Republic of the Philippines
OFFICE OF THE PRESIDENT
COMMISSION ON HIGHER EDUCATION

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No. 19
Series 2007

SUBJECT: MINIMUM POLICIES AND STANDARDS FOR BACHELOR OF SCIENCE IN MATHEMATICS AND BACHELOR OF SCIENCE IN APPLIED MATHEMATICS

In accordance with the pertinent provisions of Republic Act (RA) No. 7722, otherwise known as the "Higher Education Act of 1994," and for the purpose of rationalizing the undergraduate mathematics programs in the country in order to keep pace with the advances in the discipline and the demands of globalization, the following rules and guidelines are hereby adopted and promulgated by the Commission.

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ARTICLE I
INTRODUCTION

Section 1

Mathematics has been referred to as the *Queen of the Sciences* by Carl Friedrich Gauss, one of the most brilliant mathematicians of all time. It is a universal discipline with a rich, diverse and dynamic theory that spans a wide range of applications.

Mathematics was borne out of the need to systematically solve real problems. It continues to evolve today because the abstractions generated for solving these problems lead not only to their applications in everyday life but also to further expansions of the abstractions.

Mathematics can be divided into two branches, pure and applied mathematics. Pure mathematics involves the study of structures, their components and the relationships among them. Applied mathematics relates mathematical knowledge to other disciplines.

Consequently, the undergraduate major in mathematics and applied mathematics can be a vital and engaging part of the preparation for many careers and for a well-informed and responsive citizenship.

The minimum policies and standards provided herein aims to guide institutions in their mission of providing the best undergraduate mathematics education possible for their majors. It seeks to ensure a reasonable level of harmony in mathematics and applied mathematics programs, covering minimum competency standards, curricular offerings, course descriptions, library and other resources, and faculty qualifications.

ARTICLE II
AUTHORITY TO OPERATE

Section 2

All private higher education institutions (HEIs) intending to offer the Bachelor of Science in Mathematics/Applied Mathematics must secure proper authority from the Commission in accordance with existing rules and regulations. State universities and colleges (SUCs), and local colleges and universities should likewise strictly adhere to the provisions stated in these policies and standards.

ARTICLE III
PROGRAM SPECIFICATIONS

Section 3 Degree Names

The degree programs herein shall be called Bachelor of Science in Mathematics and Bachelor of Science in Applied Mathematics.

Section 4 Program Description

4.1 Nature of the Program

The undergraduate degree program in mathematics or applied mathematics should be a balance between a holistic general education program and a substantial mathematics or applied mathematics curriculum.

4.2 Objectives

The Bachelor of Science in Mathematics and Bachelor of Science in Applied Mathematics programs shall have the following general objectives:

a. To provide the student with a curriculum that represents the breadth and depth of mathematics, from classical to contemporary, from theoretical to applied. The curriculum shall enhance the student's mathematical and critical thinking skills, and develop in the student a greater appreciation and understanding of the importance of mathematics in history and in the modern world.
b. To prepare the student for advanced studies in mathematics, applied mathematics or related fields;
c. To prepare the student who plans to pursue a career in the academe;
d. To prepare the student for jobs or research work that require analytical-thinking skills.

4.3 Professions/careers/ occupations or trades.

Graduates of BS Mathematics or BS Applied Mathematics often obtain jobs in education, statistics, actuarial science, operations research, finance, and information technology.

Section 5 Allied Programs

Mathematics/applied mathematics is closely related to the fields of statistics, physics, computer science and (computer, electronics and communications, industrial, electrical, civil) engineering.

ARTICLE IV
COMPETENCY STANDARDS

Section 6 Competency Standards

Graduates of a BS Mathematics/Applied Mathematics program are expected to:

a. Develop an appreciation of the power of mathematical thinking and achieve a command of the ideas and techniques in pattern recognition, generalization, abstraction, critical analysis, problem-solving and rigorous argument;
b. Acquire and develop an enhanced perception of the vitality and importance of mathematics in the modern world;
c. Apply analytical and critical reasoning skills to express mathematical ideas with clarity and coherence;
d. Know how to use problem-solving approaches to investigate and understand mathematical content;

e. Formulate and solve problems from both mathematical and everyday situations;
f. Communicate mathematical ideas orally and in writing using clear and precise language;
g. Make and evaluate mathematical conjectures and arguments and validate their own mathematical thinking;
h. Determine the truth or falsity of mathematical statements using valid forms of argument;
i. Appreciate the concept and role of proof and reasoning and demonstrate knowledge in reading and writing mathematical proofs;
j. Show an understanding of the interrelationships within mathematics; and,
k. Connect mathematics to other disciplines and real-world situations.

ARTICLE V
CURRICULUM

Section 7 Curriculum Description

The curricula for the BS Mathematics and BS Applied Mathematics programs should both contain at least 67 units of mathematics beyond the G.E. courses, broken down into 52 units of core courses and 15 units of math or applied math electives.

A BS Mathematics/BS Applied Mathematics program offering a minor or specialization must include at least 15 units of the suggested electives for the specific area of specialization. Minors or specializations may include actuarial science, computing, operations research or statistics. HEIs offering minors or specializations must possess the necessary faculty resources and facilities.

Since the mathematics departments of different schools will have their particular strengths and orientation, the elective courses will allow for flexibility and accommodate the special interests of the various departments.

HEIs may offer mathematics/applied mathematics courses beyond those specified in the recommended programs, according to their faculty and institutional resources, and thrusts.

Section 8 Curriculum Outline

The minimum requirements for a Bachelor of Science in Mathematics and a Bachelor of Science in Applied Mathematics are outlined in Table 1 below.

Table 1. Components of the BS Mathematics and BS Applied Mathematics curricula and their corresponding units.

PROGRAM	COMPONENT	UNITS
BS Mathematics		
	General Education Courses	51
	Core Courses	46*
	Math Electives	(15) 18†
	Free Electives‡	6
	Undergraduate Thesis or Special Problem	(3)
	Total	121

Table 1 continued

BS Applied Mathematics		
	General Education Courses	51
	Core Courses	46*
	Applied Math Electives	(15) 18†
	Free Electives‡	6
	Undergraduate Thesis or Special Problem	(3)
	Total	121

* The Precalculus Mathematics I and II courses listed in the core courses form part of the required 51 units for GEC B (CM 4 series 1997) as GE Mathematics.

†HEIs without a thesis/special problem requirement should have an additional 3-unit elective course.

‡A free elective is any course chosen by a student with the approval of the program adviser/s.

Section 9 General Education Courses

The list of GE courses is in Table 2.

Table 2. GE courses and corresponding units.

FIELDS OF STUDY	SPECIFIC COURSES	UNITS	
1. Language and Humanities	English	6	**21**
	Filipino	6	
	Humanities Subjects (e.g. Literature, Art, Philosophy)	9	
2. Mathematics, Natural Sciences and Information Technology	Mathematics	6	**15**
	Natural Science	6	
	Elective e.g. Information Technology/Natural Science/ Science, Technology and Society (STS)	3	
3. Social Sciences	Consist of subjects such as Political Science, Psychology, Anthropology, Economics, History and the like, provided that the following topics are taken up in appropriate subjects: Taxation and Land Reform, Philippine Constitution, Family Planning and Population Education.	12	**15**
	Life and Works of Rizal (*Mandated Subject*)	3	
	Total		**51**

Section 10 Core Courses

The following core courses (see Table 3) comprise the minimum requirements of the BS Mathematics and BS Applied Mathematics programs.

Table 3. List of core courses for the BS Mathematics and BS Applied Mathematics programs.

PROGRAM	DESCRIPTIVE TITLE	UNITS
BS Mathematics		
	a. Advanced Calculus I	3
	b. Calculus I, II, III *	13 (5,5,3)
	c. Differential Equations I	3
	d. Fundamentals of Computing I	3

	e. Linear Algebra	3
	f. Precalculus Mathematics I and II †	6 (3,3)
	g. Probability	3
	h. Statistics	3
	i. Abstract Algebra I	3
	j. Complex Analysis	3
	k. Fundamental Concepts of Mathematics	3
	l. Modern Geometry	3
	m. Advanced Course in Analysis/Algebra ‡	3
	Total	52
BS Applied Mathematics		
	a. Advanced Calculus I	3
	b. Calculus I, II, III*	13 (5,5,3)
	c. Differential Equations I	3
	d. Fundamentals of Computing I and II	6
	e. Linear Algebra	3
	f. Precalculus Mathematics I and II †	6 (3,3)
	g. Probability	3
	h. Statistics	3
	i. Operations Research I	3
	j. Discrete Mathematics	3
	k. Numerical Analysis	3
	l. Theory of Interest	3
	Total	52

* Calculus I, II, III may be offered as a series of courses with a total of 12-15 units provided all the topics in the recommended syllabi are covered.

† Precalculus Mathematics I and II may be offered as a one-semester 5-unit course with the descriptive title: College Algebra and Trigonometry.

‡ This course may be one of the following: Advanced Calculus II, Real Analysis, Topology, or Abstract Algebra II.

Section 11 Suggested Electives

Electives may be chosen from the recommended list of courses below (see Table 4). Programs with (without) a thesis/special problem should have at least 15 (18) units of electives.

Table 4. List of elective courses for the BS Mathematics and BS Applied Mathematics programs.

PROGRAM	DESCRIPTIVE TITLE	UNITS
BS Mathematics		
	a. Abstract Algebra II	3
	b. Actuarial Mathematics I	3
	c. Actuarial Mathematics II	3
	d. Graph Theory and Applications	3
	e. Differential Equations II	3
	f. Discrete Mathematics	3
	g. Fundamentals of Computing II	3
	h. Mathematical Modeling	3
	i. Number Theory	3

PROGRAM	DESCRIPTIVE TITLE	UNITS
	j. Numerical Analysis	3
	k. Operations Research I	3
	l. Operations Research II	3
	m. Real Analysis	3
	n. Set Theory	3
	o. Topology	3
	p. Statistical Theory	3
	q. Theory of Interest	3
BS Applied Mathematics		
	a. Actuarial Mathematics I	3
	b. Actuarial Mathematics II	3
	c. Mathematical Finance	3
	d. Risk Theory	3
	e. Applied Multivariate Analysis	3
	f. Sampling Theory	3
	g. Statistical Theory	3
	h. Time Series Analysis	3
	i. Linear Models	3
	j. Computational Complexity	3
	k. Data Structures and Algorithms	3
	l. Automata and Computability Theory	3
	m. Theory of Databases	3
	n. Simulation	3
	o. Operations Research II	3
	p. Operations Research III	3
	q. Mathematical Modeling	3
	r. Differential Equations II	3
	s. Fundamental Concepts in Mathematics	3
	t. Graph Theory and Applications	3

Section 12 Free Electives

Free electives are any academic courses offered in the HEI chosen by a student in consultation with the program adviser. They comprise six (6) units of the curricula for the BS Mathematics and BS Applied Mathematics programs.

Section 13 Thesis or Special Problem

Institutions are encouraged to implement a 3-unit thesis or a 3-unit special problem requirement. Both the thesis and the special problem options provide opportunities for students to conduct research on a mathematics topic that builds on areas covered by the core and elective courses.

The thesis/special problem involves activities that include independent reading from mathematical literature and other sources, as well as problem solving. The final paper should contain, organize and present a body of mathematics or a solution to a mathematical problem in a detailed, coherent and original manner.

Section 14 Recommended Program of Study

Tables 5 and 6 below give the recommended programs of study. HEIs may adhere to these, or when necessary, modify the sequencing of courses.

Table 5. Recommended sequence of courses in the BS Mathematics program. (Abbreviations used Lec-Lecture, Lab-Laboratory, GE-General Education, PE-Physical Education, NSTP-National Service Training Program)

BS MATHEMATICS (121 units)								
	First Semester				Second Semester			
Year	Descriptive Title	Units			Descriptive Title	Units		
		Lec	Lab	Total		Lec	Lab	Total
I								
	Precalculus Mathematics I and II	6		6	Calculus I	5		5
	GE Course 1	3		3	Fundamentals of Computing I	3		3
	GE Course 2	3		3	GE Course 4	3		3
	GE Course 3	3		3	GE Course 5	3		3
	PE I		2	0	GE Course 6	3		3
	NSTP		3	0	PE II		2	0
					NSTP		3	0
	Total	15	5	15	Total	17	5	17
II								
	Calculus II	5		5	Calculus III	3		3
	Statistics	3		3	Probability	3		3
	Fundamental Concepts of Mathematics	3		3	Linear Algebra	3		3
	GE Course 7	3		3	Elective 1	3		3
	PE III		2	0	GE Course 8	3		3
					PE IV		2	0
	Total	14	2	14	Total	15	2	15
III								
	Abstract Algebra I	3		3	Modern Geometry	3		3
	Differential Equations I	3		3	Advanced Calculus I	3		3
	Elective 2	3		3	Elective 3	3		3
	GE Course 9	3		3	GE Course 11	3		3
	GE Course 10	3		3	GE Course 12	3		3
	Total	15	0	15	Total	15	0	15
IV								
	Complex Analysis	3		3	Elective 5	3		3
	Advanced Calculus II*	3		3	Free Elective 2	3		3
	Elective 4	3		3	GE Course 14	3		3
	Free Elective 1	3		3	GE Course 15	3		3
	GE Course 13	3		3	Thesis/Special Problem or Elective 6	3		3
	Total	15	0	15	Total	15	0	15

*This course may be one of the following: Advanced Calculus II, Real Analysis, Topology, or Abstract Algebra II

Note: GE courses in the Languages, Humanities, and Social Sciences (including Life and Works of Rizal).

NSTP and PE courses are not included in the total number of units.

Table 6. Recommended sequence of courses in the BS Applied Mathematics program. (Abbreviations used: GE-General Education; PE-Physical Education; NSTP-National Service Training Program)

BS APPLIED MATHEMATICS (121 units)								
	First Semester				Second Semester			
YEAR	Descriptive Title	Units			Descriptive Title	Units		
		Lec	Lab	Total		Lec	Lab	Total
I								
	Precalculus Mathematics I and II	6		6	Calculus I	5		5
	GE Course 1	3		3	Fundamentals of Computing I	3		3
	GE Course 2	3		3	GE Course 4	3		3
	GE Course 3	3		3	GE Course 5	3		3
	PE I		2	0	GE Course 6	3		3
	NSTP		3	0	PE II		2	0
					NSTP		3	0
	Total	15	5	15	Total	17	5	17
II								
	Calculus II	5		5	Calculus III	3		3
	Statistics	3		3	Probability	3		3
	Fundamentals of Computing II	3		3	Linear Algebra	3		3
	GE Course 7	3		3	Elective 1	3		3
	PE III		2	0	GE Course 8	3		3
					PE IV		2	0
	Total	14	2	14	Total	15	2	15
III								
	Discrete Mathematics	3		3	Numerical Analysis	3		3
	Differential Equations I	3		3	Operations Research I	3		3
	Elective 2	3		3	Elective 3	3		3
	GE Course 9	3		3	GE Course 11	3		3
	GE Course 10	3		3	GE Course 12	3		3
	Total	15	0	15	Total	15	0	15
IV								
	Theory of Interest	3		3	Elective 5	3		3
	Advanced Calculus I	3		3	Free Elective 2	3		3
	Elective 4	3		3	GE Course 14	3		3
	Free Elective 1	3		3	GE Course 15	3		3
	GE Course 13	3		3	Thesis/Special Problem or Elective 6	3		3
	Total	15	0	15	Total	15	0	15

Note: GE courses in the Languages, Humanities, and Social Sciences (including Life and Works of Rizal).
NSTP and PE courses are not included in the total number of units.

ARTICLE VI
COURSE SPECIFICATIONS

Section 15

The Commission has determined the minimum content of the courses included in the BS Mathematics/BS Applied Mathematics programs as provided in the course outlines below.

15.1 ABSTRACT ALGEBRA I

Course Description: This course covers groups, subgroups, cyclic groups, permutation groups, abelian groups, normal subgroups, quotient groups and homomorphisms and isomorphism theorems, rings, integral domains, fields, ring homomorphisms, ideals, and field of quotients.

Credit: 3 units

Prerequisite: Fundamental Concepts of Mathematics

Topics	**Time Allotment**
a. Preliminaries (Review) • Sets • Equivalence relations • Functions • Binary operations • Division Algorithm in Z (integers) and modular operations	4 hours
b. Groups • Definition and elementary properties • Group tables • Order of a group • Subgroups • Isomorphism of groups	6 hours
c. Cyclic Groups and Cosets • Definition, Order of an element • Structure of cyclic groups • Cosets • Lagrange's Theorem	5 hours
d. Permutation Groups • Permutations, cycles, transpositions • The symmetric and alternating groups • Dihedral group • Cayley's Theorem	4 hours

e. Direct Product and Generating Sets — 4 hours
- The direct product
- Subgroup generated by a subset
- Fundamental theorem of finitely generated abelian groups

f. Quotient Groups and Homomorphisms — 6 hours
- Normal subgroup
- Quotient group
- Homomorphism, kernel, image, basic properties
- Isomorphism theorems

g. Rings — 6 hours
- Definition and basic properties
- Subring
- The group of units of a ring
- Ideal
- Quotient ring

h. Ring Homomorphisms, Integral Domains, Fields — 7 hours
- Basic properties of ring homomorphism
- Ring isomorphism theorems
- Zero divisors, integral domains
- Fields
- Field of quotients of an integral domain

Suggested texts/references

a. Fraleigh. A First Course in Abstract Algebra
b. Galllian. Contemporary Abstract Algebra
c. Herstein. Abstract Algebra

15.2 ABSTRACT ALGEBRA II

Course Description: This course covers rings of polynomials, fundamental theorem of field theory, extension fields, algebraic extensions, finite fields, geometric constructions, fundamental theorem of Galois theory, illustrations of Galois theory.

Credit: 3 units

Prerequisite: Abstract Algebra I

Topics — **Time Allotment**

a. Introduction — 2 hours
- Historical background
- Solution of quadratic, cubic, quartic equations

b. Rings — 4 hours
- Review of basic concepts on rings
- Characteristic of a ring
- Prime subfield
- Prime ideal

- Maximal ideal
- Principal ideal

c. Rings of Polynomials 6 hours
- Division algorithm in F[x] (F a field)
- Ideal structure in F[x]
- Divisibility conditions in ideal form
- Irreducible polynomials
- Tests for irreducibility

d. Extension Fields 6 hours
- Fundamental theorem of field theory (Kronecker)
- Algebraic and transcendental elements
- Irreducible polynomial of an algebraic element
- Extension fields as vector spaces

e. Finite Fields 4 hours
- Cyclic structure of group of units
- Subfield structure
- Frobenius automorphism

f. Special Extension Fields 6 hours
- Finite extensions
- Algebraic extensions
- Splitting fields
- Algebraically closed fields, algebraic closure

g. Geometric Constructions 3 hours
- Constructible numbers
- Trisecting an angle, doubling the cube

h. Some Important Theorems 3 hours
- Primitive element theorem
- Isomorphism extension theorem

i. The Fundamental Theorem of Galois Theory 8 hours
- The Galois group
- The Galois correspondence (sketch of proof)
- Normal extensions
- Illustrations of Galois theory: finite fields, cyclotomic fields
- *Insolvability of the quintic*

Suggested text/references:

a. Fraleigh. A First Course in Abstract Algebra
b. Gallian. Contemporary Abstract Algebra
c. Herstein. Abstract Algebra

Note: Italicized items are optional topics

15.3 ACTUARIAL MATHEMATICS I

Course Description: This course covers the mathematical theory of life contingencies involving single-life functions, mortality, life annuities and insurances, and reserves.

Credit: 3 units

Prerequisites: Probability, and Theory of Interest

Topics	**Time Allotment**
a. Survival Distributions and Life Tables	9 hours
b. Life Insurance	8 hours
c. Life annuities	8 hours
d. Benefit Premiums	10 hours
e. Reserves	7 hours

a. Survival Distributions and Life Tables — 9 hours
- Probability for age-at-death
  - The survival function
  - Time-until-death for a person age x
  - Curtate-future-lifetimes
- Force mortality
- Life tables
- Deterministic survivorship group
- Assumptions for fractional ages
- Some analytical laws of mortality
- Select and ultimate tables

b. Life Insurance — 8 hours
- Insurance payable at the moment of death
  - Level benefit insurance
  - Endowment insurance
  - Deferred insurance
  - Varying benefit insurance
- Insurances payables at the end of the year of death
- Relationships between insurances payables at the moment of death and the end of the year of death
- Commutation notations

c. Life annuities — 8 hours
- Continuous life annuities
- Discrete life annuities
- Life annuities with monthly payments
- *Apportionable annuities-due and complete annuities-immediate*

d. Benefit Premiums — 10 hours
- Fully continuous premiums
- Fully discrete premiums
- True monthly payment premiums
- Commutation notations for premiums
- Other factors affecting pricing
- Cammack-type formula for contract premiums
- Apportionable premiums

e. Reserves — 7 hours
- Loss random variable
- Fully continuous benefit reserves
- Other formulas for fully continuous benefit reserves

- Fully discrete benefit reserves
- Benefit reserves on a semi-continuous basis
- Benefit reserves based on true monthly benefit premiums
- Fackler's method
- Modified reserves (FPT and CRVM)
- Benefit reserves on an apportionable or discounted continuous basis

Suggested text/references:

a. Bowers, Gerber, Hickman, Jones, and Nesbitt. <u>Actuarial Mathematics</u>
b. Gerber. <u>Life Insurance Mathematics</u>
c. Jordan. <u>Life Contingencies</u>
d. Larson and Gaumnitz. <u>Life Insurance Mathematics</u>
e. Veeh. <u>Lecture Notes in Actuarial Mathematics</u>. Available at http://javeeh.net/lecnotes/actmath.pdf

Note: Italicized items are optional topics

15.4 ACTUARIAL MATHEMATICS II

Course Description: This course covers the following topics: multiple decrement theory, disability and mortality tables, monetary applications, and introduction to pension theory.

Credit: 3 units

Prerequisite: Actuarial Mathematics I

Topics	**Time Allotment**
a. Analysis of Benefit Reserves • Benefit reserves for general insurances • Recursion relations for fully discrete benefit reserves • Benefit reserves at fractional durations • Allocation of the risk to insurance years • Differential equations for fully continuous benefit reserves	6 hours

b. Multiple Life Functions
• Joint distributions of future lifetimes
• Joint-life status
• Last-survivor status
• More probabilities and expectations
• Dependent lifetime models
  o Common shock
  o Copulas
• Insurance and annuity benefits
  o Survival statuses
  o Special two-life annuities
  o Reversionary annuities
	8 hours

- Evaluation – Special mortality assumptions
  - Gompertz and Makeham Laws
  - Uniform distributions
- Simple contingent functions
- Evaluation – Simple contingent functions

c. Multiple Decrement Models — 8 hours

- Distribution of two random variables
- Random survivorship group
- Deterministic survivorship group
- Associated single decrement tables
  - Basic relationships
  - Central rates of multiple decrement
  - Constant force assumption for multiple decrements
  - Uniform distribution assumption for multiple decrements
  - Estimation issues
- Construction of a multiple decrement table

d. Applications of Multiple Decrement Theory — 6 hours

- Actuarial present values and their numerical evaluations
- Benefit premiums and reserves
- Withdrawal benefit patterns that can be ignored in evaluating premiums and reserves
- Valuation of pension plans
  - Demographic assumptions
  - Projecting benefit payment and contribution rates
  - Defined benefit plans
  - Defined-contribution plans
- Disability benefits with individual life insurance
  - Disability income benefits
  - Waiver-of-premium benefits
  - Benefit premiums and reserves

e. Insurance Models Including Expenses — 8 hours

- Expenses augmented models
  - Premiums and reserves
  - Accounting
- Withdrawal benefits
  - Premium and reserves
  - Accounting
- Types of expenses
- Algebraic foundations of accounting: single decrement model
- Asset shares
  - Recursion relations
  - Accounting
- Expenses, reserves and general insurances

f. Business and Regulatory Considerations — 6 hours
- Cash values
- Insurance option
  - Paid-up insurance
  - Extended term
  - Automatic premium loans
- Premiums and economic considerations
  - Natural premiums
  - Fund objective
  - Rate of return objective
  - Risk-based objectives
- Experience adjustments
- Modified reserve methods
- Full preliminary term
- Modified preliminary term
- Non-level premiums of benefits
  - Valuation
  - Cash values

Suggested text/references:

a. Bowers, Gerber, Hickman, Jones and Nesbitt. Actuarial Mathematics
b. Jordan. Life Contingencies
c. Veeh. Lecture Notes in Actuarial Mathematics. Available at http://javeeh.net/lecnotes/actmath.pdf

15.5 ADVANCED CALCULUS I

Course Description: Advanced Calculus I is the first of two courses that provides an introduction to mathematical analysis beyond the calculus series. Topics include the real number system, point set topology, limits and continuity, the derivatives, multivariable differential calculus, implicit functions and extremum problems.

Credit: 3 units

Pre-requisite: Calculus III

Topics — **Time Allotment**

a. R as a Complete Ordered Field — 2 hours
- Countable and uncountable sets

b. Point Set Topology — 4 hours
- Euclidean space R^n
- Open and closed sets in R^n
- Accumulation points
- Bolzano-Weiestrass Theorem
- Heine-Borel Theorem
- Compactness of R^n
- Metric spaces
- Compact subsets of a metric space
- Boundary of a set

c. Limits and Continuity — 8 hours
- Convergent sequences in a metric space
- Cauchy sequences
- Complete metric spaces
- Limit of a function
- Continuous functions
- Continuity of composite functions
- Examples of continuous functions
- Continuity and inverse images of open or closed sets
- Functions continuous on compact sets
- Topological mappings
- Uniform continuity and compact sets
- Discontinuities of real-valued functions
- Monotonic functions

d. Derivatives — 8 hours
- Derivatives and continuity
- The chain rule
- One-sided derivatives
- Rolle's theorem
- The mean-value theorem for derivatives
- Taylor's formula with remainder

e. Multivariable Differential Calculus — 12 hours
- The directional derivative
- Differential of functions of several variables
- Jacobian matrix
- The chain rule
- Matrix form of chain rule
- The mean-value theorem for differentiable functions
- A sufficient condition for differentiability
- A sufficient condition for equality of mixed partial derivatives
- Taylor's formula for functions from R^n to R

f. Implicit Functions and Extremum Problems — 9 hours
- Functions with nonzero Jacobian determinant
- The inverse function theorem
- The implicit function theorem
- Extrema of real-valued functions of one variable
- Extrema of real-valued functions of several variables

Suggested text/references

a. Apostol. Mathematical Analysis
b. Rudin. Principles of Mathematical Analysis
c. Protter and Morrey. A First Course in Real Analysis
d. Lang. Undergraduate Analysis
e. Ross. Elementary Analysis: The Theory of Calculus

15.6 ADVANCED CALCULUS II

Course Description: This course is a continuation of Advanced Calculus I. Topics include the convergence of sequences and series of real numbers, sequences and series of functions, uniform convergence, power series, functions of bounded variation and rectifiable curves, Riemann-Stieltjes integrals, interchanging of limit operations, multiple integration, improper integrals, transformations.

Credit: 3 units

Pre-requisite: Advanced Calculus I

Topics	**Time Allotment**
a. Infinite Series • Limit superior and limit inferior of a sequence of real numbers • Infinite series • Alternating series • Absolute and conditional convergence • Tests for convergence of series • Dirichlet's test and Abel's test • Rearrangement of series • Double series and rearrangement theorem for double series • Multiplication of series	12 hours
b. Riemann-Stieltjes Integral • Functions of bounded variation • Curves and paths • Rectifiable curves and arc length • Definition of Riemann-Stieltjes integral • Sufficient and necessary conditions for the existence of Riemann-Stieltjes integrals • Differentiation under the integral sign • Interchanging the order of integration • Multiple integrals and improper integrals	9 hours
c. Sequences of Functions • Pointwise convergence of sequences of functions • Uniform convergence and continuity • Uniform convergence of infinite series of functions • Uniform convergence and Riemann-Stieltjes integration • Uniform convergence and differentiation • Power series	8 hours
d. Green's Theorem for Rectangles and Regions	3 hours
e. Review of Vector Fields	3 hours
f. Surfaces • Surface area • Integrals over curves and surfaces	7 hours

- Stokes' Theorem, Gauss' Theorem

Suggested text/references

a. Apostol. Mathematical Analysis
b. Rudin. Principles of Mathematical Analysis
c. Protter and Morrey. A First Course in Real Analysis
d. Lang. Undergraduate Analysis
e. Ross. Elementary Analysis: The Theory of Calculus

15.7 APPLIED MULTIVARIATE ANALYSIS

Course Description: This course is concerned with statistical methods for describing and analyzing multivariate data. Topics include dependence and interdependence techniques for data reduction and analysis. In-class lectures and discussions are supplemented by computer hands-on sessions with statistical software.

Credit: 3 units

Prerequisites: Statistical Theory, and Linear Models

Topics	**Time Allotment**
a. Introduction to Multivariate Analysis • Statistical concepts, vector and matrix operations • Types of multivariate techniques	3 hours
b. Multivariate Analysis of Variance (MANOVA) • Objectives of MANOVA • Assumptions of ANOVA and MANOVA • Applications of MANOVA	5 hours
c. Multiple Discriminant Analysis and Logistic Regression • Comparison of Discriminant Analysis (DA) and Logistic Regression with Regressions and MANOVA • Objectives and Assumptions of DA and Logistic Regression • Applications of DA and Logistic Regression • Alternative Statistical Tools	9 hours
d. Canonical Correlation Analysis • Objectives of canonical correlation • Assumptions of canonical correlation • Limitations of canonical correlation	3 hours
e. Principal Component Analysis (PCA) • Objectives of PCA • Assumptions of PCA • Applications of PCA	6 hours
f. Factor Analysis • Purpose of factor analysis • Factor analysis decision diagram • Naming of factors	9 hours

- How to select surrogate variables for subsequent analysis
- How to use factor scores
- Differentiating principal components analysis and factor analysis

g. Cluster Analysis — 8 hours
- Objectives of cluster analysis
- How does cluster analysis work
- Types of clustering techniques
- Applications of cluster analysis

h. Multidimensional Scaling and Correspondence Analysis

Suggested text/references:

a. Morrison. Multivariate Statistical Methods
b. Hair, Anderson, Tatham, and Black. Multivariate Data Analysis
c. Johnson and Wichern. Applied Multivariate Statistical Analysis
d. Tabunda. Applied Multivariate Analysis

Note: *Italicized item is an optional topic.*

15.8 AUTOMATA AND COMPUTABILITY THEORY

Course Description: This course covers finite automata and regular expressions, context-free grammars and pushdown automata, Turing machines, undecidability, and Gödel's Incompleteness Theorem.

Credit: 3 units

Prerequisite: Discrete Mathematics

Topics — **Time Allotment**

a. Finite Automata and Regular Languages — 18 hours
- Strings and sets
- Finite automata and regular sets
- Nondeterministic finite automata
- The subset construction
- Pattern matching and regular expressions
- Regular expressions and finite automata
- Kleene algebra and regular expressions
- Homomorphisms
- Limitation of finite automata
- Pumping lemma
- DFA state minimization
- The Myhill-Nerode Theorem
- Two-way finite automata

b. Context Free Languages and Push-Down Automata — 10 hours
- Context free grammar and languages

- Balanced parentheses
- Normal forms
- The pumping lemma for CFLs
- Pushdown automata
- PDAs and CFGs
- The Coke-Kasami-Younger algorithms

c. Turing Machines and Computability — 6 hours
- Turing machines and effective computability
- Equivalent models
- Universal machines and diagonalization

d. Undecidability — 8 hours
- Decidable and undecidable problems
- Reduction
- Rice's theorem
- Undecidable problems about CFLs
- Gödel's Incompleteness Theorem

Suggested texts/references:

a. Kozen. Automata and Computability
b. Hopcroft, Motwani and Ullman. Introduction to Automata Theory, Languages and Computation

15.9 CALCULUS I

Course Description: This course is an introduction to calculus with analytic geometry. It covers lines, circles, conic sections, special functions, limits, continuity, derivatives and their applications, differentials, antiderivatives, definite integrals and their applications.

Credit: 5 units

Prerequisite: Precalculus Mathematics II

Topics — **Time Allotment**

a. Review of the Real Number System as an Ordered Field — 3 hours
- Representation on the real line
- Inequalities and intervals
- Absolute values

b. The 2-Dimensional Coordinate System — 3 hours
- Graph of equations
- Graph of inequalities
- Distance between two points and midpoint of a line segment

c. Lines and Circles — 8 hours
- Slope of a line
- Standard and general equations of a line
- Parallel and perpendicular lines
- *Angle between two lines*

- *Distance from a point to a line*
- Equation of a circle in center-radius form
- General equation of a circle

d. Conics — 6 hours
- Parabola
- Ellipse
- Hyperbola

e. Functions — 4 hours
- Domain and range
- Special functions (absolute value, step, constant, linear, quadratic, greatest integer, simple)
- Graphs of functions
- Operations on functions

f. Limits of Functions — 8 hours
- Intuitive motivation for limits
- Formal definition (epsilon-delta)
- Theorems on limits
- One-sided limits
- Infinite limits and limits at infinity
- Asymptotes

g. Continuity — 4 hours
- Definition
- Removable and essential discontinuities
- Theorems on continuity

h. Derivatives — 9 hours
- The tangent line to a curve (slope as instantaneous rate of change of y with respect to x)
- Instantaneous velocity in rectilinear motion
- Definition and notations of derivatives
- Geometric interpretation of derivatives of a function
- Differentiability of a function
- Differentiability and continuity
- Basic rules on differentiation
- The chain rule: composite functions
- Implicit differentiation

i. Applications of Derivatives — 8 hours
- Derivatives as a rate of change
- Related rates
- Derivatives of higher order
- Relative maximum and minimum values of a function
- Absolute maximum and minimum values of a function in a closed interval
- Extreme-value problems
- Rolle's Theorem and the Mean Value Theorem

- Sketch of the graph of a function (increasing/decreasing; relative extrema; concavity; points of inflection)

j. Differential of a Function — 3 hours
- Geometric interpretation
- Applications
- Differential forms of differentiation formulas

k. Antidifferentiation — 4 hours
- Formulas
- Integration by substitution (chain rule)
- *Differential equations with separable variables*
- Applications to rectilinear motions and economics

l. The Definite Integral — 5 hours
- Approximation of area of a region under the curve of a function
- Summation notation and the Riemann sum
- Definition of the definite integral
- Properties of the definite integrals
- The Mean Value Theorem for Integrals
- The Fundamental Theorem of Calculus

m. Applications of the Definite Integral — 5 hours
- Area of a region in a plane
- Volume of a solid of revolution
- *Work*
- *Center of mass of a plane region*
- *Center of mass of a solid of revolution*
- *Length of arc of a plane curve*

Suggested text/references:

a. Leithold. The Calculus with Analytic Geometry
b. Purcell. Calculus with Analytic Geometry
c. Thomas. Calculus with Analytic Geometry
d. Edwards and Penney. Calculus with Analytic Geometry
e. Anton. Calculus With Analytic Geometry

Note: *Italicized topics may be skipped or postponed for the next course, Calculus II.*

15.10 CALCULUS II

Course Description: This course covers the derivatives and integrals of transcendental functions, techniques of integration, approximations of definite integrals, polar coordinate system, vectors, and curves and surfaces in 3-dimensional space.

Credit: 5 units

Prerequisite: Calculus I

Topics	**Time Allotment**
a. The Natural Logarithmic Function; The Exponential Function	8 hours
• Properties and graphs	
• Review of inverse functions	
• Inverse Function Theorem	
• Derivatives and integrals	
• Applications (laws of decay and growth)	
b. The Circular Functions; Inverse Circular Functions	6 hours
• Properties and graphs	
• Derivatives and integrals	
• Applications to problems	
c. Techniques of Integration	10 hours
• Integration by parts	
• Integration of powers of trigonometric functions	
• Integration by trigonometric substitutions	
• Integration of rational functions by partial functions	
• Miscellaneous substitutions	
d. Approximations of Definite Integrals	4 hours
• Trapezoidal rule	
• Simpson's rule	
e. Improper Integrals and Indeterminate Forms	6 hours
• Indeterminate forms	
• Improper integrals with infinite limits of integration	
• Improper integrals with discontinuous integrands	
f. Polar Coordinate System	8 hours
• Polar functions	
• Polar graphs	
• Slope and tangent lines in polar curves	
• Area of regions in polar coordinates	
g. Vectors in the Plane	8 hours
• Properties of vectors	
• Addition and scalar multiplication	
• Dot product	
• Vector-valued functions and parametric equations	
• Calculus of vector-valued functions	
• Length of an arc	

- Plane motion
- The unit tangent vectors, unit normal vectors and arc length as a parameter
- Curvature

h. Vectors in 3-Dimensional Space R^3 — 8 hours
- The 3-dimensional space R^3
- Vector
- Sums, multiplication by a scalar of vectors in space
- The dot product
- Planes
- Lines in R^3
- The cross product

i. Curves and Surfaces in 3-Dimensional Space — 10 hours
- Cylinders and surfaces of revolution
- Quadric surfaces
- Curves in R^3

Suggested text/references:

a. Leithold. The Calculus with Analytic Geometry
b. Purcell. Calculus with Analytic Geometry
c. Thomas. Calculus with Analytic Geometry
d. Edwards and Penney. Calculus with Analytic Geometry
e. Anton. Calculus With Analytic Geometry

15.11 CALCULUS III

Course Description: This course covers calculus of functions of several variables, sequences, infinite series and power series.

Credit: 3 units

Prerequisite: Calculus II

Topics — **Time Allotment**

a. Functions of More Than One Variable — 4 hours
- Properties and graphical or geometrical representations
- Limits
- Continuity

b. Partial Derivatives — 6 hours
- Definitions
- Differentiability and the total differential
- The chain rule
- Higher-order partial derivatives

c. Geometric Applications of Partial Derivatives in 3-Dimensional Space — 6 hours
- Directional derivatives and gradients

- Tangent plane and normal to surfaces
- Extrema of functions of two variables
- Obtaining a function from its gradient

d. Integrals — 10 hours
- Line integrals
- Path-independence of line integrals
- Double integrals
- Evaluation of double integrals by iterated integration
- Applications to center of mass and moments of inertia
- Double integrals in polar coordinates
- Area of a surface
- Triple integrals
- Triple integrals in cylindrical and spherical coordinates

e. Infinite Series — 8 hours
- Sequences
- Monotonic bounded sequences
- Infinite series of constant terms
- Infinite series of positive terms
- Tests for convergence/divergence of an infinite series

f. Power Series — 8 hours
- Region of convergence of a power series
- Differentiation of power series
- Integration of power series
- Taylor's formula
- Taylor's series
- Binomial series

g. Introduction to differential equations

Suggested text/references:

a. Leithold. The Calculus with Analytic Geometry
b. Purcell. Calculus with Analytic Geometry
c. Thomas. Calculus with Analytic Geometry
d. Edwards and Penney. Calculus with Analytic Geometry
e. Anton. Calculus With Analytic Geometry
f. Anton, Bivens and Davis. Early Transcendentals

Note: *Italicized topic may be added for a 5-unit course*

15.12 COMPLEX ANALYSIS

Course Description: This course involves a study of the algebra of complex numbers, analytic functions, elementary complex functions, complex integration, and the residue theorem and its applications.

Credit: 3 units

Prerequisite: Advanced Calculus 1

Topics	**Time Allotment**
a. Complex Numbers • The algebra of complex numbers • Geometric representation of complex numbers • Polar coordinates • Powers and roots • Stereographic projection	5 hours
b. Analytic Functions • Functions of a complex variable • Limits and continuity • Derivatives and differentiation formulas • Necessary and sufficient conditions for differentiability • Cauchy-Riemann equations: Cartesian and polar form • Analytic functions • Harmonic functions	7 hours
c. Elementary Complex Functions • The exponential function and its properties • Trigonometric functions and their properties • Hyperbolic functions • Logarithmic functions and their properties • Multiple-valued functions and their branches • Complex exponents • Inverse trigonometric functions	7 hours
d. Mapping of Elementary Functions • Linear functions • The function $1/z$ • Linear fractional transformations • Special linear fractional transformations • The functions z^n • The transformation $w = \exp z$ • Successive transformations	6 hours
e. Complex Integration • Contours • Line integrals • The Cauchy-Goursat Theorem • Simply and multiply-connected domains	9 hours

- The Cauchy integral formulas
- Derivatives of functions
- Morera's Theorem
- Maximum moduli of functions

f. Residues and Poles — 8 hours

- Residues and the Residue Theorem
- Laurent series
- The principal part of a function
- Poles
- Quotients of analytic functions
- Improper integrals
- Improper integrals involving trigonometric functions
- Integration around a branch point

g. Conformal Mapping

Suggested text/references:

a. Pennisi. Elements of Complex Variables
b. Churchill, Brown, and Verhey. Complex Variables and Applications
c. Lang. Complex Analysis
d. Spiegel. Theory and Problems of Complex Variables

Note: *Italicized item is an optional topic.*

15.13 COMPUTATIONAL COMPLEXITY

Course Description: This course covers fundamental concepts of complexity including Church's thesis, recursive and recursively enumerable languages, universal Turing machines and computability, complexity measures, time and space bounded computations, P versus NP, NP-completeness, intractability.

Credit: 3 units

Prerequisite: Discrete Mathematics, or Automata and Computability Theory

Topics	**Time Allotment**
a. Mathematical Preliminaries	6 hours

- Sets, relations and functions
- Proof techniques
- Graphs
- Alphabets, words and languages

b. Turing Machines — 9 hours

- Turing machines (TM)
- Computable languages and functions
- Techniques for TM construction
- Church's thesis
- Turing machines as enumerator
- Restricted TMs equivalent to basic model

c. Undecidability — 9 hours
- Recursive and recursively enumerable languages
- Universal Turing machines
- Rice's Theorem
- Undecidability of Post's correspondence problem
- Valid and invalid computations of TMs
- Oracle computations

d. Complexity — 6 hours
- Definition of computational complexity
- Hierarchy theorems
- Relations among complexity measures
- Properties of general complexity measures

e. Complexity classes — 6 hours
- Polynomial time and space
- Some NP-complete problems
- The class co-NP
- PSPACE-complete problems

f. Intractability — 3 hours
- Some provably intractable problems
- The P=NP question for TMs with oracles

Suggested texts/references:

a. Hopcroft, Motwani and Ullman. Introduction to Automata Theory, Languages and Computation
b. Bovet and Crescenzi. Introduction to Theory of Complexity
c. Zhou, Du, and Ko. Theory of Computational Complexity
d. Papadimitriou. Computational Complexity

15.14 DATA STRUCTURES AND ALGORITHMS

Course Description: This course covers the different ways of representing and storing data, including stacks, queues, trees and graphs. It includes the study of algorithms used to create, update and access these data structures. Discussions may be done using pseudocodes, and implementation may use C++ or Java or other languages that support these structures. In-class lectures and discussions are supplemented by computer hands-on sessions.

Credit: 3 units

Prerequisite: Fundamentals of Computing II

Topics — **Time Allotment**

a. Introduction — 3 hours
- Overview: Data, algorithm and data structures
- Analysis of programs and time complexity

b. Arrays, Pointers and Lists — 4 hours
- Representation of arrays

- Singly- and doubly-linked lists
- Circularly-linked lists

c. Linear Data Structures — 6 hours
- Stacks
- Queues
- Implementation of stacks and queues using arrays and linked lists
- Operations on stacks and queues

d. Recursion — 3 hours

e. Trees — 10 hours
- Tree representation
- Binary trees
  - Binary tree traversal
  - Expression trees: prefix, infix, postfix
- Search trees
  - Binary search trees
  - AVL trees
  - Operations on binary search trees
- M-way search trees
  - B-, B*- and B'-trees
  - Operations on m-way search trees

f. Heaps — 4 hours
- Heap algorithms
- Priority queues

g. Sorting — 7 hours
- Exchange sort
- Selection sort
- Insertion sort
- Merge sort
- Heap sort

h. Graphs — 6 hours
- Graph representations: adjacency matrix, adjacency list
- Graph operations
- Spanning trees
- Shortest path algorithms

Suggested text/references:

a. Gilberg and Forouzan. Data Structures: A Pseudocode Approach With C++
b. Hubbarb and Huray. Data Structures With Java
c. Horowitz and Sahni. Fundamentals of Data Structures Using Pascal

15.15 DIFFERENTIAL EQUATIONS I

Course Description: This is an introductory course in ordinary differential equations (ODEs). It focuses primarily on techniques for finding explicit solutions to linear ODEs. Topics include first order ordinary differential equations, linear differential equations, linear equations with constant coefficients, nonhomogeneous equations, undetermined coefficients and variation of parameters, linear systems of equations; the existence and uniqueness of solutions.

Credit: 3 units

Prerequisite: Calculus III

Topics	**Time Allotment**
a. Ordinary Differential Equations of Order One • Existence of solutions • Separation of variables • Homogeneous functions • Equations with homogeneous coefficients • Exact equations • The general solution of a linear equation	6 hours
b. Linear Differential Equations • The general linear equation • Existence and uniqueness theorem • Linear independence of solution • Using the Wronskian to determine linear independence • General solution of a homogeneous equation • General solution of a nonhomogeneous equation	6 hours
c. Linear Equations with Constant Coefficients • The auxiliary equation (distinct roots, repeated roots and imaginary roots)	6 hours
d. Nonhomogeneous Equations (Undetermined Coefficients) • Construction of a homogeneous equation from a specified solution • Solution of a nonhomogeneous equation o Method of undetermined coefficients o Solution by inspection o Variation of parameters	9 hours
e. Linear System of Equations • First-order systems with constant coefficients • Solution of a first-order system • Complex eigenvalues • Repeated eigenvalues • Nonhomogeneous systems • Laplace transform	12 hours

Suggested text/references

a. Rainville and Bedient. Elementary Differential Equations
b. Edwards and Penney. Elementary Differential Equations with Boundary Value Problems
c. Polking, Boggess, and Arnold. Differential Equations and Boundary Value Problems
d. Arnold and Polking. Ordinary Differential Equations using Matlab

15.16 DIFFERENTIAL EQUATIONS II

This may either be an introduction to partial differential equations or a course in non-linear dynamics.

15.16.1 DIFFERENTIAL EQUATIONS II (INTRODUCTION TO PARTIAL DIFFERENTIAL EQUATIONS)

Course Description: This course covers first-order linear partial differential equations, initial and boundary conditions, the wave equation, the diffusion (heat) equation, boundary problems, Fourier series solutions, and Laplace's equation.

Credit: 3 units

Prerequisites: Differential Equations I, and Advanced Calculus I

Topics	Time Allotment
a. First Order Linear Partial Differential Equations	6 hours
• Linear homogeneous equations • Methods of characteristics • Linear non-homogeneous equations • Simple physical examples: transport, diffusion and vibration • Initial and boundary value problems • Well-posed problems	
b. Second-Order Partial Differential Equations	2 hours
• Hyperbolic, parabolic and elliptic	
c. The Wave Equation and Diffusion Equation	10 hours
• Wave and diffusion equations on the whole real line • The wave equation: coordinate method and geometric method of characteristics • Causality and Energy o Domain of Dependence o Domain of Influence o Uniqueness and stability o Weak Solutions • Maximum principle for the diffusion equation o Uniqueness and stability of solutions • Derivation of the solution of the diffusion equation	

- Comparison of wave and diffusion equations

d. Boundary Problems — 10 hours
- Problems on the Half Line
    - Diffusion
        - Dirichlet condition
        - Neumann condition
    - Wave Equation
        - Dirichlet condition
        - Neumann condition
- Problems on Finite Intervals
    - Dirichlet at both ends
    - Neumann at both ends
    - Dirichlet and Neumann

e. Fourier Series — 6 hours
- Orthogonality and completeness of Fourier series
- Convergence theorems
- Eigenfunction expansions
- *Bessel functions and Legendre functions*

f. Harmonic Functions — 6 hours
- Laplace's equation: examples
- Maximum principle: uniqueness and stability of solutions
- Laplace equation on rectangles and cubes
- Poisson's formula for solutions of boundary value problems
- *Laplace equation on circles, wedges and annuli*
- *Green's functions*

g. Some Physical Examples — 2 hours

Suggested textbook/references:

a. Strauss. Partial Differential Equations: An Introduction
b. John. Partial Differential Equations
c. Evan. Partial Differential Equations

Note: *Italicized items are optional topics.*

15.16.2 DIFFERENTIAL EQUATIONS II (NONLINEAR DYNAMICS)

Course Description: This course includes first-order differential equations and their bifurcations, one-dimensional maps, logistic map, Lyapunov exponent, universality and renormalization methods, phase-plane analysis, limit cycles and their bifurcations and Poincare-Bendixson Theorem.

Credit: 3 units

Prerequisite: Differential Equations I

Topics	**Time Allotment**
a. One-dimensional Systems and Elementary Bifurcations	12 hours
• One-dimensional systems • Fixed points and stability • Saddle-node bifurcation • Transcritical bifurcation • Super-critical pitchfork bifurcation • Sub-critical pitchfork bifurcation • Imperfect bifurcations and catastrophes • Insect outbreak	
b. One-Dimensional Maps	9 hours
• Fixed points and cobwebs • Logistic map • Chaos and chaotic maps • Lyapunov exponent • Universality and renormalization methods	
c. Two-Dimensional Systems and Limit Cycles	12 hours
• Linear systems • Classification of linear systems • Jordan canonical forms • Phase plane analysis • Limit cycles and periodic orbits • Poincare-Bendixson Theorem	
d. Bifurcations in Two-Dimensional Systems	12 hours
• Saddle-node, transcritical and pitchfork bifurcations • Hopf bifurcation • Homo-clinic bifurcation • Center manifold reduction • Poincare maps	

Suggested references:

a. Strogatz. <u>Nonlinear Dynamics and Chaos: with Applications to Physics, Biology, Chemistry and Engineering.</u>
b. Verhulst. <u>Nonlinear Differential Equations and Dynamical Systems.</u>
c. Wiggins. <u>Introduction to Applied Nonlinear Dynamical Systems and Chaos.</u>

15.17 DISCRETE MATHEMATICS

Course Description: This is a course that covers the fundamentals of logic and sets, the fundamental principles of counting, algorithms and some concepts in graph theory.

Credit: 3 units

Prerequisite: Precalculus Mathematics II

Topics	**Time Allotment**
a. Fundamentals of Logic • Propositions • Logical operators • Rules of replacement • Proofs of validity/invalidity • Quantifiers • Quantification rules	8 hours
b. Sets • Basic concepts • Set operations and algebra of sets	6 hours
c. Fundamental Principles of Counting • Permutations • Combinations • The principle of inclusion-exclusion • Pigeonhole principle	8 hours
d. Generating Functions and Recurrence Relations • Linear homogeneous recurrence relations • Nonhomogeneous recurrence relations • Generating functions	6 hours
e. Algorithms • Basic concepts and notations • The Euclidean algorithm • Recursive algorithms • Complexity of algorithms • Analysis of algorithms	8 hours
f. Introduction to Graph Theory • Basic concepts • Trees • Optimization and matching	8 hours

Suggested text/references:

a. Rosen. Discrete Mathematics and Applications
b. Grimaldi. Discrete and Combinatorial Mathematics
c. Ross. Discrete Mathematics
d. Johnsonbaugh. Discrete Mathematics

15.18 FUNDAMENTAL CONCEPTS OF MATHEMATICS

Course Description: This course covers sets, principles of logic, methods of proof, relations, functions, integers, binary operations, complex numbers, matrices and matrix operations, and an introduction to mathematical systems.

Credit: 3 units

Prerequisite: Precalculus Mathematics II

Topics	**Time Allotment**
a. Sets • Basic definitions and notation • Set operations, algebra of sets • Venn diagrams • Counting properties of finite sets	4 hours
b. Principles of Logic • Statements, logical connectives • Validity, truth table • Tautologies • Quantifiers	6 hours
c. Methods of Proof • Direct proof • Indirect proof • Proof by specialization and division into cases • Mathematical induction	8 hours
d. Relations • Definition • Equivalence relations • Equivalence classes and partitioning • Partial ordering	3 hours
e. Functions • Injection, surjection, bijection • Image, inverse image • Inverse function • Cardinal number of a set • Counting principles • Countable and uncountable sets	8 hours
f. Integers • Divisibility • Division algorithm • Euclidean algorithm • Fundamental Theorem of Arithmetic	6 hours
g. Binary Operations • Definition • Modular operations • Operations on matrices • Operations on complex numbers	4 hours
h. Introduction to Mathematical Systems • Semigroup • Group • Ring • Field	6 hours

Suggested Texts/References

a. Morash. Bridge to Abstract Mathematics
b. Gerstein. Introduction to Mathematical Structures and Proofs
c. Rotman. Journey to Mathematics
d. Kurtz. Foundations of Abstract Mathematics
e. Sundstrom. Mathematical Reasoning: Writing and Proofs
f. Chartrand, Polimeni and Zhang. Mathematical Proofs: A transition to advanced mathematics

15.19 FUNDAMENTALS OF COMPUTING I

Course Description: This course introduces fundamental programming constructs: types, control structures, functions, I/O, basic data structures using the C programming language. In-class lectures and discussions are supplemented by computer hands-on sessions.

Credit: 3 units

Topics	**Time Allotment**
a. Introduction to Computer Programming • Basic components of a computer • Overview of programming languages • Number systems and conversions • Overview of command shell • Problem-solving on a computer	4 hours
b. Introduction to C Language • Syntax and semantics • Elements of a C program • Basic I/O: printf, scanf	3 hours
c. Basic Data Types	3 hours
d. Identifiers, Keywords, Variables, Constants	3 hours
e. Operators and Precedence • Arithmetic • Boolean • Relational • Increment/Decrement	3 hours
f. Type Conversions	3 hours
g. Control Structures • Statements and blocks • Conditional: if-else, switch, ternary operator • Looping: while, do-while, for • Others: break, continue	4 hours
h. Functions and Procedures	6 hours
i. Arrays and Strings	3 hours
j. Pointers	3 hours
k. User-Defined Data Types	3 hours

l. Manipulating Files — 3 hours

m. Searching and Sorting — 3 hours
   - Linear search
   - Binary search
   - Bubble sort

Suggested references:

a. Kernighan and Ritchie. The C Programming Language
b. Kelly and Pohl. C by Dissection-The Essentials of C Programming
c. Goldstein and Gritz. Hands-on Turbo C

15.20 FUNDAMENTALS OF COMPUTING II

Course Description: This course covers advanced programming concepts and techniques using Java, C++ or other suitable object-oriented programming languages. Topics include recursion, abstract data types, advanced path structures, programming interfaces, object-oriented programming, inheritance, polymorphism, event handling, exception handling, API programming. In-class lectures and discussions are supplemented by computer hands-on sessions.

Credit: 3 units

Pre-requisite: Fundamentals of Computing I

Topics	**Time Allotment**
a. Introduction to Java	2 hours
b. Programming Fundamentals • Comments, statements, blocks, identifiers, keywords, literals, primitive data types, variables • Operators (arithmetic, relational, logical, conditional)	4 hours
c. Control Structures • Decision control structures • Repetition control structures • Branching statements (break, continue, return)	3 hours
d. Java Arrays	2 hour
e. Command Line Arguments	2 hours
f. Working with Java class library • Encapsulation • Classes and objects • Class variables and methods • Casting, converting and comparing objects	4 hours
g. Object-Oriented Programming • Defining your own classes • Overloading methods • Packages	6 hours

- Access modifiers

h.	Inheritance	2 hours
i.	Polymorphism	2 hours
j.	Exceptions and assertions	2 hours
k.	Advanced Programming Techniques • Recursion • Abstract data types: stacks, queues, linked lists • Java collections	5 hours
l.	Sorting Algorithms • Insertion sort • Selection sort • Merge sort • Quick sort	4 hours
m.	GUI Event Handling	2 hours
n.	Threads	2 hours
o.	Applets	2 hours

Suggested text/references:

a. Horstmann. Computing Concepts with Java Essentials
b. Deitel and Deitel. Java: How To Program
c. Cornell and Horstmann. Core Java

15.21 GRAPH THEORY AND ITS APPLICATIONS

Course Description: This course is an introduction to concepts in graph theory, networks, graph algorithms and their applications.

Credit: 3 units

Prerequisite: Discrete Mathematics

Topics		**Time Allotment**
a.	Graphs • Basic concepts • Paths, cycles, complete graphs, bipartite graphs • Digraphs • Operations on graphs	8 hours
b.	Connectivity • Connected graphs • Vertex connectivity and edge connectivity • Blocks • The connector problem	6 hours
c.	Covering Circuits and Graph Coloring • Eulerian graphs and digraphs • Hamiltonian graphs and digraphs	10 hours

- Weighted graphs
- The Traveling Salesman Problem
- Graph coloring and the chromatic number
- Storage problem

d. Trees — 8 hours
- Basic properties of trees
- Search trees and spanning trees
- Shortest paths and Dijkstra's Algorithm
- Minimal spanning trees

e. Networks — 6 hours
- Cuts and flows
- The Max Cut-Min Flow Theorem
- Feasible flows

f. Matchings — 6 hours
- Matchings and coverings
- Perfect matchings
- The Assignment Problem

Suggested text/references:

a. Bondy and Murty. Graph Theory With Applications
b. Chartrand and Lesniak. Graphs and Digraphs
c. Tucker. Applied Combinatorics

15.22 LINEAR ALGEBRA

Course Description: This course covers matrices, systems of linear equations, vector spaces, linear independence, linear transformations, determinants, eigenvalues and eigenvectors, diagonalization, and inner product spaces.

Credit: 3 units

Prerequisite: Fundamental Concepts of Mathematics

Topics — **Time Allotment**

a. Matrices over a Field — 4 hours
- Definition
- Matrix operations and their properties
- Transpose of a matrix
- Special types of square matrices

b. Row/Column Operations — 6 hours
- Echelon form of a matrix
- Solution of systems of linear equations
- Elementary matrices
- Row equivalence
- Rank of a matrix
- Inverse of a matrix
- Determinants

c. Vector Spaces over a Field — 6 hours
- Definition and examples
- Subspaces, their sum and intersection
- Spanning sets, linear combination

d. Linear Independence — 5 hours
- Definition
- Basis and dimension
- Isomorphism of vector spaces

e. Linear Transformations — 7 hours
- Definition and examples
- Kernel, range, nullity and rank
- Nonsingular linear transformations
- Algebra of linear transformations
- Matrix of a linear transformation and similarity

f. Eigenvectors and Eigenvalues — 7 hours
- Characteristic polynomial
- Eigenvalues, eigenvectors and eigenspaces
- Hamilton-Cayley Theorem
- Diagonalization

g. Inner Product Spaces — 6 hours
- Inner product
- Gram-Schmidt orthogonalization
- Diagonalization of symmetric matrices
- Quadratic forms

h. Isometries
- *Types of isometries*
- *Products of isometries*
- *Application of isometries to the solution of some geometric problems*

Suggested text/references

a. Kolman. Elementary Linear Algebra
b. Finkbeiner. Introduction to Matrices and Linear Transformations
c. Herstein. Topics in Algebra
d. Lang. Linear Algebra

Note: *Italicized items are optional topics.*

15.23 LINEAR MODELS

Course Description: This course is concerned with various linear statistical models for regression, analysis of variance and experimental designs that arise in practice. Topics include the multivariate normal distribution, quadratic forms, general linear models, estimation and tests of hypothesis about linear hypotheses and design matrices. Extensive use of statistical software is made throughout the course.

Credit: 3 units

Prerequisites: Probability, Statistics, and Statistical Theory

Topics	**Time Allotment**

a. Simple Linear Regression
• The problem and motivation behind curve fitting
• Least squares estimates
• Maximum likelihood estimates
• Inferences on regression model:
  o Inferences concerning the slope parameter
  o Inferences concerning the intercept
  o Interval estimation of the mean
  o Prediction of new observations
• Correlation: Inference and relationship to simple linear regression
	12 hours

b. Model Validation
• Regression assumptions: Linearity, Independence, Homoscedasticity, Normality of Errors
• Validating error normality with plots
  o P-P plots of residuals
  o Residual histograms
• Further residual analysis
  o Outlier detection
  o Detecting heteroscedasticity
• Transformations
  o Transforming nonlinear models to linear models
  o Box-Cox transformation
	8 hours

c. Multiple Regression • Matrix representation of multiple regression model • Estimation of parameters • Algebraic and geometric interpretations of multiple regression • Tests and confidence intervals based on the T-distribution	12 hours
d. Variable Selection and Model Building • Criteria for selecting appropriate models: MSE, Mallow's Cp, adjusted R^2, AIC, BIC • Forward selection, backward elimination and stepwise routines • Detecting multicollinearity • Polynomial regression	6 hours

- Use of dummy and interaction variables

e. Other Linear Models — 6 hours
- Analysis of variance
  - Single-factor ANOVA
  - Multiple comparisons
  - Multifactor ANOVA
- Experimental designs
  - Randomized block designs
  - Latin squares
- Analysis of covariance

Suggested references:

a. Draper and Smith. Applied Regression Analysis.
b. Seber and Lee. Linear Regression Analysis.
c. Montgomery and Peck. Introduction to Linear Regression Analysis

15.24 MATHEMATICAL MODELING

Course Description: The course introduces students to the process of modeling real-world phenomena using the tools of mathematics. In-class lectures and discussions are supplemented by computer hands-on sessions.

Credit: 3 units

Prerequisites: Differential Equations I, Statistics, and Linear Algebra

Topics	**Time Allotment**
a. Introduction to Modeling • Examples of models • Why we model	3 hours
b. Modeling approaches • Discrete models • Continuous models • Deterministic models • Stochastic models	6 hours
c. The modeling process • Iteration between modeling and model validation	3 hours
d. Modeling with dimensional analysis	6 hours
e. Modeling with ordinary differential equations	9 hours
f. Empirical modeling with data fitting • Using graphs to fit a model to data • Model fitting and extrapolation • Weighted least squares	6 hours
g. Some examples • Population growth • Predator-prey • Enzyme kinetics	9 hours

- Sun-moon-earth system
- 1-dimensional heat equation

h. Modeling with partial differential equations

Suggested text/references:

a. Giordano, Weir and Fox. A First Course in Mathematical Modeling
b. Mooney and Swift. A Course in Mathematical Modeling

Note: *Italicized item is an optional topic.*

15.25 MATHEMATICAL FINANCE

Course Description: This course covers the truth in lending act and its applications, introduction of financial instruments, determinants of the interest rate levels, stochastic interest rates, option pricing model, Cox-Ross-Rubenstein Model for stock models, conditional expectation, and European and American options.

Credit: 3 units

Prerequisite: Theory of Interest

Topics	**Time Allotment**
a. The Truth in Lending Act • Real estate mortgages • Approximation methods for finding the APR • Depreciation methods • Short sales • Financial instruments	10 hours
b. Advance Financial Analysis • Determinants of interest rate levels • Recognition of inflation • Reflecting risk and uncertainty • Yield rates • Interest rate assumptions • Duration • Immunization • Matching assets and liabilities	10 hours
c. Stochastic Interest Rates • Independent and dependent rates of interest • The capital asset pricing model • Option pricing model • Single period binomial lattice model • Multi-period option pricing model (CRR model)	10 hours
d. Mathematics of Finance • The Sigma field • Conditional probability and expectation • A stochastic view of the pricing model	12 hours

- European vanilla options
- European exotic options
- American options

Suggested text/references:

a. Kellison. Theory of Interest
b. Shreve. Stochastic Calculus for Finance Volume I: The Binomial Asset Pricing Model
c. Bass. The Basics of Financial Mathematics

15.26 MODERN GEOMETRY

This may either be an introduction to Euclidean and Non-Euclidean Geometry or a course in Projective Geometry.

15.26.1 MODERN GEOMETRY (EUCLIDEAN AND NON-EUCLIDEAN GEOMETRY)

Course Description: The first part of the course focuses on Euclidean and affine geometry on the plane. The second half may continue with Euclidean geometry on the sphere; alternatively, an introduction to finite geometries and to the non-Euclidean hyperbolic and elliptic geometries may be given. This course interrelates and makes use of tools from Geometry, Linear Algebra and Abstract Algebra.

Credit: 3 units

Prerequisites: Linear Algebra, and Abstract Algebra I

Topics	**Time Allotment**
a. Plane Euclidean Geometry	16 hours

- Review
  - Coordinate Plane
  - The Vector Space R^2
  - The Inner-Product Space R^2
  - The Euclidean Plane E^2
- Lines
- Orthonormal pairs
- Equation of a line
- Perpendicular lines
- Parallel and intersecting lines
- Reflections
- Congruence and isometries
- Symmetry groups
- Translations
- Rotations
- Glide reflections
- Structure of the isometry group
- *Fixed points and fixed lines of isometries*

b. Affine Transformations in the Euclidean Plane	8 hours

- Affine transformations

- Fixed lines
- The 2-dimensional affine group
- Fundamental theorem of affine geometry
- Affine reflections
- Shears
- Dilatations
- Similarities
- Affine symmetries

c. Geometry on the Sphere* 16 hours

- Preliminaries from 3-dimensional Euclidean space
- The cross-product
- Orthogonal bases
- Planes
- Incidence geometry of the sphere
- The triangle inequality
- Parametric representation of lines
- Perpendicular lines
- Motions of the sphere
- Orthogonal transformations of
- Euler's theorem
- Isometries
- *Fixed points and fixed lines of isometries*

d. Finite Geometries* 8 hours

- Introduction to finite geometries
    - Axiomatic systems
    - Four-line and four point geometries
- Finite geometries of Fano and Young
- Finite geometries of Pappus and Desargues
- Finite geometries as linear spaces
    - Near-linear and linear spaces
    - Incidence matrices
    - Numerical properties
- Finite projective planes and projective spaces
- *Finite affine spaces*

e. Other Modern Geometries* 8 hours

- Euclid's Fifth Postulate
- Introduction to hyperbolic geometry
    - Fundamental postulate of hyperbolic geometry
    - Ideal points and omega triangles
    - Quadrilaterals and triangles
- Introduction to elliptic geometry
    - Characteristic postulate of elliptic geometry
    - Quadrilaterals and triangles

Section c may be replaced by sections d and e.

Suggested text/references

a. Ryan. Euclidean and Non-Euclidean Geometry (for sections a, b and c)
b. Wald. Geometry: An Introduction
c. Greenberg. Euclidean and Non–Euclidean Geometries: Development & History
d. Batten, Combinatorics of Finite Geometries (for section d)
e. Smart, Modern Geometries (for section e)

Note: *Italicized items are optional topics.*

15.26.2 MODERN GEOMETRY (PROJECTIVE GEOMETRY)

Course Description: This course covers projective planes, projectivities, analytic projective geometry, cross ratio and harmonic sequences, geometric transformations, and isometries.

Credit: 3 units

Prerequisite: Linear Algebra

Topics	**Time Allotment**
a. Introduction and Historical Background • From Euclidean geometry to non-Euclidean geometry • Some geometries: hyperbolic, elliptic, inversive and projective	3 hours
b. The Projective Plane • Axioms of the projective plane • Principle of duality • Number of points/lines n a finite projective plane • Applications	5 hours
c. Triangles and Quadrangles • Definitions • Desarguesian plane • Harmonic sequence of points/lines	3 hours
d. Projectivities • Central perspectivity • Projectivity • Fundamental theorem of projective geometry • Theorem of Pappus	5 hours
e. Analytic Projective Geometry • Projective plane determined by a three-dimensional vector space over a field • Homogeneous coordinates of points/lines • Line determined by two points • Point determined by two lines • Collinearity, concurrency	8 hours
f. Linear Independence of Points/Lines • Definition	3 hours

- Analytic proof of some theorems like Desargues' Theorem

g. The Real Projective Plane — 2 hours
- Ideal points
- Ideal line

h. Matrix Representation of Projectivities — 6 hours
- Derivation of matrix representation
- Fundamental theorem of projective geometry (analytic approach)

i. Geometric Transformations — 4 hours
- Affine transformations and the affine plane
- Similarity transformation
- Homothetic transformation

j. Isometries — 6 hours
- Types of isometries
- Products of isometries
- Application of isometries to the solution of some geometric problems

Suggested text/references

a. Coxeter and Greitzer. Geometry Revisited
b. Smart. Modern Geometry
c. Hughes and Piper. Projective Planes

15.27 NUMBER THEORY

Course Description: This course covers integers and divisibility, congruences, linear diophantine equations, residues, number theoretic functions, primitive roots, quadratic residues, quadratic reciprocity law, and the Legendre symbol.

Credit: 3 units

Topics — **Time Allotment**

a. The Integers — 3 hours
- Basic properties of integers
- The Well-Ordering Principle
- Divisibility and the division algorithm

b. The Greatest Common Divisors and Prime Factorization — 8 hours
- The greatest common divisor and the least common multiple
- The Euclidean algorithm
- Prime and composite numbers
- The Fundamental Theorem of Arithmetic

c. Congruences — 10 hours
- Definitions and properties
- *Divisibility Tests*
- Solutions to linear congruences
- Linear diophantine equations
- The Chinese Remainder Theorem
- Wilson's Theorem and Fermat's Little Theorem
- Euler's Theorem

d. Multiplicative Functions — 3 hours
- The Euler-phi function $\phi(n)$
- The sum and number of divisors ($\sigma(n)$ and $\tau(n)$)

e. Primitive Roots — 6 hours
- The order of an integer and primitive roots
- Existence of primitive roots
- Primality tests using primitive roots

f. Quadratic Residues and Reciprocity — 9 hours
- Quadratic congruences
- Quadratic residues and nonresidues
- Gauss Lemma
- Quadratic reciprocity
- *The Jacobi Symbol and the Reciprocity Law for Jacobi Symbols*

g. Applications — 4 hours
- Cryptosystems
- RSA and public key cryptography

Suggested text/references:

a. Rosen. <u>Elementary Number Theory and Its Applications</u>
b. Burton. <u>Elementary Number Theory</u>
c. Barnett. <u>Elements of Number Theory</u>
d. Niven and Zuckermann. <u>Introduction to Number Theory</u>
e. Ore and Oystein. <u>Number Theory and Its History</u>

Note: *Italicized items are optional topics.*

15.28 NUMERICAL ANALYSIS

Course Description: This is an introductory course that covers error analysis, solutions of linear and nonlinear equations, numerical integration and differentiation, and numerical solutions of ordinary differential equations. In-class lectures and discussions are supplemented by computer hands-on sessions.

Credit: 3 units

Prerequisites: Differential Equations I, and Linear Algebra

Topics	**Time Allotment**
a. Error Analysis • Floating point numbers • Error • Accuracy • Convergence • Order	7 hours
b. Solutions of Nonlinear Equations • Bracketing methods (bisection, regula falsi) • Fixed point iteration • Newton's method	5 hours
c. Solutions of Linear Systems • Gaussian elimination • LU-decomposition • Gauss-Seidel method • Gauss-Jacobi method	6 hours
d. Numerical Interpolation • Lagrange interpolation • Divided differences • Interpolation at equally spaced points o Newton's forward and backward differences o Gauss' forward, backward and central formulas • Cubic splines	9 hours
e. Numerical Integration and Differentiation • Newton's formulas • Trapezoidal rule • Simpson's rule • Gaussian integration • Finite differences	8 hours
f. Numerical Solution of Ordinary Differential Equations • One-Step methods o Euler's method o Taylor series method o Runge-Kutta method • Multi-step methods o Adams' formulas o Milne's method	7 hours

Suggested text/references:

a. Atkinson. Elementary Numerical Analysis
b. Gerald and Wheatley. Applied Numerical Analysis
c. Kreysig. Advanced Engineering Mathematics
d. Sastry. Introductory Methods of Numerical Analysis
e. Scheid. Theory and Problems of Numerical Analysis

15.29 OPERATIONS RESEARCH I

Course Description: This course is an introduction to linear programming. It covers basic concepts, problem formulation, graphical solution for two-variable problems, simplex algorithm and other algorithms for special LP problems, duality and sensitivity analysis. In-class lectures and discussions are supplemented by computer hands-on sessions.

Credit: 3 units

Prerequisite: Linear Algebra

Topics	**Time Allotment**
a. Overview of Operations Research	3 hours
b. Linear Programming	7 hours
c. Geometry of LP in Two Variables	2 hours
d. Review of Linear Algebra	2 hours

a. Overview of Operations Research — 3 hours
- Definition of OR
- The general optimization problem
  - Survey of applications/Intro to some classical LP models
  - The product mix problem
  - The diet problem
  - The transportation problem
  - The fluid bending problem
  - The caterer's problem

b. Linear Programming — 7 hours
- Definition of linear programming
- Formulation of verbal problems into LPs
- Assumptions/Limitations:
  - Proportionality
  - Additivity
  - Divisibility
  - Nonnegativity
  - Certainty
  - Single objective

c. Geometry of LP in Two Variables — 2 hours
- Graphing of linear inequalities
- The feasible region as a convex polyhedral area
- Geometric interpretation of convex combination
- The extreme points
- The objective function as a family of parallel lines

d. Review of Linear Algebra — 2 hours
- Systems of linear equations
- Canonical forms
- Basic solutions
- Basic feasible solution
- Degenerate solutions
- Inconsistent systems
- Pivoting as a sequence of elementary row operations or a sequence of algebraic substitutions

e. Equivalent Formulations of an LP — 2 hours
   - The use of slacks and surpluses
   - How to handle variables with no sign restrictions
   - The symmetric forms
   - The standard form of an LP
   - The adjoined form
   - The canonical forms
   - The feasible canonical forms
   - Tableau conventions and notation
   - Conversion from maximization to minimization

f. The Simplex Algorithm — 4 hours
   - A simple illustration
   - The Fundamental Theorem of LP and its proof
   - Details of the algorithm
   - Possible entrance rules
   - The exit rule (minimum ratio test)
   - Test of optimality
   - Questions of uniqueness
   - The need for the nondegeneracy assumption

g. The Two-Phase Simplex Method — 2 hours
   - Artificial variables
   - Phase I as a test of feasibility
   - Phase I and algebraic redundancy
   - The Big M method

h. Revised Simplex Method — 3 hours

i. Duality in LP — 3 hours
   - The concept of duality
   - Dual linear programs in symmetric form
   - Duality theorems
   - Solving an LP problem from its dual

j. Sensitivity Analysis — 2 hours

k. Parametric Programming — 2 hours

l. Integer Programming — 4 hours

m. Special Purpose Algorithms — 4 hours
   - Transportation problem
   - Assignment problem
   - *Maximal flow problem*
   - *Traveling salesman problem*

n. Computer Applications — 2 hours

Suggested text/references:

a. Taha. Operations Research: An Introduction
b. Gass. Linear Programming (Methods and Applications)

c. Gillet. Introduction to Operations Research (A Computer-Oriented Approach)

Note: *Italicized items are optional topics.*

15.30 OPERATIONS RESEARCH II

Course Description: The course introduces the students to nonlinear programming and its applications. Topics include unconstrained/constrained optimization, quadratic and convex programming, Kuhn-Tucker conditions, gradient search and method of steepest ascent. In-class lectures and discussions are supplemented by computer hands-on sessions.

Credit: 3 units

Prerequisite: Operations Research I

Topics	**Time Allotment**
a. Overview of Nonlinear Programming • Definition of nonlinear programming • Applications	3 hours
b. Basic Concepts in NLP • Classical optimization techniques • Partial differentiation • The gradient vector • The Hessian matrix • Quadratic forms • Definite, semidefinite and indefinite matrices • Concave and convex functions • Recognizing saddle points • Differences between linear and quadratic problems	8 hours
c. Unconstrained Multivariate Problems • Concave objective functions, general objective function • Conjugate gradient methods • Gradient search method • Method of steepest ascent • Fletcher-Powell method • Hooves-Jeeves pattern search • Newton-Raphson method	16 hours
d. Constrained Multivariate Problems • Standard form • Lagrange multipliers • Penalty functions • Kuhn-Tucker conditions • Quadratic programming • Convex programming	16 hours

Suggested text/references:

a. Avriel. Nonlinear Programming, Analysis and Methods
b. Hillier and Lieberman. Introduction to Operations Research
c. Taha. Operations Research: An Introduction
d. Wagner. Principles of Operations research
e. Taylor. Management Science

OPERATIONS RESEARCH III

Course Description: The course introduces the students to dynamic programming and its applications. It includes deterministic and stochastic programming, allocation problems, inventory problems, forward and backward algorithms, and Markov chains. In-class lectures and discussions are supplemented by computer hands-on sessions.

Credit: 3 units

Prerequisite: Operations Research II

Topics	**Time Allotment**
a. Overview of Dynamic Programming (DP)	6 hours
b. Deterministic DP	15 hours

a. Overview of Dynamic Programming (DP) — 6 hours
- Preliminary concepts and definitions
- Applications

b. Deterministic DP — 15 hours
- Shortest path problems: prototype deterministic DP
  - Optimal value
  - Optimal decision
- Principle of optimality
  - Forward algorithm
  - Backward algorithm
- Features of DP problems
  - Stages
  - State spaces
  - Action/Decision spaces
  - Transitions
  - Reward/Cost structure
  - Objective functionals
- Problems with Discrete States and Spaces
  - Simple discrete allocation problem
  - General nonlinear allocation problem
  - Special linear problem: Knapsack problem
  - Traveling salesman problem
  - Equipment replacement problem
  - Periodic review inventory problem
- Problems with Continuous/Infinite States and Spaces
  - DP and linear programming
  - DP and nonlinear programming
- Limits of DP
  - Curse of dimensionality

c. Stochastic DP — 15 hours
- Path problems
- Inventory problems
- DP over a Markov chain

d. Introduction to queuing theory and simulation — 6 hours

Suggested text/references:

a. Dreyfus and Law. Art and Theory of Dynamic Programming.
b. Hillier and Lieberman. Introductions to Operations Research
c. Denardo. DP Models and Applications
d. Derman. Finite State Markov Decision Processes
e. White. Finite Dynamic Programming
f. Beckman. DP of Economic Decisions
g. Hartley. OR : A Managerial Emphasis
h. Sasieni, Yapan and Friedman. Methods and Problems in Operations

15.31 PRECALCULUS MATHEMATICS I (COLLEGE ALGEBRA)

Course Description: The course covers the real number system, algebraic expressions, the one- and two- dimensional coordinate systems, functions, equations and inequalities, word problems, and variation and progressions.

Credit: 3 units

Topics	**Time Allotment**
a. Sets	3 hours
• Definitions and basic notations	
• Subsets and counting	
• Operations on sets	
b. Number Systems	3 hours
• Counting numbers	
• Integers	
• Rational and irrational numbers	
• Real numbers and their properties	
c. Algebraic Expressions	6 hours
• Definition of terms	
• Addition and subtraction of algebraic expressions	
• Multiplication and division of algebraic expressions	
• Special Products	
• Factors and factoring	
d. Rational Expressions	4 hours
• Simplification of rational expressions	
• Addition and subtraction of rational expressions	
• Multiplication and division of rational expressions	
• Complex fractions	

e. Radicals — 4 hours
- Integral and zero exponents
- Rational exponents
- Simplification of radicals
- Addition and subtraction of radicals
- Multiplication and division of radicals

f. The Coordinate Systems — 3 hours
- Order axioms for the real numbers
- 1-dimensional coordinate system
- 2-dimensional coordinate system
- The distance formula
- Definition and formula for slope

g. Functions and Relations — 4 hours
- Basic definitions
- Domain and range
- Graphical representation of functions and relations
- Definition and graph of linear functions

h. Solutions of Equations and Associated Word Problems — 9 hours
- Solving linear equations
- Solving quadratic equations
- Relation between zeroes and coefficients of quadratic functions
- Equations in quadratic form
- Equations involving radicals
- Theorems on roots of equations (Factor Theorem, Rational Root Theorem etc.)
- Polynomial equations
- Systems of two linear equations
- Systems of three linear equations
- Systems of one linear and one quadratic equation

i. Inequalities — 3 hours
- Solving linear inequalities
- Solving nonlinear inequalities
- Inequalities with absolute values
- Graphical solutions of inequalities in two variables

j. The Exponential and Logarithmic Functions — 3 hours
- The exponential function
- The logarithmic function

k. Variations and Progressions — 3 hours
- Variation
- Arithmetic progression
- Geometric progression

Suggested text/references

a. Leithold. College Algebra and Trigonometry
b. Vance. Modern College Algebra and Trigonometry
c. Reyes and Marasigan. College Algebra
d. Rees, Spark and Rees. College Algebra

15.32 PRECALCULUS MATHEMATICS II (TRIGONOMETRY)

Course Description: This course covers circular functions, circular functions identities, solutions of equations involving circular functions, inverse circular functions, circular functions of angles, and applications of circular functions.

Credit: 3 units

Corequisite: Precalculus Mathematics I

Topics	Time Allotment
a. Review of Functions • Polynomial functions • Exponential functions • Logarithmic functions	4 hours
b. Angles and Circular Functions • Angles • The unit circle and arc length • The terminal point associated with a real number • Circular functions • The sine and cosine functions • Behavior of the sine and cosine functions	5 hours
c. The Other Four Circular Functions • Definitions • The fundamental circular function identities • Values of the circular functions of special real numbers	4 hours
d. Formulas Involving Circular Functions • Circular functions of sums and differences of angles • Double angle formulas • Half angle formulas • Conversions of sums and products • General reduction of formulas	6 hours
e. Graphs of the Circular Functions	4 hours
f. Solving Equations Involving Circular Functions	2 hours
g. Inverse Circular Functions • Inverse functions • Inverse circular functions • Operations involving inverse circular functions	5 hours
h. The Law of Sines and the Law of Cosines	3 hours

i. Applications — 9 hours
   - Solution of oblique triangles
   - Solution of right triangles
   - Complex numbers and the geometric use of angles in complex numbers
   - Powers and roots of complex numbers (De Moivre's theorem)

Suggested text/references:

a. Leithold. College Algebra and Trigonometry
b. Vance. Modern College Algebra and Trigonometry

Note: Precalculus Mathematics I and II may be offered as a one-semester 5-unit course with the descriptive title: College Algebra and Trigonometry.

15.33 PROBABILITY

Course Description: This is an introductory course in probability covering axiomatic probability space, discrete and continuous random variables, special distributions, mathematical expectation, conditional probability and independence, multivariate distributions, Laws of Large Numbers, and the Central Limit Theorem.

Credit: 3 units

Corequisite: Calculus III

Topics	**Time Allotment**
a. Probability	6 hours
• Sample spaces and events	
• Methods of assigning probability	
• Axiomatic approach to probability	
• Calculating probabilities	
• Conditional and independence of events	
• Bayes' rule	
b. Random Variables, Distribution Functions and Expectations	8 hours
• The notion of a random variable	
• The distribution function	
o Definition of a distribution function	
• Properties of a distribution function	
• Classification of a random variable	
o Discrete random variable	
o (Absolutely) continuous random variables	
o *Other types of random variables*	
• Mathematical expectation	
c. Some Special Distributions	8 hours
• Discrete probability distributions	
o Discrete uniform distributions	
o Bernoulli/binomial distribution	

- Poisson distribution
- Other special discrete distributions: hypergeometric, geometric, negative binomial

- Continuous probability distribution
  - Continuous uniform distribution
  - The normal distribution
  - Exponential/gamma distribution
  - Other special continuous distributions: Beta, Weibull, Cauchy

d. Functions of Random Variables — 6 hours
- Mathematical formulation
- Distribution of a function of random variables
  - CDF technique
  - MGF technique
  - Method of transformations
- Expectation of functions of random variables

e. Joint and Marginal Distributions — 6 hours
- The notion of random vector
- Joint distribution functions
- Marginal distributions
- Mathematical expectations

f. Conditional Distribution and Stochastic Independence — 6 hours
- Conditional distributions
- Stochastic independence
- Mathematical expectation

g. Sampling and Sampling Distributions — 4 hours

h. Laws of Large Numbers and Central Limit Theorem — 4 hours

Suggested text/references

a. Ross. A First Course in Probability
b. Hogg, Craig and McKean. Introduction to Mathematical Statistics
c. Mood, Graybill and Boes. Introduction to the Theory of Statistics

Note: *Italicized item is an optional topic.*

15.34 REAL ANALYSIS

Course Description: This course provides an introduction to measure and integration theory. It develops the theory of Lebesgue measure and integration over the real numbers. The course covers topics like the real number system, measurable functions, measurable sets, convergence theorems, integrals of simple and nonnegative measurable functions, and Lebesgue integral.

Credit: 3 units

Prerequisite: Advanced Calculus I

Topics	**Time Allotment**
a. Introduction • Comparison between Lebesgue and Riemann integral • Countable and uncountable sets • The extended real number system • Infinite limits of sequences	4 hours
b. Measurable functions • Measurable sets • Measurable functions	9 hours
c. Measures • Lebesgue measure • Measure spaces	2 hours
d. Integrals • Simple functions and their integrals • The integral of a non-negative extended real-valued measurable function • The monotone convergence theorem • Fatou's lemma and properties of integrals	9 hours
e. Integrable functions • Integrable real-valued functions • The positivity and linearity of the integral • The Lebesgue dominated convergence theorem	10 hours
f. Modes of convergence • Relations between convergence in mean • Uniform convergence • Almost everywhere convergence • Convergence in measure • Almost uniform convergence • *Egoroff's Theorem* • *Vitali Convergence Theorem*	6 hours
g. The Lebesgue spaces Lp • *Normed linear spaces* • *The Lp spaces* • *Holder's inequality* • *The completeness theorem* • *The Riesz's representation theorem for Lp*	

Suggested references:

a. Bartle. Elements of Integration and Lebesque Measure
b. Chae and Soo Bong . Lebesgue Integration
c. Royden. Real Analysis

Note: *Italicized items are optional topics.*

15.35 RISK THEORY

Course Description: This course covers economics of insurance and financial instruments, utility and loss theory, risk formulation, stochastic models, and applications.

Credit: 3 units

Prerequisite: Probability

Topics	**Time Allotment**

a. Introduction
• Economics of insurance and financial instruments
• Stochastic models
• Elements of decision theory
  o Loss function
  o Expectation and risks
  o Decision rules
	4 hours

b. Utility and Loss • Utility theory • The utility of money • Loss function • Loss distributions • Credibility theory	6 hours
c. Risk Formulation • Frequentist risks • Bayesian risks • Some applications	7 hours
d. Stochastic Models • Markov process • Independent increment process • Brownian motion • Martingales • ARIMA models • ARCH models	12 hours
e. Applications • Premium calculation, retention and reserves • Dividend policy • Option pricing of financial derivatives • Asset management • Forecasting exchange rates and interest rates	13 hours

Suggested text/references:

a. Berger. Statistical Decision Theory and Bayesian Analysis
b. Dunis. Forecasting Financial Markets: Exchange Rates, Interest Rates and Asset Management
c. Klugman, *et al.* Loss Models: From Data to Decisions

15.36 SAMPLING THEORY

Course Description: This course provides a discussion of the basic principles behind probability sampling and estimation. It includes the steps and assumptions undertaken in conducting sample surveys, as well as a discussion of simple random sampling, stratification, systematic sampling, cluster and multi-stage sampling, and probability proportional to size sampling, estimation procedures using these designs, ratio and regression estimation, sample size estimation as well as variance estimation.

Credit: 3 units

Prerequisites: Statistics, and Probability

Topics	**Time Allotment**
a. Introduction • Brief history of survey research • Examples of surveys in the Philippines • Census versus sample surveys • Basic principles and assumptions in sampling • Probability versus non-probability samples	2 hours
b. Simple Random Sampling • Definition and purpose • Notations • Sample selection • Estimators of means and totals • Variance of estimators • Finite population correction factor • Estimation of means over subpopulations • Sampling proportions and percentages • Sample size estimation	9 hours
c. Systematic Sampling • Definition and purposes of systematic sampling • Notations • Sample selection procedure • Linear systematic sampling • Circular systematic sampling • Variance estimation • Issues on systematic sampling	6 hours
d. Probability Proportional to Size Sampling • Definition and purposes of PPS sampling • Notations • Sample selection procedure	3 hours
e. Stratified Sampling • Definition and purpose of stratified sampling • Notations • Sample selection procedure • Mean and variance estimators and their properties • Allocation of sample size into strata	8 hours

    - Equal allocation
    - Proportional allocation
    - Optimum allocation
  - Construction of strata
  - Relative precision over simple random sample
  - Sample size estimation

f. Cluster Sampling and Multistage Sampling — 8 hours
  - Definition and purpose of cluster sampling
  - Notations
  - Sample selection procedure
  - Two stage sampling
  - Three stage sampling
  - Unequal cluster sampling

g. Ratio and Regression Estimators — 6 hours
  - Ratio estimator and its properties
  - Regression estimator and its properties
  - Ratio and regression estimators in large samples
  - Sample sizes
  - Interval estimates
  - Ratio and regression estimators in stratified sampling
  - Ratio and regression estimators in two-stage sampling

Suggested text/references:

a. Cochran. Sampling Techniques
b. Kish. Survey Sampling
c. Barnett. Sample Survey Principles and Methods
d. Groves, et al. Survey Methodology
e. Lohr. Sampling:Design and Analysis

15.37 SET THEORY

Course Description: The course covers the Zermelo-Fraenkel axioms of set theory, algebra of sets, relations and functions, natural numbers, cardinal numbers, Axiom of Choice, and ordinals.

Credit: 3 units

Prerequisite: Fundamental Concepts of Mathematics

Topics — **Time Allotment**

a. Review of Logic — 3 hours
  - Sentencial connectives
  - Truth tables
  - Tautologies

b. Cantor's Algebra of Classes — 3 hours
  - Class construction axiom
  - Class operations
  - Russel's Paradox

c. Zermelo-Fraenkel Axioms — 6 hours
- Existence
- Extent
- Specification
- Separation
- Empty set
- Pairing
- Union
- Power set

d. Algebra of Sets — 3 hours
- Union, intersection, relative complement
- Theorems on sets
- Indexed families of sets
- Axiom of replacement
- Cartesian products

e. Relations and Functions — 8 hours
- Relations
  - Domain, range
  - Equivalence relations
  - Partitions
- Functions
  - Injection, surjection, bijection
  - Inclusion, restriction maps
  - Inverse of a function
  - Characteristic functions

f. Natural Numbers — 6 hours
- Successors, inductive sets, induction principle
- Axiom of infinity and successor sets
- Peano's axioms
- Transitive sets
- Recursion Theorem
- Arithmetic of natural numbers
- Ordering on natural numbers

g. Cardinal Numbers — 5 hours
- Equinumerosity
- Finite, infinite, countable and uncountable sets
- Arithmetic of cardinal numbers
- Ordering of cardinal numbers

h. Axiom of Choice — 2 hours
- Continuum hypothesis

i. Construction of the Real Numbers — 3 hours
- Integers
- Rational numbers
- Real numbers

j. Ordering and Ordinals — 4 hours

- Partial ordering and well ordering
- Replacement axioms
- Ordinal numbers
- Transfinite induction

Suggested text/references:

a. Halmos. Naive Set Theory
b. Suppes. Axiomatic Set Theory
c. Enderton. Elements of Set Theory
d. Leung and Chen. Elementary Set Theory (Parts I and II)

15.38 SIMULATION

Course Description: The course discusses basic discrete event simulation, input and output analysis of simulations, and simulation development via programming in a programming language. Simulation of queuing systems is emphasized. Topics include probabilistic aspects of simulation experiments, statistical methodology for designing simulations and interpreting their output, random process generation, and efficiency improvement techniques. In-class lectures and discussions are supplemented by computer hands-on sessions.

Credit: 3 units

Prerequisites: Statistics, Probability, and Fundamentals of Computing I

Topics	**Time Allotment**
a. Review of Probability and Random Variables • Sample space and events • Probability axioms • Conditional probability and independence • (Discrete and Continuous) Random variables, mean, and variance • Conditional distributions	4 hrs
b. Random Numbers • Pseudo random number generation: Linear congruential method • Examples of simulation applications: Monte Carlo integration, queueing systems	5 hrs
c. Generating Discrete Random Numbers • Inverse transform method • Acceptance rejection technique • Composition approach • Generating Poisson and binomial random variables	6 hrs
d. Generating Continuous Random Numbers • Inverse transform • Rejection technique • Generating normal random variables • Generating Poisson process	6 hrs

- Generating a nonhomogenous poisson process

e. Discrete Event Simulation — 9 hrs
- Simulation via discrete events
- Single server queues
- More complicated queues

f. Variance Reduction Techniques — 9 hrs
- Use of antithetic variables
- Use of control variates
- Stratified sampling
- Importance sampling

g. Analysis of Simulated Data — 6 hrs
- Sample mean and sample variance
- Confidence interval for the mean
- Bootstrapping for estimating mean squared errors
- Goodness of fit

h. Markov Chain, Monte Carlo

Suggested text/references

a. Ross. Simulation
b. Law and Kelton. Simulation Modeling and Analysis

Note: *Italicized item is an optional topic.*

15.39 STATISTICAL THEORY

Course Description: This course focuses on the basic theory of statistical inference. It covers basic random sampling, sampling distributions, point and interval estimation, and hypothesis testing.

Credit: 3 units

Prerequisites: Statistics, and Probability

Topics — **Time Allotment**

a. Sampling and Sampling Distributions — 6 hours
- Elementary sampling theory
- Results derived from the normal distribution
  - Chi-square distribution
  - F distribution
  - Student's T distribution
- Asymptotics
  - Laws of Large Numbers
  - Central Limit Theorem

b. Point Estimation Preliminaries — 9 hours
- Desirable properties of estimators
- Sufficiency
  - Rao-Blackwell and Lehmann Scheffe Theorems
  - Cramer-Rao Inequality

- Exponential family of distributions

c. Methods of Estimation — 15 hours
- Method of moments
- Maximum likelihood
- *Solving nonlinear likelihood equations*
  - *Fisher scoring*
  - *EM algorithm*
- Bayes estimator

d. Interval Estimation — 2 hours
- Definition of confidence intervals
- Applications

e. Hypothesis Testing — 12 hours
- Preliminaries
- Neyman Pearson Lemma and the Most Powerful Test
- Generalized Likelihood Ratio Tests
- Confidence intervals and hypothesis tests

Suggested text/references

a. Hogg, Craig and McKean. <u>Introduction to Mathematical Statistics</u>
b. Mood, Graybill and Boes. <u>Introduction to the Theory of Statistics</u>.

Note: *Italicized item is an optional topic.*

15.40 STATISTICS

Course Description: This course is an introduction to statistics and data analysis. It covers the following: reasons for doing Statistics, collection, summarization and presentation of data, basic concepts in probability, point and interval estimation, and hypothesis testing.

Credit: 3 units

Topics — **Time Allotment**

a. Introduction — 2 hours
- Description and history of statistical science
- Population and sample

b. Collection and Presentation of Data — 3 hours
- Preliminaries
- Methods of data collection
- Probability and non-probability sampling
- Tabular and graphical presentations
  - The frequency distribution
  - The stem-and-leaf display
  - Cross tabulations
  - Histograms

c. Measures of Central Tendency and Location — 2 hours
- Notations and symbols

- The arithmetic mean
- The median
- The mode
- Measures of location (Fractiles)

d. Measures of Dispersion and Skewness — 2 hours
- Measures of absolute dispersion
- Measures of relative dispersion
- Measures of skewness
- The Boxplot

e. Probability — 3 hours
- Random experiments, sample spaces, events
- Properties of probability

f. Probability Distribution — 5 hours
- Concept of random variable
- Discrete and continuous probability distributions
- Expected values
- The normal distribution
- Other common distribution

g. Sampling Distributions — 3 hours

h. Estimation — 6 hours
- Basic concepts of estimation
- Estimating the mean
- Estimating the difference of two population means
- Estimating proportions
- Estimating the difference of two proportions
- Sample size determination

i. Tests of Hypothesis — 8 hours
- Basic concepts of statistical hypothesis testing
- Testing a hypothesis on the population mean
- Testing the difference of two population means
- Testing a hypothesis on proportions
- Testing the difference between to proportions
- Test of independence

j. Regression and Correlation — 8 hours
- Correlation coefficient
- Testing the correlation coefficient
- Simple linear regression

k. Analysis of Variance — 3 hours

Suggested text/references

a. Walpole. Introduction to Statistics
b. Freedman, Pisani and Purves. Statistics
c. Devore. Probability and Statistics

15.41 THEORY OF DATABASES

Course Description: This course provides an introduction to database analysis, design and implementation techniques and includes the following topics: data organization, relational algebra, functional dependencies, normalization and query optimization. In-class lectures and discussions are supplemented by computer hands-on sessions.

Credit: 3 units

Prerequisite: Data Structures and Algorithms

Topics	Time Allotment
a. Introduction to File Structures • File organization and file access • Storage devices • Indexing mechanisms • File compression	3 hours
b. Introduction to Database Systems • Database systems architecture • System development life cycle	3 hours
c. Data Modeling: Entity-Relationship Model	3 hours
d. Relational Database Model • Relational database management system • Relational data objects • Relational data integrity (DDL level)	3 hours
e. Database Design • Functional dependencies • Normalization	3 hours
f. Relational Algebra	3 hours
g. Relational Calculus • Tuple-oriented and domain oriented	3 hours
h. SQL • DDL: creating a database, creating tables • DQL: retrieval, restricting and sorting, subqueries • DML: insert, update, delete	9 hours
i. Database Administration • Data protection • Backup and recovery • Security	9 hours

- Integrity

j. Survey of RDBMS — 3 hours

Suggested text/references

a. Date. An Introdution to Database Systems
b. Ramez and Shamkant. Fundamentals of Database Systems
c. Korth and Silberschatz. Database System Concepts

15.42 THEORY OF INTEREST

Course Description: This course covers measures of interest, present and future values, equations of value, annuity-certains, general annuity certains, yield rates, extinction of debts, and bonds and securities.

Credit: 3 units

Prerequisite: Calculus I

Topics	**Time Allotment**
a. Measures of Interest • Accumulation and amount functions • Effective rate of interest • Simple and compound interest • Present and future values • Nominal rates of interest and discount • Force of interest	6 hours
b. Equations of Value • Present and future values equation • Current value equation • Unknown time • Unknown interest	4 hours
c. Annuity-Certains • Annuity immediate • Annuity due	6 hours
d. General Annuities • Annuities payable less frequently than interest is convertible • Annuities payable more frequently than interest is convertible • Continuous annuities • Basic varying identities • More general varying identities	6 hours
e. Yield Rates • Discounted cash flow analysis • Definition of yield rates • Uniqueness of the yield rate • Reinvestment rates	6 hours

- Interest measurement of a fund
- Dollar-weighted rate of interest for a single period
- Time-weighted rates of interest
- Portfolio methods
- Investment year methods

f. Extinction of Debts 6 hours
- Loan Extinction
- Computation of the outstanding balance
- Amortization method
- Sinking fund method
- Generalization of the amortization and sinking fund methods

g. Bonds and Securities 8 hours
- Basic financial securities
- Bonds and stocks
- Price of a bond (the FRANK formula)
- Other formulas for the bond
- Premium and discount
- Valuation between coupon payment dates
- Determination of yield rates and the Bond Salesman's Formula
- Callable bonds
- Serial bonds and stocks

Suggested text/references:

a. Hart. Mathematics of Investment
b. Kellison. The Theory of Interest
c. Shao and Shao. Mathematics for Management and Finance

15.43 TIME SERIES ANALYSIS

Course Description: This course deals with different methods of forecasting stationary and non-stationary time series data. The theoretical and model building issues of classical smoothing techniques, seasonal decomposition, and the use of Univariate Box-Jenkins statistical models are discussed. Other modern statistical models, such as ARCH, GARCH, transfer function, vector auto regression are also illustrated. In-class lectures and discussions are supplemented by computer hands-on sessions with statistical software.

Credit: 3 units

Prerequisites: Statistics, Probability, and Statistical Theory

Topics **Time Allotment**

a. Introduction 2 hours
- Definition of terms
- Typical components of a time series
- Overview of forecasting methods

b. Statistical Fundamentals — 4 hours
   - Summary statistics used in forecasting
   - Measuring errors
   - Model fitting
   - Review of linear regression
   - Autocorrelation function
   - White noise behavior

c. Simple Smoothing Methods — 6 hours
   - Moving averages
   - Simple exponential smoothing
   - Seasonal moving averages and simple exponential smoothing

d. Decomposition Methods and Seasonal Indices — 4 hours
   - Additive and multiplicative seasonality
   - Classical decomposition
   - Decomposition using regression

e. Trend-Seasonal Smoothing Methods — 8 hours
   - Estimating trend using first differences
   - Double moving average
   - Brown's double exponential smoothin
   - Holt's two-parameter trend model

f. Univariate ARIMA Modeling — 10 hours
   - Autoregressive process
   - Moving average process
   - Autoregressive integrated moving average process
   - Use of autocorrelation functions and partial autocorrelation functions
   - Parameter estimation
   - Model checking
   - Model validation

g. Seasonal ARIMA Modeling — 6 hours
   - Seasonal differencing
   - Seasonal ARIMA models

h. Overview of Advanced Models — 4 hours
   - ARCH and GARCH
   - Transfer Function models
   - Vector Autoregression

Suggested references:

a. De Lurgio. Forecasting Principles and Applications
b. Pankratz. Forecasting with Univariate Box-Jenkins Models: Concepts and Cases
c.

c. Enders. Applied Econometric Time Series
d. Wei. Time Series Analysis

15.44 TOPOLOGY

Course Description: This course is an introduction to topology. It includes topics fundamental to modern analysis and geometry like topological spaces and continuous functions, connectedness, compactness, countability axioms, and separation axioms.

Credit: 3 units

Prerequisite: Advanced Calculus I

Topics	Time Allotment
a. Review of Fundamental Concepts of Set Theory and Logic	3 hours
b. Topological Spaces and Continuous Functions • Topological spaces • Basis for a topology • Continuous functions and homeomorphisms • Construction of subspace, product, quotient, and sum topologies • Closed sets and limit points • The metric topology and the metrization problem	16 hours
c. Connectedness and Compactness • Connected spaces • Connected sets in the real line • Compact spaces • Tychonoff's Theorem • Compact sets in the real line • Limit point compactness	16 hours
d. Countability and Separation Axioms • The countability axioms • The separation of axioms and characterization of various spaces • *The Urysohn Lemma: Tietze Extension Theorem* • *The Urysohn Metrization Theorem*	6 hours

Suggested references:

a. Munkres. Topology: A First Course
b. Simmons. Topology and Modern Analysis
c. Engelking and Sieklucki. Introduction to Topology
d. Jänich. Topology
e. Kahn. Topology, An Introduction to the Point-Set and Algebraic Areas
f. Dixmier. General Topology

Note: *Italicized items are optional topics.*

ARTICLE VII
GENERAL REQUIREMENTS

Section 16 Program Administration

The minimum qualifications of the head of the unit that implements the degree program are the following:

16.1 Dean of the unit/college

The dean of a unit/college must be at least a master's degree holder in any of the disciplines for which the unit/college offers a program; and a holder of a valid certificate of registration and professional license, where applicable.

16.2 Head of the mathematics unit/department

The head of the unit/department must be at least a master's degree holder in the discipline for which the unit/department offers a program or in an allied field (cf. Section 5).

Section 17 Faculty

17.1 Qualification of faculty

a. Faculty teaching in a BS Mathematics/BS Applied Mathematics program must be at least a master's degree holder in mathematics or in an allied field (cf. Section 5).
b. All undergraduate mathematics courses in the recommended program of study for the BS Mathematics/BS Applied Mathematics program starting from the 2nd year must be taught by at least an MS degree holder in Mathematics/Applied Mathematics. Specialized courses in the program (e.g. actuarial science, computing, operations research, and statistics) must be taught by at least an MS degree holder in the appropriate field, or by an expert with equivalent qualifications (e.g. Fellow/Associate of the Actuarial Society of the Philippines).

17.2 Full time faculty members

The institution shall maintain at least 50% of the faculty members teaching in the BS Mathematics/Applied Mathematics program as full time.

17.3 Teaching Load

Teaching load requirements for the BS Mathematics/Applied Mathematics program shall be as follows:

a. Full time faculty members should not be assigned more than four (4) different courses/subjects within a semester.
b. In no instance should the aggregate teaching load of a faculty member exceed 30 units per semester (inclusive of overload and teaching loads in other schools).
c. Teaching hours per day should not exceed the equivalent of 6 lecture hours.

17.4 Faculty Development

The institution must have a system of faculty development. It should encourage the faculty to:

a. Pursue graduate studies in mathematics/applied mathematics especially at the PhD level;
b. Undertake research activities and to publish their research output;
c. Give lectures and present papers in national/international conferences, symposia and seminars; and,
d. Attend seminars, symposia and conferences for continuing education.

The institution must provide opportunities and incentives such as:

a. Tuition subsidy for graduate studies;
b. Study leave with pay;
c. Deloading to finish a thesis or carry out research activities;
d. Travel grants for academic development activities such as special skills training and attendance in national/ international conferences, symposia and seminars; and,
e. Awards & recognition.

Section 18 Library

18.1 Policy

Libraries service the instructional and research needs of the staff and students making it one of the most important service units within an HEI. It is for this reason that libraries should be given special attention by HEI administrators by maintaining it with a wide and up-to-date collection, qualified staff, and communications and connectivity portals.

18.2 Library Staff

The Head Librarian should: 1) have an appropriate professional training; 2) be a registered librarian; and 3) have a Master's degree.

The library should be: 1) staff with one full time professional librarian for every 1,000 students and 2) a ratio of 1 librarian to 2 staff /clerks should be observed.

18.3 Library Holdings

Library holdings should conform to existing requirements for libraries. For the BS Mathematics/Applied Mathematics program, the libraries must provide at least 5 book titles for each core/elective course found in the curriculum at a ratio of 1 volume per 15 students enrolled in the program. Preferably, these titles must have been published within the last 5 years.

The HEI is strongly encouraged to maintain periodicals and other non-print materials relevant to mathematics/applied mathematics to aid the faculty and students in their academic work. CD-ROMs could complement a library's book collection but should otherwise not be considered as replacement for the same.

18.4 Internet Access

Internet access is encouraged but should not be made a substitute for book holdings.

18.5 Space Requirements

At least 126 m^2 or approximately 2 classrooms shall be required for the library. It should include space for collections, shelving areas, stockroom, office space for staff and reading area

The library must be able to accommodate 5% of the total enrollment at any one time.

18.6 Finance

All library fees should be used exclusively for library operations and development for collections, furniture and fixtures, equipment and facilities, maintenance and staff development.

18.7 Networking

Libraries shall participate in inter-institutional activities and cooperative programs whereby resource sharing is encouraged.

18.8 Accessibility

The library should be readily accessible to all.

18.9 Office Hours

The library should be open to serve the needs of the users.

Section 19 Facilities and Equipment

19.1 Laboratory requirements

Laboratories should conform to existing requirements as specified by law (RA 6541, "The National Building Code of the Philippines" and Presidential Decree 856, "Code of Sanitation of the Philippines"). List of required and recommended equipment are listed in the course specifications above.

19.2 Classroom requirements (Class Size)

a. For lecture classes, ideal size is 30 students per class, maximum is 50.
b. For laboratory and research classes, class size shall be 20-25 students per class.
c. Special lectures with class size more than 50 may be allowed as long as the attendant facilities are provided.

19.3 Educational Technology Centers

The institution should provide facilities to allow preparation, presentation and viewing of audio-visual materials to support instruction.

ARTICLE VIII
ADMISSION AND RETENTION REQUIREMENTS

Section 20 Admission and Retention

The basic requirement for eligibility for admission of a student to any tertiary level degree program shall be graduation from the secondary level recognized by the Department of Education. Higher education institutions must specify admission, retention and residency requirements. They should ensure that all students are aware of these policies.

ARTICLE IX
TRANSITORY, REPEALING AND EFFECTIVITY PROVISIONS

Section 21 Transitory Provision

HEIs that have been granted permit or recognition for Bachelor of Science Mathematics/Bachelor of Science in Applied Mathematics program are required to fully comply with all the requirements in this CMO, within a non-extendable period of five (5) years after the date of its effectivity. State Universities and Colleges (SUCs) and Local Colleges and Universities (LCUs) shall also comply with the requirements herein set forth.

Section 22 Repealing Clause

All CHED issuances, rules and regulations or parts thereof, which are inconsistent with the provisions of this CMO are hereby repealed.

Section 23 Effectivity Clause

This CMO shall take effect fifteen (15) days after its publication in the Official Gazette, or in two (2) newspaper of national circulation. This CMO shall be implemented beginning Academic Year 2008-2009.

For strict compliance.

Pasig City, Philippines March 30, 2007.

FOR THE COMMISSION

CARLITO S. PUNO, DPA
Chairman