BUREAU OF SECONDARY EDUCATION
DEPARTMENT OF EDUCATION

DISTANCE LEARNING MODULE
MATHEMATICS 1

UNIT I

REAL NUMBER SYSTEM, MEASUREMENT AND SCIENTIFIC NOTATION

The picture below shows the mathematics at work. operations in the real number. Other photo shows a balance scale for measurement. Students measure the length and width of a table.

Mark, Jomar, and Raul wanted to treat Christine to a Burger house. Each of them gave P85.00 for a total of P255.00. They bought 3 or 4 breakfast value meals which cost P75.00 but a value-added tax (VAT) of P22.50 was charged.

How much change was received by each of Mark, Jomar, and Raul?

Hence, each of them paid P75.50. Computing gives:

P75.50+ P75.50 + P75.50 + P22.50 = P249.00

↓ ↓ ↓ ↓

Mark Jomar Raul VAT

What happened to the other P2.00?

After this unit, you will be able to answer this question. This unit deals with the development of the Real Number System. Properties of real numbers and the laws governing operations on real numbers are presented so that you have the basic tools necessary to understand the concepts of real numbers.

Almost everyday, you are confronted with questions like “How far is your house from the school? How big is the ball? How long is the edge of the table? How many books do you have? To answer these questions you need to possess some measurement skills.

Measurement of length, weight, volume and temperature are important to everyone's life. Other measurements that are of equal importance are related to water and electric consumptions. These are measured with special units.

Measuring is never exact and you will need to decide how accurate the measurement should be. You will make use of some standard unit of measurement. However you need to learn not only to measure with the proper unit but also to estimate with some degree of accuracy. It is true many people go through life without knowing much about measurement. Would you want to be one of them?

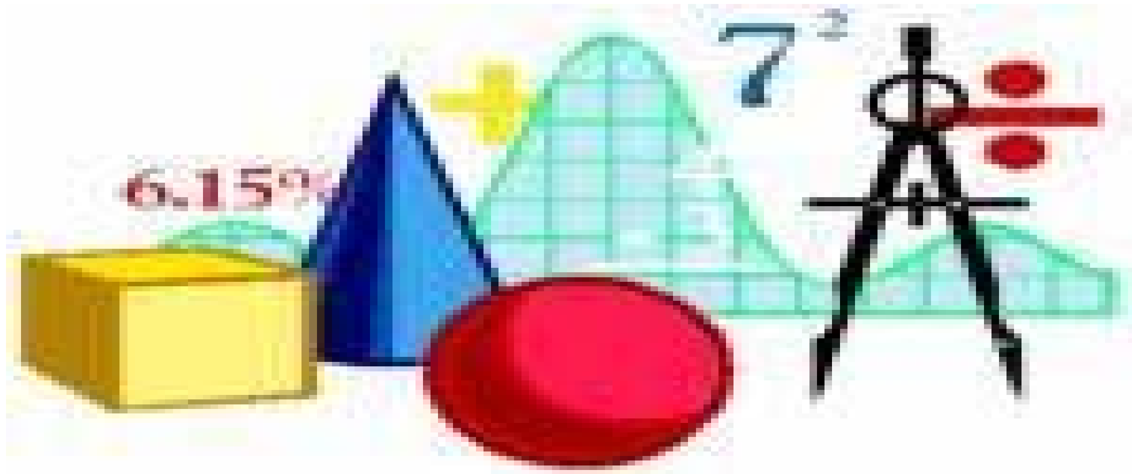
Measurement is also discussed in this unit in order for you to acquire the measurement skills and use them in your daily life.

Did you watch the boxing encounter between Manny Pacquiao and Eric Morales where the National Anthem was being played live and also broadcast over a radio system? You heard it over a radio sooner than you did in person. This is because radio waves travel 300,000,000 meters per second.

Large numbers like 300,000,000 can be written in a much easier and shorter way. This is by expressing the number in scientific notation.

Scientific notation is very important to engineers and scientists as well as to astronomers as it saves one from unconsciously omitting a zero or misplacing the decimal point in writing large or small numbers.

Real Number System

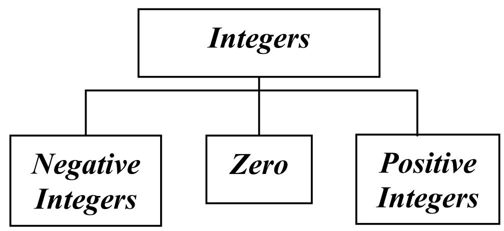


Natural numbers or counting numbers are the numbers you have learned first. These are the numbers 1, 2, 3, …. Later you learned how to add, subtract, multiply, and divide these numbers. However, dividing these numbers such as 20 ÷ 4 or 42 ÷ 6 resulted in whole numbers as quotients. To name the result when a number is subtracted from itself, the ***whole numbers***, 0, 1, 2, 3, … were introduced.

In arithmetic, whole numbers and zero are considered problems in this system of numbers. You cannot subtract a large number from a smaller number to get an answer that is zero or greater. To solve this problem, mathematicians introduced negative numbers or numbers less than zero. For every positive number there exists a number that is the negative of the positive number, more popularly known as the opposite.

Thus, to name the result when a bigger number is subtracted from a smaller number, the ***integers*** were introduced.

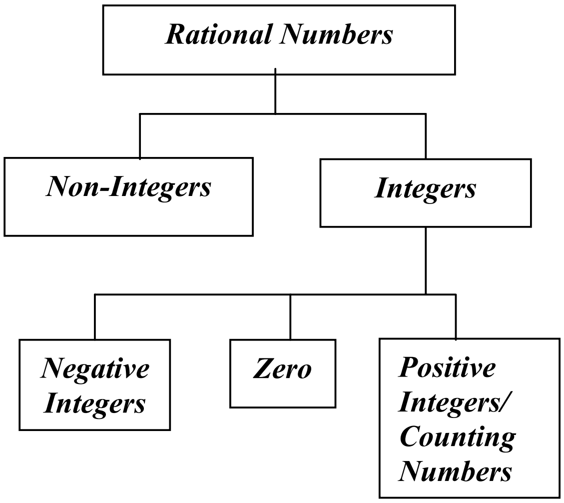
If you consider the corresponding positive and negative numbers, you have the ***Set of Integers***. Below is a diagram of the set of integers:



Knowing the natural numbers, whole numbers and integers, dividing numbers such as 8 ÷ 3 or 15 ÷ 4 is still a problem in the number system since these numbers will not result in whole numbers as quotients. Thus, a new set of numbers known as ***fractions*** was introduced to give meaning to the result of such division.

If you include the entire set of integers and positive and negative fractions you have the ***Set of Rational Numbers***.

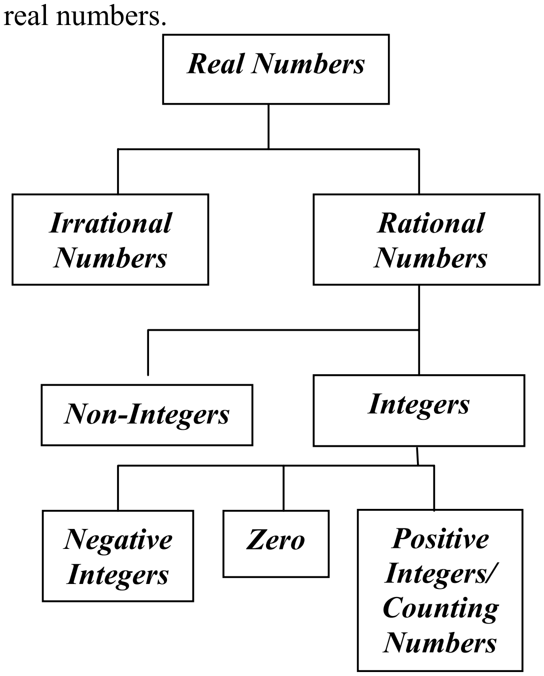
Below is a diagram of the set of rational numbers.



A ***rational number*** is a number that can be expressed as the quotient of two integers such as -3, $\frac{1}{3}$, 0.5.

The counterpart of the set of rational numbers is the set of irrational numbers. ***Irrational Numbers*** are those which cannot be expressed as the ratio of two integers such as $\pi, \sqrt{2}$, 3.012….

The rational numbers together with the set of irrational numbers comprise the entire ***Set of Real Numbers***. Below is a diagram of a set of real numbers.



Rational Numbers

A number that can be written as a quotient of two integers where the denominator is not zero is a rational number.

Example 1. Express each rational number as a quotient of two integers.

a) 12

b) $3\frac{6}{11}$

c) 25%

d) 0.7

Example 2. Express each rational number as a quotient of two integers.

e) 4 apples to a dozen apples

f) 80 minutes to 2 hours

Solution:

a) Any whole number can be written as a fraction with a denominator of 1.

$$12 = \frac{12}{1}$$

b) Any mixed number can be written as a fraction.

$$3\frac{6}{11} = \frac{39}{11}$$

Multiply 11 to 3 to get 33 then add the numerator 6.

Recall

c) Percent means by the hundreds. Any percent can be expressed as a fraction.

To change percent to fraction, write the given number over 100, then simplify

$25\% = \frac{25}{100}$ write over 100 and remove the % sign

$= \frac{1}{4}$ simplify by dividing both numerator and denominator by 25 d). A decimal number can be expressed as a fraction. The number of decimal places in the given number indicates the number of zeros in the denominator

$0.7 = \frac{7}{10}$ write 7 over 10 if the given is tenth

$0.07 = \frac{7}{100}$ write 7 over 100 if the given is hundredth

$0.007 = \frac{7}{1000}$ write 7 over 1000 if the given is thousandth

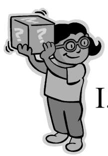
Recall:
Example 2:

d) One dozen = 12

$\frac{4apples}{12apples} = \frac{4}{12} = \frac{1}{3}$

e) 60 minutes = 1 hour
120 minute = 2 hours

$\frac{80}{120} = \frac{8}{12} = \frac{2}{3}$



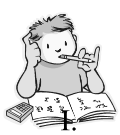
<u>Let's practice for mastery 1</u>

I. Express each number as a quotient of two integers and determine if each is rational. Explain your answer.

1. 42
2. $-6\frac{1}{7}$
3. -2.6
4. 23%
5. 0.3

II. Express the following relationships as fractions.

6. 5 oranges to a dozen oranges
7. fifty centavos to one peso
8. 3 days to one week
9. 45 minutes to 1 hour
10. 3 months to 1 year



<u>Let's check your understanding 1</u>

I. I. Express each number as a quotient of two integers and determine if each is rational. Explain your answer.

1. 8
2. $-8\frac{3}{4}$
3. 6.7
4. 67%
5. 0.18

II. Express the following relationships as fractions.

6. 12 mangoes to one dozen mangoes
7. 90 centavos to two pesos
8. 12 days to one week
9. 35 minutes to two hours
10. 12 months to 3 years

- *After answering the test, check your answers with those on the answer key page.*
- *If your score is 7 or higher, you may proceed to the next lesson; otherwise, read the lesson once for the missed items.*

Irrational Numbers

Numbers that cannot be written as a quotient of two integers are called ***irrational numbers***. In decimal form, these numbers are nonterminating and nonrepeating.

Examples:

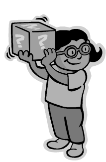
1. $\sqrt{2}$ no exact value for $\sqrt{2}$ since there is no number when multiplied by itself will give exactly 2. $\sqrt{2}$ is read as "the square root of two".

2. $\sqrt{17}$ no exact value for $\sqrt{17}$

3. 3.666… } non repeating /
4. 7.012… } non terminating decimals

Numbers like $\sqrt{3}$, $\sqrt{5}$, and $\sqrt{7}$ are irrational numbers because there is no number when multiplied by itself equals 3, 5 and 7. Hence, their exact values cannot be expressed as either terminating or repeating decimals. However, you can use a calculator or the table of square roots to find their decimal approximation.

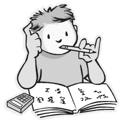
Numbers whose roots cannot be extracted are not the only irrational numbers. For example, π is an irrational number which is approximately 3.1415926.



<u>Let's practice for mastery 2</u>

Determine if the following numbers are irrational or rational.

1. $\sqrt{48}$
2. $\sqrt{20}$
3. $\sqrt{64}$
4. $\sqrt{72}$
5. $\sqrt{121}$



<u>Let's check your understanding 2</u>

Determine if the following numbers are irrational or rational.

1. $\sqrt{32}$
2. $\sqrt{576}$
3. $\sqrt{24}$
4. $\sqrt{80}$
5. $\sqrt{\frac{1}{4}}$

- *After answering the Test, check your answers with those on the answer key page.*
- *If your score is 3 or higher, you may proceed to the next lesson; otherwise, read the lesson once more for the missed items.*

Decimal Form of Rational Numbers

Since every rational number can be expressed as a/b or $a \div b$, where a and b are integers and $b \neq 0$, we can divide a and b to obtain a ***decimal number***. If the remainder is zero the decimal is called a ***terminating decimal***.

Example 1:

Let us write $\frac{3}{8}$ as a decimal.

Solution:

```
   0.375
8 |3.000     Move the decimal point
   24         straight up add as many
    60        zeros as necessary.
    56
     40
     40
      0
```

The division shows that $\frac{3}{8}$ can be expressed as a terminating decimal 0.375.

If the remainder zero is not reached when dividing the numerator by the denominator, continue to divide until the remainders begin to repeat.

Example 2:

Express each rational number as a decimal.

a) $\frac{7}{6}$ b) $\frac{12}{99}$ c) $\frac{23}{7}$

Solution:

a) $\frac{7}{6}$

```
   1.166
6 |7.000
   6
   1.0
     6
     40
     36
     40
```

Therefore $\frac{7}{6} = 1.1666\ldots$

b) $\frac{12}{99}$

```
     0.1212
99 |12.0000
    99
     210
     198
     120
      99
     210
     198
     120
```

Therefore, 12/99 = 0.1212…

c) $\frac{23}{7} = 7\overline{)23.}$

Solution: 3.2857

```
     3.28571
  7)23.00000
    21
    ----
     20
     14
     ----
      60
      56
      ----
       40
       35
       ----
        50
        49
        ----
         10
```

Therefore, $\frac{23}{7} = 3.285714285714\ldots$

The dots shown in the quotient indicate that the decimals continue without end. Hence, the decimal quotients are ***repeating or non-terminating decimals*** because the same digit or block of digits repeats indefinitely. A ***bar*** is used to indicate the block of digits that repeat.

a) $\frac{7}{6} = 1.1666\ldots$

$= 1.1\overline{6}$

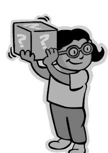
b) $\frac{12}{99} = 0.121212\ldots$

$= 0.\overline{12}$

c) $\frac{23}{7} = 3.285714285714\ldots$

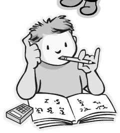
$= 3.\overline{285714}$

Let's practice for mastery 3



Write each rational number as a decimal and determine if it is a terminating or repeating decimal.

1. $\frac{1}{4}$ 2. $\frac{2}{3}$ 3. $\frac{3}{11}$ 4. $\frac{7}{10}$ 5. $\frac{5}{12}$



Let's check your understanding 3

Write each rational number as a decimal and determine if it is a terminating or repeating decimal.

1. $\frac{89}{100}$ 2. $\frac{4}{11}$ 3. $\frac{9}{2}$

- *After answering the Test, check your answers with those on the answer key page.*
- *If your score is 2 or higher, you may proceed to the next lesson; otherwise, read the lesson once more for the missed items.*

Decimal Form to Fractional Form

In the preceding lesson, we changed fractions to decimals. How about the reverse process, that is changing decimals to fractions? Does a terminating decimal represent a rational number? Can decimals be written as fractions? Find out.

Example 1:

Write each terminating decimal as a fraction.

1. 0.9 3. 8.2
2. 0.536 4. 2.25

Solution:

1. $0.9 = \frac{9}{10}$
2. $0.536 = \frac{536}{100}$ or $\frac{67}{125}$
3. $2.25 = \frac{225}{100}$ or $\frac{9}{44}$

If you notice from the examples, if a terminating decimal is given, move the decimal point to the right to make it an integer. This will be the numerator. The denominator will be 10, 100, 1000, and so on, where the number of zeros depends upon the number of decimal places the decimal point is moved to the right.

Example

Fractions should always be reduced to its lowest term. To reduce fraction to its lowest term, divide both numerator and denominator by their greatest common factor (GCF).

Example:

Write $\frac{12}{18}$ in lowest term.

Solution:

Find the GCF of the numerator and denominator.

The GCF of 12 and 18 is 6.

$$\frac{12}{18} = \frac{12 \div 6}{18 \div 6} = \frac{2}{3}$$

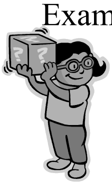
Greatest Common Factor

If the factors of the numbers 30 and 42 are listed, the numbers 1, 2, 3 and 6 appear in both lists.

Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30

Factors of 42: 1, 2, 3, 6, 7, 14, 21, 42

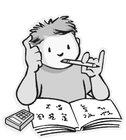
These numbers are called common factors of 30 and 42. The number 6 is the GCF and is therefore called the greatest common factor of the two numbers. We write, GCF (30, 42) = 6 to denote the greatest common factor of 30 and 42.



Let's practice for mastery 4

Express each terminating decimal as a fraction.

1. 0.4 2. 0.45 3. 0.225
4. 0.63 5. 0.984

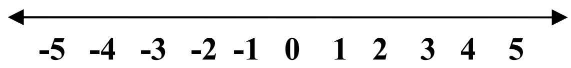


Let's check your understanding 4

Express each terminating decimal as a fraction.

1. 0.15 2. 0.6 3. 0.473
4. 0.2 5. 0.08

Set of Integers



Joan is a first year student. On weekends, She earns a little pocket money by selling ice candies to her neighbor. On some days when it is too warm, some ice candies melt. In order that they will not be wasted completely she and her family consume or eat the ice candies, thereby incurring some losses.

Below is the record of her sales of ice candies for three weekends

Date	Sales
April 9	+20
April 10	+15
April 16	-18
April 17	-12
April 23	+25
April 24	+20

Notice that Joan used the number symbols +20, +15, -18, -12, and +25 which are called integers. She used +20, +15, +25 to represent the gains she made, and -18, -12 indicate the losses she incurred.

The ***set of integers*** consists of the ***positive numbers***, the ***negative numbers***, and ***zero***.

Uses of Integers

Integers can be associated with many real-world problems and situations. How long before 1992 A.D. were the pyramids of Egypt built? You can think of B.C. years as negative and A.D. years as positive. The following examples will help us represent concepts in many fields.

Here are some examples.

1. deposit of 85 pesos as +85 or simply 85
2. withdrawal of 84 pesos as -84
3. rise of 5in temperature as +5 or simply 5
4. drop of 5 in temperature as -5
5. distance of 45 km traveled going east as +45 or simply 45
6. distance of 45 km traveled westward as -45
7. 40m above sea level as +40 or simply 40
8. 40m below sea level as -40

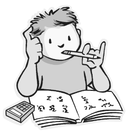
Let's practice for Mastery 5

Represent the following in integers. Use the (+) sign and the (-) sign.

_____ 1. profit of P700.00
_____ 2. loss of P150.00
_____ 3. gain of 15 grams in weight
_____ 4. reduction of 50 kilowatts in the electric meter reading
_____ 5. overtime pay of P1,000.00

Give an answer for each.

______ 6. If 32 represents a deposit of P32.00, what does –32 represent?
______ 7. If 280 means 280 feet above sea level, what does -280 mean?
______ 8. If -25 represents 25 km south, what does 25 represent?
______ 9. If -20 represents 20 seconds before departure, what does 20 represent?
______ 10. If -P5,000.00 represents liabilities, what does P5,000.00 represent?



Let's check your understanding 5

Represent the following in integers. Use the (+) sign and the (-) sign.

______ 1. A growth of 5 meters
______ 2. A reduction of 2 kg in weight
______ 3. The elevation is 1,125 m
______ 4. It is 40 minutes after takeoff
______ 5. The jet is flying at 33,750 feet

Give an answer for each.
______ 6. If -30 represents 30 minutes late, what does 30 mean?
______ 7. If 20 cm represents an increase in height, what does -20 represent?
______ 8. If -12 is to the left of zero, where is 12 located with regards to 0?
______ 9. If 5 km represents going upstream, what does -5 mean?
______ 10. If P250.00 represents earning, what does –P250.00 represent?

Order of Real Numbers

A number line can be used to order numbers. Note that as you move to the right of zero along the number line, the numbers increase in value and as you move to the left of zero, the numbers decrease in value. One number is greater than the second if the first lies to the right of the second on the number line.

Let us now study how to determine when a rational number is greater than another.

Comparing Fractions

The following models of fractions help you determine which fractions are greater when two or more fractions are compared.

1. Consider the fractions, $\frac{5}{12}$, $\frac{1}{12}$, $\frac{7}{12}$, $\frac{11}{12}$. *(Pls. insert visuals of the fractions)*

a. What do you notice about their denominators?

b. Which fraction is the greatest?

c. Which fraction is the smallest?

When fractions have the same **denominator,** the greater the numerator, the greater the value of the fraction.

For example,

a) $\frac{5}{8} > \frac{3}{8}$ b) $\frac{7}{15} < \frac{12}{15}$

$\frac{1}{2}$ x $\frac{3}{3}$ = $\frac{3}{6}$

$\frac{4}{6} > \frac{3}{6}$

2. Consider the fractions $\frac{5}{8}$, $\frac{5}{9}$, $\frac{5}{12}$, $\frac{5}{18}$.

a. What do you notice about their numerators?

b. Which fraction is the greatest?

c. Which fraction is the smallest?

When fractions have the same ***numerator***, the greater the denominator the smaller is the value of the fraction

For example,

a) $\frac{5}{12} < \frac{5}{15}$ b) $\frac{3}{5} > \frac{3}{7}$

Comparing fractions with different numerators and different denominators is easy if you change them first to equivalent fractions with the same denominators.

Consider the fractions $\frac{3}{5}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{4}{5}$

What do you notice about their denominators?

Equivalent fractions are fractions having the same denominators.

For example,

Which is greater $\frac{1}{2}$ or $\frac{2}{3}$?

(Pls. insert visuals of equivalent fractions)

Multiply both numerator and denominator of each fraction by a certain number to make them equivalent fractions.

You can choose whatever method that is convenient for you.

More Examples:

Determine which is greater $\frac{3}{4}$ or $\frac{5}{6}$.

$\frac{2}{3}$ x $\frac{2}{2}$ = $\frac{4}{6}$

Therefore $\frac{2}{3} > \frac{1}{2}$

Another way to compare them is to change them to decimal number.

$\frac{1}{2} = 0.5$

$\frac{2}{3} = 0.6666\ldots$

So, 0.6666… is greater than 0.5.

Therefore $\frac{2}{3} > \frac{1}{2}$.

Solution:

$\frac{3}{4} = 0.75$

$\frac{5}{6} = 0.833\ldots$

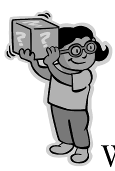
Therefore $\frac{5}{6} > \frac{3}{4}$.

1. Write <, =, or > on the blank to make a true statement.

a) $\frac{3}{5}$ _>_ $\frac{4}{7}$ b) $\frac{7}{4}$ _<_ $\frac{9}{4}$

↓ ↓

0.6 0.57142



Let's practice for mastery 6

Which fraction in each pair is greater?

_____ 1. $\frac{2}{5}, \frac{3}{5}$ _____ 3. $\frac{2}{3}, \frac{3}{4}$ _____ 5. $\frac{2}{3}, \frac{4}{7}$

_____ 2. $\frac{5}{7}, \frac{5}{9}$ _____ 4. $\frac{1}{2}, \frac{4}{5}$

Write <, =, or > on the blank to make a true statement.

1. $\frac{3}{7}$ ____ $\frac{2}{7}$ 3. $\frac{7}{9}$ ____ $\frac{6}{9}$ 5. $\frac{8}{3}$ ____ $\frac{9}{4}$

2. $\frac{6}{11}$ ____ $\frac{6}{12}$ 4. $\frac{8}{8}$ ____ $\frac{12}{12}$

Problem Solving:

The UST High School basketball team won 10 out of its 13 games. The Araullo High School basketball team won 9 out of 11 games. Which school has the better record?

Let's check for your understanding 6

Which fraction in each pair is lesser?

_____ 1. $\frac{5}{12}, \frac{3}{5}$ _____ 2. $\frac{7}{9}, \frac{8}{9}$

Write <, =, or > on the blank to make a true statement.

3. $\frac{8}{12}$ ____ $\frac{3}{15}$ 4. $\frac{5}{6}$ ____ $\frac{7}{8}$

Problem Solving:

Adamson High School won $\frac{5}{8}$ of the baseball games that it played. Magsaysay High School won $\frac{3}{4}$ of the total number of its baseball games. Which school had a better record? Why?

- *After answering the Test, check your answers with those on the answer key page.*
- *If your score is 3 or higher, you may proceed to the next topic; otherwise, read the lesson once more for the missed items.*

Comparing and Ordering Integers

-5 -4 -3 -2 -1 0 1 2 3 4 5

Consider the given number line.

a. Where can we locate 5 with respect to 2? (5 is located at the right of 2.)
b. Which is greater 5 or 2? (5 is greater than 2.)

In comparing two integers, we compare their positions on the number line. For any two integers on a number line, the number farther to the right is greater.

Looking at the number line above, 5 > 2; 10 > 3.

Consider the numbers 4 and -1 in the number line, which do you think is greater? (4 is greater than -1.)

Any positive integer is greater than any negative integer like 4 > -1; 1 > -4; and 3 > -5.

Zero is greater than any negative integer for example, 0 > -1; 0 > -4.

How can we compare two negative integers? Which is greater (-3) or (-5)?

From the number line you will notice that (-3) is to the right of (-5), so -3 > -5.

Which is greater (-2) or (-4)?

Of two positive or two negative integers, the one located to the right of the other in the number line has the greater value.

More Examples:

1. Draw a number line to illustrate the following.
   a. -1 > -5 b. -6 < -3
   c. -2 > -4

2. List 8, -4, -12, and -1 in order from least to greatest.

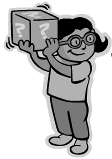
Solution:

Think of the given numbers on the number line. The farther to the right the greater the number. Thus, you have -12, -4, -1, 8.

3. List -5, 4, 0, -2, 10, -6 from the greatest to the least.

Answer:

10, 4, 0, -2, -5, -6



Let's practice for mastery 7

Pick out the greater number in each pair.

_____ 1. (-12), (-18)   _____ 2. (-4), (10)   _____ 3. 0, 5

Compare these integers. Use < or > in the ☐.

4. 8 ☐ -16   5. -9 ☐ - 4   6. -72 ☐ -64

Arrange the following integers in increasing order.

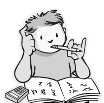
7. -13, -15, -19, -12, -1 ____________________

8. -17, 18, 0, -8, -5 ____________________

Arrange the following integers in decreasing order.

9. -6, 3, -8, 6, -7, -9, 10 ____________________

10. Who is closer to the ground, a man who is on a hill 207 m above sea level or a man who is in a mine shaft 210 m below sea level?



Let's check your understanding 7

Pick out the greater number in each pair.

_____ 1. (-3), (8)   _____ 2. (-9), (-15)   _____ 3. (-5), (5)

Compare these integers. Use < or > in the ☐.

4. -14 ☐ -34   5. -8 ☐ -13   6. -7 ☐ 7

Arrange the following integers in increasing order.

7. 25, -10, -32, -41, 15 ____________________

8. -4, -2, -9, 4, 9, 6, -5 ____________________

Arrange the following integers in decreasing order.

9. -15, 15, 16, -16, -17, 17 ____________________

10. Who did better, a student whose score was 35 below the passing score or one whose score was 20 above the passing score? What is the difference in their scores?

- *After answering the Test, check your answers with those on the answer key page.*
- *If your score is 7 or higher, you may proceed to the next topic; otherwise, read the lesson once more for the missed items.*

Basic Properties of Real Numbers

Recall several basic properties in addition and multiplication of real numbers.

A. Closure Property

Each pair of real numbers has a unique (one and only) sum which is also a real number.

Examples: 1) $7+6=13$
2) $19+8=27$

Each pair of real numbers has a unique product which is also a real number.

Examples: 1) $9\times5=45$
2) $6\times5=30$

B. Commutative Property

Adding two real numbers will give the same sum no matter in what order the numbers are added.

Examples: 1) $6+8=8+6$
$14=14$
2) $12.5+6.2=6.2+12.5$
$18.7=18.7$

Multiplying two real numbers will give the same product no matter in what order the numbers are multiplied.

Examples: 1) $9\times3=3\times9$
$27=27$

2) $6\times8=8\times6$
$48=48$

Examples: 1) $1\times8=8$
2) $12\times1=12$

C. Associative Property

Adding three or more real numbers will give the same sum no matter how the numbers are grouped.

Examples: 1) $(2+5)+9=2+(5+9)$
$7+9=2+14$
$16=16$

2) $4+(8+3)=(4+8)+3$
$4+11=13+3$
$15=15$

Multiplying three or more real numbers will give the same product no matter how the numbers are grouped.

Examples: 1) $\left(9\times\frac{1}{3}\right)\times18=9\times\left(\frac{1}{3}\times18\right)$
$3\times18=9\times6$
$54=54$

2) $3\times(2\times4)=(3\times2)\times4$
$3\times8=6\times4$
$24=24$

D. Identity Property

Any number added to 0 is equal to the given number. Zero is called the ***additive identity***.

Examples: 1) $0+9=9$
2) $28+0=28$

Any number multiplied by 1 is equal to the given number. **1** is called the ***multiplicative identity***.

2) $3(5+8)=(3\times5)+(3\times8)$
$3(13)=15+24$
$39=39$

E. Inverse Property

The sum of a real number and its opposite is 0. The number opposite the given real number is called the ***additive inverse***.

Examples 1:

1) -8 is the additive inverse of 8.

2) 12 is the additive inverse of -12.

Examples 2:

1) $5+(-5)=0$

2) $-10+(10)=0$

The product of a real number and its reciprocal is 1. The reciprocal of the given number is called the ***multiplicative inverse.***

Examples 1:

1) $\frac{1}{4}$ is the multiplicative inverse of 4.

2) -8 is the multiplicative inverse of $-\frac{1}{8}$.

Examples 2:

1) $12 \times \frac{1}{12}=1$

2) $-\frac{1}{5} \times (-5) = 1$

G. Distributive Property

Multiplication is distributive with respect to addition.

Examples:

1) $7(6\times 9) = (7\times 6)+(7\times 9)$

$7(15) = 42 + 63$

$105 = 105$

Operations with Real Numbers

Fractions

A fraction whose numerator is greater than or equal its denominator is called *improper fraction.* A *proper fraction* is a fraction whose numerator is less than its denominator

Examples:

Proper Fractions	Improper Fractions
$\frac{1}{4},\frac{2}{3},\frac{5}{9},\frac{10}{12},\frac{17}{18}$	$\frac{5}{2},\frac{8}{3},\frac{12}{5},\frac{18}{15},\frac{17}{4},\frac{5}{5}$

A number that is expressed as the sum of a whole number and the fraction is called a *mixed number.*

Examples:

$4\frac{1}{2},\ 5\frac{1}{4},\ 7\frac{3}{4}$

These are sometimes referred to as numbers in mixed forms.

To change an improper fraction to mixed number in simplest form, we divide the numerator by the denominator and expressed the remainder as a fraction.

In general, the result of dividing can be written as follows:

$\frac{P}{D} = Q + \frac{R}{D}$, where P = Any real number except zero

D = Divisor

Q = Quotient

R = Remainder

Example:

Change $\frac{14}{3}$ to a mixed number.

Solution:

$\frac{14}{3} = 14 \div 3 = 4\ \textit{remainder}\ 2$

Here it is written as $4\ \frac{2}{3}$

Therefore, $\frac{14}{3} = 4\frac{2}{3}$ in mixed number

To change mixed number to an improper fraction , rewrite it as an indicated sum as shown in the following examples .

Example : Change $1\frac{3}{8}$ to an improper fraction

Solution:

$$1\frac{3}{8} = 1 + \frac{3}{8} = \frac{8}{8} + \frac{3}{8} = \frac{11}{8} \text{ since } 1 = \frac{8}{8}$$

Another solution is shown below,

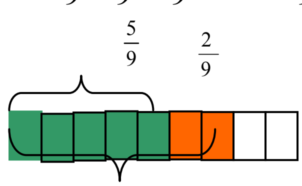
$$1\frac{3}{8} = \frac{(8)(1)+3}{8} = \frac{8+3}{8} = \frac{11}{8}$$

Notice that this is done by multiplying the denominator by the whole number and adding to it the numerator , then copy the numerator

Addition/Subtraction

The diagram below illustrates that

$$\frac{5}{9} + \frac{2}{9} = \frac{7}{9}, \qquad \frac{7}{9} - \frac{2}{9} = \frac{5}{9}$$



This example suggests the following rules for adding and subtracting fractions with the same denominator

To add/subtract fractions/mixed number with the same denominators, add/ subtract the numerators and write over the given denominator.

Example

a) $\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$

c) $3\frac{4}{9} + 1\frac{7}{9} = 4\frac{11}{9}$ or

$4 + 1\frac{2}{9} = 5\frac{2}{9}$

b) $\frac{5}{8} - \frac{2}{8} = \frac{3}{8}$

To add/subtract fractions/mixed numbers with different denominators, change them to fractions having the same denominators by finding the least common denominator LCD, then apply the rules in adding/subtracting fractions with the same denominators.

Example:

1. $\frac{5}{8} + \frac{2}{3}$ LCM of 8 and 3 = 24

$\frac{5}{8} + \frac{2}{3} = \frac{15+16}{24} = \frac{31}{24}$ or $1\frac{7}{24}$

24 ÷ 8 = 3 3 x 5 = 15

24 ÷ 3 = 8 8 x 2 = 16

2. $\frac{6}{7} - \frac{3}{28} = \frac{24-3}{28} = \frac{21}{28}$ or $\frac{3}{4}$

LCM = 28

28 ÷ 7 = 4 x 6 = 24

28 ÷ 28 = 1 x 3 = 3

3. $5\frac{3}{10} + 7\frac{7}{15}$

Solution: LCM = 30

$\frac{30}{10} = 3; \ 3(3) = 9$

$\frac{30}{15} = 2; \ 2(7) = 14$

$5\frac{3}{10} + 7\frac{7}{15} = 12\frac{9+14}{3} = 12\frac{23}{30}$

$5\frac{3}{10} + 7\frac{7}{15}$

What is the difference between least common denominator and least common multiple?

Given:

a. 3, 6, 9

The least common multiple of 3, 6, and 9 is 18.

The least common multiple (LCM) of two or more numbers is the least or smallest number that is the a multiple of two or more non-zero given number

b. $\frac{1}{3}, \frac{1}{6}, \frac{1}{9}$

Multiplying Fractions

To multiply fractions, write the product of the numerators over the product of the denominators then simplify

Example:

1. $\frac{2}{3} x \frac{5}{6} = \frac{10}{18}$ or $\frac{5}{9}$

When the numerator and denominator of either fraction have a common factor, you can simplify before you multiply.

For example,

1. $\frac{12}{14}$ x $\frac{7}{36} = \frac{1}{6}$

The common factor of 12 and 36 is 12 so divide both by 12.

The common factor of 14 and 7 is 7 so divide both by 7.

Then get the product of the numerators and write over the product of the denominators

2. $\frac{15}{24}$ x $= \frac{5}{24} = \frac{6}{18}$

The common factor of 6 and 24 is 6 and the common factor of 15 and 18 is 3.

Therefore, the product is $\frac{5}{24}$ in simplest form.

3. $(5\frac{3}{4})\left(4\frac{2}{3}\right) = \frac{23}{4} X \frac{14}{3}$.

$= 1\frac{161}{6}$ or $26\frac{5}{6}$

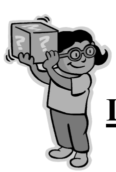
Dividing Fractions

To divide fractions, multiply the dividend by the reciprocal of the divisor. You can use cancellation if you wish to.

Example:

1. $\frac{3}{4} \div \frac{2}{3} = \frac{3}{4}$ x $\frac{3}{2} = \frac{9}{8}$ or $1\frac{1}{8}$

2. $2\frac{2}{3} \div 10\frac{2}{3} = \frac{8}{3} \div \frac{32}{3} = \frac{8}{3} \times \frac{3}{32} = \frac{1}{4}$



<u>Let's practice for mastery 8</u>

A. Perform the indicated operation. Simplify your answers.

1. $\frac{5}{12} + \frac{1}{12}$ = ______
2. $\frac{5}{6} - \frac{1}{6}$ = ______
3. $\frac{1}{9} + \frac{2}{3}$ = ______
4. $\frac{3}{5} + \frac{1}{7}$ = ______
5. $\frac{7}{8} - \frac{3}{16}$ = ______
6. $\frac{1}{2}$ x $\frac{1}{4}$ = ______
7. $\frac{3}{8}$ x $\frac{4}{7}$ = ______
8. $\frac{3}{4}$ x 8 = ______
9. $\frac{2}{3} \div \frac{5}{6}$ = ______
10. $\frac{1}{2} \div \frac{7}{16}$ = ______

11. $3\frac{3}{5} + 1\frac{4}{5} =$ ______

12. $5\frac{7}{8} - 1\frac{3}{8} =$ ______

13. $6\frac{5}{9}$
$+ \frac{7}{9}$
____

14. $3\frac{1}{2} \times \frac{4}{7} =$ ______

15. $\frac{3}{4} \div 2\frac{1}{2} =$ ______

B. :Perform the indicated operations:

1. 8 +3-9 x 2 ÷3, in this type of problem where expressions are written without grouping symbols, to simplify this we use Rule of MDAS. Do all multiplications and divisions in order from left to right then do all additions and subtractions from left to right.

Solution

$8 + 3 - 9 \times 2 \div 3$
$= 8 + 3 - 18 \div 3$
$= 8 + 3 - 6$
$= 11 - 6$

$= 5.$

Try the following:

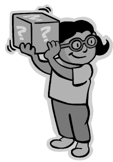
2.) $72 - 24 \div 3$
3) $8 + 3 - 9 \times 2 \div 3$
4) $18 - 3 \times 4 \div 3$
5) $9 \div 3 \times 2 + 8$

<u>Let's check your understanding 8</u>

Perform the indicated operation. Simplify your answers.

1. $\frac{6}{18} + \frac{1}{18} =$
2. $\frac{12}{15} - \frac{5}{15} =$
3. $\frac{2}{3} + \frac{3}{4} =$
4. $\frac{5}{12} + \frac{2}{3} =$
5. $\frac{4}{9} - \frac{1}{6} =$
6. $\frac{5}{8} \times \frac{4}{8} =$
7. $\frac{6}{8} \times \frac{12}{24} =$
8. $\frac{4}{7} \times 7 =$
9. $\frac{3}{7} \div \frac{3}{7} =$
10. $\frac{5}{6} \div \frac{3}{5} =$
11. $3\frac{1}{5}$
$+ 2\frac{4}{5}$
12. $5\frac{3}{4}$
$- 2\frac{1}{4}$
13. $6\frac{2}{5} + 1\frac{1}{5} =$
14. $3\frac{1}{2} \times 9\frac{1}{3} =$
15. $1\frac{2}{3} \div \frac{1}{6} =$

- *After answering the Test, check your answers with those on the answer key page.*
- *If your score is 7 or higher, you may proceed to the next lesson; otherwise, read the lesson once more for the missed items.*



Let's practice for mastery 9

Problem Solving:

Solve the following problems using the operation of fractions. Read and analyze the problem before solving to determine what operation on fractions should be applied.

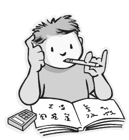
1. Last year was a bad year for Mang Jojo. Typhoon Juaning ruined 3/8 of his cornfields and a few days later another storm, Milenyo destroyed an additional $\frac{1}{10}$ of his cornfields. What fractional part of Mang Jojo's cornfields was destroyed by the two typhoons?

2. Last Monday, John was on duty for $9\frac{3}{4}$ hours. During that time he drove for $6\frac{1}{2}$ hours. How long was he not driving while on duty?

3. Elsa found $\frac{2}{3}$ of a cake in the refrigerator and ate $\frac{1}{2}$ of it. What part of the whole cake did she eat?

4. How many boxes will be needed to contain 3 kilograms of butter if a box holds $\frac{1}{4}$ kilogram?

5. Ryan is making gold and silver earrings. The length of the silver wire hook is $\frac{7}{16}$ inch. The gold earring is $\frac{5}{8}$ inch long. Find the total length of each earring.



Let's check your understanding 9

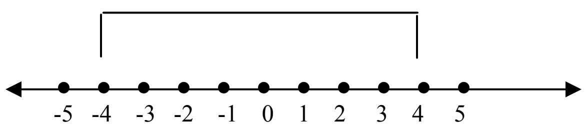
Solve the problems.

1. From a piece of ribbon $\frac{5}{6}$ m long, a $\frac{1}{5}$-meter piece was cut. How much ribbon was left?

2. Wilma is making curtains. She needs two curtains for each window. Each curtain requires $\frac{3}{4}$ m of materials. How many curtains can she finish with 9m material?

Operations on Integers

Absolute Value



The distance from 0 to the graph of a number is called the ***absolute value*** of the number thus 4 and -4 have the same absolute value, 4. The symbol for the absolute value of a number *n*, is $|n|$. We write $|4| = 4$ or $|-4| = 4$. The absolute value of zero is zero

The graph of -4 and 4 are the same distance from zero, but in opposite directions. We call such a pair of numbers <u>opposites</u>. Thus -4 is the opposite of 4 and 4 is the opposite of 4.

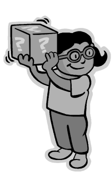
Addition/Subtraction of Integers

To add two integers having the same signs, get the sum of their absolute values and prefix the common sign.

$(5) + (8) = 13$
$(-6) + (-4) = -10$

To add two integers with different signs, get the difference of their absolute values and prefix the sign of the number having the greater absolute value.

$(-10) + (3) = -7$
$(15) + (-6) = 9$



<u>Let's practice for understanding 10</u>

Perform the indicated operation.

_____ 1. 8 + (-12)

_____ 2. -20 + (-13)

_____ 3. Subtract (-10) from (-28)

_____ 4. From 49, subtract (-18)

To subtract two integers, add the opposite of the subtrahend.

$(12) – (4) = 12 + (-4) = 8$
$(-15) – (-3) = -15 + (3) = -12$
$(18) – (-6) = 18 + (6) = 24$
$(-20) – (5) = -20 + (-5) = -25$

Multiplication and Division of Integers

The product of two integers with the same sign is positive.

a. $(5)(6) = 30$ c. $(-2)(-3)(-4) = -24$
b. $(-7)(-8) = 56$ d. $(5)(2)(3) = 30$

The product of two integers with different sign is negative.

a. $(-6)(8) = -48$ c. $(-3)(4)(6) = -72$
b. $(9)(-7) = -63$ d. $(2)(-4)(-3) = 24$

The quotient of two integers with same sign is positive.

$(24) \div (8) = 3$
$(-36) \div (-4) = 9$

The quotient of two integers with different sign is negative.

$(-72) \div (8) = -9$
$(35) \div (-7) = -5$

What should be in the box to make the statement true?

5. (-3) + ☐ = 8

6. (-6) (-4) = ☐

7. (32) ÷ (-8) = ☐

8. (-54) ÷ ☐ = -6

9. An elevator is on the ground floor. It goes up 8 floors, then 5 down, and then 4 up. What is its final position from the ground floor?

10. Ryan weighed 65 kg. He got sick and lost 4 kg on the first week, then 2 kg more on the second week. When he got well he gained 3 kg on the third week and 1 kg on the fourth week. What was his weight after one month?

11. A deep-sea diver plunged 25 m under water, then 23 m deeper. Then the diver rose 15 m. How many meters is the diver below water level?

12. How much greater is (-15) than (-25)?

13. Which is greater -19 or 6? How much is it greater?

<u>Let's check your understanding 10</u>

Perform the indicated operation.

_____ 1. (-15) + (12)

_____ 2. (-16) + (-9)

_____ 3. Subtract (-6) from 14

_____ 4. From 24, take away (-6)

What should be in the box to make the statement true?

5. ☐ – 23 = 9

6. (-15) – ☐ = 0

7. (-5) (9) = ☐

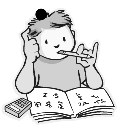
8. An airplane flying at 4,260 m from the ground ascends 1,059 m to avoid a storm. Then it drops 2,115 m and finally ascends 780 m. What is the final altitude?

9. Ma. Jo Ann deposited P2000 in a bank. She withdrew P1050 on the first month, P450 on the second month and deposited P1380 on the third month. How much money did she have in the bank after the third month?

10. At 5 p.m. the temperature in Baguio City is 18°C. The temperature in Manila is 27°C. What is the difference in temperature in:

a) Baguio City to Manila?
b) Manila to Baguio City?

- *After answering the Test, check your answers with those on the answer key page.*
- *If your score is 7 or higher, you may proceed to the next lesson; otherwise, read the lesson once more for the missed items.*



Unit Test I

Answer the following. Solve if necessary.

1. A number that can be expressed as a quotient of two integers is __________.
2. Express "18 oranges to a dozen" in fraction.
3. Express the rational number $\frac{7}{9}$ in decimal.
4. Express 0.012 in fraction.

Represent the following in integers. Use the (+) sign and (-) sign.

5. 5° below freezing point on the Celsius scale
6. An increase of 6 points in the PBA game
7. A 9-yard loss in football
8. A bank deposit of P500

Write <, =, or > on the blank to make a true statement.

9. $\frac{5}{6} \square \frac{3}{4}$ 10. $\frac{7}{8} \square \frac{9}{10}$ 11. $-15 \square -18$ 12. $-30 \square 25$

Perform the indicated operation.

13. $\frac{3}{7} + \frac{2}{7} = \square$ 14. $\frac{5}{6} - \frac{3}{4} = \square$ 15. $\frac{5}{12} \times \frac{3}{5} = \square$ 16. $\frac{8}{12} \div \frac{2}{3} = \square$

17. What is the sum of (-5) and (8)?

18. Is the product of (-8) (9) equal to (-6) (12)?

19. What must be added to -18 to get -3?

20. If -6 is subtracted from -15, the difference is __________?

21. The product of two numbers is -48. If one number is -12, what is the other number?

22. One number is 18 and the other is -5. What is the product?

23. Mario lost 37.5 lbs. If he lost 2.5 pounds each week, how long has he been dieting? Represent and solve the problem using signed numbers.

24. For 5 consecutive months, Pedro withdraws P20,000 from his deposit of P150,000 in the bank to finance his monthly bills. What signed number represents his final financial condition?

25. In the city of Baguio, the temperature was 9°C above zero at dawn. At noon the temperature rose by 11 degrees. What was the temperature at noon?