examples:

1. Find the value of $2x-4$ for x= 3.

Solutions:

$2x-4=2(3)-4$ Substitute 3 for x

$=6-4$ Multiply 2 and 3

$=$ **2** Subtract 4 from 6

: . $2x-4=2$ (for x= 3)

2. Evaluate 2(y-4) +3 for y= -2

Solutions:

2 (y-4) +3 = 2(-2-4) +3 Substitute 2 for y

= 2(-6)+3 Add -2 and -4

= -12 +3 Multiply -6 and 2

= -9 Add the integers -12 and 3

: . 2(y-4)+3=-9 (for y=-2)

3. Evaluate $xy-\frac{x}{y}$ for x= 6 ; y = -12

Solutions:

$xy-\frac{x}{y}$ = 7(-12) - $\frac{-12}{6}$ Substitute x for 6 and y for -12

= -84 – (-12) Multiply 7 and -12 Divide -12 by 6

= -84 +2 Multiplication of integers

= **-82** Addition of integers

: . $xy-\frac{x}{y}$ **= -82 (** for x = 6 and y = -12)

4. $3\{ 4x +[-3x- (2x+6)]\}$ for $x = 2$

Solutions:

$3\{4x +(-3x – [2x +6])\}=$

$= 3\{ 4(2) + (-3(2) – [2(2) +6])\}$ Substitute $x = 2$

$= 3\{8 + (-6– [4 + 6])\}$ Multiply 2 by 2

$= 3\{8 + (-6 - [10])\}$ Add 4 and 6

$= 3\{8 +(-6 -10)\}$ Remove the[] grouping sign

$= 3\{8 + (-16)\}$ Add -3 and -10

$= 3\{ 8 + -16\}$ Remove the ( ) grouping sign

$= 3 \{ -8\}$ Add algebraically 8 and -13

$= -24$ Multiply 3 and -5

$\therefore 3$ **$\{4x +(-3x –[2x +6])\} = -24$** for $x = 2$

Another solution:

$3\{4x +(-3x – [2x +6])\}=$

$=3\{4x + (-3x – 2x -6)\}$ Remove the grouping signs[ ] by distributive property and since it is preceded by negative sign, change the signs of each term

$= 3\{4x + (-5x -6)$ Combine similar terms

$= 3\{ 4x + -5x -6\}$ Remove the grouping sign ( ) by distributive property and since it is preceded by plus sign retain the signs of each term

$= 3\{ -x -6)$ Combine similar terms

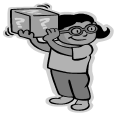
$= -3x -18$ Remove the grouping signs[ ] by distributive property

$= -3(2) – 18$ Replace x by 2

$= -6 -18$ Multiply -3 and 2

$= -24$ Subtract algebraically -6 and 18

$\therefore 3\{4x+(-3x-[2x+6])\}=-24 \; for \;\; x=2$



ACTIVITY I

A. Translate the following phrases into algebraic expression:

1.	The sum of 6 and x .	
2.	A number decreased by 8.	
3.	One-third of a number increased by 4.	
4.	The product of x and 4 decreased by 2	
5.	Five less than one-half of a number.	
6.	Three plus the product of a number and thirteen	
7.	Two –fifths times the quantity C plus four .	
8.	The product of negative two times a certain number a and the quantity a plus nine.	
9.	The difference between 5 times a number and eight	
10.	Three plus the product of a number and 5	

B. Translate the following algebraic expressions to their equivalent word expressions

1. $9x-4 =$ ____

2. $\frac{-3}{4}x-4=$ ____

3. $\frac{3}{5}\left(x^2+2\right)=$ ____

4. $4a^3-7=$ ____

5. $z^2\left(z-\frac{1}{4}\right)=$ ____

C. Complete each statement with a variable expression.

1. Lita is 2 cm taller than Cecile. If Cecile's height is P cm, then Lita's height is _____ cm.

2. Josie has twice as much money as Carlos. If Carlos has W pesos, then Josie has _____ pesos.

3. The sum of two numbers is 12. If one number is Q, then the other number is _______

4. Marie is 2 years older than Vicky. If Vicky is G years old now, then Marie is _______ years old.

5. The product of two numbers is 15. If one number is y, then the other number is ______.

D. Find the value of the following expressions:

1. $3x+2y-1$ *for* $x=2$; $y=-2$
2. $\{4x-2(x+4)\}$ *for* $x=1$
3. $[10z+4\{2z+6y\}-1]$ *for* $z=1$; $y=2$
4. $\dfrac{x+y}{10}$ *for* $x=2$; $y=8$
5. $(12x-y)-(5x+3y)$ *for* $x=-1; y=-1$

E. Solve the following problems:

1. Myrna jogs on weekends. The sum of the distance she covered for one month is (24x – 2)km. What is the actual distance covered if x = 6.

2. A gardener uses water sprinkler to water the plants as well as the garden lawns. The area covered by the sprinkler defined by A = πr^2 *with* π = 3.14, r = 1.5 meters. What is the actual area watered by the sprinkler?

3. Mrs. Campos ,a Health teacher, measured the weight of her 4 elementary students. She found out that Marie is heavier than Cecile by 1 kg. while Cora is twice as heavy as Marie . Tessie and Cecile have the same weight denoted by *x*. What must be the actual weight of Marie and Cora if *x = 20 kg.* ? What must be the sum of the weights of the four students?

TEST 1

A. Fill in the blanks: (2 points each)

1. An algebraic expression for "10 less than d" is ____________.
2. The value of the expression $4m^2 - n(s-m)$ if n = 3 , s=5 and m = -2 is ______.
3. The expression "$\frac{n}{4}+6$" can be written as ________.
4. Lilia has two more brothers than sisters. If Lilia has Q brothers, then she has ______ sisters.
5. The dimensions of a rectangular window are 3x meters long and 2x meters wide. If George needs to enclose the window with moldings, how many meters of molding would he need to enclose it? The expression that represents the answer is ______.

B. Write the mathematical phrase for each word phrase.

1. Two more than twice a number s.

2. The number of centimeters in g meters.(100 cm = 1 meter)
3. The quotient when a number *r* is divided by five.
4. A number D increased by four.
5. The difference when 10 is subtracted from the sum of *q and r.*

C. Evaluate the following expressions if *x = 10 ; y = 20 and z=30*

1. 2x - 3y + z
2. 123 + y - 12 x
3. 3xy + 2z
4. z + 12 + 18y
5. y – 83 + x + y

Scoring:

- After answering the test, check your answers with those on the answer key page
- If your score is 4 or 5, you may proceed to the next lesson; otherwise, read the lesson once more and do the test again.
- If you did not make it for the second time, consult your teacher.

Laws on Integer Exponents

An expression of the form x^n is a power. The base is x and the exponent is n.

Consider the table below which shows the powers of 2.

2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}
2	4	8	16	32	64	128	256	215	1024

Observe the pattern in the products below

$$4 \cdot 32 = 128 \qquad 8 \cdot 128 = 1024$$
$$\downarrow \ \downarrow \quad \downarrow \qquad \downarrow \ \downarrow \quad \downarrow$$
$$2^2 \cdot 2^5 = 2^7 \qquad 2^3 \cdot 2^7 = 2^{10}$$

How do we get the exponent of the product, in the preceding examples?

Do you see that the exponents of the factors with the same base are added to get the exponent of the product? Observe that :

2+5 = 7 and 3 + 7 = 10

Product of Powers

For any number a and all integers m and n , $a^m \cdot a^n = a^{m+n}$

Examples:

1) $2^3 \cdot 2^2 = 2^{3+2} = 2^5$
2) $2^3 \cdot 2^7 = 2^{3+7} = 2^{10}$
3) $x^4 \cdot x = x^{4+1} = x^5$
4) $b^3 \cdot b^7 = b^{3+7} = b^{10}$
5) $\left(4x^2yz^2\right)\left(2x^3y^4z\right)\left(3xy^2z^3\right)$

Applying the commutative and associative properties repeatedly, we have

$$= (4 \cdot 2 \cdot 3)\left(x^2x^3x\right)\left(y \cdot y^4 \cdot y^3\right)\left(z^2z \cdot z^3\right)$$
$$= 24x^{2+3+1} \cdot y^{1+4+3} \cdot z^{2+1+3}$$
$$= 24x^6y^8z^6$$

Consider the following examples

$$\left(2^3\right)^2 = \left(2^3\right)\left(2^3\right) = 2^{3+3} = 2^6$$
$$\left(x^6\right)^3 = x^6 \cdot x^6x^6 = x^{6+6+6} = x^{18}$$

Based on the above examples, we can say that to find the power of a power, we simply multiply the exponents.

Power of a Power

For any number a and all integers m and n

$$(a^m)^n = a^{mn}$$

Another case for integral exponents which we will encounter in our study is illustrated by the following examples:

1. $(xyz)^3 = (xyz)(xyz)(xyz)$
$$= (x \cdot x \cdot .x)(y \cdot y \cdot y)(z \cdot z \cdot z) = x^3y^3z^3$$

2. $\left(2ab^3c\right)^2 = \left(2ab^3c\right)^2$
$$= (2 \cdot 2)(a \cdot a)\left(b^3 \cdot b^3\right)(c \cdot c)$$
$$= 4a^2b^6c^2$$

The above examples suggest that the power of a product is the product of the powers.

Below are some extensions to the integral powers

Power of a Product

For all numbers a and b and any integers m ,

$$(ab)^m = a^m \cdot b^m$$

Power of a Monomial

For all numbers a and b and any integers m, n and p

$$\left(a^mb^n\right)^p = a^{mp}b^{np}$$

Now, consider the following quotients. Each number can be expressed as power of a number:

Powers of 2

1. $\frac{32}{4} = 8$ $\downarrow \ \downarrow$ $\frac{2^5}{2^2} = 2^{5-2}$ $= 2^3$

2. $\frac{256}{32} = 27$ $\downarrow \ \downarrow$ $\frac{2^8}{2^5} = 2^{8-5}$ $= 2^3$

Powers of 3

1.

$$\frac{81}{9} = 9$$

$\downarrow \quad \downarrow$

$$\frac{3^4}{3^2} = 3^2$$

$$3^{4-2} = 3^2$$

$$3^2 = 3^2$$

2.

$$\frac{243}{9} = 27$$

$\downarrow \quad \downarrow$

2. $$\frac{3^5}{3^2} = 3^3$$

$$3^{5-2} = 3^3$$

$$3^3 = 3^3$$

What will you do with the exponents of the dividend and the divisor to get the exponent of the quotient?

To get the exponent of the quotient you simply subtract the exponent of the divisor from the exponent of the dividend,

1. a) $\frac{2^5}{2^2} = 2^3$ b) $\frac{2^5}{2^2} = 2^3$

2. a) $\frac{3^4}{3^2} = 3^{4-2} = 3^2$ b) $\frac{3^5}{3^2} = 3^{5-2} = 3^3$

Quotient of Powers

For all integers m and n, and any non-zero number a, $\frac{a^m}{a^n} = a^{m-n}$

Zero Exponent

For any non-zero number a, $a^0 = 1$

Consider the following fractions:

$$\frac{2}{2} = \frac{3}{3} = \frac{7}{7} = \frac{d}{d} = \frac{h^2}{h^2} = \frac{y^a}{y^a} = 1$$

What do you observe about the numerators and the denominators of the fractions?

Applying the quotient of powers what will happen to the exponents of each fractions?

What conclusion can you give about the fractions with the same numerator and denominator? What about if the expression has a zero exponent?

When the numerator and denominator of a fraction are equal the value of the fraction is always equal to 1 and any expressions whose exponent is equal to zero then it is equal to 1.

Hence , $a^0 = 1$ *for any* $a \neq 0$.

Next, consider the example below. Simplify

$$\frac{10^7}{10^9} = 10^{7-9}$$

$$10^{-2} = \frac{1}{10^2}$$

This example illustrates the law for negative exponents.

Negative Exponent

For any non-zero number a and any integer n,

$$a^{-n} = \frac{1}{n}$$

An exponent is said to be simplified if it does not contain a zero exponent or a negative exponent

In simplifying an algebraic expression, you write an equivalent expression that has positive exponent and no powers of powers, each base should appear only once and all fractions should be expressed in simplest form

Example: Simplify

1) $$\frac{16b^4c}{-4bc^3} = \left(\frac{16}{-4}\right)\left(\frac{b^4}{b}\right)\left(\frac{c}{c^3}\right)$$

$$= -4b^{4-1}c^{1-3}$$

$$= -4b^3c^{-2}$$

$$= \frac{-4b^3}{c^2}$$

2) $$\frac{22a^2b^5c^7}{11abc^2} = \left(\frac{22}{11}\right)\left(\frac{a^2}{a}\right)\left(\frac{b^5}{b}\right)\left(\frac{c^7}{c^2}\right)$$

$$= 2a^{2-1}b^{5-1}c^{7-2}$$

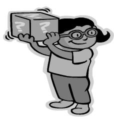
$$= 2ab^4c^5$$

3) $$\frac{-125x^6y^4z}{-25x^3y^6} = \left(\frac{-125}{-25}\right)\left(\frac{x^6}{x^3}\right)\left(\frac{y^4}{y^6}\right)z$$

$$= 5x^{6-3}y^{4-6}z$$

$$= 5x^3y^{-2}z$$

$$= \frac{5x^3z}{y^2}$$



ACTIVITY 2

I Solve the problems and write the letter above each matching answer to decode the message.

A. $6^2 =$ ______ J. $10^2 =$ ______ S. $7^1 =$ ______

B. $12^2 =$ ______ K. $5^1 =$ ______ T. $4^3 =$ ______

C. $5^3 =$ ______ L. $2^2 =$ ______ U. $1^7 =$ ______

D. $10^3 =$ ______ M. $11^1 =$ ______ V. $9^2 =$ ______

E. $10^1 =$ ______ N. $3^2 =$ ______ W. $3^3 =$ ______

F. $2^5 =$ ______ O. $0^3 =$ ______ X. $5^2 =$ ______

G. $4^2 =$ ______ P. $6^3 =$ ______ Y. $11^2 =$ ______

H. $7^2 =$ ______ Q. $3^5 =$ ______ Z. $3^1 =$ ______

I. $20^2 =$ ______ R. $2^3 =$ ______

6 #

< =

¼ ±

______ ______ ______ ______ ______ ______ ______ ______ ______ ______ ______ ______ ______ ______
(2)(2) (9)(4) (9)(3) (8 – 1) (1-1) (8)(4) (40)(10) ((8 +1) (16)(4) (2)(5) (8)(2) (4)(2) (3)(12) (4)(1)

______ ______ ______ ______ ______ ______ ______ ______ ______
(2)(5) (5)(5) (36)(6) (7 -7) (5 +4) (7+3) (6 +3) (4)(4)(4) (3+3+1)

II Classify each statement as true or false. If it is false, modify the right side of the equality to obtain a true statement.

1. $3^4 \cdot 3^2 = 3^8$
2. $2^5 \cdot 2^2 = 4^7$
3. $\left(x^{-3}\right)^2 = x^{-6}$
4. $\left(2^0\right)^3 = 2^3$
5. $\dfrac{6^2}{6^2} = 1$
6. $a^2 + 2a^2 = 3a^2$
7. $\left(a^2 b\right)^3 = a^6 b^3$
8. $a^2 + a^2 = 2a^4$
9. $\dfrac{b^{-5}}{b^3} = b^{-2}$
10. $\left(\dfrac{2}{3}\right)^4 = \dfrac{2^4}{3^4}$

III. Simplify the following :

1. $(a^5)(a)(a^7)$

2. $(5a^2)^3$

3, $(3y^3z)(7y^4)$

4. $-3(ax^3y)^2$

5. $\left(\frac{-1}{8}a\right)\left(\frac{-1}{6}\right)(b)(48c)$

6. $(-27ay^3)\left(\frac{-1}{3}ay^3\right)$

7. $\frac{7x^3z^5}{4z^{15}}$

8. $\left(\frac{r^{-4}k^5}{5k^2}\right)^2$

9. $\left(\frac{7m^{-1}n^3}{n^3r^{-2}}\right)^{-1}$

10. $\frac{30x^4y^7}{-6x^{-12}y^2}$

11. $y^2 \cdot y^b$

12. $x^{2a}x^{3a}x^{5a}$

13. $(3^{2x+6})(3^{3x-4})$

IV. Answer each question. You may replace the variables with a number and evaluate the results.

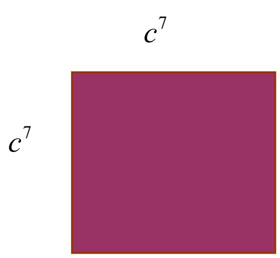
1. For all numbers, is $\left(\frac{a}{b}\right)^m = \left(\frac{a^m}{b^m}\right)$ a true statement?

2. Is $(a+b)^m = a^m + b^m$ a true statement for all numbers a and b and any integer m ?

3. In the quotient of powers property, $\frac{a^m}{a^n}$, why must a be non –zero?

V. Find the measure of the area of each rectangle and the measure of the volume of each rectangular solid.

1.



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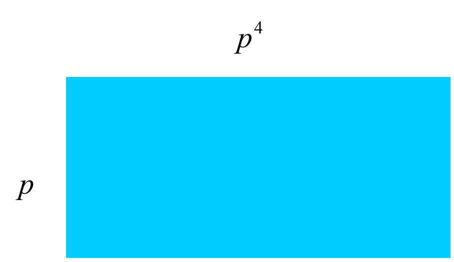

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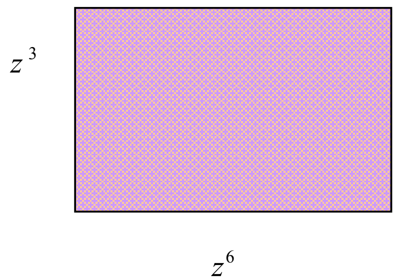

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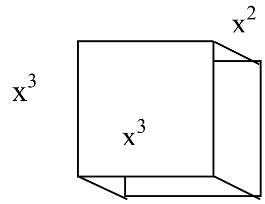

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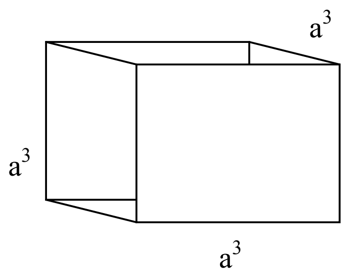


VI. Write each expression in simplified form.

1. $\left(-2a^2b\right)^4$
2. $\left(-2x^3y\right)^2\left(-3x^2y^2\right)^3$
3. $\left(3ab^4\right)\left(-2a^3b^2\right)$
4. $\dfrac{\left(x^2y\right)^4}{(xy)^2}$
5. $\dfrac{(x-2y)^6}{(x-2y)^2}$
6. $\left(\dfrac{x^{-1}}{y^3}\right)^{-4}$
7. $\left(2x^3y^2\right)^0$
8. $\left(\dfrac{a^{-1}b^2}{c^{-4}d^0}\right)^2$

VII. Answer the following :

1. Is $x^5 \cdot x^3 = x^{15}$? Why?
2. Simplify $\left(a^{10}\right)^3$. What property did you apply ?
3. If you were offered two jobs, one that pays 3x pesos and the other pays 3^x pesos for x hours of work, which pay scale would you choose ? Why?

4. After working 8 hours, how much more could you get for working 3 hours if your hourly rate is 3^x pesos ? (x = number of hours overtime)

TEST 2

Answer the following:

1. Is $\frac{1}{2^{-3}} = -2^3$? Yes or No? Why?

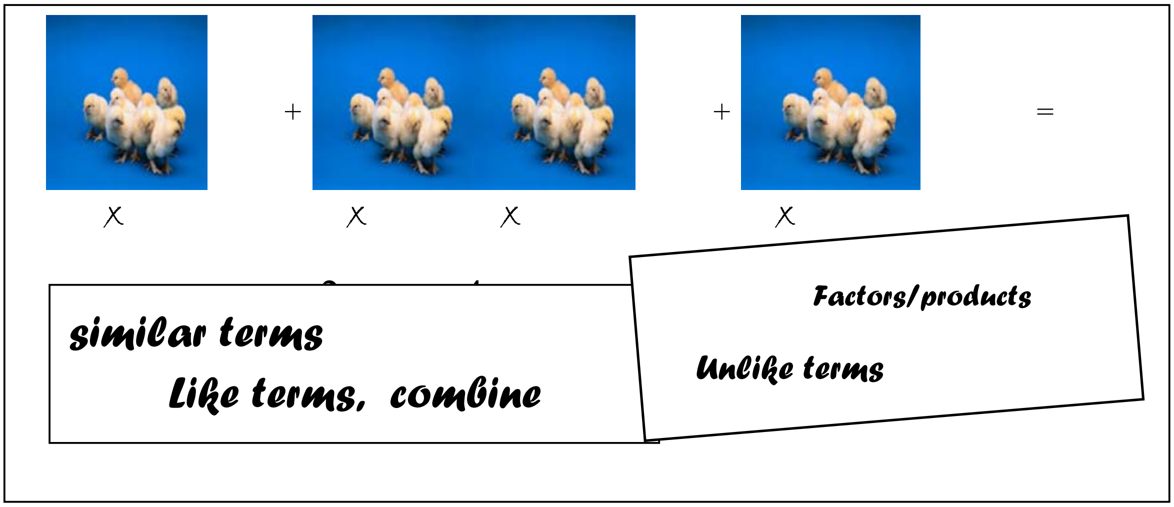
2. Evaluate each expression below when a = 1 and b=2
   a) $(a^2)^3$
   b) $(a^2b^2)^5$
   c) $b^3 \cdot b^4$
   d.) $(b^2 \cdot b^3)(b^2)^5$
   e) $(a^3 \cdot b)^3$

3. Simplify:
   a) $(-3)^2(-2)^3$
   b) $\frac{5x^0y^{-2}}{x^{-1}y^{-2}}$
   c) $\left(\frac{3a^2}{b^3}\right)^2\left(\frac{-2a}{3b}\right)^2$
   d) $\left[(-7)^2(-3)^2\right]^{-1}$

Scoring:

- After answering the test, check your answer with those on the answer key page
- If your score is 7 or higher , you may proceed to the next lesson, otherwise, read the lesson once again and do the test again
- If you did not make it for the second time, consult your teacher.

OPERATIONS WITH ALGEBRAIC EXPRESSIONS



Simplifying Algebraic Expressions

An *algebraic expression* is a collection of numbers, variables, operations and grouping symbols.

Expressions separated by + or - are called terms. For example the expression $9x^4 - x^3 + 5x^2 + 3x + 1$ has five terms. In the expression $9x^4$, 9 is called the numerical coefficient and the x is called the literal coefficient. If the terms have the same literal coefficient and the same exponents then they are called *like terms* or *similar terms.* For example, 2xy, 4xy and xy are called like or similar terms , while $2x^2$ *and* y^2 are *unlike terms* or *dissimilar terms*.

In simplifying algebraic expressions , we reduce algebraic expressions to equivalent expressions that does not contain terms or factors with zero or negative exponent .

To simplify expressions by addition or by subtraction, start with removing grouping symbols the innermost symbol first and work from inside out. Combine similar terms as they appear.

Examples: Simplify the following:

1. $6x + 4y + 5x + 3y$
   $= 6x + 5x + 4y + 3y$ by CPA
   $= (6x+5x)+(4y+3y)$ Combining like terms
   $= 11x + 7y$

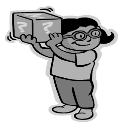
2. $3x - 4y - 5x - 8y$
   $= 3x - 5x - 4y - 8y$
   $= (3x - 5x) + (-4y - 8y)$
   $= -2x - 12y$

3. $9k - 4 - 3(2 - 5k)$
   $= 9k - 4 - 6 + 15k$
   $= 9k + 15k - 4 - 6$
   $= 24k - 10$

4. $3x - 5 - (2x - 4)$
   $= 3x - 5 - (2x - 4)$
   $= 3x - 5 - 2x + 4$
   $= 3x - 2x - 5 + 4$
   $= x - 1$

5. $\{3ab-[2ab-2+(6-3ab)]\}-(ab+1)$
$=\{3ab-[2ab-2+(6-3ab)]\}-(ab+1)$
$=\{3ab-[2ab-2+6-3ab]\}-ab-1$
$=\{3ab-2ab+2-6+3ab\}-ab-1$
$=\{4ab-4\}-ab-1$
$=4ab-4-ab-1$
$=3ab-5$

6. $4--[3(3x-3)-6]+5(4y-2)$
$=4-[9x-9-6]+20y-10$
$=4-[9x-15]+20y-10$
$=4-9x+15+20y-10$
$=4-9x+20y+5$
$=-9x+20y+9$



ACTIVITY 3

A. 1. Give the numerical coefficients of each term:

a. $3d^2$
b. $\frac{2}{3}x^5$
c. qt^6
d. $-25mn$
e. $-.28kl^2$

2. Tell whether the following group of terms are like or unlike

a. 4r, 2r, -5r
b. $t,-t,\frac{2}{5}t$
c. $4h^5,-4h^{-5},-5h^{-4}$
d. $3c^3,4c^{-3},c$
e. $3x^2y,-5x^2y,9x^2y$

B. Simplify the following algebraic expressions:

1. $4x^2-5x+6x^2-2x$
2. $-5(6y^3-4y^2+y-3)$
3. $2(3u-4c)-(5u-3c)$
4. $4(-t^2+3st-2s^2)-6(7t^2-st-s^2)$
5. $-(x-10)-4[2x-3(6+2x)-5]$
6. $3(3h+3e)-2[5(2h+e)+8]$
7. $2x+[5y+3(2x-2y)-(3y-x)]+4y$
8. $5q^3 2q\{q^2-3[2q-5(q+4)+3q]-q^3\}$
9. $3(2a+5b)-6[4(3a+b)+6]$
10. $7x-\{3y-[9x-(2y-3x)]\}-(3x+6y)$

C. 1. Complete the following table by evaluating the expression $x+1$ for the given values of x

x	0	1	2	3
$x+1$				

Based on your results, what can you say about the value of x + 1 as the value of x increases

2. Repeat the process in # 1 for the expressions $a.(x+2)$, b. $x+3$. Describe the results.

3. Repeat the process in # 1 for the expressions $x-1$ and $x-2$. Write a brief description of the results obtained.

Test 3

Simplify the following: (2 points each)

1. $3x-2(2x+3y)-(2x-3y)$
2. $-(-4d+5e)-[-2e+(4d-3e)]-5(e+d)$
3. $7x-\{3y+[8x-(2y+4x)]\}-(4x+3)$
4. $(5w-2x)-\{(4x-3w)-[10w-(3x+w)+5x]\}-\{-[(18w+4w)-(2w+3x)]\}$
5. $3c-\{2a-[(3b+c)-4(-2c+a)]-[-(a+b)-(2a-3c)]\}-4[(5a-3c)-4b]$

Scoring:

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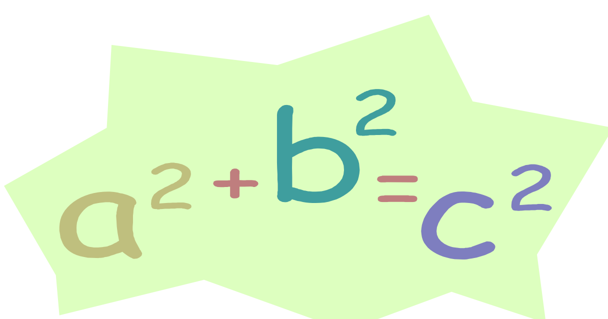


Polynomials

A special type of algebraic expressions is the *polynomial*.

Definition:

A *polynomial* is an algebraic expression whose variables have exponents that are non-negative integers. $2x^3 + 3x^2 - 5x + 12x + 1$ is an example of a polynomial, notice the exponent of each term are non-negative or positive also with the exponent of the constant which is equal to zero it is non- negative.

However, in the expression $\frac{5}{x^5} + 2$, we can say that this is not a polynomial since x^5 is in the denominator which denotes that the exponent is negative applying the previous concept on negative exponent.

Examples: Tell whether the expression is a polynomial or not, if not why

1. $8x^3 + 4xy$ a polynomial
2. 3x a polynomial
3. $\frac{2}{x^2} + 5$ not a polynomial since $\frac{2}{x^2}$ denotes negative exponent
4. $4x^{\frac{1}{2}} + 2$ not polynomial since the exponent is not an integer but a fraction

Kinds of Polynomials

1. *Monomial*- a polynomial with one term, a product of a number and variables
   Examples: 2x , 4xy and $3x^3y^2$
2. *Binomial* - a polynomial of two terms
   Examples: x+3y , 2x-4yz and $2x^2+1$
3. *Trinomial* – a polynomial of three terms
   Example: $3x^2-2x-1$

The *degree of the polynomial* refers to the greatest exponent or sum of the exponents of the variables in each term.

Examples:

Monomial	Degree
$5x^2$	2
$4ab^3c^4$	1+3+4 = 8
8	0

The degree of a polynomial, is the same as the highest/greatest degree of any of its terms.

Examples:

a) $3x^4 - 2x^3 + 2x^2 + x - 7$

How many terms are there in this polynomial? What are these terms? What is the degree of each term?

Notice that the terms are $3x^4, -2x^3$, $2x^2$, x *and* 7. with corresponding degrees are 4,3,2, and 0, respectively . The greatest degree is 4 . Hence the degree of the polynomial is 4.

b) $6x^3y + 3x^2y^2z^2 - 2x^2yz^2$

This polynomial has 3 terms. Their degrees are 4,6 and 5 respectively. Therefore, the degree of the polynomial is 6.

The terms of the polynomial are usually arranged so that the powers of the variable are either arranged in ascending or descending order.

Examples of polynomials arranged in ascending order are as follows:

1) $7 - x + 2x^2 - 2x^3 + 3x^4$
2) $1 + x^2 + x^3 + x^4 - 2x^5$

Examples of polynomials arranged in descending order are shown below.

1) $3x^4 - 2x^3 + 2x^2 - x + 7$
2) $-2x^5 + x^4 + x^3 + x^2 + 1$

Let us try these exercises.

A. Find the degree of each polynomial below

1. $5x^2 - 2x^5$
2. $7x^3 + 4xy + 3xz^3$
3. $13s^2t^2 - 4st + 5s^3t^3$
4. $32xyz - 11x^2y + 17xz^2$
5. $4x^3y + 3xy^4 - x^2y^4 - x^2y^3 + y^4$

B. Arrange the terms of each polynomial so that the powers of x are in descending order.

1. $-6x + x^5 + 4x^3 - 10x^4 - 20$
2. $\frac{3}{4}x^3y + 3xy^4 - x^2y^3 + y^4$

Addition and Subtraction of Polynomials

We can perform operations in polynomial, just like the operations with numerals. However, we should always remember that we can only add or subtract similar terms. Otherwise, we simply copy them.

Polynomials can be added in different ways: we can use the associative and the commutative properties of addition or we can use the vertical or column format where similar terms are aligned and their coefficients are added or subtracted as the case maybe.

Let us try the following examples:

1. Add $(4x^3 - 10x^2 + 5x + 8)\, and\, (12x^2 - 9x - 1)$

Method 1: Combine like terms (applying the Associative and the Commutative Properties of Addition)

$$(4x^3 - 10x^2 + 5x + 8) + (12x^2 - 9x - 1)$$
$$= 4x^3 + (12x^2 - 10x^2) + (5x - 9x) + (8 - 1)$$
$$= 4x^3 + 2x^2 - 4x + 7$$

Or Method 2: Using the vertical or column format

$$(4x^3 - 10x^2 + 5x + 8) + (12x^2 - 9x - 1)$$

$$\begin{array}{r} 4x^3 - 10x^2 + 5x + 8 \\ +\quad 12x^2 - 9x - 1 \\ \hline 4x^3 + 2x^2 - 4x + 7 \end{array}$$

The *additive inverse* of any number a is –a such that $a + (-a) = 0\ and\ (-a) + a = 0$.

On the other hand, you can subtract polynomials by adding the additive inverse of the subtrahend to the minuend.

To find the additive inverse of a polynomial, replace each term with its additive inverse.

Polynomials	Additive inverse
$x^2 + 3y$	$-x^2 - 3y$
$3x^2 - 2xy + y^2$	$-3x^2 + 2xy - y^2$
$-2x + 5xy - 7z$	$2x - 5xy + 7z$
$12x^2 - 7x + 9$	$-12x^2 + 7x - 9$

Example 2 : Subtract : $(12t^2 - 9t - 1)$ from $(4t^3 - 10t^2 + 5t + 8)$

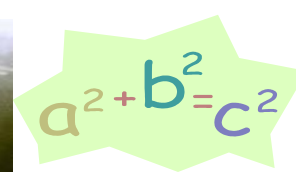
Method 1: Grouping like terms

$$(4t^3 - 10t^2 + 5t + 8) - (12t^2 - 9t - 1)$$
$$= (4t^3 - 10t^2 + 5t + 8) + (-12t^2 + 9t + 1)$$
$$= 4t^3 - 10t^2 + 5t + 8 - 12t^2 + 9t + 1$$
$$= 4t^3 + (-10t^2 - 12t^2) + (5t + 9t) + 8 + 1$$
$$= 4t^3 - 22t^2 + 14t + 9$$

Method 2: Subtraction in column form

$$(4t^3 - 10t^2 + 5t + 8) - (12t^2 - 9t - 1)$$

$$\begin{array}{rcr} 4t^3 - 10t^2 + 5t + 8 \Rightarrow & & 4t^3 - 10t^2 + 5t + 8 \\ -\quad 12t^2 - 9t - 1 \Rightarrow & + & -12t^2 + 9t + 1 \\ \hline & & 4t^3 - 22t^2 + 14t + 9 \end{array}$$



Example 3: Given ΔABC, the perimeter of ΔABC is $P = 12x^2 - 7x + 9$ and the two sides have lengths $3x^2 + 12x - 1$ and $5x^2 - 8x + 5$. Find the measure of the third side.

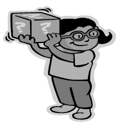
Solution:

Perimeter is the sum of the measures of the three sides of the triangle. Let S represent the measure of the third side of the triangle.

$$P = S + (3x^2 + 2x - 1) + (5x^2 - 8x + 5)$$
$$P - (3x^2 + 2x - 1) - (5x^2 - 8x + 5) = S$$

$$S = 12x^2 - 7x + 9 - 3x^2 - 2x + 1 - 5x^2 + 8x - 5$$
$$S = (12x^2 - 3x^2 - 5x^2) + (-7x - 2x + 8x) + (9 + 1 - 5)$$
$$S = 4x^2 - x + 5$$

Therefore , the measure of the third side of the triangle is $S = 4x^2 - x + 5$



ACTIVITY 4

A. State whether each expression is a polynomial. If it is so, identify the kind of polynomial. If it is not a polynomial , explain why.

1. $4x^2 - 2xy$
2. $\frac{6}{x^2} - \frac{2}{x} + 1$
3. $x^{\frac{1}{2}} + 2x$
4. $\frac{a^2}{3}$
5. $x^2 - \frac{x}{2} + \frac{1}{3}$

B. Find the degree of each term of the polynomial and the degree of the polynomial

1. $2x^2 - 3x^3$
2. $10x^2y^4 - 2x^4y^2 - x^3y^3$
3. $-5xyz^4 + 10x^2y^4z$
4. $6x^2y^3 - 5x^3y^2 - x^4y^2$
5. $17bx^2 + 11b^2x - x^2$

C. Arrange the terms of each polynomial so that the power of x are in descending order.

1. $-6x + x^5 + 4x^3 - 20$
2. $4x^3y + 3xy^4 - x^2y^3 + y^4$

Test 4

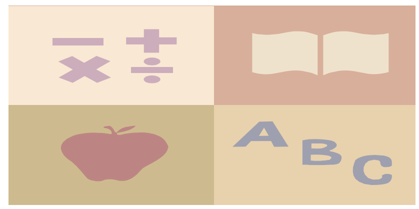
Answer the following:

1. Is $\frac{2}{5}x^3 + 3x^2 - 2x^0$ *a* polynomial? why?
2. Using the expression in no 1 problem find the degree of each term.
3. Are the expressions $3x^2, 3y^2, 3z^2$ similar terms? why?

Activity 5

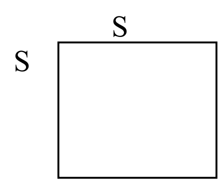
Answer the following:

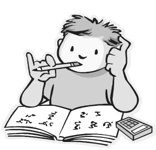
1. What is the first step when adding or subtracting polynomials in column form?
2. What is the best way to check on the subtraction of polynomials?
3. Give the additive inverse of each polynomial:
   a. $x^2 - 4x + 7$
   b. $-4h^2 + 6hk - 2k^2$
   c. $5ab^2 + 6a^2b + 1$
   d. $4x^2 + 2x - 1$



4. Find each sum or difference
   a. $(3x^2 5x - 5) + (5x^2 - 7x + 9)$
   b. $(4x^3 + x^2 + 2x - 4) - (2x^2 - 3x + 5)$
   c. $(a^3 - b^3) + (3a^3 + 2a^2b - b^2 + 2b^3)$
   d. $(5ax^2 + 3a^2x - 5x) + (2ax^2 - 5a^2x + 7x)$
   e. $(3x - 7y) + (3y + 4x)$
   f.
$$\begin{array}{rl} & 4a + 5b - 8c - d \\ & 2a + 7b - 2c + 8d \\ + & \underline{2a - b \qquad + 6d} \end{array}$$

5. Find the measure of the side S of the square if the perimeter of the square is P = 4x-4y .





Test 5

Find the sum or difference of the following:

1.
$$\begin{array}{rr} & 12m^2n^2 + 3mn - 11 \\ - & \underline{5m^2n^2 - 8mn + 7} \end{array}$$

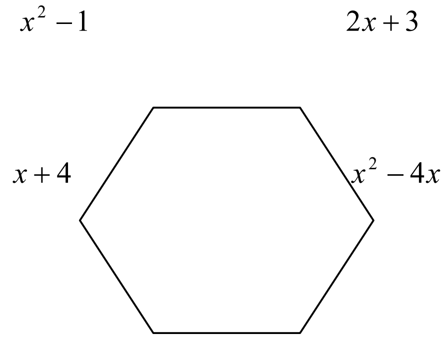
2.
$$\begin{array}{rr} & 2x^2 - 5x + 7 \\ & 5x^2 + 7x - 3 \\ + & \underline{x^2 - x + 11} \end{array}$$

3. $(n^2 + 5n + 4) + (3n^2 + 8n + 8) =$

4. $\left(3+2a+a^2\right)-\left(5+8a+a^2\right)=$

5. Given the perimeter P of the hexagon and the measure of the five sides of the hexagon
Find the measure of the side S.

$P = 11x^2 - 29x + 10$ $\qquad\qquad$ $x^2 + 3x + 24$



6. Mario decided to construct a square casing for his component if one side measures $\left(4x^2 + 2x + 8\right)$cm , what will be the perimeter of the square casing.

Scoring:

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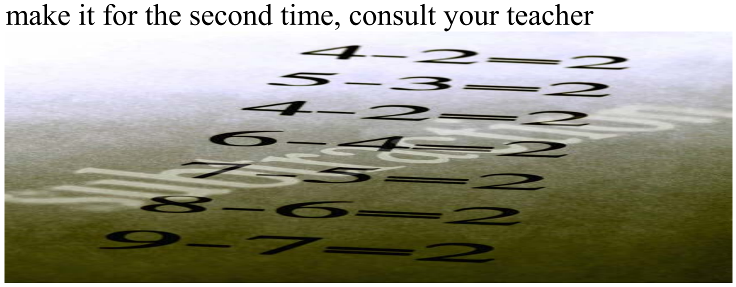


Multiplication of Polynomials by a Monomial

The distributive property can be used in multiplying a polynomial by a monomial.

Examples:

1) $3x(2x+4)=3x(2x)+3x$

$= 6x^2 + 12x$

2) $2a(4a^2 - 3a + 5) = 2a(4a^2) + 2a(-3a) + 2a(5)$

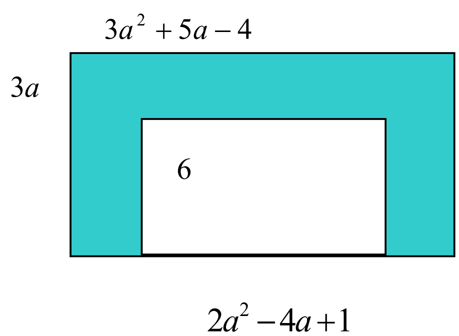
$= 8a^3 - 6a^2 + 10a$

3) $3xy(2x^2y - 3xy^2 - 7y^3)$

$= 3xy(2x^2y) + 3xy(-3xy^2) + 3xy(-7y^3)$

$= 6x^3y^2 - 9x^2y^3 - 27xy^4$

4. Find the measure of the area of the shaded region in simplest terms.



Solution:

Subtract the area of the smaller rectangle from the measure of the area of the larger rectangle

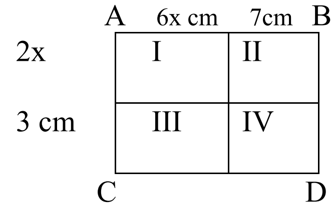
(Area of big rectangle – Area of small rectangle)

$3a(3a^2 + 5a - 4) - 6(2a - 4a + 1)$

$= 9a^3 + 15a^2 - 12a - 12a^2 + 24a - 6$

$= 9a^3 - 3a^2 + 12a - 6$

1. Product of two binomials

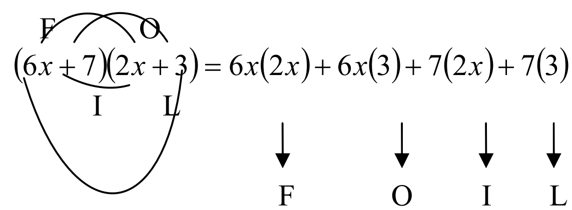


The area of the rectangle is the product of the length and width . You can multiply $(6x+7)cm$ and $(2x+3)cm$ to find the area of the rectangle.

Notice that this big rectangle ABDC is composed of four smaller rectangles I,II, III and IV , Therefore, to find the area of the big rectangle we get sum of the area of the four small rectangles.

$$(6x+7)(2x+3) = 6x(2x) + 6x(3) + 7(2x) + 7(3)$$
$$= 12x^2 + 18x + 14x + 21$$
$$= 12x^2 + 32x + 21$$

The above solution illustrates the short cut method for multiplying binomials ,that is, the FOIL method (first terms, outer terms, inner terms, last terms)



FOIL METHOD :

To multiply binomials using FOIL Method, find the sum of the products of

F- the First Terms

I – the Inner Terms

O – the Outer Terms

L – the Last Terms

Examples:

Find the indicated product:

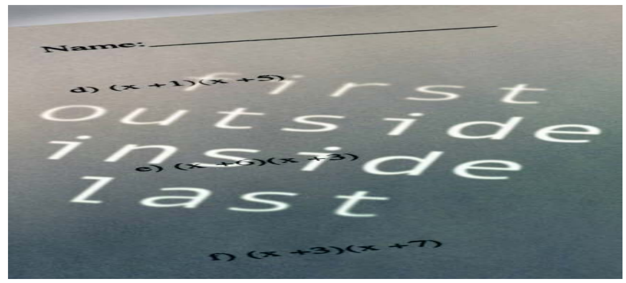
1. $(x+5)(x+6) = x^2 + 6x + 5x + 30$
$= x^2 + 11x + 30$

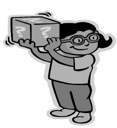
2. $(2x+3)(5x+8)$
$= (2x)(5x) + (2x)(8) + 3(5x) + 3(8)$
$= (10x^2) + 16x + 15x + 24$
$= 10x^2 + 31x + 24$

3. $(x^2 + 5x - 4)(2x^2 + x - 7) - 4(2x^2 + x - 7)$

$= x^2(x^2 + 5x - 4) + 5x(2x^2 + x - 7) - 8x^2 - 4x + 28$

$= x^4 + 5x^3 - 4x^2 + 5x^3 + 5x^2 - 35x - 8x^2 - 4x + 28$
$= x^4 + 10x^3 - 7x^2 - 39x + 28$





ACTIVITY 6

1. Find the sum of the product of the inner terms and the product of the outer terms:

   a) $(5b-3)(2b+1)$
   b) $(2a+1)(3a-2)$
   c. $(4a+3)(2a-1)$
   d. $(5r-7s)(4r+3s)$

2. Find the product
   a) $(c+2)(c-8)$
   b) $(2a+3b)(3a-2b)$
   c. $(x^2 + 7x - 9)(4x - 3y)$
   d. $(5x^2 - 6x + 9)(4x^2 + 3x + 11)$

3. Complete the product of the following :
   a . $(x-2y)(2x-y) = 2x^2 +$ _______ $+ y^2$
   b. $(3a+2c)(2b-3c) = 6b^2 +$ _____ $- 6c^2$

   c. $(q-2y)(q^2+y) =$ _____ $+ qy - 2q^2y - 2y^2$
   d. $(12x^2 + 3y)(x-y) = 12x^3 +$ _____ $+$ _____ $- 3y^2$

4. Ben owns a fishpond whose length is 4 meters more than the width. A path of 2 meters surrounds the fishpond with a total area of 216 m^2. What are the dimensions of the fish pond?

5. Marlo wishes to construct a perimeter fence around his farm. If the dimensions of the farm are $(2x+4y)$ by $(3x-5y)$ respectively, find the polynomial that will represent the length of the wire that will be needed to fence his farm.

6. Refer to the above figure :
   a) Find the polynomial that represents the area of the rectangle.
   b) Assuming that the area of the rectangle is 150 meter^2, find the length of the rectangle.
   c) Suppose the rectangle is your living room and you want to change the floor tiles into bigger tiles. If it costs P65 per square meter, how much will it cost you to tile the entire living room?

Test 6

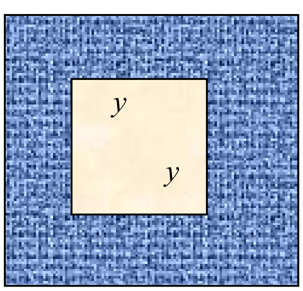
Find the product:

1. $(7y-1)(2y-3)$
2. $(6x^2 - 5xy + 9y^2)(5x - 2y)$
3. $-8d^2(4d+m)(4d-m)$
4. If R= 2x-1, S=3x+2, and T = -3x^2, find each of the following:

   a) $R \cdot S$ b) $R(S+T)$ c) $S \cdot T$ d) $T(R+S)$

5. Find the polynomial that represents the area of the shaded region .

   a.



Scoring:

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Division of Polynomials

1. Division of Monomials by Monomials

Dividing monomials by monomials is done by considering the variables and their exponents. In this case, we have to follow the laws of integral exponents.

Let us recall the rule on Quotient of Powers

1. $\frac{a^m}{a^n} = a^{m-n}$ *if* $m>n$,

   Example $\frac{a^3}{a^2} = a^{3-2} = a$

2. $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$ *if n>m,*

   Example $\frac{a^2}{a^3} = \frac{1}{a^{3-2}} = \frac{1}{a}$

3. $\frac{a^m}{a^n} = 1$ if m=n,

   Example : $\frac{a^3}{a^3} = a^{3-3} = a^0 = 1$

Here are the steps in dividing a monomial by another monomial:

Step 1. Divide the numerical coefficients following the rules for dividing integers.

Step 2. Divide the variables by subtracting the exponents of each like terms.

Example 1. Divide $6x^3$ by $2x^2$

Solution: $\frac{6x^3}{2x^2} = 3x$

Example 2. $12a^4b^6 \div 4a^2b^2$

Solution: $\frac{12a^4b^6}{4a^2b^2} = 3a^2b^4$

2. Division of Polynomial by a Monomial

To divide a polynomial by a monomial we simply write the division as product, and use the distributive property and simplify each resulting fraction.

Example1: Divide $(12x^2 + 4x)$ by 2x

Solution: $\frac{12x^2 + 4x}{2x} = \frac{12x^2}{2x} + \frac{4x}{2x} = 6x + 2$

2. Divide $x^2 - 2x + 1$ by $x + 1$

   Solution:

Example 2: Divide $18a^4b^2 - 12a^2b + 6ab$ by $3ab$

Solution : $\frac{18a^4b^2}{3ab} - \frac{12a^2b}{3ab} + \frac{6ab}{3ab}$

$= 6a^3b - 4a + 2$

Here is the standard form of writing the division of Polynomials

$\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}$, where P(x) is the given polynomial, D(x) is the Divisor, Q(x) is the quotient and R(x) is the remainder if there is.

3. Division of Polynomial by a Binomial

Steps in dividing polynomial by a Binomial

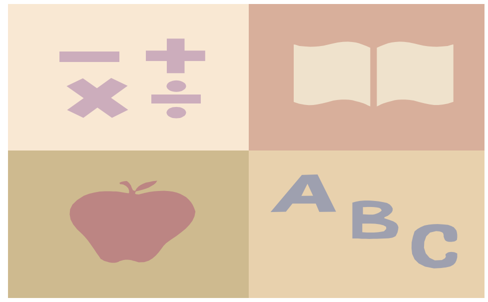
1. Arrange the polynomial in descending order of the exponents of common variable.
2. Using the long division, divide the first term of the dividend by the first term of the quotient. Multiply the divisor by the divisor and then subtract the product from the dividend.
3. Using the remainder as the new dividend, repeat the process to obtain the next term of the quotient.
4. Do the same procedure until the remainder is zero or a polynomial with degree less than the degree of the divisor.

Example

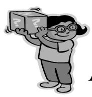
1. Divide $42c^3 + 46c^2d - 22cd^2 + 30d^3$ by $6c + 10d$

$$
\begin{array}{r}
7c^2 - 4cd + 3d^2 \\
6c + 10d \overline{)\,42c^3 + 46c^2d - 22cd^2 + 30d^3} \\
-(42c^3 + 70c^2d) \qquad\qquad\qquad \\
\hline
-24c^2d - 22cd^2 \qquad\quad \\
-\ (-24c^2d - 40cd^2) \qquad\quad \\
\hline
18cd^2 + 30d^2 \\
-\quad 18cd^2 + 30d^2 \\
\hline
0
\end{array}
$$

$$
\begin{array}{r}
x-3 \quad \text{r } 4 \\
x+1\overline{)\,x^2-2x+1} \\
-\ (x^2+x) \\
\hline
-3x+1 \\
-\ (-3x-3) \\
\hline
4
\end{array}
$$



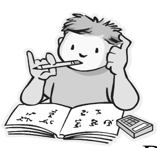
Answer: $(x-3)+\dfrac{4}{x+1}$



ACTIVITY 7

Find the quotient:

1. $\dfrac{27a^2-8a^4}{9a^2}$
2. $\dfrac{15r^6y^6-20r^3y^4+10r^8y^7}{5r^2y^3}$
3. $\dfrac{2x^3+x^2y+xy^2+4y^3}{x+y}$
4. $4^{2x}\div\left(-4^{3x}\right)$
5. $\dfrac{15x^2-10x^3}{5x}$
6. $6e^y\div3e^y$
7. The area of a rectangular garden is $2d^3+7d^2-5d-4$ sq. meters, Find the length if the width is $2d+1$



Test 7

Divide :

1. $\dfrac{16r^4s^2-8r^3s}{4rs}$ (2 points)
2. $\dfrac{20x^5-12x^4y-2x^3y^2 10x^2y^3-4xy^4+8y^5}{4x^2-2y^2}$ (2 points)
3. A vegetable garden has $3h^3-2h-1$ vegetables planted in h-1 rows , with equal number of vegetables in each row . How many vegetables are planted in each row. (3 points)
4. Show that $(x+y)$ is a factor of $\left(x^3+y^3\right)$ (3 Points)

Scoring:

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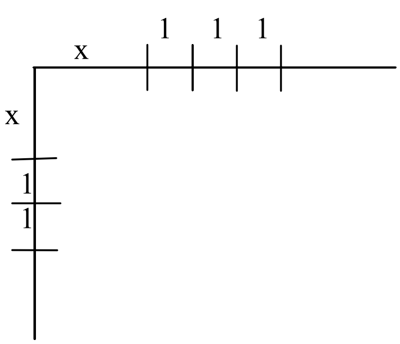
SPECIAL PRODUCTS

Discovering Patterns in Special Product

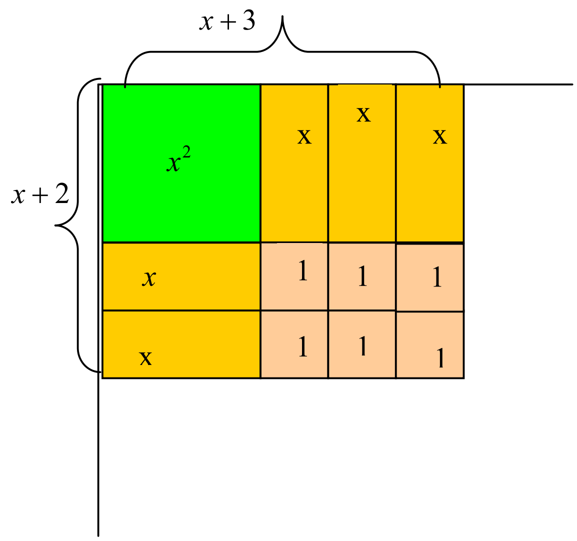
Some products of binomials occur so often in the study of Algebra that we easily notice or identify patterns.
Try to discover the patterns in order to get the product of the following binomials

A. Use algebra tiles to find the product $(x+2)(x+3)$.

1. The rectangle has a width of x+2 and length of x+3. Mark off the dimension on the product mat



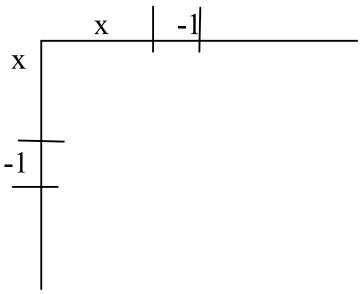
2. Using the marks on the product mat as guide, make the rectangle with algebra tiles



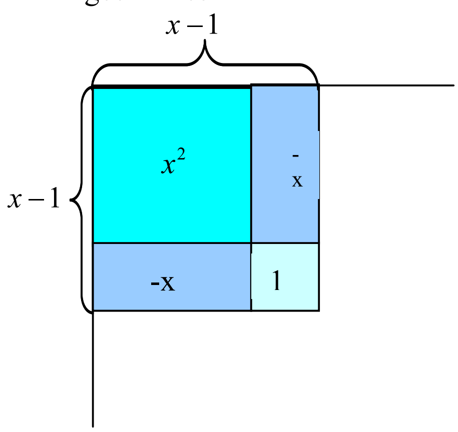
3. The rectangle has one x^2 –tile, and five x-tiles and six 1-tiles. The area of the rectangle then is x^2+5x+6. Thus, $(x+2)(x+3)=x^2+5x+6$

B. Find the product $(x-1)(x-1)$

1. The rectangle has a width of x – 1 and the length of x-1 units.



2. Using the marks on the product as guide make the rectangle using the algebra tiles



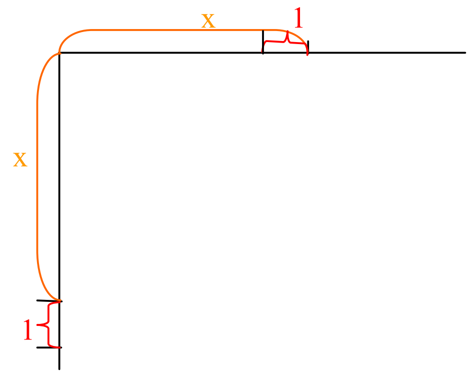
3. The rectangle has one x^2 – tile, two x – tiles, and one one unit-tile. The area of the rectangle is x^2-2x+1 .

4. Hence

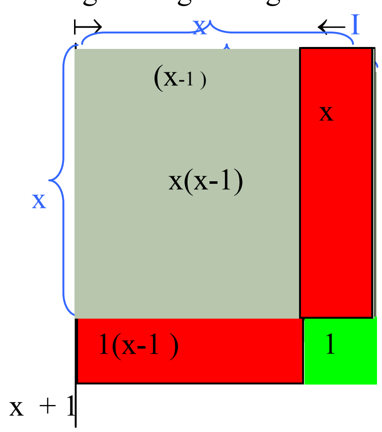
$$(x-1)(x-1)=x^2-2x+1$$

C. Find the product $(x+1)(x-1)$

1. The rectangle has a length of x +1 and width x -1 units



2. Using the marks as guide to make the rectangle using the algebra tiles



3. The rectangle has one x(x-1) – tile , one(x-1) – tile. The one x-tile and one one unit tile are to be subtracted from the resulting figure The area of the rectangle is x(x-1) +(x-1) = x^2 -1

4. The product of (x+1)(x-1) = x^2 -1

Multiplication using algebra tiles illustrates special products

Let us have more exercises on special products:

Example 1. *Product of Two Binomials*

$$(2x+y)(3x-2y) = 6x^2 - 4xy + 3xy - 2y^2$$
$$= 6x^2 - xy - 2y^2$$
$$(3b+2c)(2b+3c) = 6b^2 + (9+4)bc + 6c^2$$
$$= 6b^2 + 13bc + 6c^2$$

What pattern have you discovered? How do you find the product of this kind of binomial?

Example 2. (*Square of Binomials*)

a) $(2x-y)(2x-y)$
$$= (2x)^2 - 2(2x)(y) + y^2$$
$$= 4x^2 - 4xy + y^2$$

b) $(x+3y)(x+3y)^2$
$$= x^2 + 2(x)(3y) + (3y)^2$$
$$= x^2 + 6xy + 9y^2$$

c) $(4b+c)^2 = (4b)^2 + 2(4b)(c + c^2)$
$$= 16b^2 + 8bc + c^2$$

Is there a shorter way of squaring the binomial?

Squaring Binomials,
square the first terms and also the last term and add to these twice the product of the two terms.

Example 3. (*Sum and Difference of Two Terms)*

a) $(a-b)(a+b) = a^2 - b^2$

b) $(4x-y)(4x+y) = (4x)^2 - y^2$
$$= 16x^2 - y^2$$

c) $(abc+2)(abc-2) = (abc)^2 - (2)^2$
$$= a^2b^2c^2 - 4$$

The *product of the sum and difference of two terms* is equal to the difference of the squares of terms

Example 4. (*Cube of a Binomial*)

a) $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

b) $(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

Cube of Binomial The product is equal to the cube of the first term plus /minus three times the product of the square of the first term and the last term plus the 3 times the first term and the square of the last term minus/plus the cube of the last term.

Example 5: *Squaring a trinomial*- this is an extension when you square a binominal

a) $(2a-3b-c)^2 = [(2a-3b)-c]^2$
$$= (2a-3b)^2 - 2c(2a-3b) + c^2$$
$$= (2a)^2 - 2(2a)(3b) + (3b)^2 - 2c(2a-3b) + c^2$$
$$= (2a)^2 + (-3b)^2 + (-c)^2 + 2(2a)(-3b) + 2(2a)(-c) + 2(-3b)(-c)$$
$$= 4a^2 + 9b^2 + c^2 - 12ab - 4ac + 6bc$$

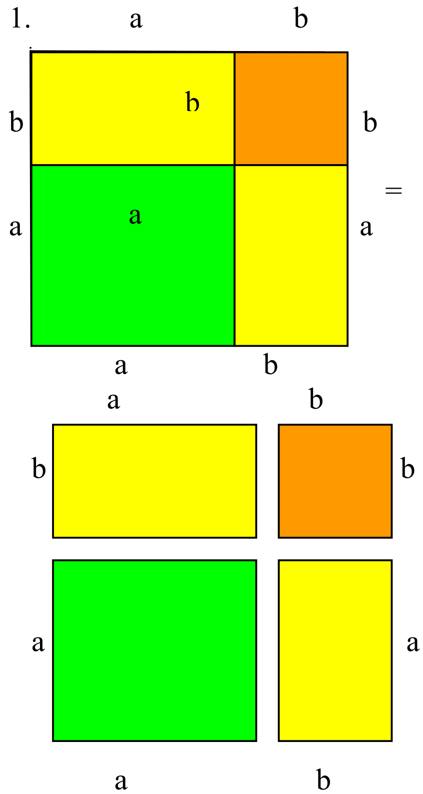
b) $(x+2y+3z)^2 =$
$$= (x)^2 + (2y)^2 + (3z)^2 + 2(x)(2y) + 2(x)(3z) + 2(2y)(3z)$$
$$= x^2 + 4y^2 + 9z^2 + 4xy + 6xz + 12yz$$

$x-y$

Let us try the following

Each picture illustrates an algebraic fact, complete the statement

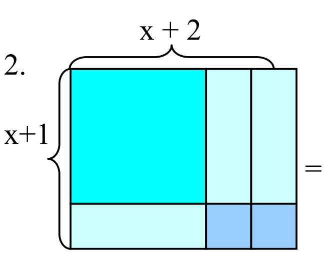
1.

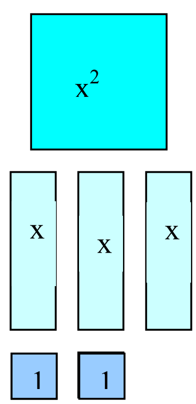


$(a+b)(a+b) =$ ____________

$(a+b)^2 =$ ______________

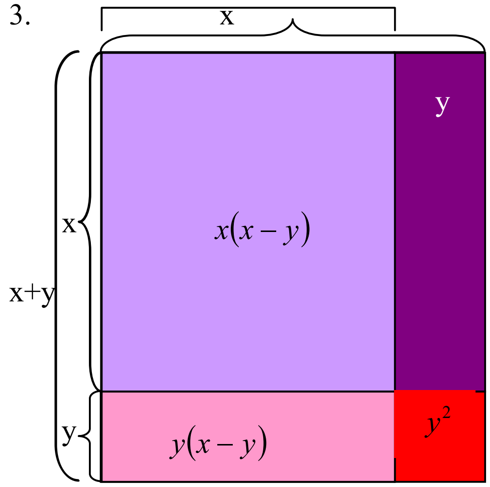
2.





(x+1) (x+2) = _________

3.



$(x+y)(x-y) =$ ______________

In summary, the different special products:

1. Product of Two Binomials

$$(ax+b)(cx+d) = acx^2 + (ad+bc)x + bd$$

2. Product of a Sum and Difference of Two Terms

$$(x+y)(x-y) = x^2 - y^2$$

3. Square of a Binomial

$$(x+y)^2 = x^2 + 2xy + y^2$$

$$(x-y)^2 = x^2 - 2xy + y^2$$

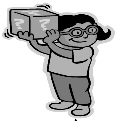
4. Cube of a Binomial

$$(x+y)^3 = (x+y)(x^2 - xy + y^2)$$

$$(x-y)^3 = (x-y)(x^2 + xy + y^2)$$

5. Square of a Trinomial

$$(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2xz + 2yz$$



Activity 8

A. Find the indicated product

1. $(2a-7b)(3a+5b)$

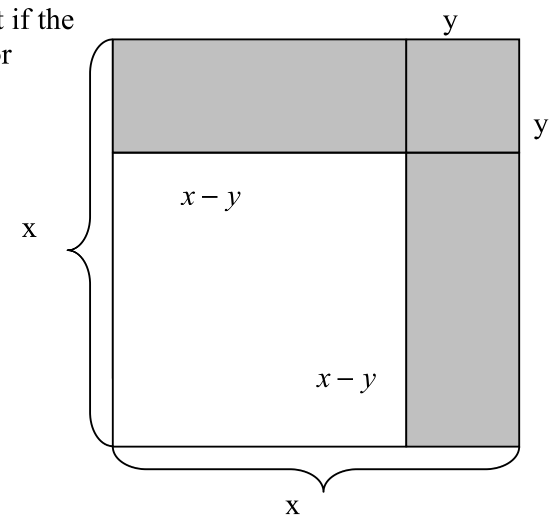
2. $(4x-3)(x+4)$

3. $(x-6)(x+6)$

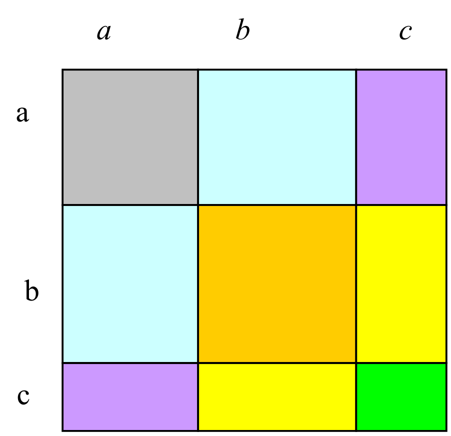
4. $(2x+y)^2$

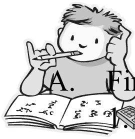
5. $(2b+2c+d)^2$

B. What does the diagram at the right represent if the shading represents regions to be removed or subtracted.



C. The diagram at the right represents $(a+b+c)^2$. Use the diagram to write the polynomial that will represent the area of the figure.





Test 8.

A. Find the product of the following

1. $(x-3)(x+4)(x+3)(x-4)$
2. $(7a^2+b)(7a^2-b)$
3. $\left(\frac{1}{2}a+k\right)^2$
4. $(4x-5)(4x-5)(x-4)$.
5. $\left(\frac{4}{3}d^2-h\right)\left(\frac{4}{3}d^2+h\right)$

B. Find the missing term.

1. $(2x+y)^2 = 4x^2 +$ _____ $+ y^2$
2. $(3x-2y)^2 =$ ______ $-12xy+4y^2$
3. (2x – 4y)(x+y) = 2x^2 _______ - 4y^2
4. $(2y)^3-(3z)^3 = (2y-3z)($ _____________ $)$
5. $(a-b)^2-(c+d)^2 =$ _________ $[(a-b)+(c+d)]$

Scoring:

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Identifying and Finding Special Products

In the previous lessons you have learned and discovered the patterns on finding special products.

Given another problem,can you apply the formulas in the special products to solve it?

1. Find a pattern for the fourth power of the binomial $(a+b)^4$.

How are you going to get the product indicated? What special product formula are you going to use?

Solution:

In this type of problem you can apply the square of a binomial

$$(a+b)^4 = (a+b)^2(a+b)^2$$

$$= (a^2 + 2ab + b^2)(a^2 + 2ab + b^2)$$
$$= a^4 + 2a^3b + a^2b^2 + 2a^3b + 4a^2b^2 + 2ab^3 + a^2b^2 + 2ab^3 + b^4$$
$$= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

2. Using problem 1 , check the answer by using $(a+b)^3(a+b)$.

Solution:

$$(a+b)^3(a+b) =$$
$$= (a^3 + 3a^2b + 3ab^2 + b^3)(a+b)$$
$$= a^4 + 3a^3b + 3a^2b^2 + ab + a^3b + 3a^2b^2 + 3ab^3 + b^4$$
$$= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

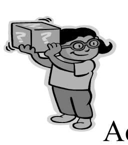
3. Find the product of $(x^2+y)^4$

Solution :

$$(x^2+y)^4 = (2x^2+y)^2(2x^2+y)^2 \text{ or } [(2x^2+y)^2]^2$$

$$= (4x^4 + 4x^2y + y^2)^2$$

$$= 16x^8 + 16x^4y^2 + y^4 + 16x^6y + 4x^4y^2 + 4x^2y^3$$

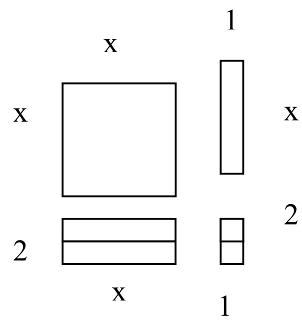


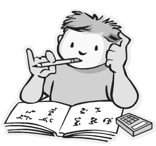
Activity 9

A. Use special product formulas to find the following:

1. $(2g-s)(2g+s)$
2. $(m^3+3f^2)(m^3-3f^2)$
3. $(y-5)(2v+r)(y+5)(2v-r)$
4. $(9a-2q)^2$
5. $[(2a-c)-5f][(2a+c)+5]$
6. $(3x^2-4)^2$
7. $\left(\frac{4}{3}x^2-y\right)\left(\frac{4}{3}x^2+y\right)$
8. $[(8a-2b)+3]^2$
9. $[(p^2-3q2)-2q^2]^3$
10. $[(a+b)+c+(d-e)]^2$

B. The figure below shows the model for $(x+2)(x+1)$ separated into four parts. Write a paragraph explaining how the model shows the use of distributive property





Test: 9

A. Tell whether each statement is true or false. Justify your answer with a drawing

1) $(x+2)(x-2)=x^2-4$

2) $(x+3)(x-2)=x^2-x-6$

B. Find the missing term:

1. $(2x+2y)^2 = 4x^2 +$ _____ $+4y^2$
2. $(2x-7)^2 = 4x^2 +$ _____ $+49$

C. Find the indicated product :

1. $(u^2+7v)^3$
2. $[(x^5-x^3)+(x^2-x)][(x^5-x^3)-(x^2-x)]$
3. $[(2x+3y)-(5x-4y)]^2$
4. $(3p+2r^2)^4$

Scoring:

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FACTORING

The living room is the place where you and your family may possibly bond together. Stories are shared among members of the family and sometimes kept secret within the four walls of the room.

This is also a place where you seem to be comfortable especially in entertaining people. Arrangement of the furniture and appliances is important to make the room cozy and comfortable thus the area of the room is important in determining the size and the kind of furniture the room should have.

The application of the formula for finding the area of the rectangle, A = lw , in solving problems leads to a trinomial expression of degree 2. In this unit, you will learn the reverse operation of multiplying binomials , that is, you are going to look for the expressions to be multiplied to get the given product, this is called *factoring* .

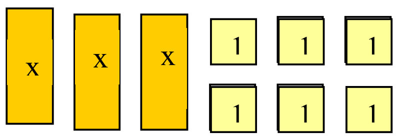
Factoring can be best illustrated by using Algebra Tiles

A. Factoring using the Distributive Property

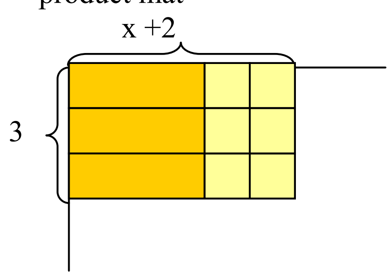
Using the algebra tiles model , you know the area of the rectangle and are asked to find the length and the width.

1. Use algebra tiles to factor 3x + 6

   a. Model the Polynomial 3x +6



   b. Arrange the tiles into a rectangle on the product mat

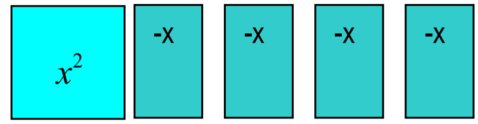


   c. The rectangle has a width of 3 and a length of $x+2$. Therefore

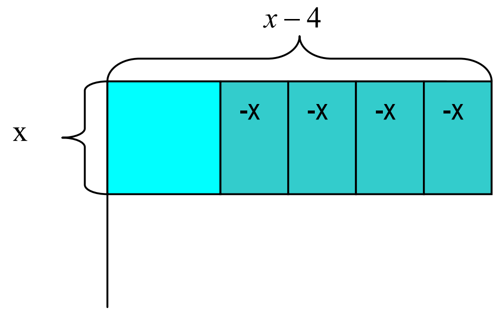
   $3x+6=3(x+2)$. Hence the factors of 3x + 6 are 3 and $(x+2)$

2. Use algebra tiles to factor x^2-4x

   a. Model the Polynomial x^2-4x



   b. Arrange the tile into a rectangle



   c. Therefore the rectangle has a width of x units and length of x - 4

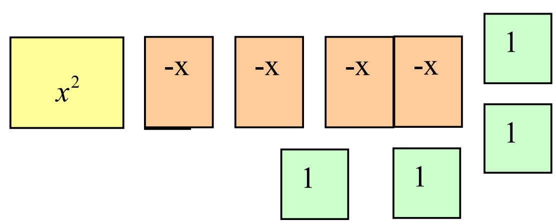
   A = lw

   $= x^2-4x = x(x-4)$

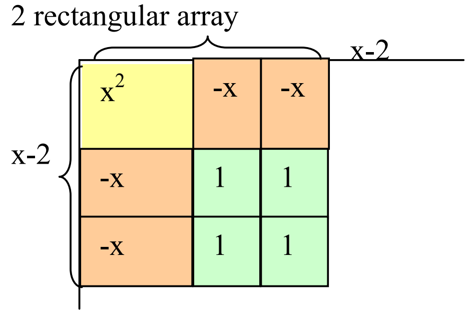
   Factors of x^2-4x are x and x+4

4. Use algebra tiles to factor x^2-4x+4

   a. Model the polynomial x^2-4x+4



   b. Place the x^2 – tile to the corner of the product mat and arrange the 1 – tiles into 2 by 2 rectangular array



In the previous lessons you multiplied two binomials to find the product. Here we are to study factoring

Factors	Product
$(x+y)(x-y)=$	$x^2 - y^2$
$(x+y)^2 =$ or $(x+y)(x+y)=$	$x^2 + 2xy + y^2$
$(x-y)^3 =$	$x^3 - 3x^2y + 3xy^2 - y$
$(x+y)^3 =$	$x^3 + 3x^2y + 3xy^2 + y$
$(x+y+z)^2 =$	$x^2 + y^2 + z^2 + 2xy +$

The factors are multiplied together to get the product. This time you will do the reverse operation that is given the product , you are going to look for the factors. The process of doing it is called *factoring* or finding the factors.

FACTORING- is the process of expressing the given number or expression in terms of its prime factors

Types of Factoring

A. Greatest Common Factor –

To find the greatest common factor of two or more expressions, determine the product of prime factor common to the given expression.

Example: Find the factors of $25x^2y^2 - 5xy$.

Solution: Notice that the term common to both is 5xy. Find out the other factor by dividing each term by 5xy.

$$\frac{25x^2y^2}{5xy} - \frac{5xy}{5xy} = 5xy - 1$$

Checking:

$$5xy(5xy-1) = 25x^2 - 5xy$$

Therefore, the factors of $25x^2y^2 - 5xy$ are: 5xy and 5xy -1. Hence,

$$25x^2y^2 - 5xy = \quad 5xy(5xy-1).$$

Another way of finding the GCF of the factors

$$25x^2y^2 = 5 \cdot 5 \cdot x \cdot x \cdot y \cdot y$$
$$5xy = 5 \quad \cdot x \quad \cdot y$$
$$\text{GCF} = 5 \quad \cdot \quad x \quad \cdot y \quad = 5xy$$

Example2: Factor the expression $18x^2 + 12x$.

What factor is common to the two terms of the given expression?

Solution:

The common factor is 6x. Therefore the factors are 6x and $3x+2$. Thus,

$$18x^2 + 12x = 6x(3x+2)$$

Example 3: Find the factors of $12x^2y^2 + 3x^4y + 6x^3y^3$

Notice that $3x^2y$ is the greatest common factor. Therefore the other factor is $(4x\ \ y + x^2 + 2xy)$. Thus,

$12x^2y^2 + 3x^4y + 6x^3y^3 =$
$3x^2y(4x\ \ y + x^2 + 2xy)$

B. Factoring Special Products

Difference of two squares- Factors of this product are two binomials which are the sum and difference of the square root of the two squares.

To recognize that it is the difference of perfect square for the variables raised to the second power.

Examples:

1) $x^2 - y^2 = (x+y)(x-y)$
2) $4a^2 - b^2 = (2a-b)(2a+b)$
3) $49a^4 - 4b^2 = (7a^2 - 2b)(7a^2 + 2b)$
4) $8j^4 - 18k^4 = 2(4j^4 - 9k^4)$

What do you notice in problem number 4?

One of the factors $(4j^4 - 9k^4)$ can still be factored since this is still the difference two squares. Therefore, the complete factorization is:

$$2(2j^2 - 3k^2)(2j^2 + 3k^2)$$

2. Perfect Square Trinomial

Example: Factor $x^2+2xy+y^2$, this is a perfect square trinomial. How many terms are there ? What do you notice about the first and the last terms? What can you say about the middle term?

To recognize a perfect square trinomial, the first and the last terms must be positive and perfect squares, and the middle term is twice the product of the square roots of the two terms.

Product	Factors
$x^2+2xy+y^2$	$=(x+y)(x+y)$ or $(x+y)^2$

How do you factor a perfect square trinomial?

Get the square root of the first term and the third term. Then copy the sign of the middle term raised this binomial to the second power.

More examples: Factor

1. $x^2-4x+4=(x-2)^2$
2. $9x^2-6x+1=(3x-1)^2$
3. $x^2+16x+64=(x+8)^2$

3 Sum or Difference of Two Cubes

a. Is $x^3-y^3=(x-y)(x^2+xy+y^2)$?

Verify the product of $(x-y)(x^2+xy+y^2)$.

Solution:

$$
\begin{array}{r}
x^2+xy+y^2 \\
x-y \\
\hline
-x^2y-xy^2-y^3 \\
x^3+x^2y+xy^2 \qquad\quad \\
\hline
x^3 \qquad\qquad\quad -y^3
\end{array}
$$

This shows that $(x-y)(x^2+xy+y^2)$ are the factors of x^3-y^3.

Examples: Factor:

1. $m^3-n^3=(m-n)(m^2+mn+n^2)$
2. $8a^3-b^3=(2a-b)(4a^2+2ab+b^2)$
3. $g^3-27h^3=(g^3-3^3h^3)$
   $=(g-3h)(g^2+3gh+h^2)$

How do you factor the difference of two cubes?

1. Get the difference of the cube roots of the first and second terms, This is the first binomial factor.
2. To get the trinomial factor square the first term of the binomial factor plus the product of the cube root of the two terms and square the last term.

Now, if we are asked to factor x^3+y^3,

The factors are:

b. $x^3+y^3=(x+y)(x^2-xy+y^2)$

Examples: Factor:

1. $d^3+h^3=(d+h)(d^2-dh+h^2)$
2. $125q^3+27t^3=(5q)^3+(3t)^3$
   $=(5q+3t)(25q^2-15qt+9t^2)$
3. $27k^3+1=(3k)^3+1$
   $=(3k+1)(9k^2-3k+1)$

4. Factoring Quadratic Trinomials

$$ax^2+bxy+cy^2=(px+ny)(qx+my)$$

$$a=pq$$

where $b=pm+nq$

$$c=nm$$

Example: Factor the following completely:

1. $4x^2 - 35xy - 9y^2 = (x-9)(4x+1)$
2. $x^2 + 6x + 8 = (x+4)(x+2)$
3. $x^2 - 5x + 6 = (x-3)(x-2)$

5. Factoring by Grouping

In your previous lessons have tried factoring binomials also with trinomials, what if you will be asked to factor polynomials with four terms, what will you do?

Example:

The area of a rectangular picture frame is (3ab-21b+5a-35) cm^2 what are its dimensions?

You need to find the length and the width of the rectangular picture frame and since Area = length x width , so you need to factor the polynomial in order to get the dimensions.

Solution:

$3ab - 21b + 5a - 35 =$

(*Group terms that have common monomial factor*)

$= (3ab - 21b) + (5a - 35)$

(*Factor*)

$= 3b(a-7) + 5(a-7)$

(*a-7 is the common factor by distributive property)*

$= (3b+5)(a-7)$

Therefore ,the dimensions of the picture frame are $(3b+5)$ and $(a-7)$

Other examples:

1.Factor: $8x^2y - 5x - 24xy + 15$

Solution:

$8x^2y - 5x - 24xy + 15$

$= (8x^2y - 24xy) + (-5x + 15)$

$= 8xy(x-3)$-5 $(x-3)$

$= (8xy - 5)(x-3)$

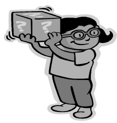
2. Factor $2f^2 + 2gc + fg + 4fc$

Solution:

$= (2f^2 + fg) + (2gc + 4fc)$

$= f(2f + g) + 2c(g + 2f)$

$= (f + 2c)(2f + g)$



ACTIVITY 10

A. Factor the following completely:

1. $6a^7b^3 - 32a^8b^4$
2. $144b^2 - 100$
3. $3cb^3 - 81cd^3$
4. $x^2 + 2x - 24$
5. $10(x-2y)^2 + 19(x-2y) + 6$
6. $9k^2 + 30km + 25m^2$
7. $64a^3b^3 - 27c^3$
8. $12ax + 20bx + 32cx$
9. $c^2 + 2c - 3$
10. $5x^2 - 13x - 6$
11. $2ax + 6xc + ba + 3bc$
12. $a^2 - 2ab + a - 2b$
13. $5a^2 - 4ab + 12b^3 - 15ab^2$
14. $28a^2b^2c^2 + 21a^2bc^2 - 14abc$

B. When you factor 15a +6a^2, what property did you use?

C. The perimeter of a square is $16y^2 + 8y$. Find the area of the square.

Test 9

Fill in the blanks: (2 points each)

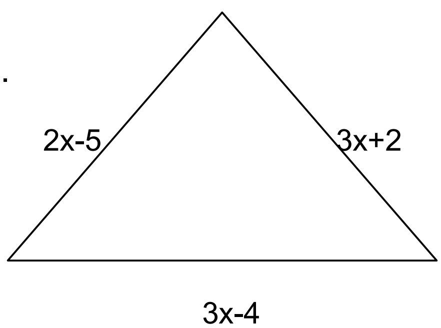
1. The factors of the trinomial $x^2 - 5x + 3$ are $(x+2)($_____$)$.
2. The middle term of the perfect square trinomial $4x^2 +$ ____$+ 9y^2$ is equal to _____.
3. The factors of $y^2 + 12y + 27$ are _______ and _______.
4. Factor completely. $b^9 - c^9$.

Scoring:

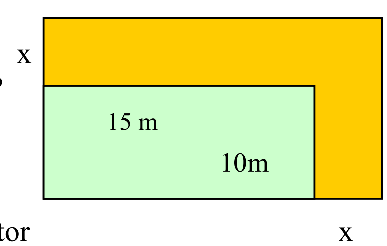
- After answering the test, check your answer with those on the answer key page
- If your score is 7 or higher , you may proceed to the next lesson, otherwise, read the lesson once again and do the test again
- If you did not make it for the second time, consult your teacher

UNIT TEST

I Fill in the blanks.

1. _______________ refers to the expression whose value does not change.
2. The degree of the polynomial $4xy^3 + x^2y$ is _____________
3. _______________ must be added to $7m^2 - 2mn + 5n^2$ to get $8m^2 + mn - 2n^2$.
4. The middle term of the product of $(3x-5)(5x+2)$ *is* ______________.
5. The factored form of $16m^3n^2 + 2mn - 32m^2n^4$ is _____________
6. The value of the polynomial $4x^3 - 3x^2y + 3xy^2 + 2y^3$ when x= 1 and y = -1 is ________________
7. The product of the expression $(ax+b)^3$ *is* _____________
8. 

   Given are measures of the sides of the triangle, The perimeter of the triangle is _____________
9. The simplified form of $\dfrac{36x^4y^6z^{10} - 12x^2y^2z^3}{6xyz^2}$ is _______________
10. The equivalent mathematical expression of the phrase " The quotient of eight times the product of the square of m and cube of n and the sum of m and n" is ______________.
11. One factor of $3a^2 + a - 2$ is $(3a-2)$ the other factor is _______________.
12. Find the dimensions of the rectangle whose area is $(9z^2 - 12z + 4)m^2$ is ____________.
13. The two sides of a school garden of uniform width are utilized as sidewalks as shown in the figure. If the area of this side walk is 82 m^2, What is its width ? Width = _______



14. In the expression $(3a+b)a^2 - (3a+b)b^2$ the common factor is _____________.
15. In symbols " The average of M,N,O, and P" is ____________

II A. Perform the indicated operation and simplify.

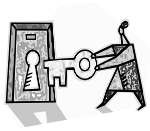
1. Add $4p^3q+8p^2q-2pq$, $2p^3q-2p^2q+8pq$, $6p^3q+12p^2q+4pq$
2. Subtract $4a-6b+c$ from the sum of $(2a-6b+3c)$ and $(3a-2b-c)$
3. Find the sum of $7x^3+2x^2+3x+1$, $-2x^3+3x^2-2x+3$, $4x^2-2x+2$
4. $-(3x-2y)(2x+3y)$
5. Divide $(18x^3y^3-12x^2y^2+6xy)$ *by* $3xy$

B. Find the indicated product

1. $(2x+3)(2x-3)$
2. $(4y^2-2z)^2$
3. $4ab(8ab-2a+c)$
4. $[(x+y)-(a+b)][(x+y)+(a+b)]$
5. $(x+9)(x+3)$

C. Factor the completely.

1. $a^2-10a+21$
2. $4g^2+20g+25$
3. $49m^4-16$
4. $27x^3+8$
5. $4x^3+14x^2+6x$



Answer to Activity 1

A.

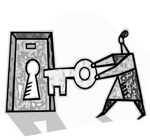
1. $6 + x$
2. $a - 8$
3. $\frac{1}{3}n + 4$ or $\frac{n}{3} + 4$
4. $4x - 2$
5. $\frac{1}{2}x - 5$
6. $13y + 3$ or 3 + 13y
7. $\frac{2}{5}(c + 4)$
8. $-2a(a + 9)$
9. $5x - 8$
10. $5x + 3$ or 3 + 5x

B.

1. Nine times a number <u>x</u> minus four.
2. Negative three-fourth *x* minus four.
3. Three –fifths times the quantity <u>x</u>-squared minus four.
4. Four times the cube of <u>a</u> minus seven.
5. Z-squared times the quantity Z minus one-fourth.

C.

1. $P + 2$
2. $2w$
3. $12 - Q$
4. $G + 2$
5. $\frac{15}{y}$



Answers to Test 1

A.

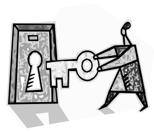
1. $d - 10$
2. -5
3. One fourth of <u>n</u> increased by 6
   Or <u>n</u> divided by 4 plus 6.

B.

1. 2s + 5
2. 100g m
3. $\frac{r}{5}$
4. D + 4
5. (q + r) - 10
4. $Q - 2$
5. $(6x + 4x)$*meters* or
   $10x$ *meters*

C.

1. 20
2. 660
3. 263
4. 402
5. -23



ANSWERS TO ACTIVITY 2

$A = 36$
$B = 144$
$C = 125$
$D = 1000$
$E = 10$
$F = 32$
$G = 16$
$H = 49$
$I = 400$
$J = 100$
$K = 5$
$L = 4$
$M = 11$

$N = 9$
$O = 0$
$P = 216$
$Q = 243$
$R = 8$
$S = 7$
$T = 64$
$U = 1$
$V = 81$
$W = 27$
$X = 25$
$Y = 121$
$Z = 3$

L A W S O F I N T E G R A L

E X P O N E N T S

IV. 1. yes
2. No
3. Division by zero is not allowed since this will lead to undefined expression

II

1. False, 3^6
2. False, 2^7
3. True
4. False, 1
5. True
6. True
7. True
8. False, $2a^2$
9. False, $\frac{1}{b^8}$
10. True

III.

1. a^{13}
2. $125a^6$
3. $21y^7z$
4. $-3a^2x^6y^2$
5. abc
6. $9a^2y^6$
7. $\frac{7x^3}{4z^{10}}$
8. $\frac{kK^6}{25r^8}$
9. $\frac{m}{7r^2}$
10. $-5x^{16}y^5$
11. y^{2+b}
12. x^{10a}
13. 3^{5x+2}

V.

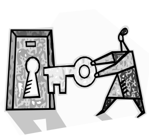
1. A = c^{14} sq. unit
2. A = p^{5} sq. unit
3. A = z^{9} sq. unit
4. V = x^{8} unit ^{3}
5. V = a^{9} unit^{3}

VI

1. $16a^8b^4$
2. $-108x^{12}y^8$
3. $-6a^4b^6$
4. x^6y^2
5. $(x-2y)^4$
6. x^4y^{12}
7. 1
8. $\dfrac{b^8c^4}{a^2}$

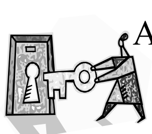
VII. 1. No, $x^5 \cdot x^3 = x^8$, by the product of Powers the exponents are added.

2. a^{30}, using the Power of Powers. i.e. $(a^m)^n = a^{mn}$
3. 3^x ,rather than 3x, because powers of three gives more pay rather than multiples of 3
4. 27.00 pesos



Answers to Test 2

1. False , $\dfrac{1}{2^{-3}} = 2^3 = 8$
2. a) 1
   b) 1,024
   c) 128
   d) 32,768
   e) 8
3. a -72
   b) 5x
   c) $\dfrac{4a^6}{b^8}$
   d) $\dfrac{1}{441}$

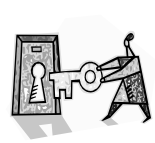


Answers to Activity 3

A. 1. a. 3
b. 2/3
c. 1
d. -25
e. -.28

2. a . like
b. like
c. unlike
d. unlike
e. like

B1. $10x^2 - 7x$
2. $-30y^3 + 20y^2 - 5y + 15$
3. $u - 5c$
4. $-46t^2 + 18st - 2s -^{2} 42t^2$
5. $-3x + 87$



Answers to Test 3

1. -3x -3y
2. -10c-5d +5e
3. –x –y -3
4. 34w -7x + 3y
5. 27c - 29a + 2b + 16ab

6. -11h – e -16

7. 9x

8. $5q^3 + 24q^2 + 120q$

9. $-66a - b - 36$

10. $16x - 11y$

C. 1. The value of x + 1increases as x increases

2. The same results is observed in x+2 also in x+3

3.for x-1 the value starts at -1 and increases by 1, while x-2 starts at -2 The same change is observed as x - 1

Answers to Activity 4

1. Arrange the terms in ascending order
2. Add the difference and the subtrahend to get the minuend
3.
   a. $-x^2 + 4x - 7$
   b. $4h^2 - 6hk + 2k^2$
   c. $-5ab^2 - 6a^2b - 1$
   d. $-4x^2 - 2x + 1$
4. a. $8x^2 - 2x + 4$
   b. $4x^3 - x^2 + 5x - 9$
   c. $4a^3 + 2a^2b - b + b^3$
   d. $7ax^2 - 2a^2x + 2x$
   e. 7x - 4y
   f. $8a + 11b - 10c + 13d$
5. S = x - y

Answers to Test 4

1. $7m^2n^2 - 5mn - 4$
2. $8x^2 + x + 15$
3. $4n^2 + 13n + 12$
4. $-2 - 6a$
5. $8x^2 - 27x - 20$

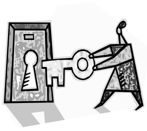
Answers to Activity 5

1. a) -b
   b) -a
   c) 2a
   d)-13rs
2. a) $c^2 - 6c - 16$
   b) $6a^2 + 5ab - 6b^2$
   c) $4x^3 + 25x^2 - 57x + 27$
   d) $20x^4 - 9x^3 + 73x^2 - 39x + 99$
3. a) -5xy
   b) -5bc
   c) q^3
   d) $-12x^2y$
4. w = 23 m l= 27m

Answers to Test 5

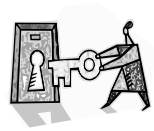
1. $14y^2 - 23y + 3$
2. $30x^2 - 37x^2y + 55xy^2 - 18y^3$
3. $-128d^4 - 8d^2m^2$
4. a) $6x^2 + x - 2$
   b) $6x^3 + 9x^2 + x - 2$
   c) $-9x^3 - 6x^2$
   d) $-15x^3 - 3x^2$
5. A = 16y +64

5. P = 10x – 2y
6. a) A= 20x +10
   b) length f the rectangle is 15 m
   c) Php 9,750.00



Answers to Activity 6

1. $3 - 2a^2$
2. $3r^4y^3 - 4ry + 2r^6y^4$
3. $2x^2 - xy + 2y^2 \quad r\,2y^3$
4. $-\frac{1}{4^x}$
5. $3x - 2x^2$
6. 2
7. $d^2 + 3d - 4$



Answers to Test 6

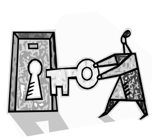
1. $4r^3s - 2r^2$
2. $5x^3 - 3x^2y + 2xy^2 + y^3 + \frac{10y^5}{4x^2 - 2y^2}$
3. $3h^2 + 3h + 1$ vegetables
4. By division of polynomials

$$\begin{array}{r l} & x^2 - xy + y^2 \\ x + y \,\big)\!\!\!\! & \overline{x^3 + ox^2y + oxy^2 + y^3} \\ - & \underline{x^3 + x^2y} \\ & -x^2y \\ & \underline{-x^2y - xy^2} \\ & xy^2 + y^3 \\ - & \underline{xy^2 + y^3} \end{array}$$

by checking:

$$(x + y)(x^2 - xy + y^2) = x^3 + y^3$$

therefore (x+y) is a factor of $x^3 + y^3$

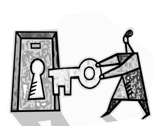


Answers to Activity 7

A. 1. $6a^2 - 11ab - 35b^2$
   2. $4x^2 + 13x - 12$
   3. $x^2 - 36$
   4. $4x^2 + 4xy + y^2$
   5. $4b^2 + 4c^2 + 4d^2 + 8bc + 4bd + 4cd$

B. 1. 4xy
   2. $9x^2$
   3. -2xy
   4. $4y^2 + 6zy + 9z^2$
   5. [(a-b) – (c-d)]

C. $a^2 + b^2 + c^2 + 2ab + 2ac + 2bd$



Answers to Test 7

A. 1. $x^4 - 25x^2 + 144$
   2. $49a^4 - b^4$
   3. $\frac{1}{4}a^2 + ak + k^2$
   4. $16x^3 - 40ax + 25x - 64x^2 + 160a$
   5. $\frac{16}{9}d^4 - h^2$

Answers to Activity 8

A. 1. $4g^2 - s^2$

2. $m^6 - 9f^4$

3. $4v^2y^2 - y^2r^2 - 100v^2 + 25r^2$

4. $81a^2 - 36aq + 4q^2$

5. $4a^2 - 4ac + c^2 - 25f^2$

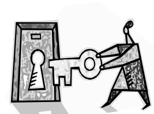
6. $9x^4 - 24x^2 + 16$

7. $\frac{16}{9}x^4 - y^2$

8. $64a^2 - 32ab + 4b^2 + 48a - 12b + 9$

9. $p^6 - 15p^4q^2 + 37p^2q^4 - 81q^6$

10. $a^2 + 2ab + b^2 + c^2 + d^2 - 2\,de + e^2 + 2ac + 2bc + 2ad - 2ae - 2bd - 2be + 2cd - 2ce$



Answers to Test 8

A.1. True

2. False, $x^2 + x - 6$

B. 1. 8xy

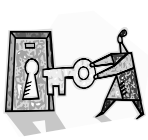
2. -28x

C. 1. $v^6 + 21u^4v + 147u^2v^2 + 343v^3$

2. $x^{10} - 2x^8 + x^6 - x^4 + 2x^3 - x^2$

3. $9x^2 - 42xy + 49y^2$

4. $81p^4 + 144p^2r^4 + 16r^8 + 216p^3r^2 + 72p^2r^4 + 96pr^6$



Answers to Activity 9

A. 1. $2a^7b^3(3 - 167ab)$

2. $4(6b - 5)(6b + 5)$

3. $3c(b - 3d)(b^2 + 3bd + 9d^2$

4. (x+6)(x-4)

5. $[2(x - 2y) + 3][5(x - 2y) + 2]$

6. $(3k + 5m)^2$

7. $(4ab - 3c)(16a^2b^2 - 12abc + 4c^2)$

8. $4x(3a + 5b + 8c)$

9. $(c - 3)(c + 1)$

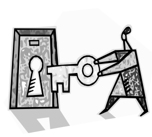
10. $(5x + 2)(x - 3)$

11. $(2x + b)(a + 3c)$

12. $(a + 2b)(a + 1)$

13. $(a - 3b^2)(5a + 4b)$

14. $7abc(4abc + 3ac - 2)$

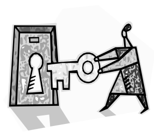


Answers to Test 9

1. (x -3)
2. 12xy
3. (y + 9) (y + 3)
4. $(b - c)(b^2 + bc + c^2)(b^6 + b^3c^3 + c^6)$

B. Distributive property

C. Area of the square = $16y^4 - 16y^3 + y^2$



Answers to Unit Test

I.

1. constant
2. 4
3. $m^2 + 3mn + 7n^2$
4. 19x
5. $2mn(8m^2n + 16mn^3 + 1)$
6. 8
7. $a^2x^2 + 2bax + b^2$
8. 8x - 7
9. $6x^3y^5z^8 - 2xyz$
10. $\frac{8m^2n}{m+n}$
11. $(3a-2)(a+1)$
12. $(z-2)$ m and $(2z-3)$ m
13. $x^2 = 82 - 25x$
14. $(3a + b)$
15. $\frac{M+N+O+P}{4}$

II A.

1. $12p^3q + 18p^2q + 10pq$
2. $a - 2b + c$
3. $5x^3 + 9x^2 - x + 6$
4. $-6x^2 - 5xy + 6y^2$
5. $6x^2y^2 - 4xy + 2$

B.

1. $4x^2 - 9$
2. $16y^4 - 16y^2z + 4z^2$
3. $32a^2b^2 - 4a^2b + 4abc$
4. $(x+y)^2 - (a+b)^2$ or $x^2 + 2xy + y^2 - a^2 - 2ab - b^2$
5. $x^2 + 12x + 27$

C.

1. $(a-7)(a-3)$
2. $(2g+5)(2g+5)$
3. $(7m^2-4)(7m^2+4)$
4. $(3x+2)(9x^2-6x+4)$
5. $2x(x+3)(2x+1)$

Common Errors / Misconceptions in Unit III

1. Application of the Law of Exponents

Example:

$$\left(3^3\right)\left(3^4\right) = 3^{12}$$

$$2x^5 + 3x^5 = 5x^{10}$$

In this kind of problem it is very common among students to multiply the exponent whenever they are multiplying expression instead of adding the exponent and copy the common base. and for the addition of algebraic expression the students usually add the exponent instead of just copying the common exponent. Thus the right answer to this problem is. 3^7 and for the second problem it should be $5x^5$

2. Special Products or multiplication of binomials

Example: $(x+y)^2 = x^2 + y^2$ here the student usually distribute the exponent to every term but this process is wrong it should be $(x+y)^2 = x^2 + 2xy + y^2$

3. Simplifying polynomials especially on operations of integers removing grouping signs

$$\left(3x^2 + 5x - 5\right) + \left(5x^2 - 7x + 9\right) = 3x^2 + 5x - 5 - 5x^2 + 7x - 9$$

This process is wrong since the grouping sign is preceded by a positive sign hence no need of changing the signs of the values inside the parenthesis. This process is applicable only if the grouping sign is preceded by a negative sign.